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# Unit-2
# TESTING OF HYPOTHESIS:

**Hypothesis** → Hypothesis is an assumption or claim or statements. By the help of testing of hypothesis we are able to test whether the hypothesis is true or not.

**Types of hypothesis:**

a) **Null Hypothesis ($H_0$)** → Null hypothesis is symbolized by $H_0$. Null hypothesis contains sign either (=) or ($\leq$) or ($\geq$).

⇒ Null hypothesis question मा दिएको हुन्छ mainly = भाको condition आउछ question मा. atmost भन्ने $\leq$ sign use गर्नि र at least भन्ने $\geq$ sign use गर्नि,

For e.g. Null Hypothesis ($H_0$): $\mu = 30$.
i.e, the population mean is 30.

Genally सबै question मा = condition हुन्छ

b) **Alternate Hypothesis ($H_1$)** → It is symbolized by $H_1$ and ~~it~~ it contains the sign either ($\neq$) or ($<$) or ($>$).

⇒ Question को आधारमा हामि sign select गर्छौं।
i.e, On the basis of question we use alternate hypothesis
either $\mu \neq$ (represents two-tailed test)
or, $\mu >$ (represents one-tailed test)
or $\mu <$ (represents one-tailed test).

(केहि condition नभए Question मा $\mu \neq$ use गर्छौं)

~~**Note:** According to question first we write null hypothesis then we select alternate (उल्टो) hypothesis as below:-~~
~~alternate of = $\neq$ (for two-tailed test)~~
~~alternate of $\leq$ is $>$ (one-tailed i.e, right tailed test)~~
~~alternate of $\geq$ is $<$ (one-tailed i.e, left tailed test)~~

For e.g.
$\mu \neq 30$ i.e, Population mean is different than 30.
$\mu < 30$ i.e, Population mean is less than 30.
$\mu > 30$ i.e, Population mean is greater than 30.

