Experimental Design Project

STA2005S - 2025

Maryam Abrahams (ABRMAR043)
Bheka Mabika (MBKBHE002)

Experimental Design and Randomization:

The two chilli farmers, Ms Hopeful and Mr Growing, compete head to head for the hearts and minds of the South African chilli market. But despite their names Ms Hopeful seems to always get higher chilli yields than Mr Growing. In Mr Growing's eyes this cannot continue.

Aim:

This experiment aims to systematically evaluate the effect of light levels, heat levels and chilli variety on crop quality and yield. With our results we will be able to give evidence-based recommendations on optimal growing conditions.

Particularly we seek to answer:

1. Which light and heat settings produce the highest quality chillies?
2. Does the above depend on variety?
3. Does quality depend on variety?
4. How big are the differences?

Priori Hypothesis:

Prior to collecting the data, we hypothesize:

- $H1$: Quality and yield will increase as light increases and with moderate heat levels, as either extreme of heat can lead to some form of denaturing of the plant's cells.
- $H2$: Although their would be natural differences between varieties of chillies the light-heat response patterns will be similar between varieties.

- $H3$: Quality of chilli would be heavily dependent on variety as taste, and look are variety-specific.

Simulating data:

Creating the design matrix in accordance to a **Randomized block design (RBD)**, with a $4 \times 4 \times 2$ factorial treatment structure.

treatments:

- Light: 4 levels (1, 2, 3, 4)
- Heat: 4 levels (1, 2, 3, 4)
- Variety: 2 levels (R = Redhot, F = Furious)

$\implies$ 32 unique treatments

blocking factors:

- Season: (1, 2)
- Greenhouse: (A, B)
- Side of greenhouse: (N = north-facing, S = south-facing)

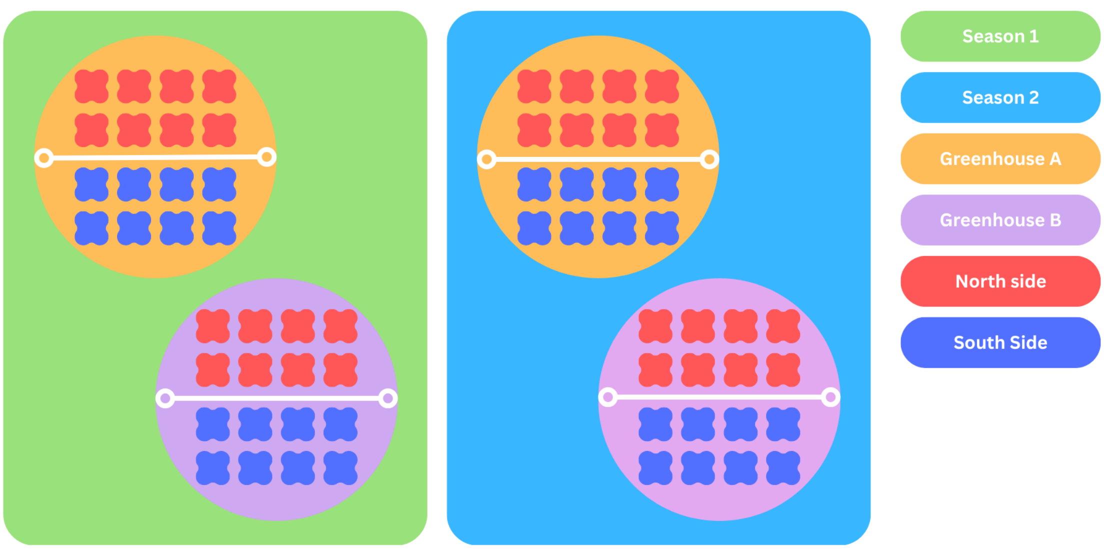


Figure 1: Blocking factors explained

Randomization:

For each season, we randomized the 32 treatment combinations independently to the 32 available plots (16 plots × 2 greenhouses):

1. Created all 32 treatment combinations using `expand.grid()`
2. For Season 1: Randomly permuted treatments and assigned to plots within each greenhouse
3. For Season 2: Independently randomized treatments again to plots
4. Set random seed (`set.seed(22)`) for reproducibility

This design ensures:

- Each treatment combination appears exactly twice (once per season)
- Complete randomization within each season-greenhouse combination
- No confounding between treatment effects and blocking factors

Obtaining observations:

Fitting the models and checking assumptions:

We fitted a linear model with quality score as the response variable:

$$Y_{ijklm} = \mu + \beta_i + \gamma_j + \delta_k + \lambda_l + \nu_m + (\delta\lambda)_{kl} + (\delta\nu)_{km} + (\lambda\nu)_{lm} + (\delta\lambda\nu)_{klm} + \epsilon_{ijklm}$$

where:

- Y_{ijklm} = quality score
- μ = overall mean
- β_i = block effect (season × greenhouse, i = 1,...,4)
- γ_j = side effect (north/south, j = 1,2)
- δ_k = light level effect (k = 1,2,3,4)
- λ_l = heat level effect (l = 1,2,3,4)
- ν_m = variety effect (m = R, F)
- Interaction terms for all two-way and three-way interactions among treatment factors
- ϵ_{ijklm} = random error, assumed $\sim N(0, \sigma^2)$

Block and side were treated as fixed effects to control for their influence, while our primary interest was in the treatment factors (light, heat, variety) and their interactions.

