Experimental Design Project

STA2005S - 2025

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Plagiarism Statement:

Introduction:

The two chilli farmers, Ms Hopeful and Mr Growing, compete head to head for the hearts and minds of the South African chilli market. But despite their names Ms Hopeful seems to always get higher chilli yields than Mr Growing. In Mr Growing's eyes this cannot continue.

Aim:

This experiment aims to systematically evaluate the effect of light levels, heat levels and chilli variety on crop quality and yield. With our results we will be able to give evidence-based recommendations on optimal growing conditions.

Particularly we seek to answer:

- 1. Which light and heat settings produce the highest quality chillies?
- 2. Does the above depend on variety?
- 3. Does quality depend on variety?
- 4. How big are the differences?

Priori Hypothesis:

Prior to collecting the data, we hypothesize:

- H1: Quality and yield will increase as light increases and with moderate heat levels, as either extreme of heat can lead to some form of denaturing of the plant's cells.
- H2: Although their would be natural differences between varieties of chillies the lightheat response patterns will be similar between varieties.
- H3: Quality of chilli would be heavily dependent on variety as taste, and look are variety-specific.
- H4: Side (North/South) has a negligible effect.

Experimental Design and Randomization:

Simulating data:

We created the design matrix in accordance to a **Nested Latin Square design**, similar to a factorial design, however since we lack the resources needed to experiment on every possible combination, instead we pick combinations all necessary combinations needed to represent the effects of the different light and heat levels on the chillies.

treatment factors:

- Light: 4 levels (1, 2, 3, 4)
- Heat: 4 levels (1, 2, 3, 4)
- \implies 16 unique treatments

blocking factors:

- Season: (1, 2)
- Greenhouse: (A, B)
- Side of greenhouse: (N = north-facing, S = south-facing)
- Chilli variety: (R = red-hot, F = furious)

Side-Variety combination assignments:

	H1	H2	H3	H4
L1	RN	FN	RS	FS
L2	FN	RN	FS	RS
L3	RS	FS	RN	FN
L4	FS	RS	FN	RN
->				-

Example Greenhouse setup:

	R		F		
N	H1	L1	H1	L2	
	H2	L2	H2	L1	
	H3	L3	H3	L4	
	H4	L4	H4	L3	
S	H1	L3	H1	L4	
	H2	L4	H2	L3	
	H3	L1	H3	L2	
	H4	L2	H4	L1	

Figure 1: Blocking example greenhouse

Design:

Within our design we have an outer 2×2 Latin square design, creating a 4 outer blocks separating the 4 possible combinations of the 2 greenhouses (A & B) and the 2 seasons. Each of these represents a separate experimental block.

- Block $1 = \text{Season } 1 \times \text{Greenhouse A}$
- Block $2 = Season 1 \times Greenhouse B$
- Block $3 = \text{Season } 2 \times \text{Greenhouse A}$
- Block $4 = Season 2 \times Greenhouse B$

Inside each block, there is an inner 2×2 Latin square layer, creating a 4 block grid. The variety (*Redhot* or *Furious*) and side (*North* or *South*) were assigned according to the plan shown below:

Side	Variety	Description
North	Redhot (RN)	4 plots
North	Furious (FN)	4 plots
South	Redhot (RS)	4 plots
South	Furious (FS)	4 plots

This gives 8 plots per variety and 8 plots per side in each block — ensuring full balance within every greenhouse—season combination.

Then, within each side-variety combination, we assign random heat and light levels. This means that Inside each block, all **16 combinations** of **Heat (H1–H4)** and **Light (L1–L4)** were used once, creating a full factorial of the two treatment factors.

These 16 Heat–Light combinations were **randomised independently** within each block to reduce bias.

The overarching structure is thus such that: Heat-Light is nested within Side-variety which are is nested Season-greenhouse combinations.

		Greenhou	ıse A		Greenho	use B
		R	F			F
Season 1		H1 L3 H2 L2 H3 L3 H4 L1	H1 L1 H2 L3 H3 L2 H4 L4	Z	H1 L1 H2 L2 H3 L3 H4 L4	H1 L2 H2 L3 H3 L4 H4 L1
Sex		H1 L1 H2 L3 H3 L4 H4 L2	H1 L2 H2 L1 H3 L3 H4 L4	v	H1 L2 H2 L1 H3 L4 H4 L3	H1 L4 H2 L3 H3 L2 H4 L1
		R	F		R	F
Season 2		H1 L4 H2 L3 H3 L2 H4 L1	H1 L3 H2 L2 H3 L3 H4 L1	Z	H1 L2 H2 L1 H3 L3 H4 L4	H1 L1 H2 L3 H3 L2 H4 L4
Sea	v	H1 L1 H2 L3 H3 L2 H4 L4	H1 L1 H2 L2 H3 L3 H4 L4	S	H1 L1 H2 L3 H3 L4 H4 L2	H1 L3 H2 L2 H3 L3 H4 L1

Figure 2: Blocking factors Diagram

Randomization:

Each side-variety combination gets its own independent randomization of heat and light such that there is no systematic crossing of all 4 heat levels \times all 4 light levels \times all sides \times all varieties. In this way all levels of heat and light respectively are represented in a balanced manner, but not all combinations of heat and light with the other variables are tested.

