CSCI 370: Stochastic Computing

Lecture#7

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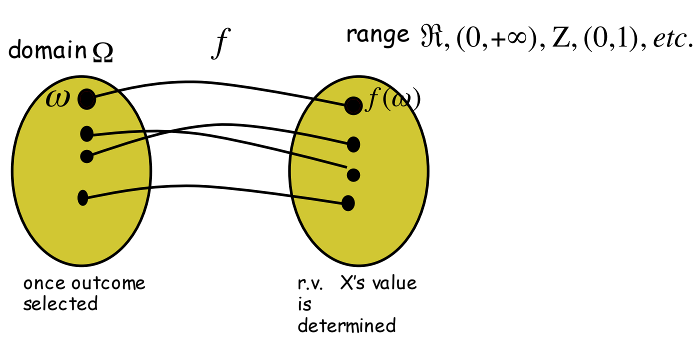
Define:

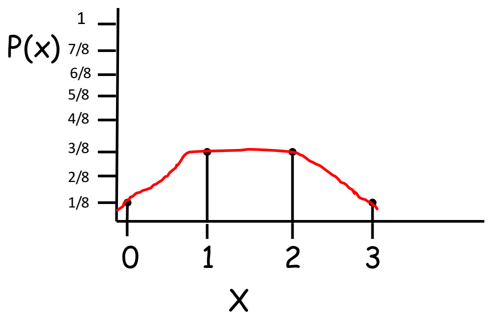
A random variable is function of outcome

$$X = f(\omega)$$



Ex:

Let X be r.v. describing the number of heads when you flip 3 fair coins



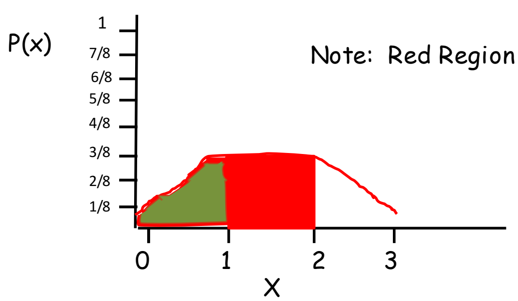
Note the shape

Issue: what about intervals?

We use the difference between cumulative distributions

$$P\{a < X \leq b\} = F(b) - F(a)$$

Note: Red Region – Green Region



For example the probability that number of heads 1 < X <=2

We compute the marginal distribution by summing along the margins of the joint distribution or **marginalizing**. This works in arbitrary dimension.

Example: I roll a fair die and flip a fair coin
Say I want P(C) or P(D)

$$P_X(x) = P\{X = x\} = \sum_y P_{(X,Y)}(x, y)$$

$$P_Y(y) = P\{Y = y\} = \sum_x P_{(X,Y)}(x, y)$$

(D,C)

	h	t	
1	1/12	1/12	1/6
2	1/12	1/12	1/6
3	1/12	1/12	1/6
4	1/12	1/12	1/6
5	1/12	1/12	1/6
6	1/12	1/12	1/6
	1/2	1/2	

Topics

- Introduction
- Warm up
- Expectation and Variance

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Suppose we played a game…

- You wager $1 and I wager $2
- I flip a coin
- You guess heads/tails
- If you are correct, you get $2,
  If you lose I keep your $1

How much will you win if you keep playing this game?

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How much will you win if you keep playing this game?

- Half the time P(heads) = 0.5 and you get $2
- Half the time P(tails)= 0.5 and you get -$1 (lose $1)

So that's….
0.5 ($2) + 0.5 (-$1) = 1 -0.5 = .50 cents

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Lets try it in MATLAB.

Both the average value and how much the value varies above and below this average describes how stochasticity "behaves" for this particular system.

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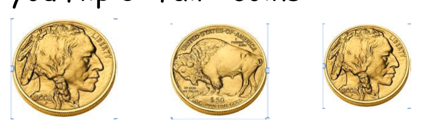
Distribution: complete accounting of r.v.

EX: you flip 3 "fair" coins



Let X be r.v. measuring the number of heads…

x	P{X=x}
0	1/8
1	3/8
2	3/8
3	1/8
total	1

This catalogue of the possible r.v. values and their probabilities is called the r.v.'s **distribution**.

Multiple descriptors in the real world…

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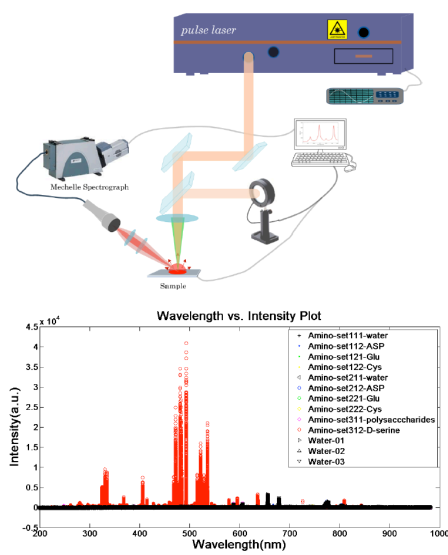

Figure 1. LIBS Experiment Setup (top). Wavelength vs. Intensity Plot (bottom).

