ECE653 Assignment 3

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Question 1

According to the rule of assignment and consequence, we can do the derivation as follows:

$$\dfrac{\dfrac{n \geq 0 \Rightarrow I[0/r, 0/i, 1/p]}{\{n \geq 0 \wedge r = 0 \wedge i = 0 \wedge p = 1\} \Rightarrow \{I\}} \quad \dfrac{\dfrac{I \wedge i \neq n \Rightarrow I[r+p/r, i+1/i, 2p/p]}{\{I \wedge i \neq n\} r := r - p; p := 2 \times p; r := r + p; i := i + 1 \{I\}}}{\{I\} \text{ while } (i \neq n) \text{ do } (r := r - p; p := 2 \times p; r := r + p; i := i + 1)\ \{I \wedge i = n\}} \quad \{I \wedge i = n\} \Rightarrow \{r = 2^n - 1\}}{\{n \geq 0 \wedge r = 0 \wedge i = 0 \wedge p = 1\} \text{ while } (i \neq n) \text{ do } (r := r - p; p := 2 \times p; r := r + p; i := i + 1)\ \{r = 2^n - 1\}}$$

In the derivation, the inductive invariant I, should satisfy three constrains:

$n \geq 0 \Rightarrow I[0/r, 0/i, 1/p]$
$I \wedge i = n \Rightarrow r = 2^n - 1$
$I \wedge i \neq n \Rightarrow I[r + p/r, i + 1/i, 2p/p]$

Let $I = (p = 2^i \wedge r = 2^i - 1 \wedge i \leq n)$, then the constrains become:

$n \geq 0 \Rightarrow 1 = 2^0 \wedge 0 = 2^0 - 1 \wedge 0 \leq n$, which is valid.
$p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i = n \Rightarrow r = 2^n - 1$, which is valid.
$p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i \neq n \Rightarrow 2p = 2^{i+1} \wedge r + p = 2^{i+1} - 1 \wedge i + 1 \leq n$, which is valid.

All constrains are valid, therefore, the program is correct.