**Remember:** assumption जहिले पनि Population को लागि हुन्छ भन्ने test statistics मा गर्छौं।

## Concept of two taild, one tailed left & one tailed right:

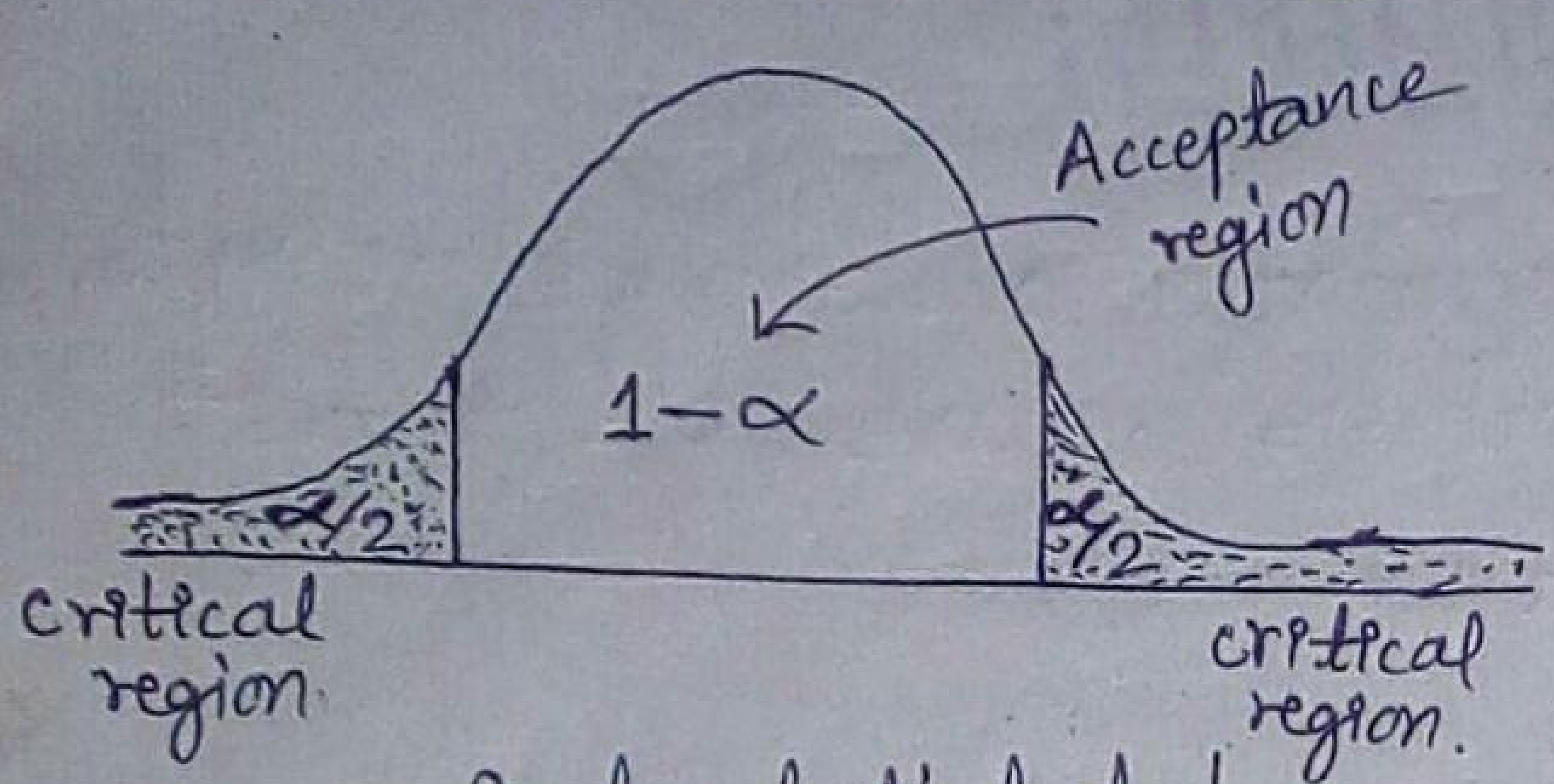


fig. two ~~taild~~ tailed

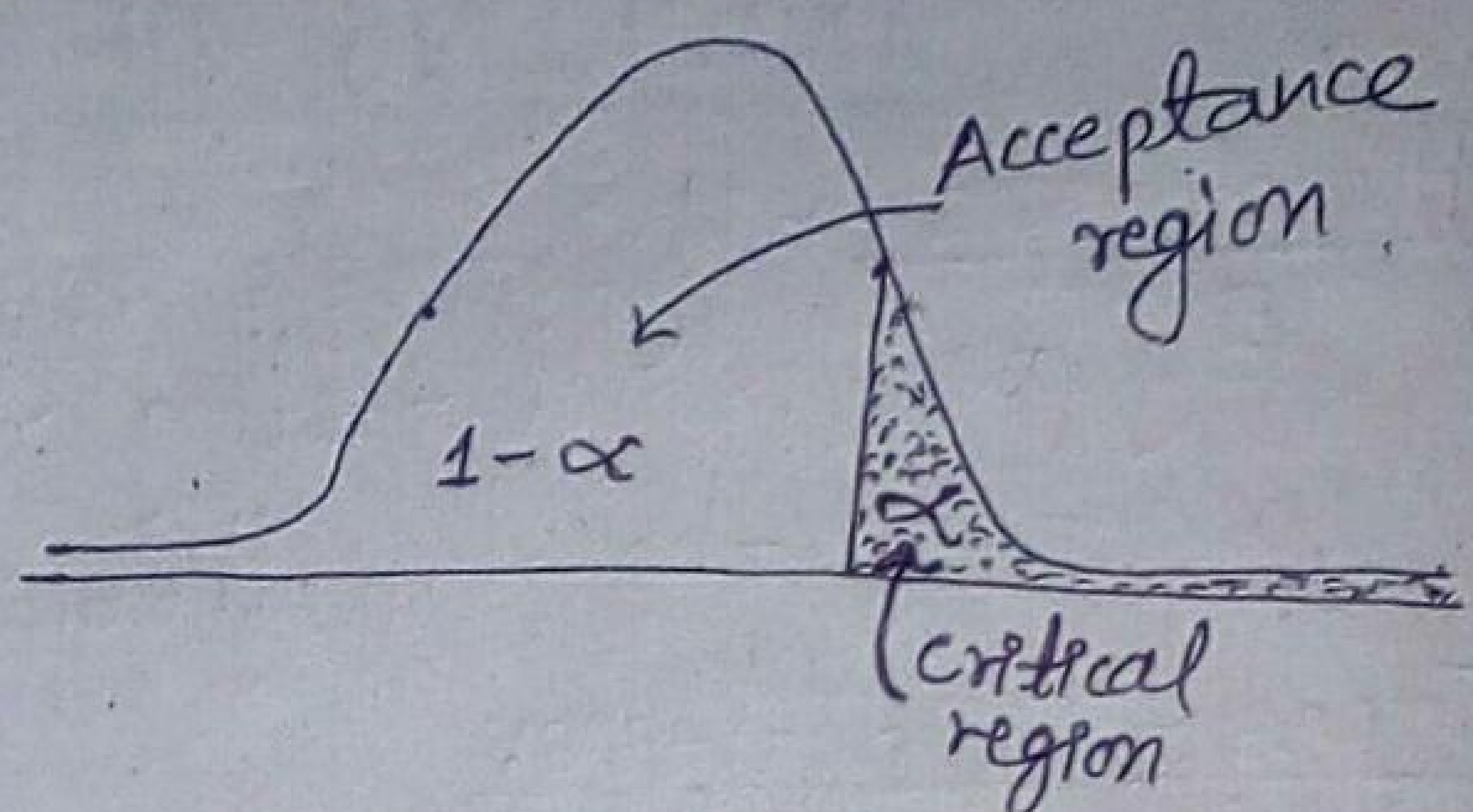


fig. One tailed right

**Note:** Critical region is also called rejection region

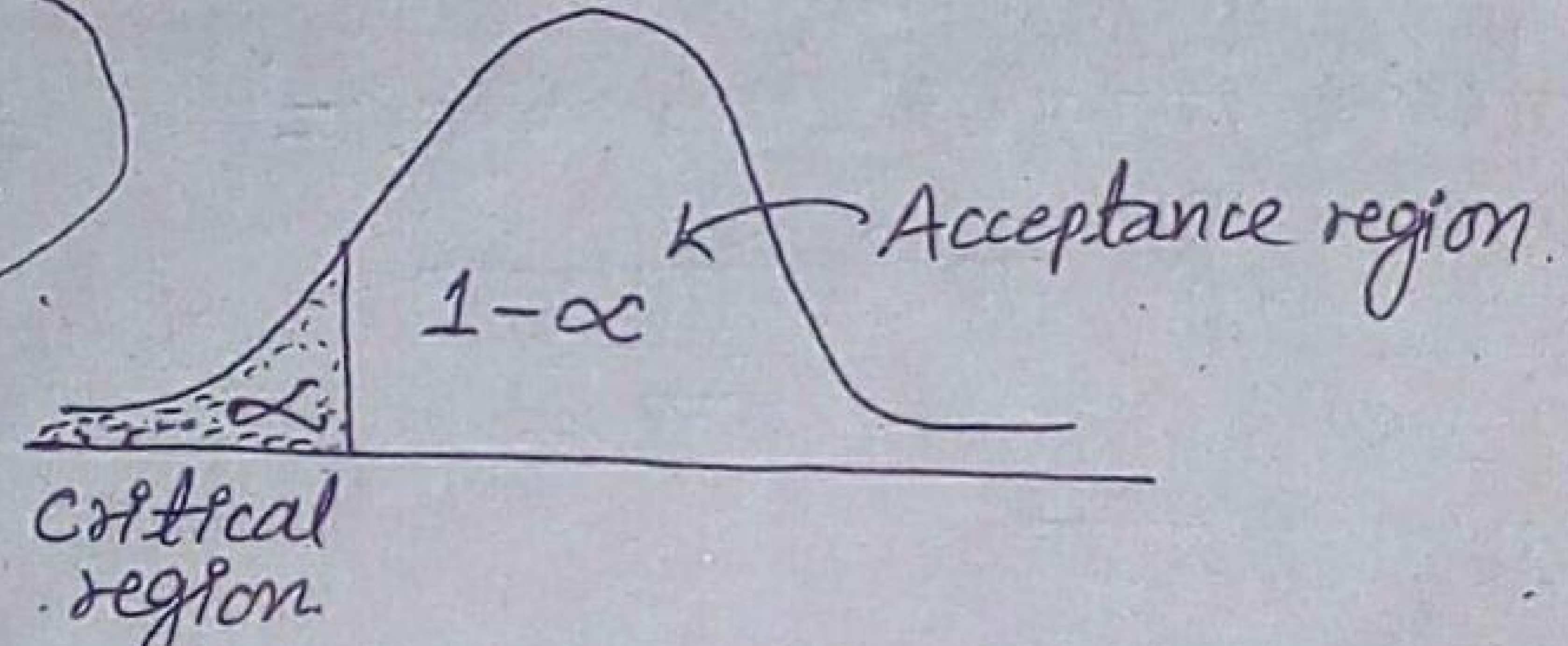


fig. One tailed left

## Errors while making decision:

| Decision | Actual situation | |
|---|---|---|
| | $H_0$ is true | $H_0$ is false |
| $H_0$ is accepted | Correct decision $(1-\alpha)$ | Type II[nd] Error $\beta = P(\text{type II}^{nd}\text{ Error})$ |
| $H_0$ is rejected | Type I[st] Error $\alpha = P(\text{type I}^{st}\text{ Error})$ | Correct decision $(1-\beta)$ |

**Note:** Level of significance ($\alpha$) Question मा दिएको भएको त्यहि value use गर्ने नत्र generally we use 5%.

Here, $\alpha$ = level of significance
$= P(H_0 \text{ is rejected} / H_0 \text{ is true})$.