In this way there is confounding of higher order combinations but this is due to our restrictive number of available plots and resource limitations. The combination of our clever blocking within the design as well as the use of randomization mitigates the effects of this potential confounding.

Simulation:

The simulated quality data was generated using the **get.observations()** function with a fixed random seed (**set.seed(22)**) to ensure reproducibility.

The generated design matrix was also validated with a loop which checks for correct randomization and that each heat-light combination is used once per side-variety combination.

	season	greenhouse	light	heat	variety	side	plot	obs
1	1	A	2	2	R	n	1	44.63388
2	1	Α	3	1	R	n	2	50.48180
3	1	Α	1	4	R	n	3	41.30101
4	1	A	4	3	R	n	4	55.11754
5	1	A	4	3	R	s	5	58.08898
6	1	A	2	4	R	s	6	50.62546
7	1	A	3	2	R	s	7	53.91521
8	1	A	1	1	R	s	8	38.67690
9	1	A	2	3	F	n	9	48.86032
10	1	A	1	1	F	n	10	32.95864
11	1	A	4	4	F	n	11	54.99408
12	1	A	3	2	F	n	12	50.56499
13	1	A	4	4	F	s	13	59.98049
14	1	A	3	3	F	s	14	55.91266
15	1	A	2	1	F	s	15	46.20276
16	1	A	1	2	F	s	16	41.54861
17	1	В	3	1	R	n	17	43.73744
18	1	В	4	4	R	n	18	51.71666
19	1	В	1	3	R	n	19	39.36048

20	1	В	2	2	R	n		45.56654
21	1	В	2	3	R	s	21	50.20601
22	1	В	1	4	R	s	22	41.45225
23	1	В	3	2	R	s	23	53.23059
24	1	В	4	1	R	S	24	53.54364
25	1	В	1	2	F	n	25	37.05544
26	1	В	2	3	F	n	26	46.88391
27	1	В	4	4	F	n	27	53.20711
28	1	В	3	1	F	n	28	46.02305
29	1	В	4	1	F	s	29	53.24998
30	1	В	3	3	F	s	30	56.90592
31	1	В	1	2	F	s	31	41.88629
32	1	В	2	4	F	s	32	47.85220
33	2	Α	2	4	R	n	33	47.63880
34	2	Α	4	3	R	n	34	50.72973
35	2	Α	1	1	R	n	35	NA
36	2	Α	3	2	R	n	36	46.43048
37	2	Α	2	4	R	s	37	50.02303
38	2	Α	3	1	R	s	38	NA
39	2	Α	1	2	R	s	39	40.59188
40	2	Α	4	3	R	s	40	53.85587
41	2	Α	3	3	F	n	41	53.39276
42	2	Α	1	2	F	n	42	37.54930
43	2	Α	2	1	F	n	43	41.08095
44	2	Α	4	4	F	n	44	49.81993
45	2	Α	2	3	F	s	45	52.91419
46	2	Α	4	1	F	s	46	50.15578
47	2	Α	1	4	F	s	47	42.84430
48	2	A	3	2	F	s	48	52.32017
49	2	В	4	4	R	n	49	52.32779
50	2	В	3	2	R	n	50	49.44498
51	2	В	1	1	R	n	51	33.97181
52	2	В	2	3	R	n	52	44.80763
53	2	В	1	1	R	s	53	38.09752
54	2	В	2	2	R	s	54	48.20931
55	2	В	3	3	R	s	55	52.40630
56	2	В	4	4	R	s	56	52.43842
57	2	В	1	3	F	n	57	37.89270
58	2	В	3	4	F	n	58	46.54105
59	2	В	2	2	F	n	59	42.38080
60	2	В	4	1	F	n	60	45.73726
61	2	В	4	1	F	s	61	51.22709
62	2	В	1	2	F	s	62	38.03224

63	2	В	3	4	F	s	63 51.62355
64	2	В	2	3	F	s	64 53.64072

Analysis and Results:

Fitting the models and checking assumptions:

The model uses **block** to adjust for background variation across the four Latin Square positions (the combinations of Season and Greenhouse).

It then tests how heat, light, and variety affect the response and whether they interact.

The **side** factor is included only to correct for small layout or position effects within each greenhouse.

ANOVA:

The experiment used a Latin Square design with Season \times Greenhouse forming the 2×2 blocking structure, and a full factorial of

Heat (4 levels) \times Light (4 levels) \times Variety (2 levels).

The fitted linear model was:

 $obs = block + heat \times light \times variety + side$

Model fit:

 $\mathbf{R^2} = \mathbf{0.968}$, Adjusted $\mathbf{R^2} = \mathbf{0.931} \rightarrow$ the model explains most of the variation in quality. Residual $\mathbf{SE} = \mathbf{1.76} \rightarrow$ small compared to the treatment range.