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So called, summary statistics

- Mean
- Variance

Expectation:

As in what do you expect or what will happen on "average"

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Defined as the probabilistic average.

If distribution is uniform (i.e. random), the probabilistic average is the mean μ

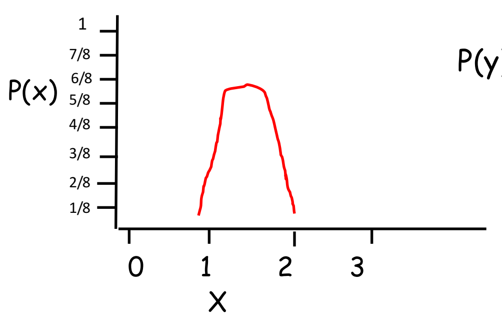
$$\mu = E(X) = \sum_x x\,P(X)$$

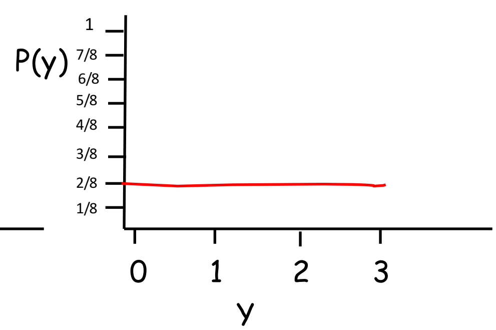
$$\mu = E(X) = \sum_{i=1}^{N} x_i\,P(x_i) \quad = \sum_{i=1}^{N} x_i\,\frac{1}{N} \quad = \frac{x_1 + \ldots + x_N}{N}$$

Variance:

How much the random variable's values are spread out or the dispersion

Easier to get something wrong if it varies widely





Suppose we had r.v. X and Y

On which one would you rather wager a bet?

Variance:

How much the random variable's values are spread out or the dispersion

- Low variance: easier to predict outcome (probability mass less disperse or spread out)
- High variance: harder to predict outcome (probability mass more disperse or spread out)

Variance:

How much the random variable's values are spread out or the dispersion

How would you define "spread out"

Spread out with respect to what?

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How would you define "spread out"

Spread out with respect to what? The "center"

Variance:

How much the random variable's values are spread out or the dispersion

Def:

The variance is the average distance or spread between each of the r.v.'s outcomes and its mean. σ^2

Not just any average, the probabilistic average

$$\sigma^2 = Var(X) = E(X - E[X])^2$$
$$= \sum_x (x-\mu)^2 \; P(X)$$

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we care about the difference but not about "direction"

the r.v.'s outcome

the r.v.'s mean

Standard Deviation:

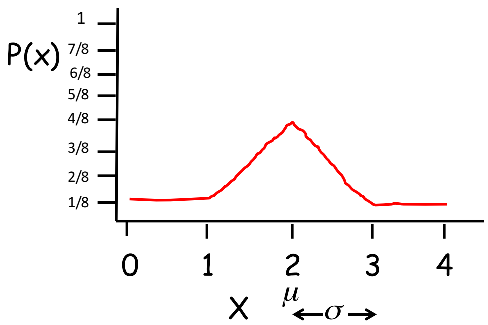
Because variance is measured as the square of some unit, it cannot be compared against x or μ

Remember we talked about the spread above and below the "center"

Def:

Standard deviation is that spread we can use to compare

$$\sigma = Std(X) = \sqrt{Var(X)}$$



Also called the location/scale parameters

$\leftarrow \mu \pm \sigma \rightarrow$

$\longleftarrow \mu \pm 2\sigma \longrightarrow$

$\longleftarrow \mu \pm 1.96\sigma \longrightarrow$

Multiple descriptors in the real world…

Measurements often coupled
- I change one measurement
- The other measurement also changes some amount

They vary together

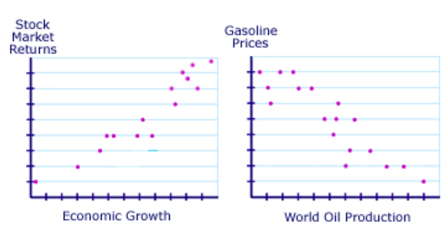
Covariance

Def:

The average way two r.v.'s spread around their center

$$Cov(X,Y) = E\{\,(X - E[X])\,(Y - E[Y])\,\}$$

$$= \sum_x \sum_y (x-\mu_x)(y-\mu_y)\,P(x,y)$$



When we compare variables that have different distributions we want a common or normalized way to compare them.

Instead of saying covariance of, say, 123.45 versus 34.7
it is helpful to express on a percentage scale.

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Correlation ρ

Def:

The normalized (percentage scale) expression of linkage

$$\rho = \frac{Cov(X,Y)}{Std(X)\,Std(Y)}$$

That is Covariance rescaled

"perfect correlation" is +1 or -1

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END

Graded material

More on summary statistics, start distributions and their properties.