So, $1-\alpha$ = level of confidence
$= P(H_1 \text{ is accepted} / H_0 \text{ is true})$

$\boxed{\text{where } P = \text{Probability}}$

& $\beta = P(\text{type II}^{nd}\text{ Error})$
$= P(H_0 \text{ is accepted} / H_0 \text{ is false})$

$1-\beta = P(H_0 \text{ is rejected} / H_0 \text{ is false})$.

## ⊛. Steps in testing of Hypothesis:

1) Identify the claims. (i.e, Question मा के find गर्नै भनेको छ त्यो उल्लेख). i.e, finding null hypothesis.

2) Setting of NULL and Alternate hypothesis. (i.e, Problem to test)
(We have already discussed Null and Alternate hypothesis).

3). <u>Test statistics:</u>

$$\text{Test statistics (i.e, Calculated value)} = \frac{\text{Sample statistics} - \text{pop}^n \text{ parameter}}{\text{Standard error}}$$

4). <u>Critical Value:</u> (Tabulated value at $\alpha$ level of significance with degree of freedom).
⇒ The no. of independent variates which make up statistic is called degree of freedom. It is simply denoted by d.f.

5) <u>Decision</u>: If calculated value is greater than critical value or tabulated value then, $H_0$ is rejected. i.e, $H_1$ will be accepted.
& If calculated value is less or equal to critical value then $H_0$ is accepted. i.e, $H_0$ will be rejected.

6) Conclusion on the basis of decision (i.e, Interpretation of result).

**Q1.** A sample of size 400 was drawn and the sample mean was found to be 99. Test whether this sample could have come from a normal population with mean 100 and standard deviation 8 at 5% level of significance.

<u>Solution:</u>

Given,

Sample size $(n) = 400$
Sample mean $(\bar{X}) = 99$
Population mean $(\mu) = 100$
standard deviation $(\sigma) = 8$.
level of significance $(\alpha) = 5\%$
$= 0.05$

We have already discussed about all these types of notations like $n, N, s, \mu$ etc. in 1st chapter revise them

Now,

<u>Problem to test</u>

Null Hypothesis $(H_0)$ : $\mu = 100$ i.e, pop$^n$ mean is 100.

Alternate Hypothesis $(H_1)$: $\mu \neq 100$
i.e, population mean is not equal to 100.

## Test Statistics:-

$$Z_{cal} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{99 - 100}{8/\sqrt{400}} = -2.5$$

$Z_{cal}$ +ve हरेल्न

$$\therefore |Z_{cal}| = 2.5$$

## Critical value:-

The tabulated value of Z at 0.05 level of significance

$$Z_{0.05} = 1.96 \text{ (two-tailed test).}$$

from value table given at back of book page no.309

## Decision:

Since $|Z_{cal}| = 2.5 > Z_{tab} = 1.96$.

So, $H_0$ is rejected. i.e, $H_1$ is accepted.

## Conclusion:

Hence we can conclude that the population mean is not equal to 100.

**Q2.** A random sample of 100 pen drive selected from a batch of 2000 pen drives shows that the average thickness of the pen drive is 0.354 with a standard deviation 0.048. Are the samples from the lot having average thickness 0.35?

## Solution:

Here, Sample size $(n) = 100$

Population size $(N) = 2000$

Sample mean $(\bar{x}) = 0.354$

Sample SD $(s) = 0.048$

Population mean $(\mu) = 0.35$

## Problem to test:

Null Hypothesis $(H_0)$: $\mu = 0.35$

i.e, average thickness of pen drive is 0.35

Alternate Hypothesis $(H_1)$: $\mu \neq 0.35$ (two-tailed)

i.e, average thickness of pen drive is not 0.35

## Test Statistics

$$Z_{cal} = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}} = \frac{0.354 - 0.35}{\frac{0.048}{\sqrt{100}}\sqrt{\frac{2000-100}{2000-1}}} = \frac{0.004}{0.0047} = 0.854$$

We use this formula in case where N (Capital N) and s are given. S.E may change according to question we already read S.E types in unit 1
standard error

## Critical Value:

Let $\alpha = 5\%$ be the level of significance then critical value for two tailed test $Z_{0.05} = 1.96$

Decision: Here, $|Z_{cal}| = 0.854 < Z_{tab} = 1.96$. So we accept $H_0$ at 5% level of significance. i.e, $H_1$ is rejected.

Conclusion: Sample are from lot having average thickness of pen drive 0.35.

**Q3.** If the mean breaking strength of a copper wire is 575 lbs with s.d. of 8.3 lbs. How many large sample must be used in order that there be one chance in 100 that the mean breaking strength of the sample is less than 572 lbs?

Solution:

Here, Population mean $(\mu) = 575$

Population SD $(\sigma) = 8.3$

P (mean breaking strength $< 572$) $= \frac{1}{100}$

Sample mean $(\bar{X}) = 572$

$\alpha = 1\%$

## Problem to test

Null Hypothesis $(H_0)$: $\mu = 575$

i.e, Mean breaking length of copper wire is 575 lbs.

Alternative Hypothesis $(H_1)$: $\mu < 575$ (one-tail left)

i.e, Mean breaking length of wire is less than 575 lbs

## Test statistic

$$Z_{cal} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{572 - 575}{8.3/\sqrt{n}} = -0.3614\sqrt{n}$$

i.e $|Z_{cal}| = 0.3614\sqrt{n}$

## Critical value

At $\alpha = 1\%$
$= 0.01$ critical value for one tailed test is $Z_{tab} = 2.32$

Here,

$0.3614\sqrt{n} = 2.32$

or, $\sqrt{n} = \frac{2.32}{0.3614} = 6.419$

or, $n = 41.2$.

or, $n \approx 41$

n निकाल्नु परे critical value = tabulated value गर्ने

**Q4.** An ambulance service claims that it takes on the average 8.9 minutes to reach to its destination in emergency calls. To check the claim, the agency which licenses ambulance services has them timed on 50 emergency calls getting a mean of 9.3 minutes with s.d. of 1.6 minutes. What can they conclude at the level of significance $\alpha = 0.05$? Use the confidence limit to make conclusion.

