Robert Morris Project 2.

Mclaurin series expansion

$$\frac{d^{(i)}}{dx}f(x) = \underbrace{f(x) + f^{(1)}(x)\,r + \frac{f^{(2)}(x)}{2}r^2 + \frac{f^{(3)}(x)}{3!}r^3 + \dots \frac{f^{(i)}(x)}{(i)!}r^i}_{\sum_{i=0}^{\infty}\frac{f^{(i)}(a)}{(i)!}(x-a)^i}$$

(if $f(a) \Rightarrow r = x-a$)

Mclaurin series expansion of x.

$$e^x = \frac{d}{dx}e^x = \frac{d^2}{dx^2}e^x = \dots \frac{}{dx^n}e^x$$

$$e^x = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots a_nx^n = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$\frac{d}{dx}e^x = a_1 + 2a_2x + 3a_3x^2 + \dots na_nx^{n-1}$$

$$\frac{d^2}{dx^2}e^x = 2a_2 + (3\times 2)a_3x + \dots n(n-1)a_nx^{n-2}$$

$$\frac{d^i}{dx^i} = (i)!\,a_i + \frac{(i+1)!}{2}a_{i+1}x + \dots \frac{n!}{(n-i)!}a_nx^{n-i}$$

$$a_0 = a_1 = a_2 = \dots (i)!\,a_i$$

$$e^x = a_0 + \cancel{a_0x} + \frac{1}{2}\cancel{a_0x^2} + \frac{1}{3!}\cancel{a_0x^3} + \dots \frac{1}{n!}\cancel{a_0x^n}$$

$$e^0 = (1) + 0 + 0 + 0 + \dots + 0$$

$$e^0 = 1 \Rightarrow a_0 = 1$$

So, $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ for all x

Matlab = `y+(x.^n)./factorial(n)`

Q1. The number of terms need for the approximation to begin to look like the ground truth function $f(x) = e^x$ was around 5 to 6.

Q2. The larger the negative or positive values of x, the more terms are needed to get the approximation to resemble the $f(x) = e^x$ function.