Analysis Procedures

Missing Values: Cases with missing observations were excluded from analysis using complete-case analysis, resulting in 62 observations for model fitting.

ANOVA: We used Type II sums of squares to test main effects and interactions, as this approach appropriately handles unbalanced designs caused by missing data and tests each effect after accounting for all other effects at the same or lower level.

Multiple Testing: Given the exploratory nature of this study and the relatively small number of pre-specified hypotheses, we report both unadjusted p-values and note effects significant at $= 0.05$. For post-hoc pairwise comparisons, we apply Tukey's HSD adjustment.

Model Diagnostics: We examined residual plots (residuals vs. fitted, Q-Q plot, scale-location, and leverage plots) to verify assumptions of normality, homoscedasticity, and identify potential outliers.

Effect Estimation: We computed estimated marginal means (EMMs) with 95% confidence intervals for all treatment combinations.

```
Call:
lm(formula = obs ~ block + heat * light * variety + side, data = mydata)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.4463 -0.6996  0.0000  0.6996  2.4463 

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)    
(Intercept)            40.6888     1.3048  31.184  < 2e-16 ***
block2.A               -4.2239     0.8447  -5.001 3.35e-05 ***
block1.B               -0.8722     0.9055  -0.963 0.344299    
block2.B               -7.0022     0.7200  -9.725 3.78e-10 ***
heat2                   2.6451     1.8468   1.432 0.163990    
heat3                   5.9029     1.8447   3.200 0.003604 ** 
heat4                   6.4148     1.7694   3.626 0.001231 ** 
light2                  7.9264     1.8180   4.360 0.000182 ***
light3                 10.4088     1.8447   5.642 6.22e-06 ***
light4                 13.7101     2.2370   6.129 1.77e-06 ***
varietyR                3.7604     2.1556   1.744 0.092891 .  
```

```
sides                  -3.7571     0.6909  -5.438 1.06e-05 ***
heat2:light2            0.1763     2.5846   0.068 0.946153    
heat3:light2            1.9210     2.4962   0.770 0.448492    
heat4:light2           -1.2975     2.5128  -0.516 0.609963    
heat2:light3           -0.4486     2.6386  -0.170 0.866309    
heat3:light3           -0.5913     2.5172  -0.235 0.816126    
heat4:light3            0.7108     2.5829   0.275 0.785348    
heat2:light4            0.7325     3.0380   0.241 0.811355    
heat3:light4            0.6125     2.9599   0.207 0.837662    
heat4:light4           -5.9527     2.8112  -2.118 0.043938 *  
heat2:varietyR         -1.0632     2.8070  -0.379 0.707950    
heat3:varietyR         -6.9882     2.8177  -2.480 0.019935 *  
heat4:varietyR         -2.9391     2.7336  -1.075 0.292173    
light2:varietyR        -0.1056     2.7304  -0.039 0.969446    
light3:varietyR        -2.0584     2.8070  -0.733 0.469927    
light4:varietyR        -3.2245     3.1840  -1.013 0.320525    
heat2:light2:varietyR  -2.2747     3.6478  -0.624 0.538330    
heat3:light2:varietyR   1.6409     3.7832   0.434 0.668056    
heat4:light2:varietyR  -1.6624     3.7086  -0.448 0.657685    
heat2:light3:varietyR   0.3691     3.7086   0.100 0.921483    
heat3:light3:varietyR   6.2303     3.7792   1.649 0.111264    
heat4:light3:varietyR   1.1873     3.6602   0.324 0.748244    
heat2:light4:varietyR   1.1191     3.9885   0.281 0.781250    
heat3:light4:varietyR   2.4489     4.0410   0.606 0.549761    
heat4:light4:varietyR   7.5228     4.0410   1.862 0.073997 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.71 on 26 degrees of freedom
  (2 observations deleted due to missingness)
Multiple R-squared:  0.9688,    Adjusted R-squared:  0.9268 
F-statistic: 23.06 on 35 and 26 DF,  p-value: 1.482e-12
```

