Assignment 7

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See the code in attached files.

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See the code in attached files.

3

3.1

For an $(a, b) - tree$ with n nodes, we assume the height of it is h, then there will be $n - 1$ leaf nodes. According to definition, we have $2 \leq a \leq (b+1)/2$. Each internal node except the root has at least a children and at most b children and the root has at most b children. If $n > 0$ the root has degree at least 2 and all other nodes have degree at least a. This gives that the number of leaf nodes is at least $2a^{h-1}$. Similarly, since the degree of any node is at most b, the number of leaf nodes is less than b^h. Thus, the number of leaf nodes $n + 1$ is larger than the first case and less than the second case. We get:

$$2a^{h-1} \leq n - 1 \leq b^h$$

This equation can be modified by taking the logarithm of both sides. Then we get:

$$\log_b(n-1) \leq h \leq 1 + log_a \frac{n-1}{2}$$

Clearly, the height of the tree is bounded by $1 + log_a \frac{n-1}{2}$. Then for each nodes, there contains at most b elements which is sorted in order. By applying appropriate method, the number of comparisons will result in $\lceil \log b \rceil$ for searching in each node.
Hench, the total comparisons needed will be bounded by $\lceil \log(b) \rceil (2 + \log_a (n-1)/2)$.

3.2

According to 3.1, we can derive $\lceil \log(b) \rceil (2 + \log_a (n-1)/2)$. As $b <= 2a$, we can have:

$$\lceil \log(b) \rceil (2 + \log_a((n-1)/2)) < 2\lceil \log(b) \rceil + \lceil \log(2a) \rceil \log_a((n-1)/2))$$

As $log(2a) < 2log(a)$, we can then obtain:

$$2\lceil \log(b) \rceil + \lceil \log(2a) \rceil \log_a((n-1)/2) < 2\lceil \log(b) \rceil + 2\lceil \log(a) \rceil \log_a((n-1)/2))$$

Hence,

$$2\lceil \log(b) \rceil + 2\lceil \log(a) \rceil \log_a((n-1)/2)) < 2\lceil \log(b) \rceil + 2\lceil \log((n-1)/2)) \rceil$$

which is obviously in $O(\log b) + O(\log n)$.