

Many years ago, Mary Pardoe, then a school-teacher, discovered this excellent proof of the theorem that the internal angles of a triangle on a plane surface add up to a straight line:

Consider moving the arrow from position 1 (going through A to C) through the three rotations labelled **a** to **c**.

The arrow must end up at location 4 (going through C to A) It has turned through each of the internal angles A, B, C.

What is the total rotation of the arrow?

Does this demonstration depend on size, shape or location of the original triangle?

A variant of this proof that uses not only rotations, but also translations, is: http://www.cs.bham.ac.uk/~axs/fig/angles-segment-proof.jpg

A discussion of the assumptions is here:

http://www.cs.bham.ac.uk/research/projects/cogaff/misc/p-geometry.html

Compare:

Roger B. Nelsen.

Proofs without words: Exercises in Visual Thinking,

Mathematical Association of America, 1993

(Freely available on google books: books.google.com)

I first met Mary when she was a student (Mary Ensor) at Sussex University, where I was a lecturer. For more information contact me using the email address on my web site: http://www.cs.bham.ac.uk/~axs/ **Note:** Like the "standard proof" using parallel lines, this one cannot be used on a curved surface