**Solution:** (s दिएको तर N नदिएको बेला S.E निकाल्न $\frac{s}{\sqrt{n}}$ गर्ने (Since $s = \hat{\sigma}$) या यसरि गर्ने).

Here, Population mean$(\mu) = 8.9$, Sample size$(n) = 50$, Sample mean$(\bar{x}) = 9.3$, Sample S.D.$(s) = 1.6$, level of significance $(\alpha) = 0.05$

## Problem to test:

$H_0$: Average time taken by ambulance to reach destination in 8.9 min $(\mu = 8.9)$.

$H_1$: Average time taken by ambulance to reach destination is not 8.9 min $(\mu \neq 8.9)$. (Two-tailed).

## Critical value

At $\alpha = 0.05$ critical value for two tailed test is $Z_{tab} = 1.96$

The limits of acceptance region for $\bar{x} = \mu + Z_{tab}\frac{s}{\sqrt{n}} = 8.9 \pm 1.96 \times \frac{1.6}{\sqrt{50}}$

$= 8.9 \pm 0.443$

Taking + sign, $\bar{x} = 9.343$ & Taking − sign $\bar{x} = 8.457$

**Decision:** 9.3 lies between 8.457 and 9.343. So, $H_0$ is accepted.

**Conclusion:** The claim of ambulance service that it takes on average 8.9 minutes to reach destination is true.

## ⊛. Testing of Hypothesis by using P-value approach:

By using Probability value (P-value) approach we can also test the hypothesis. In P-value approach P-value can be obtained as follows:-

**For one-tailed test (Either left tailed or right tailed):**

$$P\text{-value} = 0.5 - P(0 \le x \le Z_{cal}).$$

**For two-tailed test:**

$$P\text{-value} = 2\{0.5 - P(0 \le x \le Z_{cal})\}$$

⇒ For making decisions p-value is compared with level of significance then, if p-value is $\le \alpha$ then $H_0$ is rejected and if P-value is $> \alpha$ then $H_0$ is accepted.

**Q1.** The mean life time of 400 laptop cells produced by a company is found to be 1570 hours with a standard deviation of 150 hours. Test the hypothesis that the mean life of the laptop cells produced by the company is 1600 hours against the alternative hypothesis that is greater than 1600 hours at 1% level of significance.

**Solution:**

Given, Sample size $(n) = 400$, mean life time $(\overline{X}) = 1570$ hours

St. dev. $(\sigma) = 150$ hours, Pop$^n$ life time $(\mu) = 1600$ hours

level of significance $= 1\% = 0.01$.

**Problem to test**

Null Hypothesis $(H_0)$: $\mu = 1600$

Alternate Hypothesis $(H_1)$: $\mu > 1600$

**Test statistics**

$$Z_{cal} = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{1570 - 1600}{150/\sqrt{400}} = -4$$

(since $S = \hat{\sigma}$)

$\therefore |Z_{cal}| = 4$

By using P-value approach

Since this is the case of one ~~tailed test. So, P-value~~

So, P-value $= 0.5 - P(0 \leq Z \leq Z_{cal})$ or $0.5 - P(0 \leq Z \leq 4)$

$= 0.5 - 0.4997$

$= 0.00003$.

value of $P(0 \leq Z \leq 4)$ from table given back of book page no 310 maxium upto 4 given we pick that value

Decision: P-value $= 0.00003 < \alpha = 0.01$.

So, $H_0$ is rejected i.e, $H_1$ is accepted.

Conclusion: Hence the population mean lifetime of laptop is greater than 1600 hours.

## Hypothesis (Test of Significance) of difference between two means:

Problem to test

Null Hypothesis ($H_0$): $\mu_1 = \mu_2$

i.e, there is no significance difference between two means.

Alternate Hypothesis ($H_1$): $\mu_1 \neq \mu_2$

i.e, there is significance difference betwn two means.

OR $\mu_1 < \mu_2$ i.e first popn mean is less than second popn mean.

OR $\mu_1 > \mu_2$ i.e first popn mean is greater than second popn mean.

Test Statistics:

$$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \left(\text{Since } s_1 = \hat{\sigma}_1 \ \& \ s_2 = \hat{\sigma}_2\right)$$

Q1. In a certain factory there are two independent processes for manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120gms with a standard deviation of 12gms, while the other process are 124 and 14 in a sample of 400 items. Is the difference between the mean weights significant at 10% level of significance. Use p value method.

Solution:

1st Sample

Sample size ($n_1$) = 250

Sample mean ($\overline{X}_1$) = 120 gms

Standard deviation ($S_1$) = 12 gms

2nd Sample

Sample size ($n_2$) = 400

Sample mean ($\overline{X}_2$) = 124 gms

Standard deviation ($S_2$) = 14 gms.

level of significance = 10% = 0.10

Problem to test:

Null Hypothesis ($H_0$): $\mu_1 = \mu_2$

i.e, there is no significance difference between the two mean weights.

Alternate Hypothesis ($H_1$): $\mu_1 \neq \mu_2$

i.e, There is significance difference between the two mean weights.

Test Statistics:

$$Z_{cal} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \quad \left( \because S_1 = \hat{\sigma}_1 \text{ \& } S_2 = \hat{\sigma}_2 \right)$$

$$= \frac{120 - 124}{\sqrt{\frac{12^2}{250} + \frac{14^2}{400}}}$$

$$= -3.874$$

$$\therefore |Z_{cal}| = 3.874$$

By using P-value:

$$\text{P-value} = 2\{0.5 - P(0 \leq Z \leq Z_{cal})\}$$

$$= 2\{0.5 - P(0 \leq Z \leq 3.874)\}$$

$$\doteq 2\{0.5 - 0.4995\}$$

value of 3.874 from table page no. 309

$$= 2 \times 0.00005$$

$$= 0.0001$$

Decision: Since P-value = 0.0001 < $\alpha$ = 0.10, So $H_0$ is rejected.

i.e, $H_1$ is accepted.

Conclusion: Hence we can conclude that there is significance difference between two mean weights.

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Q2. Two research laboratories have independently produced drugs that provide relief to arthritis suffers. The first drug was tested on a group of 90 arthritis victims and produced an average of 8.5 hours relief with a standard deviation of 1.8 hours. The second drug was tested on 80 arthritis victims producing an average of 7.9 hours of relief with a standard deviation of 2.1 hours. At 5% level of significance does the second drug provide a significantly shorter period of relief?

