Formation Control with Mobile Robots

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Abstract

Formation control in robotics is a growing topic where mainly research works are geared towards heterogenous swarm colonies under decentralized control or heterogenous colonies where some centralization is considered. In the literature, formation control is generally done for known fixed shapes that can be defined mathematically. Moreover, agents are taken into account as point robots which has no mass and volumes in the environment. In this paper, it is aimed to provide a formation control of a multi agent system to achieve a desired complex shape by interacting of each agent between themselves and reaching a global consensus about the positions in the formation.

I. Introduction

Formation control is a research branch which focuses on performing tasks collectively within a multi agent system. These tasks may require huge physical workloads or they may require some different capabilities to be performed together. In this case, formation must consist of different type of agents with different capabilities. In this case, it may not be suitable to assume that the formation is composed of homogenoues agents. On the other hand, when a swarm is requested to get a desired position there may be some complex shape requirements which cannot be identified analytically.

As a motivating example, consider it is desired to have a coverage of different radio links in a subdivision of a map and the covarage area is determined with the boundaries of a city which cannot be stated as a simple closed shapes or regular polygons like circles, triangles etc. To provide the coverage of whole city, different types of agents carrying the desired radio equipments have to be scattered in this complex shape. In this work, a method to get formation control of heterogenous agents in a decentralized manner is proposed.

The remainder of this paper is organized as follows. Section 2 states the research in the literature and Section 3 introduces the problem in detail. Section 4 introduces the artificial force based method for partitioning the complex shape into possible agent locations. Second part of this section introduces a Hungarian algorithm based method for individual decision of every agent to get the desired complex shape. Results of differet type of tasks are given in Section 5. Discussion of the conclusion of simulations is presented in Section 6.

II. Background and related work

Samitha and Pubudu has proposed an artificial force based swarm control method that is used to partition the given complex shape in this work[1]. They are concentrated on stability analysis of a swarm under these artificial forces and provide the constraints which have to be satisfied to get swarm in a desired formation. In their work, artificial force calculations have been done in continuous time including analytical statement of the formation shapes.

Minkowski Sums are used to calculate the forbidden space caused by the obstacles in the environment. This forbidden space is than extracted from the configuration space to get the free movement space of each agent in the en-

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vironment[2]. Visibility graphs and Djikstra algorithms are used to calculate the possible minimum path to a desired goal point from start point by taking into account the obstacles in the environment[3].

Agents' individual decisions for goal points in the environment is based on Hungarian algorithm which is a combinational optimization algorithm that solves the assignment problem in polynomial time.

III. Problem definition

Problem is divided into two main parts as "Partitioning Complex Shapes" and "Decision of goal States of Each Agent".

I. Partitioning the Complex Formation Shape

Desired formation shape must be represented with the possible goal states of each agent. By providing a solution to this problem, the total formation control problem will be reduced down to decision process of each agent between some potential goal states. In this context, goal states must be located homogenously in the given complex shape to cover the whole formation.

II. Decision Process Between Possible Goal States

Each agent must calculate its free space by taking the obstacles in the environment into the account. In this free space, it must calculate the optimal path to a given goal state. Costs to all goal states must be calculated and stored in agents memory.

Given goal states and cost values to these goal states, each agent must individually decide where to position in the formation. This process must be held to optimize the utility of every agent with a collaboration. It is obvious that some of the agents may want to choose the same goal point to reach, so the swarm must reach a global consensus on target points and conflict cases must be handled.

IV. Approach

I. Artificial Forces in Formation Control

This process is used to detect the positions of the goal states in a given complex shape with an offline simulation. Formation shape is assumed to be given with "k" sample points which have equal distances to its closest two neighbors. Two artificial forces, resistive and intermember forces ares used to provide a uniform distribution of the agents in the given formation. The state of the agent i is described by

$$X_i = \begin{bmatrix} z_i \\ \dot{z}_i \end{bmatrix} \tag{1}$$

where $z_i = \begin{bmatrix} x_i \ y_i \end{bmatrix}$ is the position vector of agent i in the 2D plane. Each agent have a circular zone in the 2D plane and the radius of this zone is r_i. The artificial forces are given with contour integrals[1]. Derivation of resistive forces caused by the shape boundary for agent i,

$$F_{i,r,x} = k_f \sum_{j=1}^{n} \left(\frac{x_i - x_j}{d_{ij}} \frac{1}{\left(d_{ij} - r_i\right)^2} \right) \tag{2}$$

$$F_{i,r,y} = k_f \sum_{j=1}^{n} \left(\frac{y_i - y_j}{d_{ij}} \frac{1}{\left(d_{ij} - r_i\right)^2} \right) \tag{3}$$

where d_{ij} is the distance between the agent i position and the boundary point j. Derivation of the intermember force on the agent i,

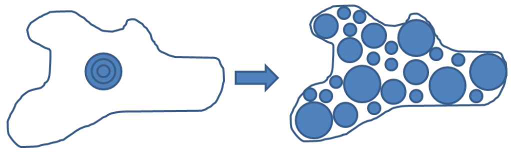
$$F_{i,m,x} = k_m \sum_{j=1, j\neq i}^{n} \left(\frac{x_i - x_j}{d_{ij}} \frac{1}{\left(d_{ij} - d_o\right)^2} \right) \tag{4}$$

$$F_{i,m,y} = k_m \sum_{j=1, j\neq i}^{n} \left(\frac{y_i - y_j}{d_{ij}} \frac{1}{\left(d_{ij} - d_o\right)^2} \right) \tag{5}$$

where $d_o = r_i + r_j$

In the offline simulation part, agents have been inserted to the desired formation with an initial state vector at the origin of the formation and zero velocities and then they are expected to scatter along the formation homogenously under the repulsive and intermember forces described above.