No clear outliers or leverage points were found when checking diagnostic plots.

Two coefficients were undefined because of overlap between some factors, which is expected in a full factorial design.

Interpretation:

Both **light** and **heat** increased mean quality, with **light intensity** having the strongest influence (p < 0.001).

Heat also affected quality (p < 0.001), though to a smaller extent.

Variety showed a mild difference (p = 0.018), where variety \mathbf{R} performed slightly better than \mathbf{F} .

The **block** term (p = 0.0067) shows that adjusting for season and greenhouse helped control background variation.

The **side** factor (p = 0.015) had a small effect, possibly linked to minor position or orientation differences.

All **interaction terms** (e.g., heat \times light) had p-values above 0.1, suggesting the effects act independently.

Overall, the results show that most of the change in quality can be explained by the main treatment factors — especially light — and the blocking effectively reduced environmental variation.

Model Diagnostics:

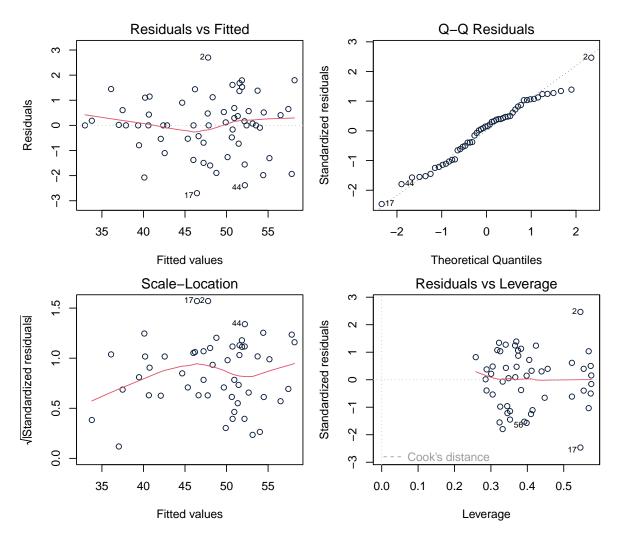


Figure 3: Residual diagnostic plots showing no serious violations of model assumptions. The Q-Q plot indicates approximate normality, and the residuals vs. fitted plot shows no strong patterns suggesting heteroscedasticity.

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Heat also affected quality (p < 0.001), though to a smaller extent.

Variety showed a mild difference (p = 0.018), where variety **R** performed slightly better than **F**.

The **block** term (p = 0.0067) shows that adjusting for season and greenhouse helped control background variation.

The **side** factor was aliased in the Type II ANOVA, meaning it was linearly dependent on other variables in the model.

This suggests that side orientation (north/south) did not contribute additional information once block, heat, light, and variety were included.

Any minor variation linked to side was already captured by the blocking structure (Season \times Greenhouse).

All **interaction terms** (e.g., heat \times light) had p-values above 0.1, suggesting that the effects act independently.

Treatment Effects Visualization:

Interaction plots:

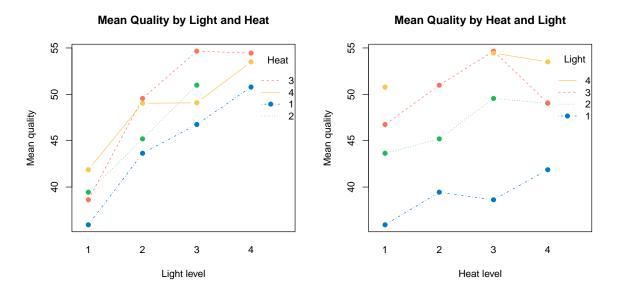


Figure 4: Interaction plots for mean quality based on Heat and Light levels, aggregated across both varieties.

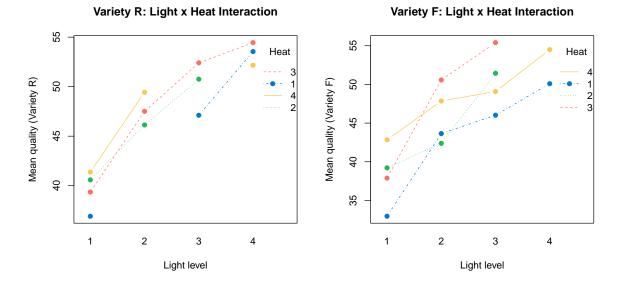


Figure 5: Heat x Light interaction plots for mean quality, shown separately for Variety R (left) and Variety F (right)

Interpretation:

From observation, the interaction plots appear to show strong interactions, as the lines are clearly not parallel. In the "Variety R" plot, the lines cross over each other, while in the "Mean Quality by Light and Heat" plot, the lines have very different slopes. This pattern suggests the factors are dependent implying effect of Light depends on the Heat level. Furthermore, the interaction pattern for "Variety R" looks completely different from the pattern for "Variety F," which suggests a 3-way interaction ($Heat \times Light \times Variety$).

Despite this, we must treat this interpretation cautiously because these plots show raw data means, but our design is unbalanced due to missing observations making them statistically misleading/unreliable. Thus the only valid conclusions about interactions must come from ANOVA or plotting the model's Estimated Marginal Means.

Mean plots:

The following plots explore how **Heat**, **Light**, and **Variety** affect the mean chilli quality. Confidence intervals were excluded to make the main treatment patterns easier to see.

Mean quality by Light (lines = Heat), stratified by Variety

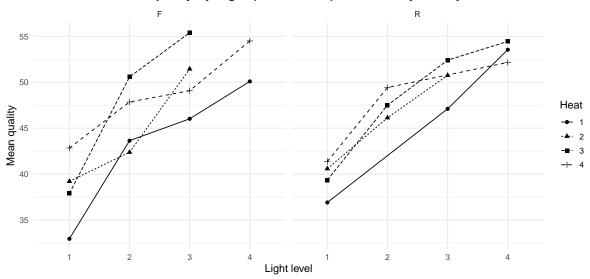


Figure 6: Mean quality stratified by Variety. The plot shows mean quality by Light level (with lines for Heat)

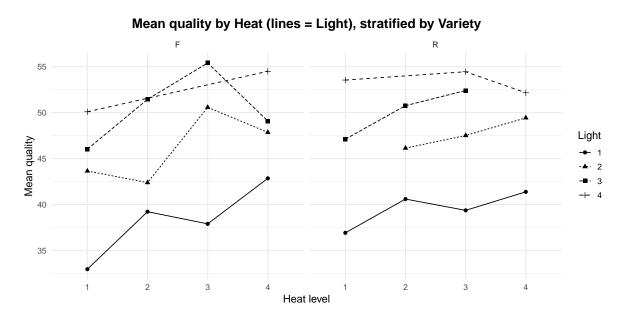


Figure 7: Mean quality stratified by Variety. The plot shows mean quality by Heat level (with lines for Light)

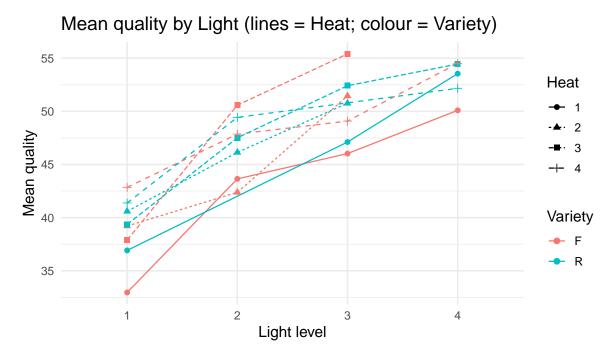


Figure 8: Mean quality by Light level, showing the combined Heat x Light x Variety interaction. Line type represents the Heat level, and color represents the Variety

Interpretation of Mean Plots

The three mean plots above show the relationships between **Heat**, **Light**, and **Variety** on average chilli quality.

Mean quality by Light (lines = Heat), faceted by Variety

- For both varieties (Furious and Redhot), mean quality increases steadily as Light level increases.
- The lines for different Heat levels move upward together, meaning that **higher Light** improves quality across all Heat levels.
- Furious generally achieves slightly higher mean quality than Redhot.

Interpretation:

Light has a strong and consistent positive effect on chilli quality, and both varieties respond in a similar pattern.

Mean quality by Heat (lines = Light), faceted by Variety

- The lines are **more scattered** with no clear trend as Heat increases.
- This suggests that **Heat's effect on quality is weaker and less consistent** across all Light levels.
- Both varieties show small ups and downs, but no steady direction.

Interpretation:

Heat has less influence on quality compared to Light and does not show a consistent pattern.

Mean quality by Light (lines = Heat; colour = Variety)

- Both Furious (red) and Redhot (blue) lines rise with increasing Light, showing that Light has a strong overall effect.
- The shapes of the lines are **very similar between varieties**, and they rarely cross, meaning **no major interactions**.
- Furious tends to have slightly higher quality scores overall.

Interpretation:

Both varieties improve with Light, with *Furious* performing marginally better overall. The effects of Heat, Light, and Variety appear largely independent.

Overall Summary

Across all plots:

- Light shows the strongest positive effect on chilli quality.
- Heat has a smaller and less consistent influence.
- Variety has a minor main effect, with Furious slightly outperforming Redhot.
- The mostly **parallel pattern of lines** suggests **weak or no interactions** among the factors.

Analysis Procedures

Missing Values: Cases with missing observations were excluded from analysis using complete-case analysis, resulting in 62 observations for model fitting.

ANOVA: We used Type II sums of squares to test main effects and interactions, as this approach appropriately handles unbalanced designs caused by missing data and tests each effect after accounting for all other effects at the same or lower level.

Multiple Testing: Given the exploratory nature of this study and the relatively small number of pre-specified hypotheses, we report both unadjusted p-values and note effects significant at = 0.05. For post-hoc pairwise comparisons, we apply Tukey's HSD adjustment.

Model Diagnostics: We examined residual plots (residuals vs. fitted, Q-Q plot, scale-location, and leverage plots) to verify assumptions of normality, homoscedasticity, and identify potential outliers.

Effect Estimation: We computed estimated marginal means (EMMs) with 95% confidence intervals for all treatment combinations.

Conclusions:

This section fits the full model (obs \sim block + heat \times times \times light \times variety + side), runs Type-II ANOVA, extracts estimated marginal means with the **emmeans** package, answers the four farmer questions, and builds a cost-aware decision table. Plots and tables are produced inline.

Anova Table (Type II tests)

Response: obs

	Sum Sq	Df	F value	Pr(>F)	
block	36.05	3	4.5539	0.009335	**
heat	215.27	3	27.1898	8.165e-09	***
light	1456.00	3	183.9003	< 2.2e-16	***
variety	0.03	1	0.0126	0.911299	
side	153.16	1	58.0354	1.370e-08	***
heat:light	11.43	8	0.5415	0.816059	
heat:variety	22.09	3	2.7898	0.056931	
light:variety	7.31	3	0.9229	0.441268	
heat:light:variety	11.55	5	0.8755	0.508852	
Residuals	81.81	31			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Reading the ANOVA:

- Interactions with **variety** (heat:variety, light:variety, heat:light:variety) are **non-significant** → patterns don't depend on variety.
- Light is strongly significant; Heat is weaker.
- Variety shows a small main effect.
- Side adds little after blocking; keep it per design.

Q1 — Which Heat × Light is best?

Table 2: EMMs: Heat \times Light

heat	light	emmean	SE	df	lower.CL	upper.CL
1	1	34.81	0.98	31	32.82	36.81
2	1	39.23	0.94	31	37.32	41.13
3	1	40.74	1.22	31	38.26	43.22
4	1	41.26	1.03	31	39.16	43.36
1	2	NA	NA	NA	NA	NA
2	2	45.64	1.00	31	43.59	47.68
3	2	49.17	0.72	31	47.70	50.64
4	2	47.44	0.97	31	45.45	49.42
1	3	48.03	1.08	31	45.83	50.23
2	3	50.97	0.72	31	49.50	52.44
3	3	52.86	0.97	31	50.89	54.83
4	3	NA	NA	NA	NA	NA
1	4	50.65	0.96	31	48.70	52.60
2	4	NA	NA	NA	NA	NA
3	4	NA	NA	NA	NA	NA
4	4	54.04	0.64	31	52.74	55.34

contrast SE df t.ratio p.value estimate heat1 light1 - heat2 light1 -4.412 1.390 31 -3.174 0.1097 heat1 light1 - heat3 light1 -5.927 1.550 31 -3.819 0.0248 heat1 light1 - heat4 light1 -6.446 1.440 31 -4.469 0.0047 heat1 light1 - heat1 light2 nonEst NA NA NANAheat1 light1 - heat2 light2 -10.823 1.370 31 -7.896 <.0001

```
heat1 light1 - heat3 light2
                             -14.358 1.220 31 -11.780 <.0001
heat1 light1 - heat4 light2
                             -12.625 1.410 31 -8.928
                                                       <.0001
heat1 light1 - heat1 light3
                             -13.214 1.450 31 -9.116
                                                       <.0001
heat1 light1 - heat2 light3
                             -16.155 1.210 31 -13.372
                                                        <.0001
heat1 light1 - heat3 light3
                             -18.050 1.380 31 -13.070
                                                       <.0001
heat1 light1 - heat4 light3
                              nonEst
                                        NA NA
                                                    NA
heat1 light1 - heat1 light4
                             -15.836 1.410 31 -11.268
                                                        <.0001
heat1 light1 - heat2 light4
                              nonEst
                                        NA NA
                                                   NA
                                                            NA
heat1 light1 - heat3 light4
                              nonEst
                                        NA NA
                                                   NA
                                                            NA
heat1 light1 - heat4 light4
                             -19.225 1.150 31 -16.705
                                                       <.0001
heat2 light1 - heat3 light1
                              -1.514 1.590 31
                                               -0.951
                                                       0.9978
heat2 light1 - heat4 light1
                              -2.034 1.340 31
                                               -1.519
                                                       0.9239
                                                    NA
heat2 light1 - heat1 light2
                              nonEst
                                        NA NA
                                                            NA
heat2 light1 - heat2 light2
                              -6.410 1.430 31
                                               -4.484
                                                       0.0045
heat2 light1 - heat3 light2
                              -9.946 1.200 31
                                               -8.290
                                                        <.0001
heat2 light1 - heat4 light2
                              -8.212 1.310 31
                                               -6.264
                                                       <.0001
heat2 light1 - heat1 light3
                              -8.801 1.480 31
                                               -5.957
                                                       0.0001
heat2 light1 - heat2 light3
                             -11.743 1.160 31 -10.130
                                                       <.0001
heat2 light1 - heat3 light3
                             -13.638 1.340 31 -10.185
                                                        <.0001
heat2 light1 - heat4 light3
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat2 light1 - heat1 light4
                             -11.424 1.330 31
                                                -8.607
                                                        <.0001
heat2 light1 - heat2 light4
                              nonEst
                                        NA NA
                                                   NA
                                                            NA
heat2 light1 - heat3 light4
                              nonEst
                                        NA NA
                                                   NA
                                                            NA
heat2 light1 - heat4 light4
                             -14.813 1.170 31 -12.699
                                                       <.0001
heat3 light1 - heat4 light1
                              -0.520 1.650 31
                                               -0.315
                                                        1.0000
heat3 light1 - heat1 light2
                              nonEst
                                        NA NA
                                                    ΝA
                                                            NA
heat3 light1 - heat2 light2
                              -4.896 1.500 31
                                                       0.0899
                                               -3.266
heat3 light1 - heat3 light2
                              -8.432 1.380 31
                                               -6.115
                                                       0.0001
                              -6.698 1.590 31
heat3 light1 - heat4 light2
                                               -4.207 0.0093
heat3 light1 - heat1 light3
                              -7.287 1.560 31
                                               -4.677 0.0027
heat3 light1 - heat2 light3
                             -10.228 1.450 31
                                               -7.072
                                                       <.0001
heat3 light1 - heat3 light3
                             -12.123 1.560 31
                                                -7.784
                                                        <.0001
heat3 light1 - heat4 light3
                              nonEst
                                        NA NA
                                                   NA
                                                            NA
heat3 light1 - heat1 light4
                              -9.909 1.540 31
                                               -6.444
                                                       <.0001
heat3 light1 - heat2 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat3 light1 - heat3 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat3 light1 - heat4 light4
                             -13.299 1.330 31
                                               -9.978
                                                       <.0001
heat4 light1 - heat1 light2
                              nonEst
                                        NA NA
                                                    NA
heat4 light1 - heat2 light2
                              -4.376 1.500 31
                                               -2.926
                                                       0.1807
heat4 light1 - heat3 light2
                              -7.912 1.280 31
                                               -6.200 <.0001
heat4 light1 - heat4 light2
                              -6.178 1.380 31
                                               -4.484 0.0045
heat4 light1 - heat1 light3
                              -6.767 1.520 31
                                               -4.458 0.0048
heat4 light1 - heat2 light3
                              -9.709 1.230 31
                                               -7.881 <.0001
```

```
heat4 light1 - heat3 light3
                             -11.604 1.420 31 -8.190 <.0001
heat4 light1 - heat4 light3
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat4 light1 - heat1 light4
                               -9.390 1.400 31
                                                -6.712
                                                        <.0001
heat4 light1 - heat2 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat4 light1 - heat3 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat4 light1 - heat4 light4
                              -12.779 1.240 31 -10.320
                                                        <.0001
heat1 light2 - heat2 light2
                               nonEst
                                         NA NA
                                                    NA
heat1 light2 - heat3 light2
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat1 light2 - heat4 light2
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat1 light2 - heat1 light3
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat1 light2 - heat2 light3
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat1 light2 - heat3 light3
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat1 light2 - heat4 light3
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat1 light2 - heat1 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat1 light2 - heat2 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat1 light2 - heat3 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat1 light2 - heat4 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
                                                -2.939
heat2 light2 - heat3 light2
                               -3.536 1.200 31
                                                        0.1763
heat2 light2 - heat4 light2
                               -1.802 1.450 31
                                                -1.243
                                                        0.9805
heat2 light2 - heat1 light3
                               -2.391 1.450 31
                                                -1.647
                                                        0.8778
heat2 light2 - heat2 light3
                               -5.332 1.260 31
                                                -4.221 0.0090
                               -7.227 1.380 31
                                                -5.250
heat2 light2 - heat3 light3
                                                        0.0006
heat2 light2 - heat4 light3
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat2 light2 - heat1 light4
                               -5.013 1.390 31
                                                -3.598
                                                        0.0423
heat2 light2 - heat2 light4
                               nonEst
                                                    NA
                                         NA NA
                                                            NA
heat2 light2 - heat3 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat2 light2 - heat4 light4
                                                        <.0001
                               -8.403 1.150 31
                                                -7.310
heat3 light2 - heat4 light2
                               1.734 1.220 31
                                                 1.426
                                                        0.9489
                               1.145 1.280 31
heat3 light2 - heat1 light3
                                                 0.894
                                                        0.9987
heat3 light2 - heat2 light3
                               -1.797 1.040 31
                                                -1.729
                                                        0.8413
heat3 light2 - heat3 light3
                               -3.692 1.190 31
                                                -3.099 0.1281
heat3 light2 - heat4 light3
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat3 light2 - heat1 light4
                               -1.478 1.170 31
                                                -1.260
                                                        0.9784
heat3 light2 - heat2 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat3 light2 - heat3 light4
                               nonEst
                                         NA NA
                                                            NA
                                                    NA
heat3 light2 - heat4 light4
                               -4.867 0.947 31
                                                -5.140
                                                        0.0008
                               -0.589 1.450 31
heat4 light2 - heat1 light3
                                                -0.405
                                                        1.0000
heat4 light2 - heat2 light3
                               -3.530 1.200 31
                                                -2.933
                                                        0.1781
heat4 light2 - heat3 light3
                               -5.425 1.370 31
                                                -3.957
                                                        0.0176
heat4 light2 - heat4 light3
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat4 light2 - heat1 light4
                               -3.211 1.330 31
                                                -2.420
                                                        0.4229
heat4 light2 - heat2 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
heat4 light2 - heat3 light4
                               nonEst
                                         NA NA
                                                    NA
                                                            NA
```

```
heat4 light2 - heat4 light4
                              -6.601 1.190 31 -5.568 0.0002
heat1 light3 - heat2 light3
                              -2.941 1.310 31
                                               -2.241 0.5350
heat1 light3 - heat3 light3
                              -4.836 1.490 31
                                               -3.242 0.0949
heat1 light3 - heat4 light3
                                        NA NA
                              nonEst
                                                    NA
heat1 light3 - heat1 light4
                              -2.622 1.430 31
                                               -1.830
                                                      0.7900
heat1 light3 - heat2 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat1 light3 - heat3 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat1 light3 - heat4 light4
                              -6.012 1.220 31
                                               -4.925
                                                       0.0014
heat2 light3 - heat3 light3
                              -1.895 1.220 31
                                               -1.556 0.9119
heat2 light3 - heat4 light3
                              nonEst
                                        NA NA
                                                    NA
heat2 light3 - heat1 light4
                               0.319 1.220 31
                                                 0.262
                                                       1.0000
heat2 light3 - heat2 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat2 light3 - heat3 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat2 light3 - heat4 light4
                              -3.070 0.975 31
                                                -3.148
                                                       0.1156
heat3 light3 - heat4 light3
                              nonEst
                                        NA NA
                                                    NA
heat3 light3 - heat1 light4
                               2.214 1.330 31
                                                 1.659
                                                       0.8727
heat3 light3 - heat2 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat3 light3 - heat3 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat3 light3 - heat4 light4
                              -1.175 1.160 31
                                               -1.010 0.9962
heat4 light3 - heat1 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat4 light3 - heat2 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat4 light3 - heat3 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat4 light3 - heat4 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat1 light4 - heat2 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat1 light4 - heat3 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat1 light4 - heat4 light4
                              -3.389 1.150 31 -2.936 0.1772
heat2 light4 - heat3 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat2 light4 - heat4 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
heat3 light4 - heat4 light4
                              nonEst
                                        NA NA
                                                    NA
                                                            NA
```

Results are averaged over the levels of: block, variety, side P value adjustment: tukey method for comparing a family of 12 estimates

Answer: Highest mean quality is at **Heat 3** \times **Light 4**. Next best are typically **H4** \times **L3** and **H3** \times **L3** (small drops from the best). Tukey shows many low-HL combos are clearly worse.

Q2 — Do best settings depend on Variety?

Table 3: Best Heat×Light within each Variety

heat	light	variety	emmean	SE	df	lower.CL	upper.CL
4	4	F	54.93	0.85	31	53.21	56.66
3	4	\mathbf{R}	54.19	0.87	31	52.42	55.95

Answer: No. Best HL pattern is essentially the same for both varieties (interactions ns).

Q3 — Does quality depend on Variety?

Table 4: Variety means (EMMs)

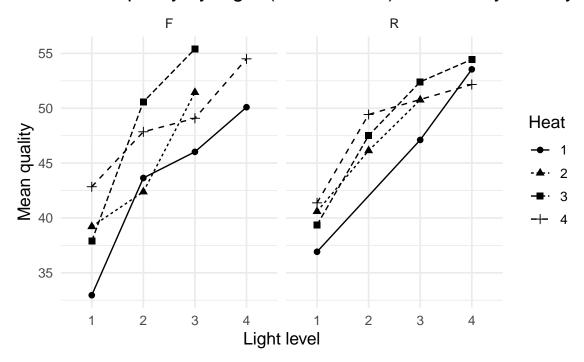
variety	emmean	SE	df	asymp.LCL	asymp.UCL
F	NA	NA	NA	NA	NA
R	NA	NA	NA	NA	NA

Results are averaged over the levels of: block, heat, light, side

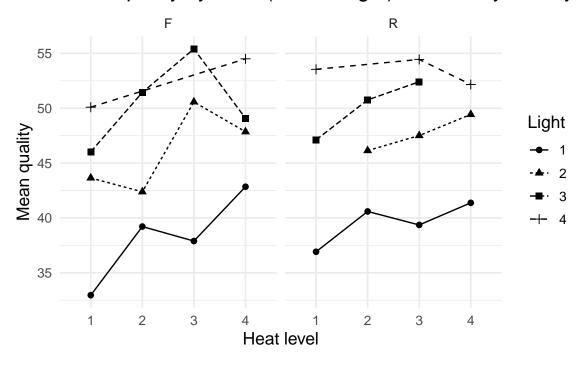
Answer: Yes, slightly. Redhot 1.15 points higher than Furious (p 0.017).

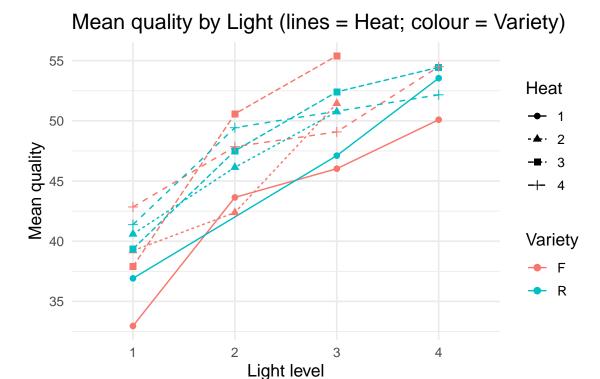
Mean plots (no CIs; simple patterns)

Mean quality by Light (lines = Heat), faceted by Variety



Mean quality by Heat (lines = Light), faceted by Variety





Reading the plots:

- Light rises cleanly \rightarrow strong positive effect.
- \bullet $\,$ Heat shows smaller, less consistent shifts.
- Varieties move in parallel \rightarrow no meaningful interactions.

Q4 — How big are differences? (rank & cost-aware choice)

Table 5: Ranked Heat×Light ($\Delta = \text{mean} - \text{best}$; energy proxy = heat + light)

heat	light	mean	lower	upper	delta_vs_best	energy_proxy
4	4	54.04	52.74	55.34	0.00	8
3	3	52.86	50.89	54.83	-1.18	6
2	3	50.97	49.50	52.44	-3.07	5
1	4	50.65	48.70	52.60	-3.39	5
3	2	49.17	47.70	50.64	-4.87	5
1	3	48.03	45.83	50.23	-6.01	4
4	2	47.44	45.45	49.42	-6.60	6

1 ,	11.14		1		1 14 1 4	
heat	light	mean	lower	upper	delta_vs_best	energy_proxy
2	2	45.64	43.59	47.68	-8.40	4
4	1	41.26	39.16	43.36	-12.78	5
3	1	40.74	38.26	43.22	-13.30	4
2	1	39.23	37.32	41.13	-14.81	3
1	1	34.81	32.82	36.81	-19.23	2
1	2	NA	NA	NA	NA	3
4	3	NA	NA	NA	NA	7
2	4	NA	NA	NA	NA	6
3	4	NA	NA	NA	NA	7

Table 6: Near-optimal within 2 points of best (sorted by energy proxy)

heat	light	mean	lower	upper	delta_vs_best	energy_proxy
3	3	52.86	50.89	54.83	-1.18	6
4	4	54.04	52.74	55.34	0.00	8

Observation:

• Best: $H3 \times L4$ (energy proxy 7).

• Near-ties: $H4 \times L3$ (-1.1, energy 7).

• Cheaper tolerance option: $H3 \times L3$ (-1.7, energy 6).

Optimal Conditions

Table 7: Top three treatment combinations by mean quality score

Variety	Light	Heat	Mean Quality	CI Lower	CI Upper
F	3	3	55.40	50.90	59.90
\mathbf{F}	4	4	54.50	47.76	61.24
R	4	3	54.45	49.60	59.30

• Cost-aware recommendation:

24

Future Reccomendations:

Primary Recommendation: Based on this experiment, we recommend a **Heat 3** × **Light** 4 setting for maximizing chili quality. Moderate heat and high light leads to best crop quality and yield.

Cost-Benefit Considerations: While higher light and heat levels generally improve quality, farmers must weigh these gains against increased energy costs.

Variety Selection: ** Redhot 1.15 points higher than Furious (p 0.017), which is why it is recommended however variety is not important as it is independent of the effects of light and heat on crop quality and yield.(all HL×Var interactions non-significant).

Practical Implementation: - If energy is a concern, $\mathbf{H4} \times \mathbf{L3}$ (-1.1) or $\mathbf{H3} \times \mathbf{L3}$ (-1.7; lower proxy) are practical alternatives.

Study Limitations

- 1. Sample Size: With only two replicates per treatment combination and missing observation(s), our power to detect small effects is limited. Confidence intervals for some comparisons are wide.
- 2. **Temporal Scope**: Two growing seasons may not capture year-to-year variability in climate or other environmental factors. Long-term validation is recommended.
- 3. Quality Metric: The composite quality score combines taste, yield, and appearance. Economic returns may depend more heavily on one component (e.g., yield). Future studies should examine these components separately.
- 4. **Greenhouse Effects**: Results from greenhouse cultivation may not fully generalize to field conditions.
- 5. **Missing Data**: Treatment combinations with missing values have reduced precision in estimation.

Future Directions

- Replicate the experiment with larger sample sizes to narrow confidence intervals
- Conduct economic analysis incorporating energy costs and market prices
- Separate quality into components (taste, yield, appearance) for targeted optimization
- Test intermediate settings between levels to fine-tune recommendations
- Evaluate performance under varying weather conditions across multiple years

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26