Solution:

| Drug I$^{st}$ | Drug II$^{nd}$ |
|---|---|
| Sample size $(n_1) = 90$ | Sample size $(n_2) = 80$ |
| Sample mean $(\overline{X_1}) = 8.5$ hours | Sample mean $(\overline{X_2}) = 7.9$ hours |
| Standard deviation $(S_1) = 1.8$ hours | Standard deviation $(S_2) = 2.1$ hours |

level of significance $(\alpha) = 5\% = 0.05$.

Problem to test:

Null Hypothesis $(H_0)$: $\mu_1 = \mu_2$ i.e, The average relief time of both drugs is same.

Alternate Hypothesis $(H_1)$: $\mu_1 > \mu_2$ i.e, First drug provide more relief time than second drug.

Test Statistics:

$$Z_{cal} = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{8.5 - 7.9}{\sqrt{\frac{(1.8)^2}{90} + \frac{(2.1)^2}{80}}} = 1.98$$

~~By using P-value~~

$$P\text{-value} = 0.5 - P(0 \le Z \le Z_{cal})$$
$$= 0.5 - P(0 \le Z \le 1.98)$$

value of 1.98 in on-tailed table

$$= 0.5 - 0.239$$
$$= 0.261$$

$$\boxed{\begin{aligned} &= 0.5 - \cancel{1.89}\ 0.239 \\ &= \cancel{-1.19}\ 0.261 \\ &= \end{aligned}}$$

~~Since one-tailed test~~
~~$\alpha = 5\% = 0.05$~~
~~$\Rightarrow 0.5 - \alpha = 0.5 - 0.05$~~
~~$= 0.45$~~
~~Now we see value of 0.45 in one-tailed probability > table (inside table) page no. 310~~

Decision: Since P-value $= 0.261 > \alpha = 0.05$. So, $H_0$ is accepted. i.e, $H_1$ is rejected.

Conclusion: Hence, the average relief time of both drugs is same.

## ⊗. Hypothesis testing of single population proportion (P):

**Problem to test:**

Null Hypothesis ($H_0$): $P = P_0$ (Specified value).

Alternate Hypothesis ($H_1$): $P \neq P_0$

OR $P < P_0$

OR $P > P_0$.

**Test Statistics:**

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

where, $p$ = sample proportion.

$P$ = Population proportion.

$Q = 1 - P$

$n$ = sample size.

**Q1.** A coin is tossed 600 times and head appear 320 times, does this result support the hypothesis.

Soln

Given,

Population proportion $(P) = \frac{1}{2}$

$$Q = 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$$

Sample size $(n) = 600$.

sample proportion $(p) = \frac{320}{600} = 0.533$

**Problem to test:**

Null Hypothesis ($H_0$): $P = \frac{1}{2} = 0.5$

i.e, coin is unbiased.

Alternative Hypothesis ($H_1$): $P \neq \frac{1}{2} = 0.5$

i.e, coin is not unbiased (i.e, biased).

**Test Statistics:**

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.533 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{600}}}$$

$$= 1.6166$$

Critical value:

The tabulated value of Z at 0.05 level of significance

$Z_{0.05} = 1.96$ (two-tailed test)

Decision: Since $Z_{cal} = 1.6166 < Z_{0.05} = 1.96$. So, $H_0$ is accepted. i.e, $H_1$ is rejected.

Conclusion: Hence the coin is unbiased and this result supports the hypothesis.

Q.2. A dice was thrown 9000 times and of these 3220 yielded a 3 or 4. Is this consistent with the hypothesis that the dice is unbiased?

Soln

Given,

no. of throws $(n) = 9000$

no. of times 3 or 4 is obtained $(X) = 3220$

Sample proportion $(p) = X/n$

$= \frac{3220}{9000}$

$= 0.358$

level of significance $(\alpha) = 5\% = 0.05$

Problem to test:

Null Hypothesis $(H_0)$: $P = \frac{1}{6} = 0.1666$

i.e, the dice is unbiased

Alternate Hypothesis $(H_1)$: $P \neq \frac{1}{6} = 0.1666$

i.e, the dice is not unbised (i.e, biased)

Test Statistics:

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.358 - 0.1666}{\sqrt{\frac{0.1666 \times 0.8334}{9000}}} = \frac{0.1914}{0.00393} = 48.7$$

Decision: Since $Z_{cal} = 48.7 > Z_{0.05} = 1.96$ (i.e, critical value). So, $H_0$ is rejected.

Conclusion: This is not consistent with the hypothesis that dice is unbiased.

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Q3. The coordinator in a college claimed that at least 98% of the students submit their assignment on time. Taking the sample of 250 students, 15 were not submitting assignment in whole semester. Test his claim at 10% level of significance.

Solution:

Given, Sample size (n) = 250

no. of student who do not submitted assignment on time = 15

no. of students who submitted assignment on time = 250 - 15 = 235

∴ Sample proportion of student who submitted the assignment on time $= x/n = 235/250 = 0.94$

level of significance($\alpha$) = 10% = 0.10.

Problem to test:

Null Hypothesis ($H_0$): $P \geq 0.98$

i.e. at least 98% of student submitted their assignment on time.

Alternate Hypothesis($H_1$): $P < 0.98$

i.e. Student submitted their assignment on time is less than 98%.

Test Statistics:

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.94 - 0.98}{\sqrt{\frac{0.98 \times 0.02}{250}}} = -4.51$$

$\therefore |Z_{cal}| = 4.51$.

Critical value: The tabulated value of Z at 0.10 level of significance is $Z_{0.10} = -1.28$ (Since this is one tailed-left so we see the value inside table)

$\therefore |Z_{0.10}| = 1.28$

Decision: $|Z_{cal}| = 4.51 > |Z_{0.10}| = 1.28$, So, $H_0$ is rejected. i.e. $H_1$ is accepted.

Conclusion: Hence we can conclude that students submitted their assignment is less than 98%.

## Hypothesis testing of difference of two population proportion:

**Problem to test:**

Null Hypothesis ($H_0$): $P_1 = P_2$

i.e, There is no. significance difference between two population proportion.

Alternate Hypothesis ($H_1$): $P_1 \neq P_2$

i.e, There is significance difference between two popⁿ proportion.

OR $P_1 < P_2$

OR $P_1 > P_2$.

**Test Statistics:**

$$Z_{cal} = \frac{P_1 - P_2}{\sqrt{\hat{P}\hat{Q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where, $P_1$ = Sample proportion drawn from $1^{st}$ population. OR $P_1 = \frac{X_1}{n_1}$

$P_2$ = Sample proportion drawn from $2^{nd}$ population. OR $P_2 = \frac{X_2}{n_2}$.

$$\hat{P} = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} \Rightarrow \frac{X_1 + X_2}{n_1 + n_2}$$

$$\hat{Q} = 1 - \hat{P}$$

**Q.** At a certain data in a large city 400 out of a random sample of 500 men were found to be smokers. After the tax on tobacco had been heavily increased, another random sample of 600 men in the same city included 400 smokers. Was the observed decrease in the proportion of smokers significant? Test at 5% level of significance.

**Solution:**

**Sample-Ist**

sample size ($n_1$) = 500

no. of smoker ($X_1$) = 400

Sample proportion of smoker ($P_1$) $= \frac{X_1}{n_1}$

$= \frac{400}{500} = 0.8$

**Sample-IInd**

Sample size ($n_2$) = 600

no. of smoker ($X_2$) = 400.

Sample proportion of smoker ($P_2$) $= \frac{X_2}{n_2}$

$= \frac{400}{600} = 0.67$

level of significance ($\alpha$) = 5% = 0.05

## Problem to test:

**Null Hypothesis ($H_0$):** $P_1 = P_2$.

i.e, There is no significance difference between the population proportion of smoker after tax is increased.

**Alternate Hypothesis ($H_1$):** $P_1 > P_2$

i.e, There is decrease in proportion of smoker after tax is increased.

## Test Statistics:

Here,
$$\hat{p} = \frac{x_1+x_2}{n_1+n_2} = \frac{400+400}{500+600} = 0.727$$

$$\hat{Q} = 1-\hat{p} = 1-0.727 = 0.273$$

Now,
$$Z_{cal} = \frac{P_1 - P_2}{\sqrt{\hat{P}\hat{Q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.8-0.67}{\sqrt{0.727 \times 0.273\left(\frac{1}{500}+\frac{1}{600}\right)}}$$

$$= 4.81$$

**Critical value:** The tabulated value of Z at 0.05 level of significance is $Z_{0.05} = 1.64$ (one-tailed test).

**Decision:** Since $Z_{cal} = 4.81 > Z_{tab} = 1.64$, So, $H_0$ is rejected. i.e, $H_1$ is accepted.

**Conclusion:** Hence we can conclude that there is decrease in number of smokers after heavily increase in tax.

## ⊛. Hypothesis testing of single population mean (for small sample):

i.e, sample size $< 30$.

**Test Statistics:**

$$t_{cal} = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

where, s is sample standard deviation
$$= \sqrt{\frac{1}{n-1}\sum(X-\bar{X})^2}$$
$$= \frac{1}{n-1}\left[\sum X^2 - n\bar{X}^2\right]$$

If we use deviation then,
$$= \frac{1}{n-1}\left[\sum d^2 - n\bar{d}^2\right]$$ where, $d = X - A$

If s is given then s is said to be biased estimate.
If s is calculated then s is said to be unbiased estimate.

and $t_{cal} = \dfrac{\overline{X} - \mu}{s/\sqrt{n-1}}$ (if s is given).

## Critical value:

The tabulated value of t at $\alpha$ level of significance with n−1 degree of freedom (d.f.) is $t_{\alpha, n-1}$.

**Q1.** Rainfall records of a particular place for last 12 years for the month of July showed that average rainfall at the place was ~~less than 500~~ 500mm and standard deviation of 30mm. Do you agree that the average rainfall at the place was less than 512mm? Use 10% level of significance.

**Solution:**

Sample size (n) = 12
Average rainfall ($\overline{X}$) = 500
Sample standard deviation (s) = 30
level of significance ($\alpha$) = 10% = 0.10
Population average rainfall ($\mu$) = 512

## Problem to test:

Null Hypotheses ($H_0$): $\mu = 512$
i.e, Average rain is 512mm.

Alternate Hypotheses ($H_1$): $\mu < 512$
i.e, Average rain is less than 512mm.

## Test Statistics:

$$t_{cal} = \frac{\overline{X} - \mu}{s/\sqrt{n-1}} = \frac{500 - 512}{30/\sqrt{11}} = -1.326$$

$\therefore |t_{cal}| = 1.326$

## Critical value

$t_{\alpha, n-1}$ form

left tailed भएर -ve राखेको

page no 318 table value

$t_{0.10,(11)} = -1.363$ (one-tailed test)

$\therefore |t_{tab}| = 1.363$

**Decision:** Since $t_{cal} = 1.326 < t_{tab} = 1.363$, So, $H_0$ is accepted. i.e $H_1$ is rejected.

**Conclusion:** Hence average rain is 512mm.

**Q2.** Ten patients are selected at random from a population of patients and their blood pressure recorded as follows;

125, 147, 118, 145, 140, 128, 155, 150, 160, 149.

Do the data support the hypothesis that the population average blood pressure of patients is 135? Use 5% level of significance.

**Solution:**

Given,

Sample size $(n) = 10$

records of blood pressure of 10 patients $(X)$; 125, 147, 118, 145, 140, 128, 155, 150, 160, 149.

level of significance $(\alpha) = 5\% = 0.05$

**Problem to test:**

Null Hypothesis $(H_0)$: $\mu = 135$

i.e, The population average of blood pressure of patient is 135

Alternate Hypothesis $(H_1)$: $\mu \neq 135$.

i.e, The population average of blood pressure of patient is not 135.

**Test statistics:**

$$t_{cal} = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

Now the value of s is not given so we calulte it using given data as follows:-

| X | $d = X - A$ $= X - 140$ | $d^2$ |
|---|---|---|
| 125 | −15 | 225 |
| 147 | 7 | 49 |
| 118 | −22 | 484 |
| 145 | 5 | 25 |
| 140 | 0 | 0 |
| 128 | −12 | 144 |
| 155 | 15 | 225 |
| 150 | 10 | 100 |
| 160 | 20 | 400 |
| 149 | 9 | 81 |
| | $\Sigma d = 17$ | $\Sigma d^2 = 1733$ |

Here,

$$\text{Mean}(\bar{X}) = A + \frac{\Sigma d}{n}$$

$$= 140 + \frac{17}{10}$$

$$= 141.7$$

and $s = \sqrt{\frac{1}{n-1}\left[\Sigma d^2 - \Sigma n\bar{d}^2\right]}$

$= \sqrt{\frac{1}{9}\left[1733 - 10\times\left(\frac{17}{10}\right)^2\right]}$ since $\bar{d} = \frac{\Sigma d}{n}$

$= 13.76.$

Now,
$$t_{cal} = \frac{141.7 - 135}{13.76/\sqrt{10}}$$
$= 1.53$

Critical value: The tabulated value of $t$ at $5\% = 0.05$ level of significance. $(n-1)$ degree of freedom is,

$t_{0.05,(9)} = 2.262$ (for two-tailed test)

Decision: Since $t_{cal} = 1.53 < t_{tab} = 2.262$. So, $H_0$ is accepted. i.e, $H_1$ is rejected.

