

Mathematics - I



23,432

Palindrome Numbers

x 12 → 21

x 432 → 234

✓ 131 → 131

✓ 1441 → 1441

✓ 5 → 5

int a = 23432

String s = a + " ";

✓ 22 → 22

[String length N → O(N)]

```

int rev = 0;
int n = 1441;

```

$n = 1441$
 $rev = 0$

```

int ld = n % 10 ;
rev = rev * 10 + ld ;
n = n / 10;

```

$ld = 1441 \% 10 = 1$
 $rev = 1$
 $n = 1441 / 10 = 144$
 \downarrow
 $ld = 144 \% 10 = 4$
 $rev = 1 * 10 + 4 = 14$
 $n = 144 / 10 = 14$
 \downarrow

N

(total no. of digits = $\log_{10} n + 1$)

$O(\log N)$

$131 \rightarrow (\log 131)$

$\frac{10}{20} \rightarrow \log_{10} 10 = 1 + 1 = \underline{2}$
 $\frac{20}{100} \rightarrow \log_{10} 20 = 1. - + 1 = 2. - \underline{n} = \underline{2}$
 $\log_{10} 100 = \underline{2} \log_{10} 10 = 2 + 1 = \underline{3}$

GCD or HCF of Two Numbers

int a = 24
int b = 36 → 12

(24, 36)
↓ ↓
 ~ ~

O($\min(a, b)$)

2, 7 → 1
5, 30 → 5
10, 15 → 5
36, 30 → 6

36 → (2, 3, 3, 3)
30 → (2, 3, 5) → $2 \times 3 = \underline{\underline{6}}$

for (int i = min(a, b); i > 1; i--) {
 if (a % i == 0 || b % i == 0)
 return i
}
return 1;

(24, 36)

24 → 23 → 22 → ... → 12

Euclid GCD

$$\left[\text{gcd}(\underline{a}, b) = \text{gcd}(\underline{a-b}, b) \right]$$

$a > b$

$$\begin{aligned}\text{- } \text{gcd}(\underline{36}, 24) &= \text{gcd}(36 - 24, 24) \\ &= \text{gcd}(\underline{12}, 24)\end{aligned}$$

$$\text{- } \text{gcd}(24, 12) = \text{gcd}(\frac{12}{\uparrow}, \frac{12}{\uparrow})$$

$$\begin{aligned}\gcd(7, 2) &= \gcd(7-2, 2) \\ &= \gcd(5, 2)\end{aligned}$$

$$\gcd(5, 2) \quad \text{and} \quad \gcd(5-2, 2) = \gcd(3, 2)$$

$$\gcd(3, 2) = \gcd(1, 2)$$

$$\text{gcd}(2, 1) = \frac{\text{gcd}(1, 1)}{1}$$

$$\rightarrow \gcd(a, b) = \underline{\underline{9}}$$

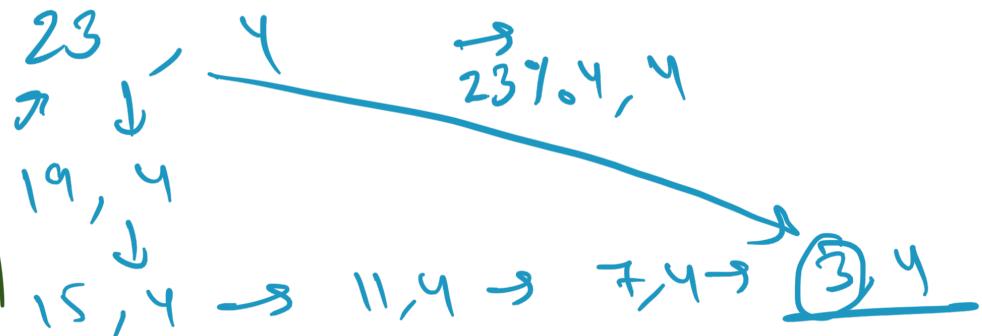
$$\begin{aligned} p * i &= a \\ p * j &= b \end{aligned}$$