Model Diagnostics

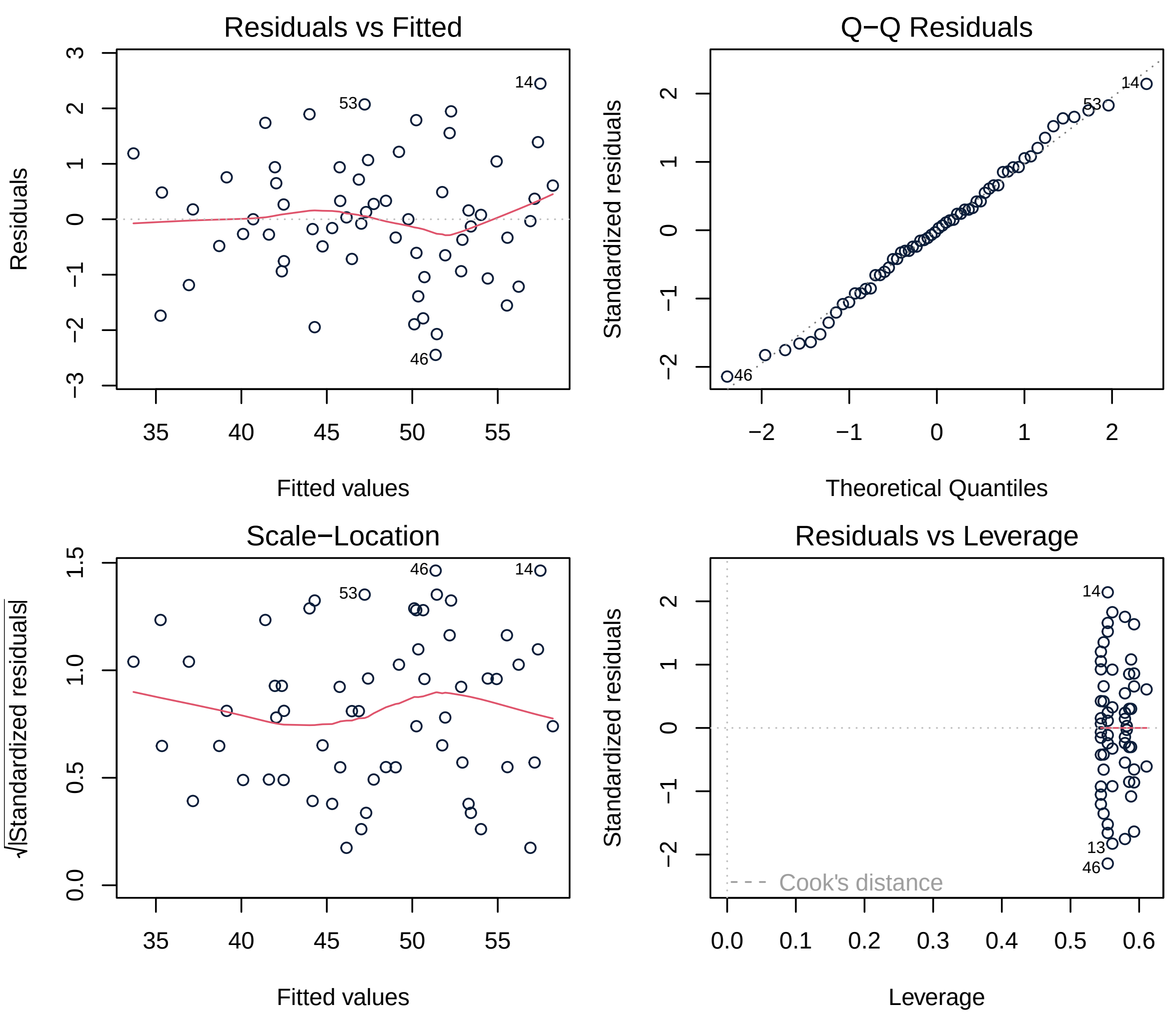
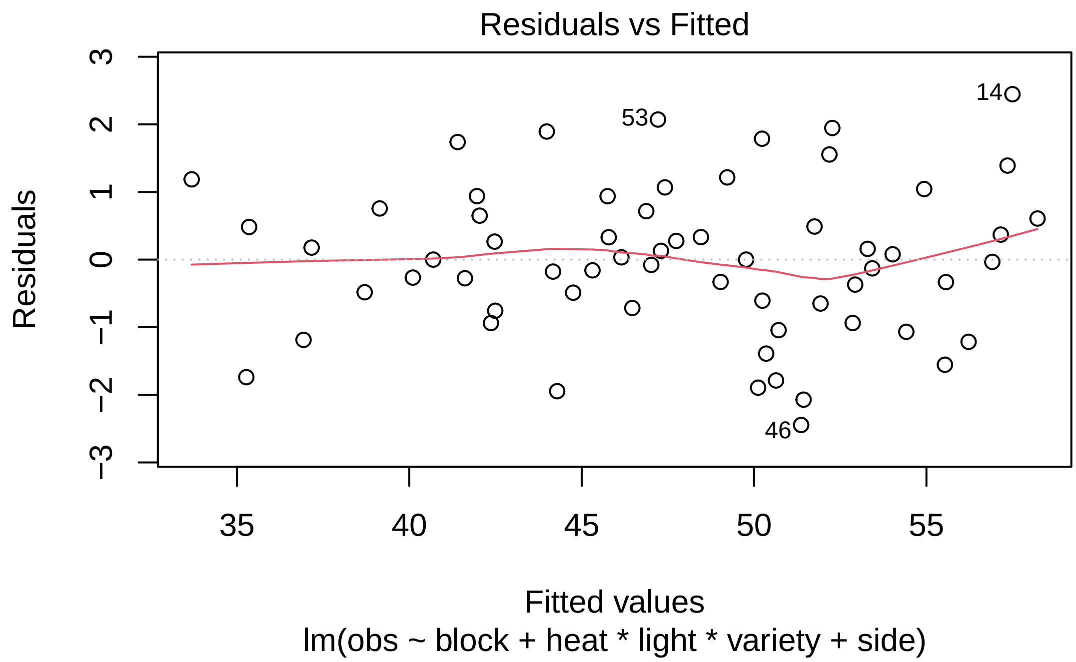


