

Mathematics - II



Prime \rightarrow 2, 3, 5, 7, 11, 13, 17

Check if Prime or Not

$$\underline{N} = \underline{31}$$

Using for loop from 2 to N-1.

$$\underline{a * b} = \underline{N} \quad (a <= b) \quad (2 \rightarrow \sqrt{N})$$

$$a * a <= N$$

$$[a <= \sqrt{N}]$$

$$\begin{array}{l} 30 \\ (1, 30), (2, 15), (3, 10) \\ (5, 6) \end{array}$$

$$\Leftrightarrow 5$$

Efficient & More Efficient solutions for isPrime

Using for loop from 2 to \sqrt{n}

$$i * i \leq n \rightarrow T.C \rightarrow O(\sqrt{n})$$

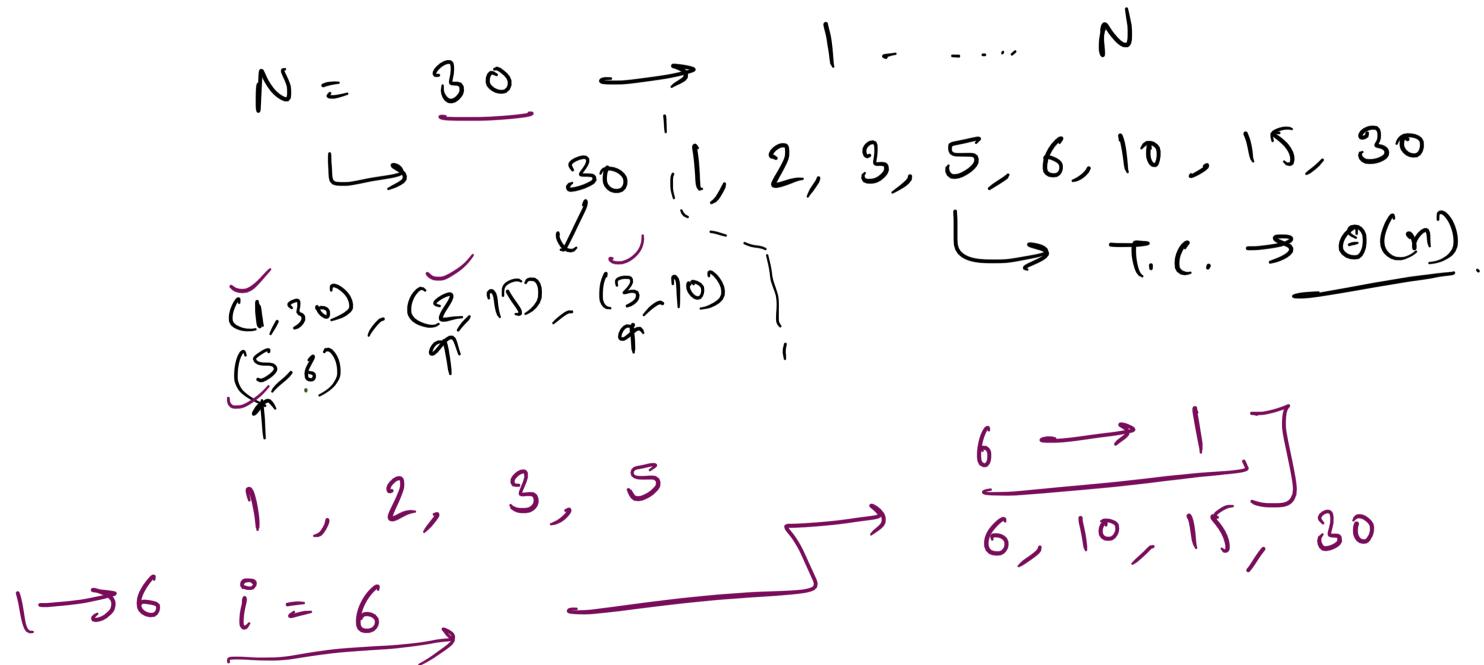
$$\boxed{i^2 \leq n}$$
$$i \leq \sqrt{n}$$

$$N = 911$$

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26
27, 28, 29, 30, 31

3 - Times faster.

Print All Divisors of N



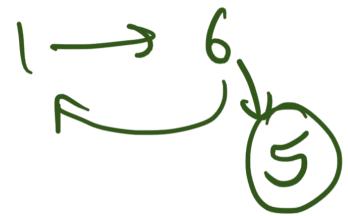
$i =$

1	2	3	4	5	6
↓	↓	↓	↓	↓	→

80
75
15
10
6

$i \neq i <= 30$ $\dfrac{++}{6}$

T.C \rightarrow $\Theta(\sqrt{n})$.