$$(a - b) = p^i - p^j = \underline{P(i-j)}$$

$$\begin{matrix} 60, 24 \\ \downarrow \\ 36, 24 \\ \downarrow \\ 12, 24 \end{matrix}$$

$$\gcd(a, b) = \gcd(a - b, b)$$

$$\gcd(a, b) = \overbrace{\gcd(a \% b, b)}$$



LCM of Two Numbers

$$\text{LCM} = \frac{a * b}{\text{lcm}}$$

$$24 \rightarrow 24, 48, \cancel{72}, 96$$
$$36 \rightarrow 36, \cancel{72}, 108,$$

$$24 = 2 * 2 * 2 * 3$$

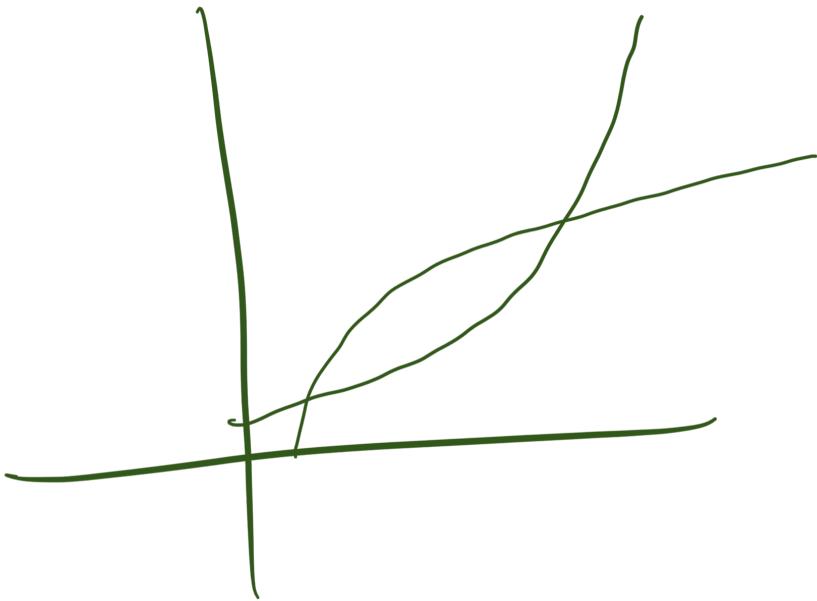
$$36 = 2 * 2 * 3 * 3$$

$$\begin{array}{r} 12 \\ \hline 144 \\ -144 \\ \hline 0 \end{array}$$

$$\begin{array}{c|cc} 2 & 24, 36 \\ \hline 2 & 12, 18 \\ 3 & 6, 9 \\ 2 & 2, 3 \\ 3 & \\ \hline & 72 \end{array}$$

$$[\text{LCM} = \frac{a * b}{\text{HCF}}] \quad \text{gcd}(a, b)$$

$$\text{LCM} = \frac{a * b}{\text{gcd}(a, b)}$$



Trailing Zeros in Factorial

$$1043 \rightarrow 0$$

$$100 \rightarrow 2$$

$$1020 \rightarrow 1$$

$$2 \times 5 = \underline{10}$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = \underline{120}$$

Ans = 1

$$10! = \cancel{10} \times \underline{9} \times \cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 = 2$$

$$15! = \cancel{15} \times \cancel{14} \cdots \overset{\uparrow}{\cancel{10}} \times \cancel{9} \cdots \overset{\uparrow}{\cancel{6}} \times \overset{\uparrow}{\cancel{5}} \times \overset{\uparrow}{\cancel{4}} \cdots \overset{\uparrow}{\cancel{1}} \\ = \underline{3}$$

$$20! = 4$$

$$5! = 1$$

$$10! = 2$$

$$15! = 3$$

$$20! = 4$$

$$14! = 2$$

n/s X

$$25! \approx 6$$

$$\begin{matrix} S & * & S \\ 25 & * & 24 \dots 20 \times \dots & 15 \dots \times & 10 & \dots & S & \times \dots & 1 \\ & | & & | & & | & & & | \\ & 5 & & 4 & & 3 & & & 1 \end{matrix}$$