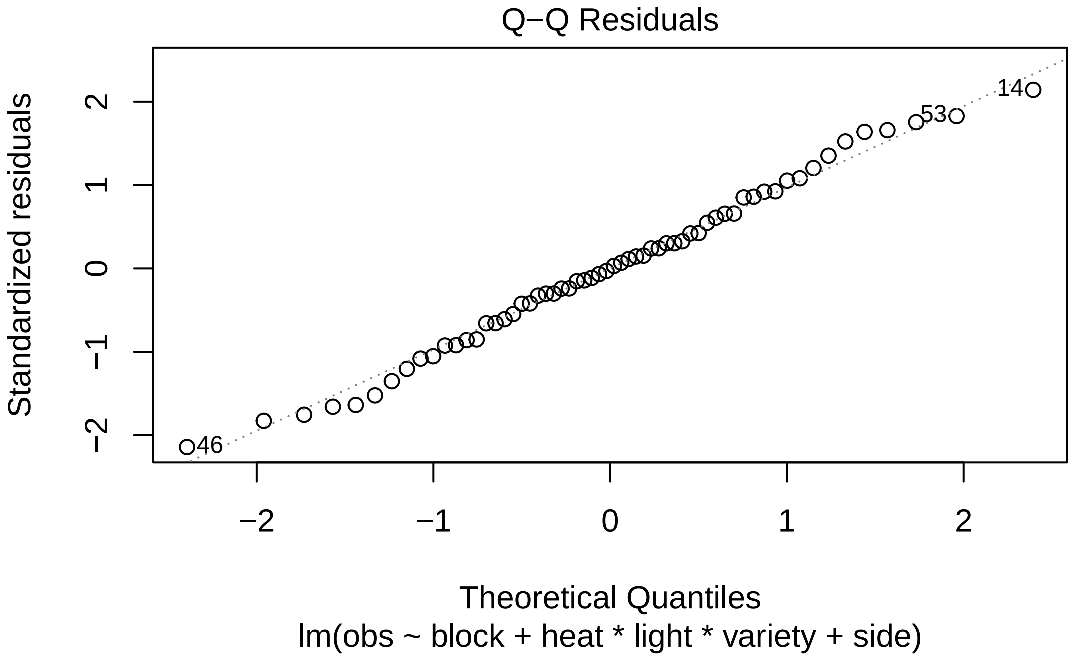
Figure 2: Residual diagnostic plots showing no serious violations of model assumptions. The Q-Q plot indicates approximate normality, and the residuals vs. fitted plot shows no strong patterns suggesting heteroscedasticity.

Analysis of Variance:

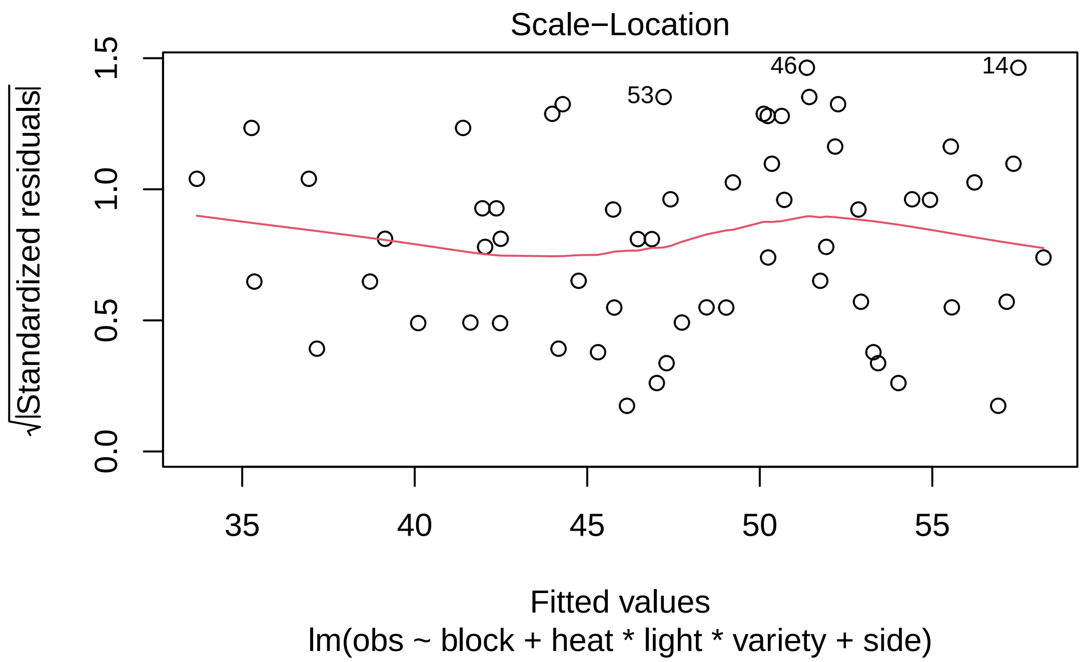
```
Analysis of Variance Table

Response: obs
                   Df  Sum Sq Mean Sq  F value    Pr(>F)
block               3  526.69  175.56  60.0083 8.096e-12 ***
```