Figure 1: *Homogenous Distribution under Artificial Forces*



This offline procedure is used to locate the potential positions of different circular sized agents in the complex formation shape.

II. Decision of Goal States

In the first part of the work, the complex formation control problem is reduced down to a problem in which every agent is expected to decide individually where to position in a given set of possible goal states $g_i \in G$

During decision process, the cost of reaching different goal states will be the main criteria for each agent. In this work, the cost of reaching a goal state is defined with the minimum shortest path in the environment. A visibility graph based approach is used to calculate the shortest possible path to a goal state.

II.1 Free Configuration Space

Assume an environment with set of obstacles $S = \{P_1, P_2, ..P_t\}$. Configuration for agent i can be described with the position of its circle center with $R = \{x_i, y_i\}$. Configuration space of i th agent is the environment itself and represented by $C(R_i)$. This configuration space is composed of two subspaces; free configuration space and forbidden configuration space of agent i.

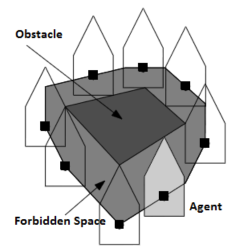
$$C(R_i) = C_{free}(R_i, S) + C_{forb}(R_i, S) \qquad (6)$$

The configuration space $C(R_i)$ of agent i is the environment itself. If the forbidden space is calculated with Minkowski Sum method, free configuration space can be derived simply by extracting the forbidden space from the environment. Let a single obstacle is described with a point set of S_1 and the agent is described with a point set of S_2. The Minkowski sum of these two sets $S_1 \subset R^2$ and $S_2 \subset R^2$ can be calculated with the following,

$$S_1 \oplus S_2 := \{p + q : p \subset S_1, q \subset S_2\} \qquad (7)$$

where $p + q$ denotes the vector sum of the vectors p and q.

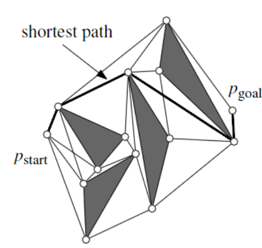
Figure 2: *Forbidden Space Caused by an Obstacle*



II.2 Visibility Graphs and Dijkstra′s Algorihtm

Consider set of obstacles in the environment is augmented with the Minkowski Sums described in the previous chapter. Let these set of augmented polygonal obstacles represented with $S_i \subset S$. The shortest path between p_{start} and p_{goal} among a set S of augmented polygonal obstacles consists of arcs of the visibility graph $\gamma_{vis}(S^*)$ where $S^* := S \cup \{p_{start}, p_{goal}\}$

Figure 3: *Shorthest Path from an initial state to a goal state*



Algorithm to calculate the visibility graph of S^*,

Data: Set of Vertices Included in S^*
Result: Visibility Graph of S^*
Initialize a graph $\gamma = (V, E)$ where V is the set of al vertices of the polygons in S and $E = \oslash$
for <*all vertices* $v \subset V$> **do**
  W = VISIBLEVERTICES(v,S);
end

Algorithm 1: VISIBILITYGRAPH(S^*)

where $VISIBLEVERTICES(v, S)$ algorithm checks whether line segments drawn from v to all vertices in S is intersecting an interior area of any obstacle in the environment. With the help of this $VISIBILITYGRAPH$ algorithm $SHORTESTPATH$ algorithm can be defined as follows.

Data: A set S of disjoint polygonal obstacles, and two points p_{start} and p_{goal} in the free space.
Result: The shortest collision-free path connecting p_{start} and p_{goal}
1) Assign γ = VISIBILITYGRAPH(S^*)
2) Assign each arc (v, w) in γ_{vis} a weight, which is the Euclidian length of segment vw
3) Use Dijkstra's algorithm to compute a shortest path between p_{start} and p_{goal} in γ_{vis}

Algorithm 2: SHORTESTPATH

Dijkstra′s algorithm computes the shortest path between two nodes in graph with k arcs, each having a non-negative weight

II.3 Collaborative Decision Process of Final Goal States

In the previous chapters, possible positions and costs of shortest paths to these positions are calculated for each agent. It is obvious that each agent will try to choose a goal state with minimum cost according to its position and the orientation in the environment. But the cases in which two or more agents are willing to reach the same goal point must be handled to optimize the overall utility of the swarm. To minimize to overall cost of whole swarm, Hungarian algorithm which is a combinational optimization algorithm that solves this assignment problem is used. To implement this algorithm, a complete bipartite graph $G = (S, T, E)$ with $n \in S$ agents and $t \in T$ goal points is constructed.In this graph, each agent have a cost which is defined by the shortest path to the destination in the environment for different goal points.

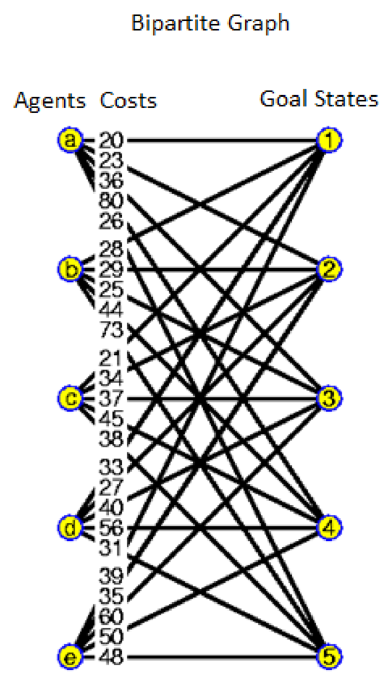


Figure 4: *Bipartite Graph Used in Assignment Problem*

V. Results

Three different tasks which consist of different formation shapes located at different places in the environment are achieved by the swarm, during evaluation process of the algorithms. These tasks are given in the Figure 5, 6 and 7. Green shapes in the figures represent the obstacles in the environment, blue shapes represent the desired formations and the red circles are the agents in the swarm.

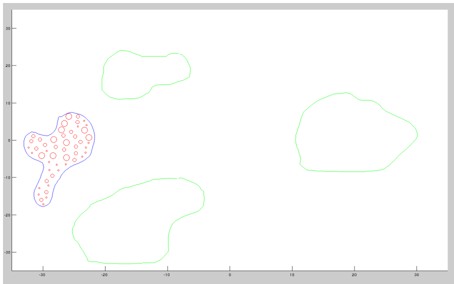


Figure 5: *Task1*

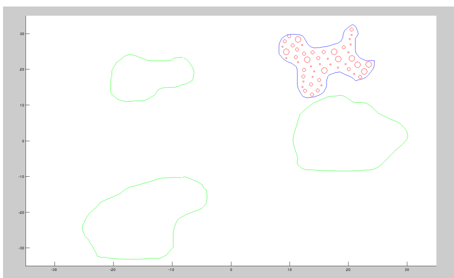


Figure 6: *Task2*

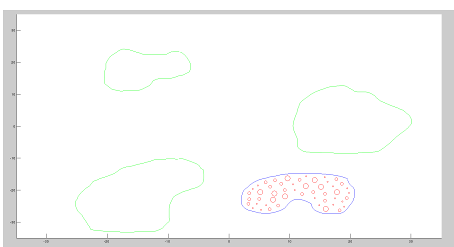


Figure 7: *Task3*

In the first part of simulations, performance of the Hungarian algorithm used in solution of assignment problem is evaluated. For this purpose, agents in the swarm are assigned to goal states with Hungarian algorithm and randomly for the same specific task, and the time spent to reach the desired shape for each method recorded.

Table 1: *Time Spent on Reaching Different Formations*

Method/Task	Task1	Task2	Task3
Random	42,4 sec	26,3 sec	35 sec
Hungarian	32 sec	15,8 sec	27,3 sec

In the second part of simulations, 20 iterations held for each task and the total number of agents which are outside of the desired shape when the swarm reached steady state are recorded.

Table 2: *Time Spent on Reaching Different Formations*

	Task1	Task2	Task3
Num.of Agents	2	0	1

VI. Conclusions

This work focuses on formation control of heterogenous mobile robots with complex formation shapes. First, the problem is reduced down by deriving the possible goal states which agents in the swarm will be scattered along the formation homogenously. At this stage the problem is converted into multiple agents′ decision process of goal states which are shared commonly by the swarm. During the decision process the main criteria is the cost of shortest paths to the goal states for each agent. To provide these cost values, Minkowski Sums , Visibility Graphs and Dijkstra′s algorithms are used. The assignment problem to optimize the overall cost of the swarm is solved by Hungarian algorithm. The results reporting that the total time spent on achieving the same specific task with random assignment method and Hungarian method, show the effects of optimal assignment technique on performance of swarm. On the other hand, results with number of iterations in which one or more agents are outside of the desired formation shape, give a good measure about the robustness of the algorithms used in formation control.

VII. Future Work

To provide a solution to the first part of the problem defined in this paper, an approach based on artificial forces is used to detect the possible goal states of agents in the desired formation shape. It is obvious that the offline simulation phase may result to some suboptimal solutions. In fact, this approach depends on the kinetic theory of gases and this theory proves the homogenous distribution of gas molecules in a closed area. In future work, it is possible to create a linkage between the kinetic theory and the solution provided in this paper, and to show how the solution is far away from the optimality. Also it is possible to introduce better methods instead of artifical forces to achieve results which are closer to optimal solution.

References

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