$$30! = 7$$

$$\frac{n}{s} + 1 \rightarrow$$

$$\underbrace{125!}_{\{5\}} \rightarrow \overline{120}$$

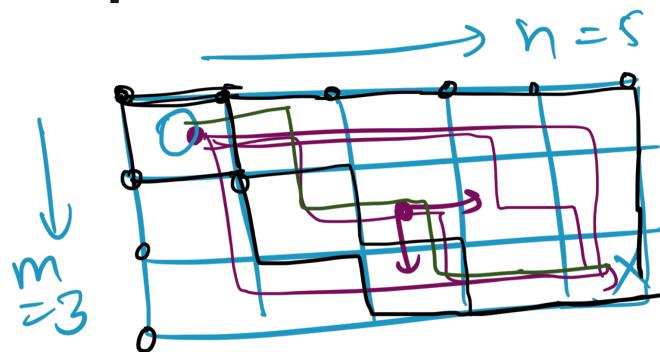
divide
 n ↓
 125 * ... 100* ... 75 * ... 50 * ... 25 * ... 20 * ... 1
 ↓
 25
 ↓
 $\boxed{5}$
 ↓
 5
 ↓
 1

20	15	10	5	4	...
4	3	2	1	0	...
0	0	0	0	0	...

* $n = \frac{125}{5} = \boxed{25}$

$\frac{25}{5} = \boxed{5}$ $\frac{5}{5} = \boxed{1}$ while $n > 0$
 $25 + 5 + 1 = \boxed{31}$ $n = \frac{n}{5}$

Unique Paths in a Grid



$m \rightarrow$ rows
 $n \rightarrow$ cols

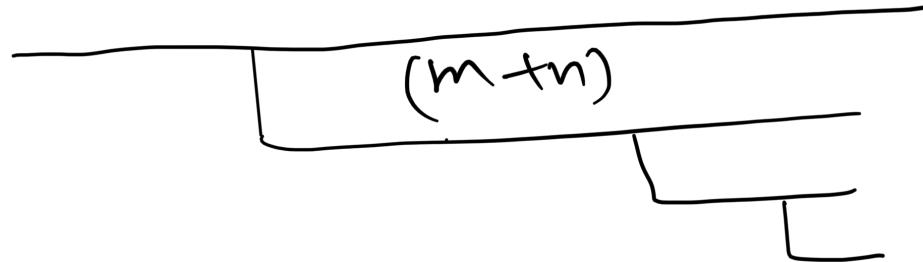
total steps $\rightarrow (m+n)$

$$\begin{aligned} & m \times n \\ & f(m, n) \\ & f(m-1, n) + f(m, n-1) \end{aligned}$$

$$2^{m \times n}$$

⑧

$S \subset 2$



in $m+n$ total steps
you need to come
down m times.

$\leftarrow P \supset C$

$$2^{m+n} = \frac{(m+n)!}{m! n!}$$

$$\begin{aligned} &= \frac{(m+n) * (m+n-1) * \dots * (m+1)}{m! * n!} \\ &= \frac{(m+1) * (m+2) * (m+3) * \dots * (m+n)}{1 * 2 * 3 * \dots * n} \end{aligned}$$

to ~~for~~ total n times

[O(n)]

✓ `int res = 1;`

`for (int i = 1; i < n; i++)`

`res = res * (m+i) / i;`

}

[Why?]

$${}^{m+n}C_m = {}^{m+n}C_n$$

Practice Problems

1. Learn about Mean, Median Mode.
2. Learn about Arithmetic & Geometric Progression.
3. Learn about Binary, Octa and Hexagonal Number systems and solve Base conversion Problems.
4. Proof of Euclidean GCD
5. Basics of Permutations & Combination.
6. Write a Program to calculate the square roots of an equation.
7. <https://www.interviewbit.com/courses/programming/math/>