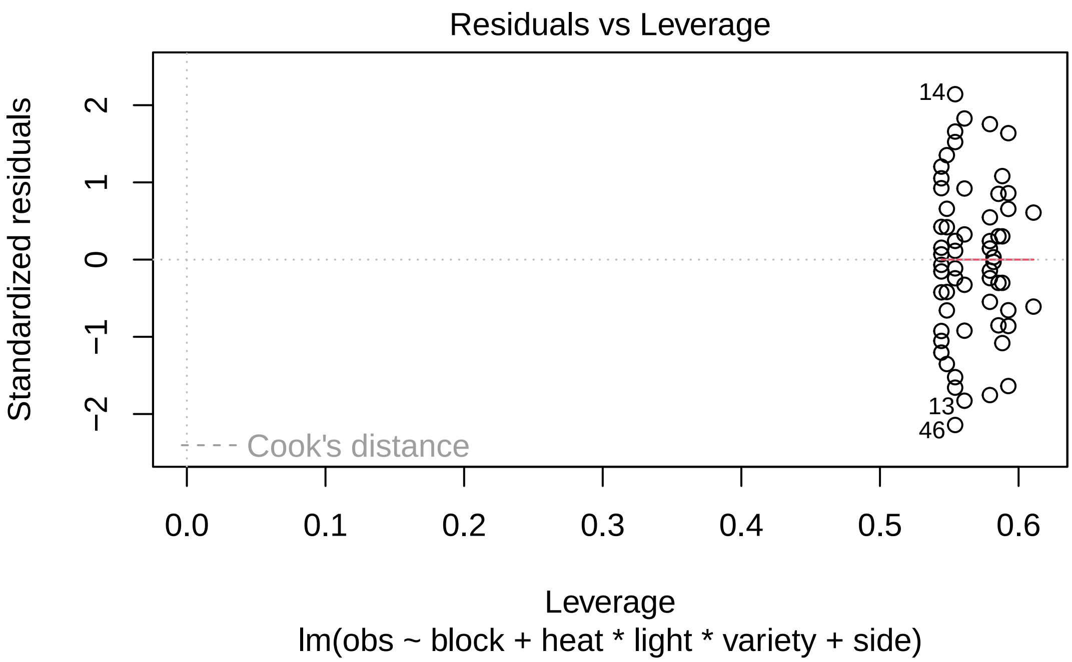
```
heat                3  142.67   47.56  16.2549 3.745e-06 ***
light               3 1460.84  486.95 166.4408 < 2.2e-16 ***
variety             1    4.22    4.22   1.4418   0.24067
side                1  103.51  103.51  35.3813 2.814e-06 ***
heat:light          9   45.27    5.03   1.7191   0.13495
heat:variety        3   38.94   12.98   4.4370   0.01203 *
light:variety       3    2.40    0.80   0.2737   0.84383
heat:light:variety  9   37.00    4.11   1.4053   0.23652
Residuals          26   76.07    2.93
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Anova Table (Type II tests)

Response: obs
                    Sum Sq Df  F value    Pr(>F)
block               450.41  3  51.3176 4.655e-11 ***
heat                176.45  3  20.1041 5.971e-07 ***
light              1295.64  3 147.6188 < 2.2e-16 ***
variety               5.16  1   1.7650   0.19554
side                 86.51  1  29.5698 1.062e-05 ***
heat:light           39.05  9   1.4830   0.20622
heat:variety         39.15  3   4.4610   0.01176 *
light:variety         2.40  3   0.2737   0.84383
heat:light:variety   37.00  9   1.4053   0.23652
Residuals            76.07 26
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```


Residuals vs Fitted
Residuals
3
2
1
0
−1
−2
−3
53
14
46
35
40
45
50
55
Fitted values
lm(obs ~ block + heat * light * variety + side)