Print all prime numbers
between 1 ... N

Sieve of Eratosthenes

$$N = 30$$

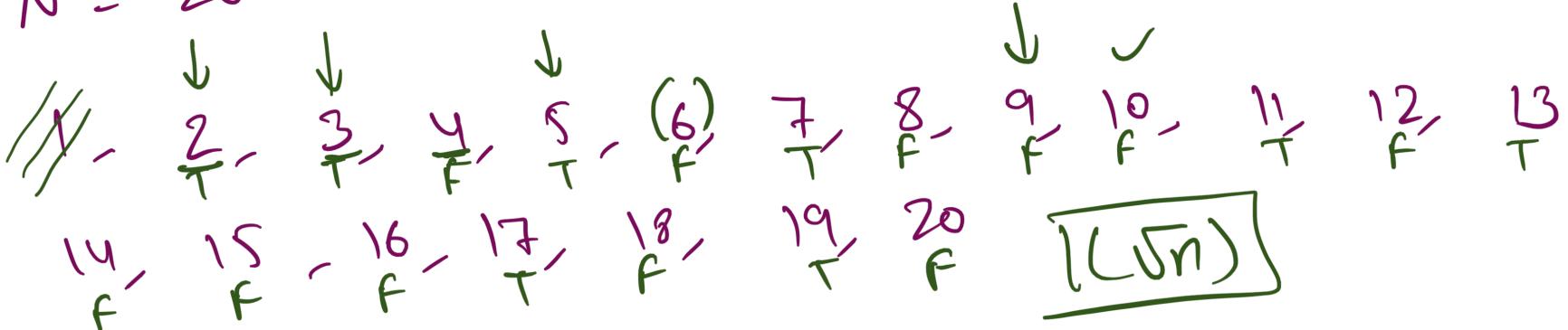
2, 3, 5, 7, 11, 13, 17, 19, 23, 29

① for loop and check if prime

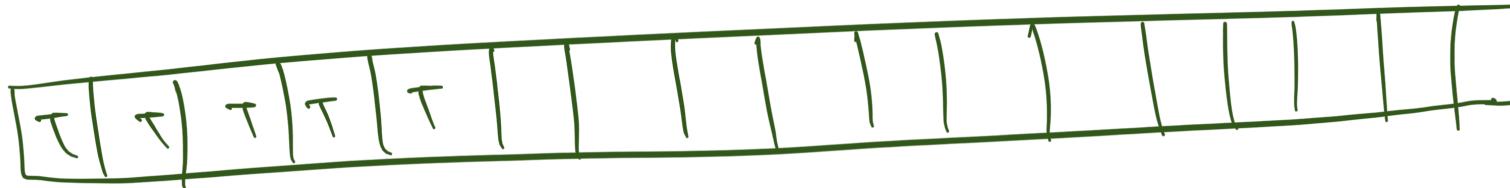
$$\hookrightarrow T.C \rightarrow O(n\sqrt{n}) \equiv O(n^{3/2})$$



$$N = 20$$



isPrime:



$$O(n \log \log n)$$

$$\frac{5 \times 5}{25}$$

(3, 4)

Fast Power, Calculate $\text{pow}(a, b)$

Math. $\text{pow}(a, b)$

$\hookrightarrow a^b$

1. for loop b times

$$3^4 = 81$$

$$3 * 3 * 3 * 3 = \underbrace{81}_{\hookrightarrow O(b)}$$

$$a^b \longrightarrow \underline{a}^{b/2} * a^{b/2} = \underline{a^b}$$

b even

$$\text{b odd} = a^{b/2} * a^{b/2} * \underline{a}$$

$$\begin{matrix} a^5 \\ \downarrow \\ (a^2) \end{matrix}$$

$$\underline{a^2} * \underline{a^2} * \underline{a} = a^5$$

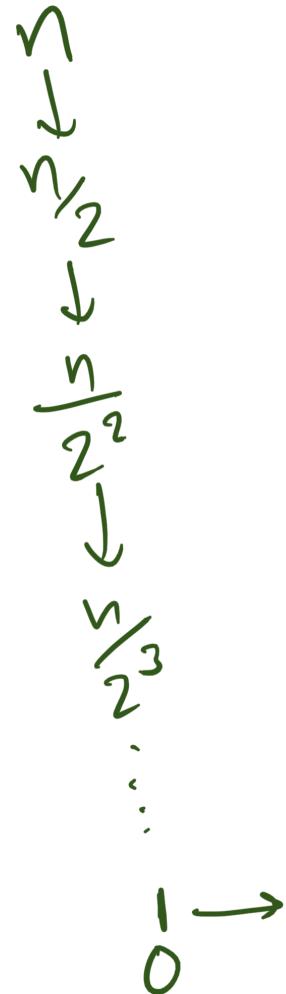
$$\begin{matrix} a^2 \\ a^4 = a^2 * a^2 = \underline{a^4} \end{matrix}$$

$$\begin{matrix} a^6 \\ \downarrow \\ (a^3) \end{matrix}$$

$$\underline{a^3} * \underline{a^3} = \underline{a^6}$$

↑ 729

~~Max steps~~



3^S

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$k = \log_2 n$$

$(3, 5)$ \downarrow $\uparrow 9$ $\rightarrow f_r = 9 * 9 = 81 * 9 = 729$

$(3, 2)$ \downarrow $\uparrow 3$ $\rightarrow f_r = 3 * 3 = 9$

$(3, 1)$ \downarrow $\uparrow 1$ $\rightarrow f_r = 1 * 1 = 1 * 3 = 3$

$(3, 0)$

T.C. $\rightarrow O(\log n)$

$$y_2 = 0$$

Practice Problems

- ✓1. Calculate $a^b \bmod n$ for large values of a & b.
- 2. Print prime factors of a given number.
- ✓3. Learn about BigInteger class in Java
- 4. Learn about Catalan Numbers and solve problems on Catalan Numbers.
- ↓5. Learn about Inclusion-exclusion principle.
- ↑6. Learn about Modular multiplicative inverse.

1. Sieve of Eratosthenes proof:

2. BigInteger class