Conclusion: The average population of blood pressure of patient is 135.

Q3. A random sample of size 25 showed a mean of 65 inches with a standard deviation of 25 inches. Determine 98% confidence intervals for the mean of the population.

Solution:

Here, Sample size $(n) = 25$, sample mean $(\bar{X}) = 65$, sample SD $(s) = 25$

confidence limit $(1-\alpha) = 98\% = 0.98$, level of significance $(\alpha) = 0.02$

Confidence limit for $\mu$ is $\bar{X} \pm t_{\alpha/2,(n-1)} \cdot \frac{S}{\sqrt{n-1}}$

$= 65 \pm 2.492 \times \frac{25}{\sqrt{25-1}}$

$= 65 \pm 12.717$

Taking -ve sign, $65 - 12.717 = 52.283$

Taking +ve sign, $65 + 12.717 = 77.717$

Hence confidence limit is 52.283 inches to 77.717 inches.

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Q4. A random sample of size 16 has the sample mean 53. The sum of the square of deviation taken from the mean value is 150. Can this sample be regarded as taken from the population having 56 as its mean at 99% confidence limit?

Solution:

Given,

The sum of square of deviation taken from mean $\Sigma(x-\bar{x})^2 = 150$.

Sample mean $(\bar{x}) = 53$.

Population mean $(\mu) = 56$.

Sample size $(n) = 16$.

Problem to test:

Null Hypo ($H_0$) : $\mu = 56$.

i.e, population average is 56.

Alternate Hypo ($H_1$) : $\mu \neq 56$ i.e, population average is not equal to 56.

Test Statistics:

$$t_{cal} = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

where, $s = \sqrt{\frac{1}{n-1}\Sigma(x-\bar{x})^2}$

$= \sqrt{\frac{150}{15}}$

$= 3.16$

$$\therefore t_{cal} = \frac{53-56}{3.16/\sqrt{16}}$$

$= -3.79$

$|t_{cal}| = 3.79$

Critical value: The tabulated value of t at 1% = 0.01 level of significance and (n-1) degree of freedom is,

$t_{0.01,(15)} = 2.947$ (two-tailed test)

Decision: $|t_{cal}| = 3.79 > t_{tab} = 2.947$, So, $H_0$ is rejected.

i.e, $H_1$ is accepted.

Conclusion:- Hence, at 99% ~~level of significance~~ confidence limits are,

$$\bar{x} - t_{0.01,(15)} \times \frac{s}{\sqrt{n}} \leq \mu < \bar{x} + t_{0.01,(15)} \times \frac{s}{\sqrt{n}}$$

or, $53 - 2.947 \times \frac{3.16}{\sqrt{16}} \leq \mu < 53 + 2.947 \times \frac{3.16}{\sqrt{16}}$

or, $50.672 \leq \mu < 55.328$

## Hypothesis testing of difference of two population mean (for small sample):

**Problem to test:**

Null Hypothesis ($H_0$): $\mu_1 = \mu_2$

Alternate Hypothesis ($H_1$): $\mu_1 \neq \mu_2$

OR $\mu_1 < \mu_2$

OR $\mu_1 > \mu_2$.

**Test Statistics**

$$t_{cal} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

where, $S_p^2$ is the ~~people~~ pooled (combined) variation and calculated as,

$$S_p^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}$$ when $s_1$ and $s_2$ are given (i.e, biased).

$$S_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$ when $s_1$ and $s_2$ are not given (i.e, unbiased).

**Critical value**

The tabulated value of $t$ at $\alpha$ level of significance with $n_1+n_2-2$ degree of freedom is; $t_{\alpha,(n_1+n_2-2)}$.

[Imp] Q. Two kinds of manure were applied to sixteen one hectare plot, other condition remaning the same. The yeilds in quintals are given below;

independent samples

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Manure I | 18 | 20 | 36 | 50 | 49 | 36 | 34 | 49 | 41 |
| Manure II | 29 | 28 | 26 | 35 | 30 | 44 | 46. | | |

Is there any significant difference between the mean yeilds? Use 5% level of significance.

**Solution:**

Given, sample size ($n_1$) = 9, Sample size ($n_2$) = 7

level of significance ($\alpha$) = 5% = 0.05

**Problem to test**

Null Hypothesis ($H_0$): $\mu_1 = \mu_2$ i.e, There is no significance difference between the mean yeilds.

Alternate Hypothesis ($H_1$): $\mu_1 \neq \mu_2$ i.e, There is significance difference between the mean yeilds.

## Test Statistics

$$t_{cal} = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

for the calculation of $s_1^2$, $s_2^2$, $\overline{x_1}$ and $\overline{x_2}$

data मा कुनै value repeat भए (त्यसैलाई assumed mean मान्ने) नत्र जुनसुकै बिचको value

| Manure I ($x_1$) | $d_1 = x_1 - A_1$ $= x_1 - 36$ | $d_1^2$ | Manure II ($x_2$) | $d_2 = x_2 - A_2$ $d_2 = x_2 - 35$ | $d_2^2$ |
|---|---|---|---|---|---|
| 18 | −18 | 324 | 29 | −6 | 36 |
| 20 | −16 | 256 | 28 | −7 | 49 |
| 36 | 0 | 0 | 26 | −9 | 81 |
| 50 | 14 | 196 | 35 | 0 | 0 |
| 49 | 13 | 169 | 30 | −5 | 25 |
| 36 | 0 | 0 | 44 | 9 | 81 |
| 34 | −2 | 4 | 46 | 11 | 121 |
| 49 | 13 | 169 | | | |
| 41 | 5 | 25 | | | |
| | $\Sigma d_1 = 9$ | $\Sigma d_1^2 = 1143$ | | $\Sigma d_2 = -7$ | $\Sigma d_2^2 = 393$ |

Now,

$$s_1 = \frac{1}{n_1 - 1}\left[\Sigma d_1^2 - \overline{d_1}^2\right] = \frac{1}{8}\left[1143 - \left(\frac{9}{9}\right)^2\right] = 142.71$$

$$s_2 = \frac{1}{n_2 - 1}\left[\Sigma d_2^2 - \overline{d_2}^2\right] = \frac{1}{6}\left[393 - \left(\frac{7}{7}\right)^2\right] = 65.33$$

$$\text{Mean}(\overline{x_1}) = A + \frac{\Sigma d_1}{n_1} = 36 + \frac{9}{9} = 37$$

& $$\text{Mean}(\overline{x_2}) = A + \frac{\Sigma d_2}{n_2} = 35 + \frac{-7}{7} = 34$$

Here,

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(9-1)\times(142.71)^2 + (7-1)\times(65.33)^2}{9 + 6 - 2}$$

$$= 109.57$$

Now,

$$t_{cal} = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{37 - 34}{\sqrt{109.5\left(\frac{1}{9} + \frac{1}{7}\right)}} = 0.568$$

**Critical value:** The tabulated value of $t$ at 0.05 level of significance with $n_1+n_2-2$ degree of freedom is;

$$t_{0.05,(14)} = 2.145 \text{ (two-tailed test).}$$

**Decision:** Since $t_{cal} = 0.568 < t_{tab} = 2.145$. So, $H_0$ is accepted. i.e, $H_1$ is rejected.

**Conclusion:** Hence we can conclude that there is no significance difference between the mean yeilds.

## ⊛. Paired t-test:

In t-test we have two independent samples but in paired t-test there are two dependent samples of size $n<30$. For testing the hypothesis we can make the following hypothesis.

Null Hypothesis ($H_0$): $\mu_x \neq \mu_y$ (OR $\mu_1 = \mu_2$) we can use any notation

Alternate Hypothesis ($H_1$): $\mu_x \neq \mu_y$

OR $\mu_x < \mu_y$

OR $\mu_x > \mu_y$.

where, $x$ is before value & $y$ is after value.

**Test Statistics:**

$$t_{cal} = \frac{\bar{d}}{sd/\sqrt{n}}$$

where, $d = X-Y$ or $Y-X$

$X \rightarrow$ represent before value

$Y \rightarrow$ represent after value.

$$\bar{d} = \frac{\sum d}{n} = \sqrt{\frac{1}{n-1}(d-\bar{d})^2}$$

$$sd = \sqrt{\frac{1}{n-1}\left[\sum d^2 - \bar{d}^2\right]}$$

& $n$ = no. of pairs.

**Critical value:**

The tabulated value of $t$ at $\alpha$ level of significance with $n-1$ degree of freedom is $t_{\alpha, n-1}$.

Q1. A certain stimlus administered to each of the 12 patients resulted in the following increase of blood pressure:

5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4 and 6.

Can it be conducted that the stimulus will in general be accompanied by an increase in blood pressure.

Solution:

Here,

$d = X - Y$

$n = 12$

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $d$ | 5 | 2 | 8 | -1 | 3 | 0 | -2 | 1 | 5 | 0 | 4 | 6 | $\Sigma d = 31$ |
| $d^2$ | 25 | 4 | 64 | 1 | 9 | 0 | 4 | 1 | 25 | 0 | 16 | 36 | $\Sigma d^2 = 185$ |

$$\bar{d} = \frac{\Sigma d}{n} = \frac{31}{12} = 2.58$$

$$sd = \sqrt{\frac{1}{n-1}\left[\Sigma d^2 - \bar{d}^2\right]} = \sqrt{\frac{1}{11}\left[185 - (2.58)^2\right]} = 3.08$$

Problem to test:

Null Hypothesis ($H_0$): $\mu_1 = \mu_2$ (or $\mu_x = \mu_2$)

Alternate Hypothesis ($H_1$): $\mu_1 < \mu_2$ (or $\mu_x < \mu_2$)

Test statistic:

$$t_{cal} = \frac{\bar{d}}{sd/\sqrt{n}} = \frac{2.58}{3.08/\sqrt{12}} = \frac{2.58 \times 3.46}{3.08} = 2.89$$

Critical value: Let $\alpha = 0.05$ be the level of significance, then critical value for one tailed test is,

$$t_{tab} = t_{0.05,(11)} = 1.8$$

Decision: $t_{cal} = 2.89 > t_{tab} = 1.8$. So, reject $H_0$ at 5% level of significance. i.e, $H_1$ is accepted.

Conclusion: The stimulus in general be accompanied by an increase in blood pressure.

[Imp] Q.2. Ten students were given a test in SPSS. Then they were given a month's training and another test was held. The marks obtained by the 10 students in the two tests are given below:

| S.N. of students | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Test I | 12 | 15 | 10 | 13 | 18 | 10 | 8 | 17 | 9 | 7 |
| Test II | 12 | 17 | 12 | 12 | 14 | 12 | 16 | 16 | 18 | 12 |

Test whether the students have benifited by the traning or not.

Solution

Here, Sample size $(n) = 10$.

| Students | Marks in test I (X) | Marks in test II (Y) | $d = X - Y$ | $d^2$ |
|---|---|---|---|---|
| 1 | 12 | 12 | 0 | 0 |
| 2 | 15 | 17 | −2 | 4 |
| 3 | 10 | 12 | −2 | 4 |
| 4 | 13 | 12 | 1 | 1 |
| 5 | 18 | 14 | 4 | 16 |
| 6 | 10 | 12 | −2 | 4 |
| 7 | 8 | 16 | −8 | 64 |
| 8 | 17 | 16 | 1 | 1 |
| 9 | 9 | 18 | −9 | 81 |
| 10 | 7 | 12 | −5 | 25 |
| | | | $\Sigma d = -22$ | $\Sigma d^2 = 200$ |

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-22}{10} = -2.2$$

$$sd = \sqrt{\frac{1}{n-1}\left[\Sigma d^2 - \bar{d}^2\right]} = \sqrt{\frac{1}{9}\left[200 - (-2.2)^2\right]} = 4.1$$

Problem to test:

Null Hypothesis $(H_0)$: $\mu_x = \mu_y$

i.e. Students have not ~~bef~~ benefited by training.

Alternate Hypothesis $(H_1)$: $\mu_x < \mu_y$. (One-tailed test)

i.e. Students have benefited by training.

$\mu_x < \mu_y$
before value ← after value
पहिलेको value अहिलेको भन्दा बढि छ त्यसैले benifited

Test Statistic:

$$t_{cal} = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{-2.2}{4.1/\sqrt{12}} = -1.69$$

i.e, $|t_{cal}| = 1.69$.

Critical value:

Let $\alpha = 5\%$ be the level of significance, then critical value for one tailed test is

$$t_{tab} = t_{0.05,(11)} = 1.83$$

Decision $|t_{cal}| = 1.69 < t_{tab} = 1.83$, So, $H_0$ is accepted at 5% level of significance.

i.e, $H_1$ is rejected.

Conclusion: The students have not benefited by the training.