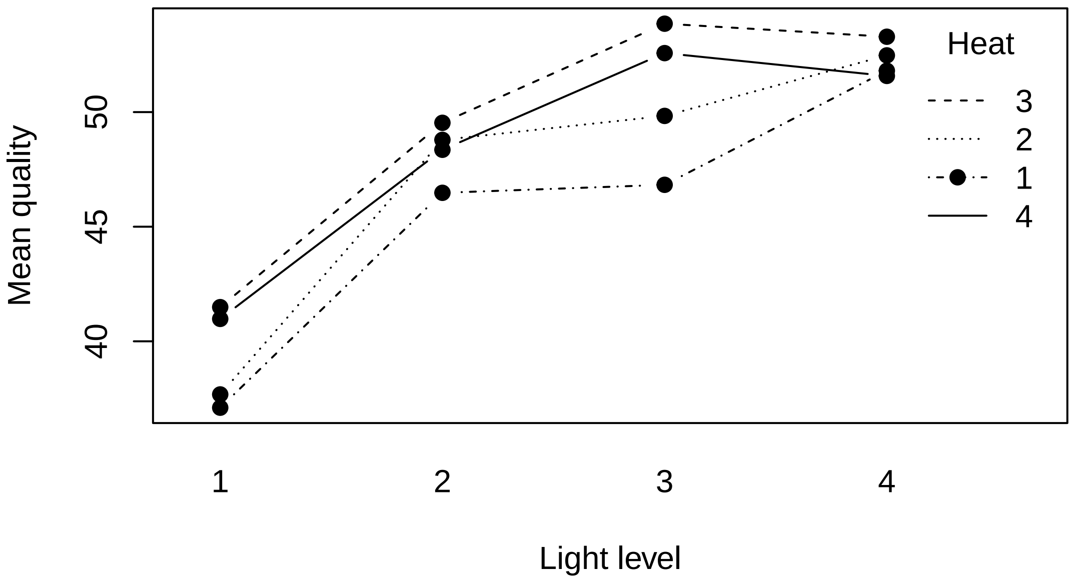
Q−Q Residuals
Standardized residuals
2
1
0
−1
−2
14
53
46
−2
−1
0
1
2
Theoretical Quantiles
lm(obs ~ block + heat * light * variety + side)

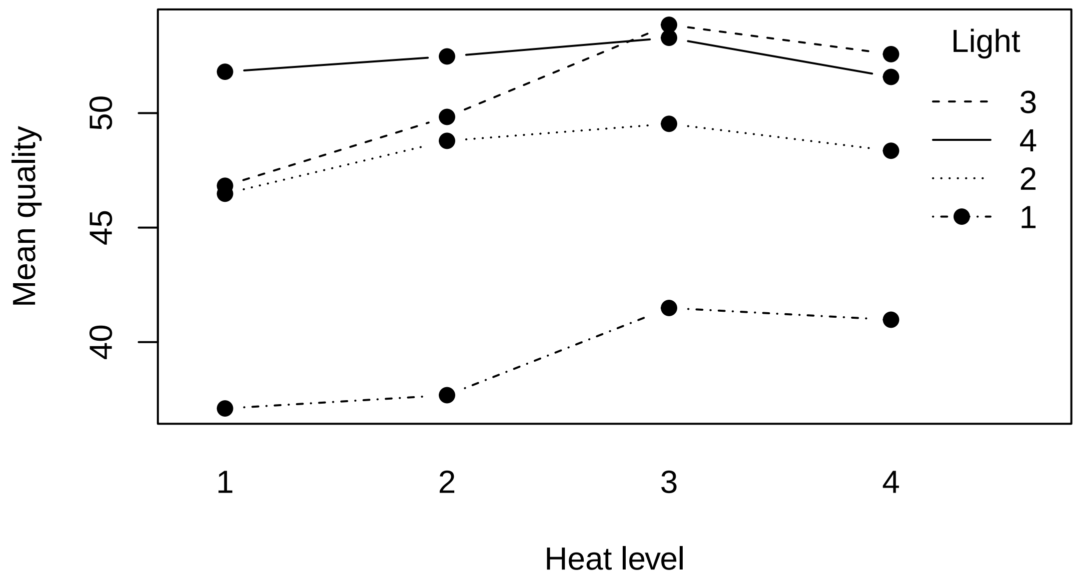

Scale−Location
46
14
53
1.5
1.0
0.5
0.0
√|Standardized residuals|
35
40
45
50
55
Fitted values
lm(obs ~ block + heat * light * variety + side)

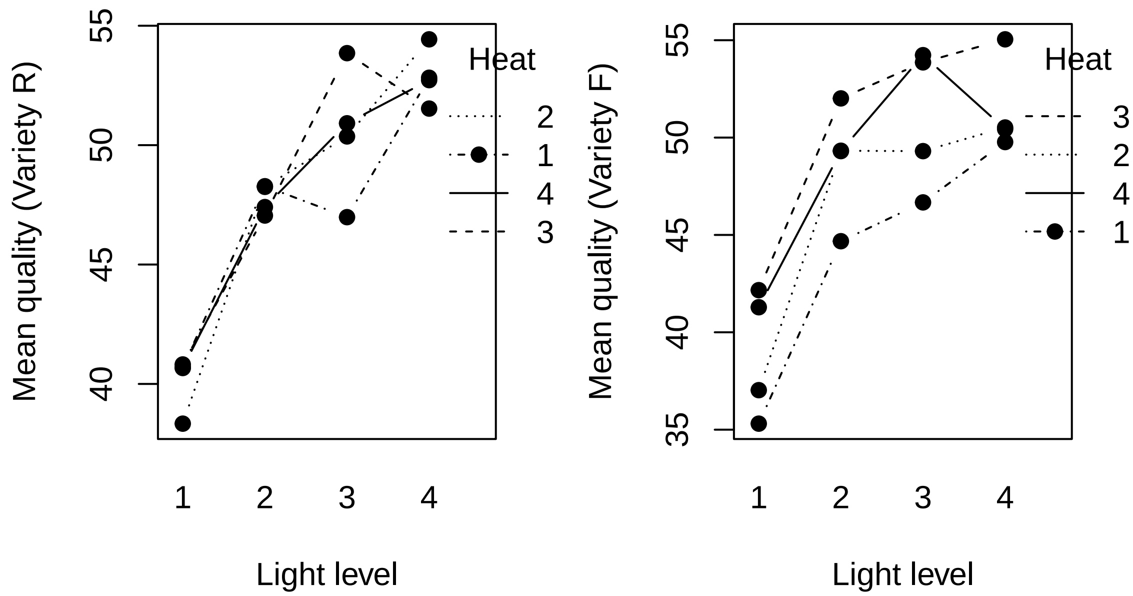

Residuals vs Leverage
Standardized residuals
2
1
0
−1
−2
14
13
46
Cook's distance
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Leverage
lm(obs ~ block + heat * light * variety + side)

Treatment Effects Visualization:

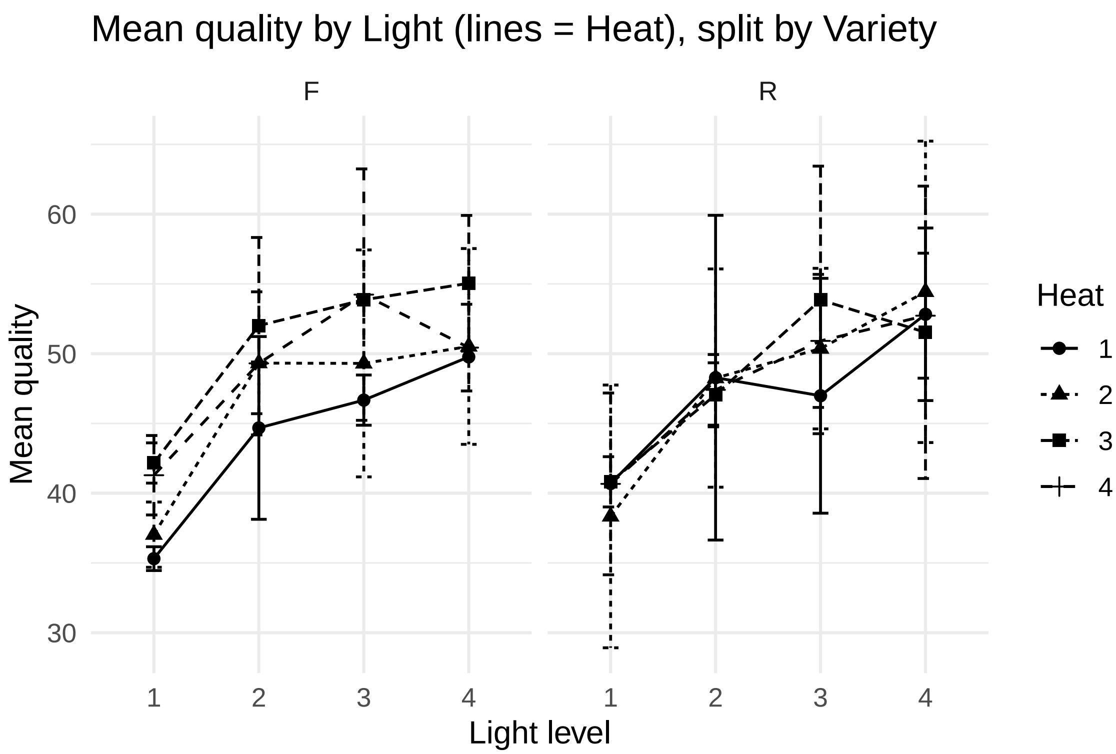
Interaction plots:

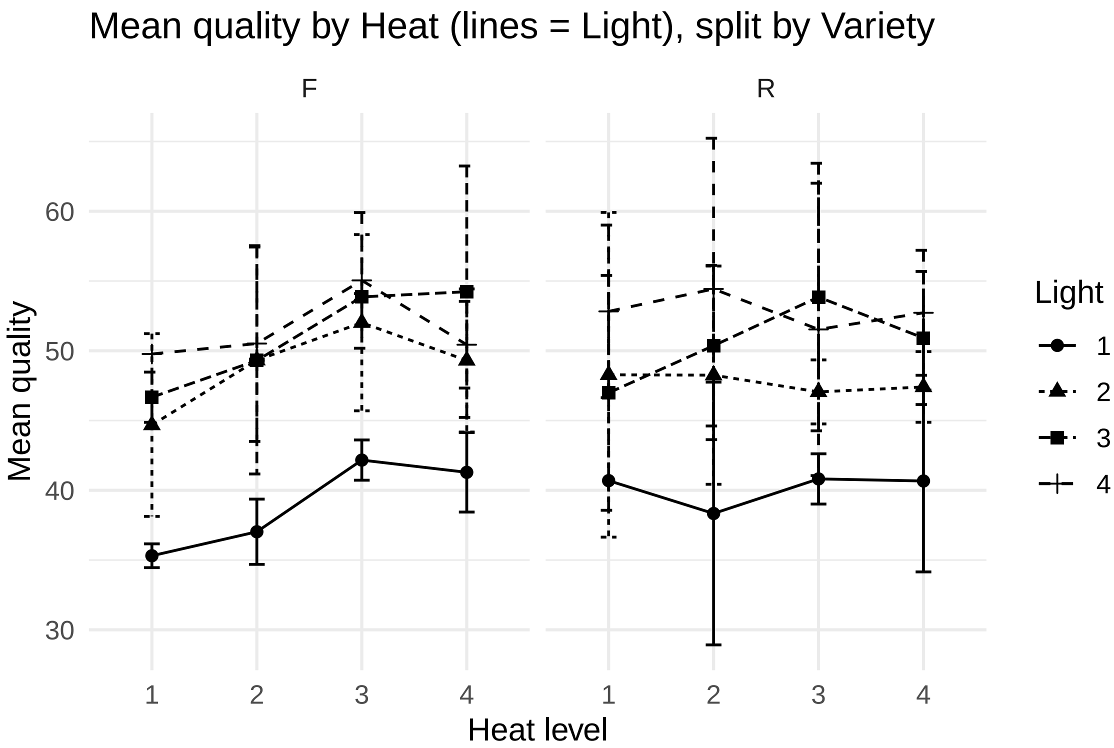


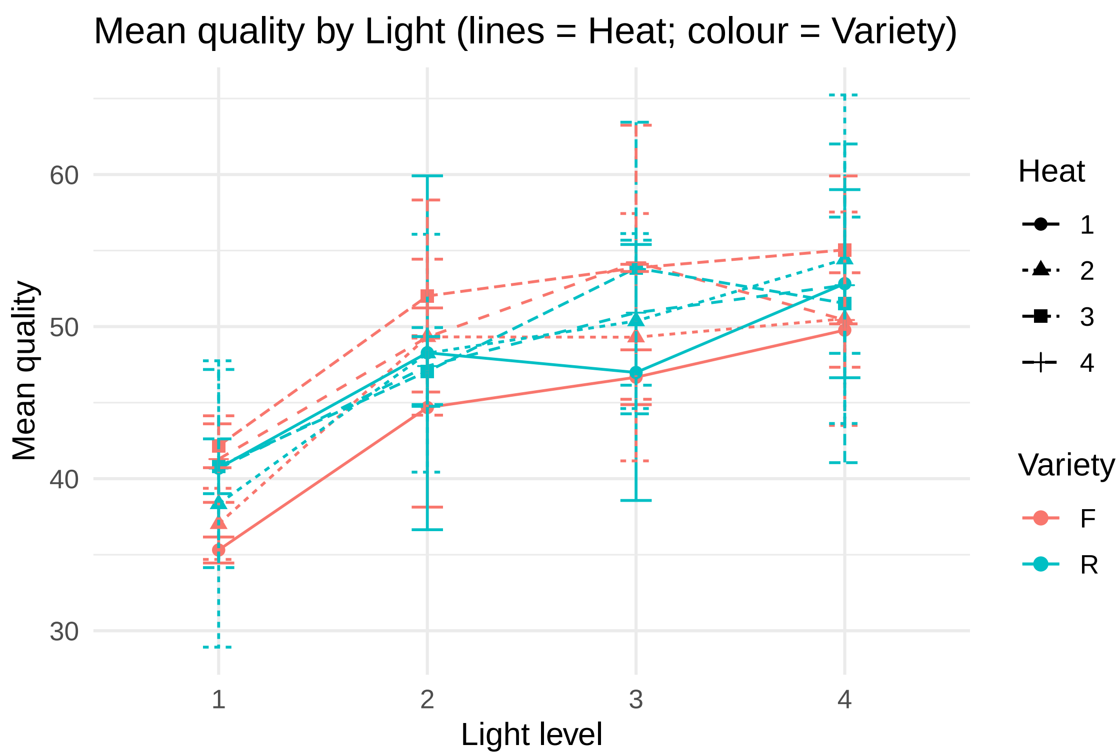

Light
3
4
2
1
50
45
40
Mean quality
1
2
3
4
Heat level


Mean quality (Variety R)
55
50
45
40
1
2
3
4
Light level
Heat
2
1
4
3
Mean quality (Variety F)
55
50
45
40
35
1
2
3
4
Light level
Heat
3
2
4
1

mean plots:


Mean quality by Light (lines = Heat), split by Variety
F
R
Mean quality
60
50
40
30
1
2
3
4
Light level
Heat
1
2
3
4


Mean quality by Heat (lines = Light), split by Variety
F
R
Mean quality
60
50
40
30
1
2
3
4
1
2
3
4
Heat level
Light
1
2
3
4



Analysis and Results:

Optimal Conditions

Table 1: Top three treatment combinations by mean quality score

Variety	Light	Heat	Mean Quality	CI Lower	CI Upper
