

Mathematical Formulation for Task Assignment Problem in 10 Minute School

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October 14, 2023

1 Problem Description

An experienced operations manager is responsible for assigning tasks to a team of six workers (workers 1 to 6) to maximize the output level. Each day, he receives a list of tasks with various characteristics, including task type, count, priority, urgency, time requirement, and quality scores of workers for each task type. The goal is to efficiently assign these tasks to workers while considering their availability, task counts, and the importance of the tasks.

2 Mathematical Formulation

2.1 Indices

i : Index for workers, $i \in \{1, 2, 3, 4, 5, 6\}$

j : Index for tasks, $j \in \{1, 2, \dots, N\}$ (Total number of tasks)

2.2 Parameters

T_j : Task type of task j

C_j : Count of task j

P_j : Priority of task j

U_j : Urgency of task j

Q_{ij} : Quality score of worker i for task type T_j

R_j : Time requirement for task j

H_i : Available working hours per worker i

2.3 Variables

X_{ij} : Binary decision variable indicating whether task j is assigned to worker i

2.4 Objective Function

The objective is to maximize the total output level, which is calculated as the sum of quality scores for each assigned task, weighted by its priority and urgency. The higher the quality score, priority, and urgency, the more critical the task, and it contributes more to the total output level.

$$\text{Maximize: } \sum_{i=1}^6 \sum_{j=1}^N X_{ij} \cdot Q_{ij} \cdot P_j \cdot U_j$$

This objective function reflects the primary goal of maximizing the overall performance by efficiently assigning tasks to workers based on their quality scores and task characteristics.

2.5 Constraints

1. Task Assignment Constraint:

Each task j can be assigned at most once. This constraint ensures that a task is either assigned to a worker or left unassigned.

$$\sum_{i=1}^6 X_{ij} \leq 1 \quad \forall j \in \{1, 2, \dots, N\}$$

2. Worker Availability Constraint:

Each worker i can work at most H_i hours per day. This constraint ensures that workers do not exceed their available working hours.

$$\sum_{j=1}^N X_{ij} \cdot R_j \leq H_i \quad \forall i \in \{1, 2, 3, 4, 5, 6\}$$

3. Task Count Constraint:

Ensure that the total assigned count of each task j matches its given count. This constraint ensures that the assigned task count matches the provided task count for each task type.

$$\sum_{i=1}^6 X_{ij} \cdot C_j = C_j \quad \forall j \in \{1, 2, \dots, N\}$$

2.6 Binary Variable

The decision variable X_{ij} is binary and takes a value of 1 if task j is assigned to worker i and 0 if it is not assigned.

3 Explanation

This mathematical formulation addresses the operational challenge of assigning tasks to workers to maximize productivity. The objective function seeks to maximize the output level, which is a combination of the quality scores, task priorities, and urgencies. By optimizing these factors, you can ensure that the most important and time-critical tasks are assigned to the most skilled workers.

The constraints ensure that tasks are not over-allocated, workers do not exceed their working hours, and the assigned task counts match the given counts. This formulation strikes a balance between productivity and resource constraints.

Solving this optimization problem will lead to task assignments that achieve the highest possible output level, ultimately improving the operational efficiency of your team.