F	4	3	55.05	50.18	59.91
R	4	2	54.43	43.63	65.23
F	3	4	54.23	45.22	63.24

Conclusion:

1. **Optimal Growing Conditions**: [State the best combination, e.g., "Light level 4 with Heat level 3 produces the highest quality chilies"]
2. **Effect Sizes**: [Quantify improvements, e.g., "The optimal combination yields quality scores approximately X points higher than standard conditions (Light 1, Heat 1), representing a Y% improvement"]
3. **Variety Differences**: [Discuss whether varieties respond differently and which performs better overall]
4. **Interaction Effects**: [Explain whether light and heat work synergistically or independently]

Future Reccomendations:

Primary Recommendation: Based on this experiment, we recommend [specific combination] for maximizing chili quality.

Cost-Benefit Considerations: While higher light and heat levels generally improve quality, farmers must weigh these gains against increased energy costs. [If results show diminishing returns, state this: "Light level 3 may offer the best cost-quality balance, as level 4 provides only marginal improvements at substantially higher cost."]

Variety Selection: [Recommend which variety to grow, or under what conditions each variety excels]

Practical Implementation: - Start with [recommended combination] for highest quality production - If costs are prohibitive, [alternative combination] offers nearly comparable quality at lower expense - Monitor results and adjust based on economic returns

Study Limitations

1. **Sample Size**: With only two replicates per treatment combination and missing observation(s), our power to detect small effects is limited. Confidence intervals for some comparisons are wide.
2. **Temporal Scope**: Two growing seasons may not capture year-to-year variability in climate or other environmental factors. Long-term validation is recommended.
3. **Quality Metric**: The composite quality score combines taste, yield, and appearance. Economic returns may depend more heavily on one component (e.g., yield). Future studies should examine these components separately.

4. **Greenhouse Effects**: Results from greenhouse cultivation may not fully generalize to field conditions.
5. **Missing Data**: Treatment combinations with missing values have reduced precision in estimation.

Future Directions

- Replicate the experiment with larger sample sizes to narrow confidence intervals
- Conduct economic analysis incorporating energy costs and market prices
- Separate quality into components (taste, yield, appearance) for targeted optimization

- Test intermediate settings between levels to fine-tune recommendations
- Evaluate performance under varying weather conditions across multiple years

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References:
