Chapter 7
Spread-Spectrum Modulation

Problems: (pp.509-511)

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7.7	7.10	7.14(选做)

Chapter 7
Spread-Spectrum Modulation

第七章 扩频调制

Chapter 7
Spread-Spectrum Modulation

- Main Topics:
  - *1. Spreading sequences* (PN sequences): properties, methods of generation
  - *2. Principles of spread spectrum modulation*
  - **3.** Two commonly used types: ***DS*** (direct sequence) & ***FH*** (frequency hopping)

7.1 Introduction

- **Spread spectrum modulation** ---- secure communication, sacrifice the efficiency of channel bandwidth
- **Primary advantage** ---- reject interference
  - 1. unintentional or intentional -- military applications
  - 2. multipath rejection & multiple access -- civilian applications

7.1 Introduction

- Definition of spread spectrum modulation
  - 1. Bandwidth >> minimum bandwidth necessary
  - 2. Spectrum spreading accomplished through the use of spreading code
  - 3. Spreading code independent of the data sequence
  - 4. Same code used in transmitter & receiver

7.1 Introduction

- Spectrum spreading
  - DS: narrowband data sequence ---> noiselike wideband signal
  - FH: carrier frequency changed in a pseudo-random manner
  - Both of the above two techniques rely on the use of a pseudo-noise sequence (**PN sequence**)

7.1 Introduction

- CDMA系统的发展历程
  - 20世纪40年代末　跳时扩频和CDMA的概念
  - 1950年　直接序列扩频
  - 70年代　广泛用于军事和卫星导航
  - 80年代　Qualcomm公司 DS-CDMA
  - 1993年　窄带CDMA的IS-95标准
  - 1996年　IS-95商用运行
  - 2000年　宽带CDMA　第三代移动通信的主要技术

7.2 Pseudo-Noise Sequences

- PN Sequence
  - *Periodic* binary sequence (period $\le 2^m$)
  - *Noiselike*
  - Generated by feedback shift register

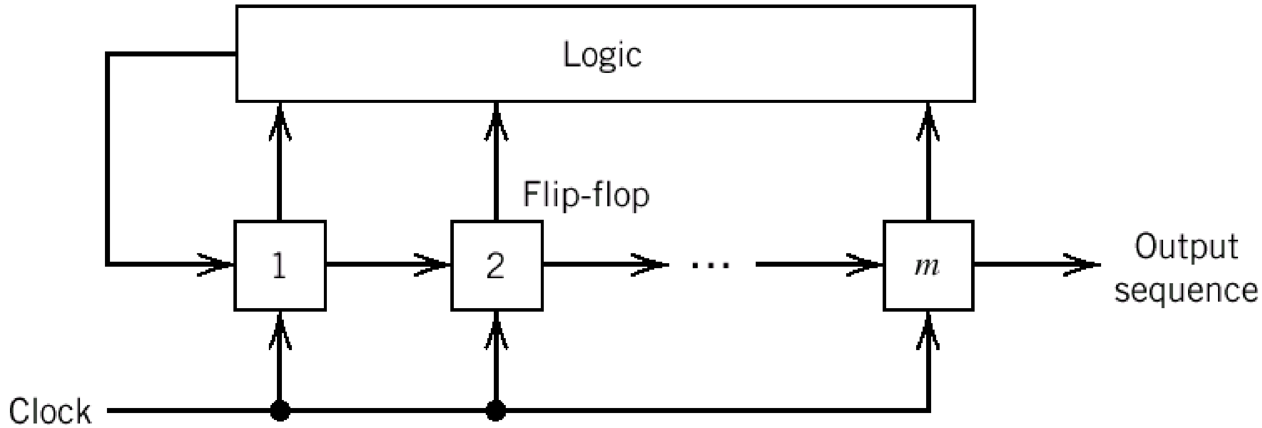


Fig 7.1 Feedback shift register

7.2 Pseudo-Noise Sequences

- PN sequence determined by
  - length *m* of the shift register
  - initial state of the register
  - feedback logic
- Period of a PN sequence ---

  at most 2^m

7.2 Pseudo-Noise Sequences

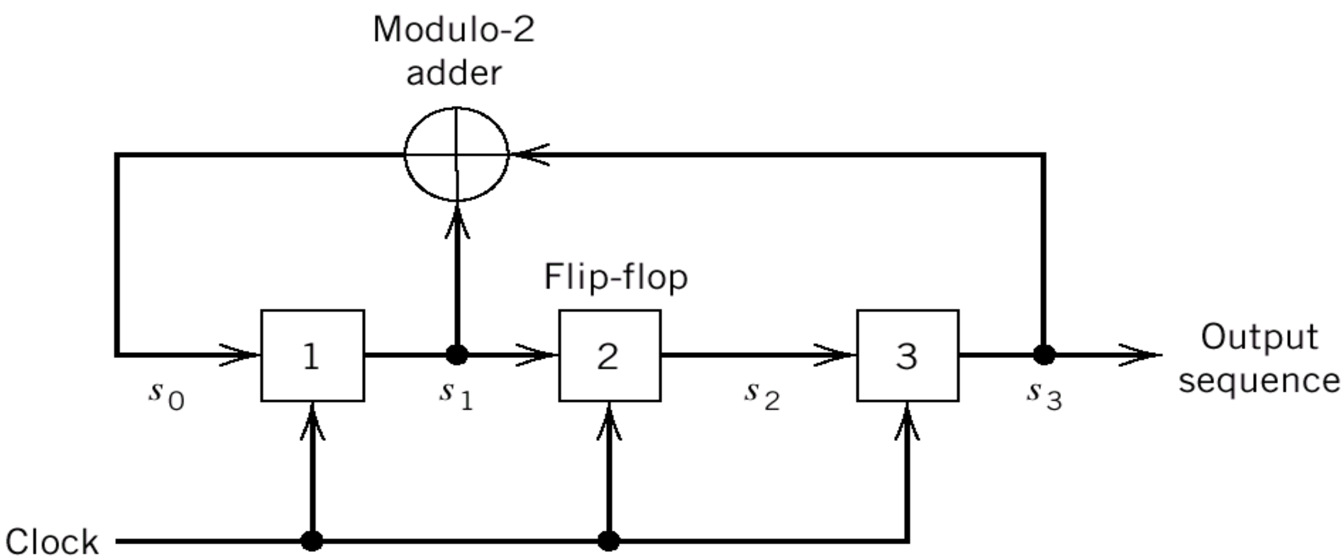
- *Generation of PN sequence* (each pulse of clock)
  - the state of each flip-flop is *shifted* to the next one
  - the logic circuit *computes* a Boolean function of the states of the flip-flops
  - the result of Boolean function is *fed back* to the first flip-flop

7.2 Pseudo-Noise Sequences

- *Maximal-length-sequence* (m-sequence)
  - produced by a ***linear*** feedback shift register (*linear means that the feedback logic consists entirely of modulo-2 adders*)
  - period $N = 2^m - 1$

7.2 Pseudo-Noise Sequences

- **Example 7.1** Maximal-length sequence generator for $m = 3$
  - period$=2^3-1=7$
  - initial state: 100, succession of states: 100, 110, 111, 011, 101, 010, 001, 100



7.2 Pseudo-Noise Sequences

- *Properties of Maximal-Length Sequences*
  - *1. Balance property:* number of 1s - number of 0s = 1
  - *2. Run property:*
    - run-- a subsequence of identical symbols
    - total number of runs = 2^{m-1}
    - number of length-i runs = $2^{m-1}/ 2^{i}$,
      $1 \leqslant i \leqslant m-2$, one half are runs of 1s and one half runs of 0s
    - number of length-m run of 1s = 1
    - number of length-(m-1) run of 0s =1

7.2 Pseudo-Noise Sequences

- *3. Correlation property:*
  - Autocorrelation function: *Page 483, Equ. (7.5)*

$$R_C(\tau) = \frac{1}{T_b}\int_{-T_b/2}^{T_b/2} c(t)c(t-\tau)dt$$

$$R_C(\tau) = \begin{cases} 1-\dfrac{N+1}{NT_C}\left|\tau - iNT_c\right|, & \left|\tau - iNT_c\right| \le T_c \\ -\dfrac{1}{N}, & \text{Others} \end{cases}$$

  - Power spectral function: *Page 484, Equ. (7.6)*

$$S_c(f) = \frac{1}{N^2}\delta(f) + \frac{1+N}{N^2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \operatorname{sinc}^2\left(\frac{n}{N}\right)\delta\left(f - \frac{n}{NT_C}\right)$$

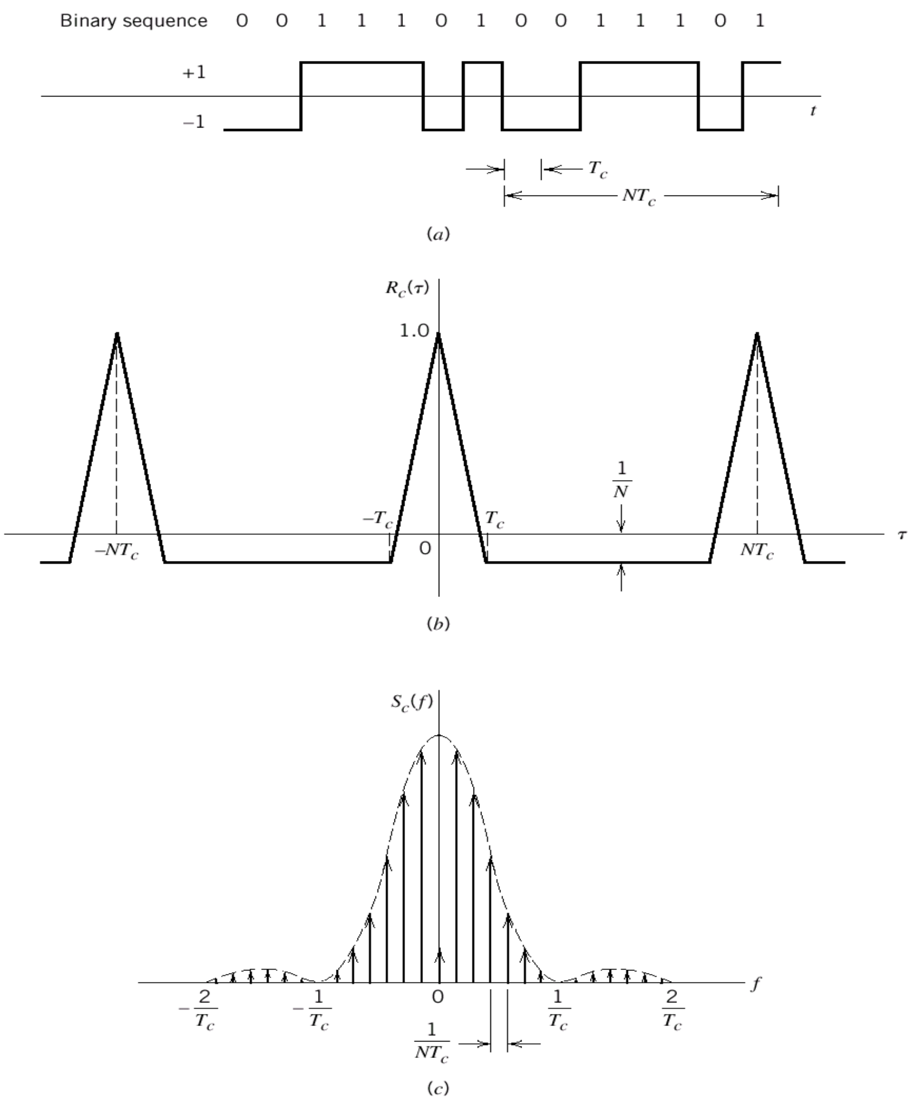


Figure 7.3

(*a*) Waveform of maximal-length sequence for length $m = 3$ or period $N = 7$.

(*b*) Autocorrelation function.

(*c*) Power spectral density.

All three parts refer to the output of the feedback shift register of Figure 7.2.

7.2 Pseudo-Noise Sequences

- *compare the m-sequence with the random sequence*
  - 1. autocorrelation functions are similar
  - 2. waveforms of the power spectral densities have the same envelope
  - 3. $N \to \infty$, the m-sequence and the random sequence become identical

7.2 Pseudo-Noise Sequences

- Choosing a Maximal-Length Sequences
  - *Key questions:* How do we find the *feedback logic* for a desired period N?
  - *Answer:*
    - feedback connection in extensive tables (table 7.1)
    - theorem covered in Chapter 10 & in 7.8 Computer Experiments: Maximal-Length and Gold Codes (primitive polynomial of degree m)

7.2 Pseudo-Noise Sequences

TABLE 7.1 *Maximal-length sequences of shift-register lengths 2–8*

Shift-Register Length, m	*Feedback Taps*
2*	[2, 1]
3*	[3, 1]
4	[4, 1]
5*	[5, 2], [5, 4, 3, 2], [5, 4, 2, 1]
6	[6, 1], [6, 5, 2, 1], [6, 5, 3, 2]
7*	[7, 1], [7, 3], [7, 3, 2, 1], [7, 4, 3, 2], [7, 6, 4, 2], [7, 6, 3, 1], [7, 6, 5, 2], [7, 6, 5, 4, 2, 1], [7, 5, 4, 3, 2, 1]
8	[8, 4, 3, 2], [8, 6, 5, 3], [8, 6, 5, 2], [8, 5, 3, 1], [8, 6, 5, 1], [8, 7, 6, 1], [8, 7, 6, 5, 2, 1], [8, 6, 4, 3, 2, 1]

Example 7.2

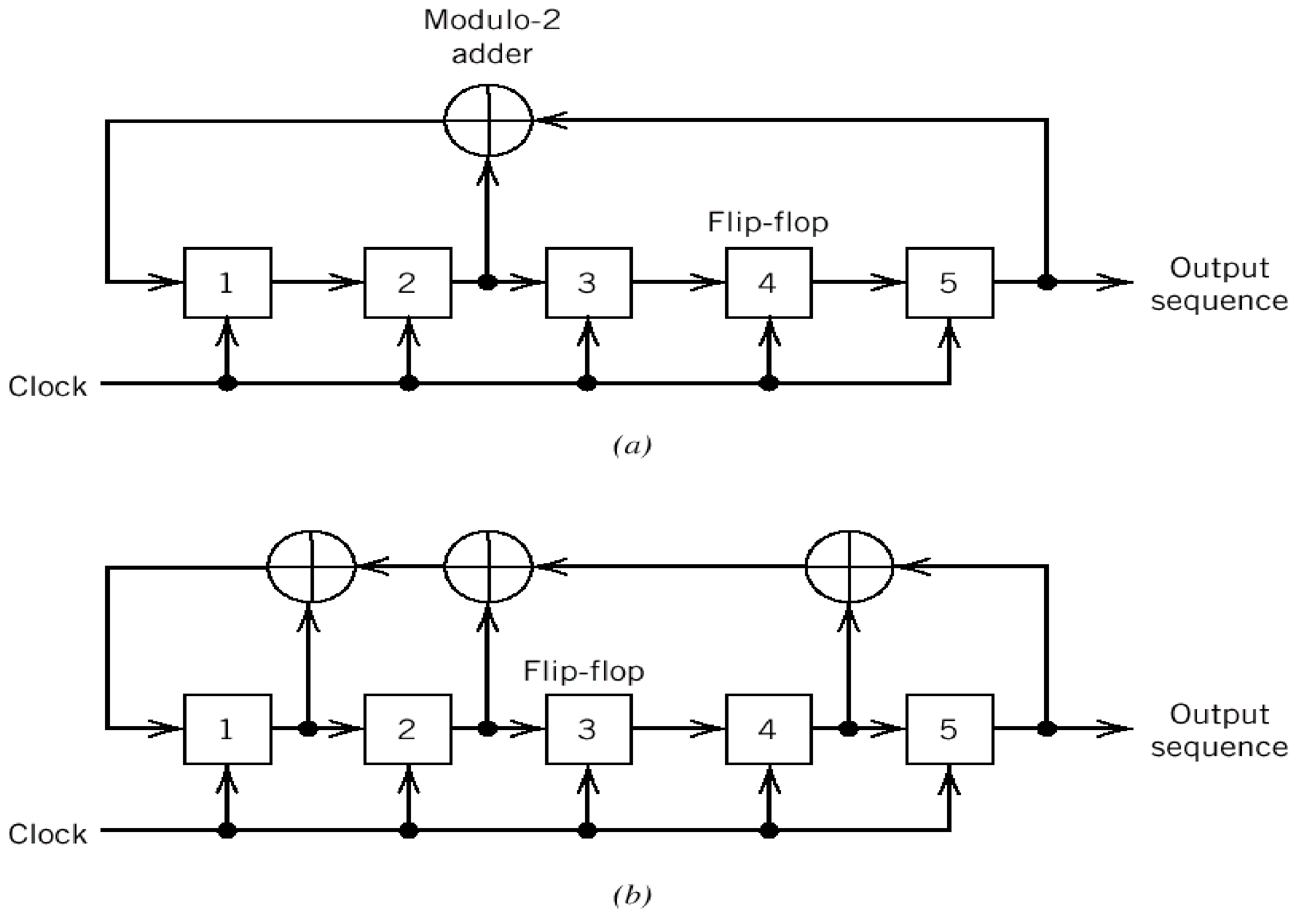


Figure 7.4 Two different configurations of feedback shift register of length $m = 5$. (*a*) Feedback connections [5, 2]. $(45)_8$ (*b*) Feedback connections [5, 4, 2, 1]. $(67)_8$

7.3 A Notion of Spread Spectrum

- *Important attribute:* Protection against interfering(jamming) signals with finite power
  - Jamming signals may be powerful broadband noise or multitone waveform
  - Protection is provided by spread spectrum modulation

7.3 A Notion of Spread Spectrum

- *Spread spectrum modulation*
  the bandwidth occupied by the information-bearing(data) signal >> the minimum bandwidth necessary to transmit it --> noiselike、undetected

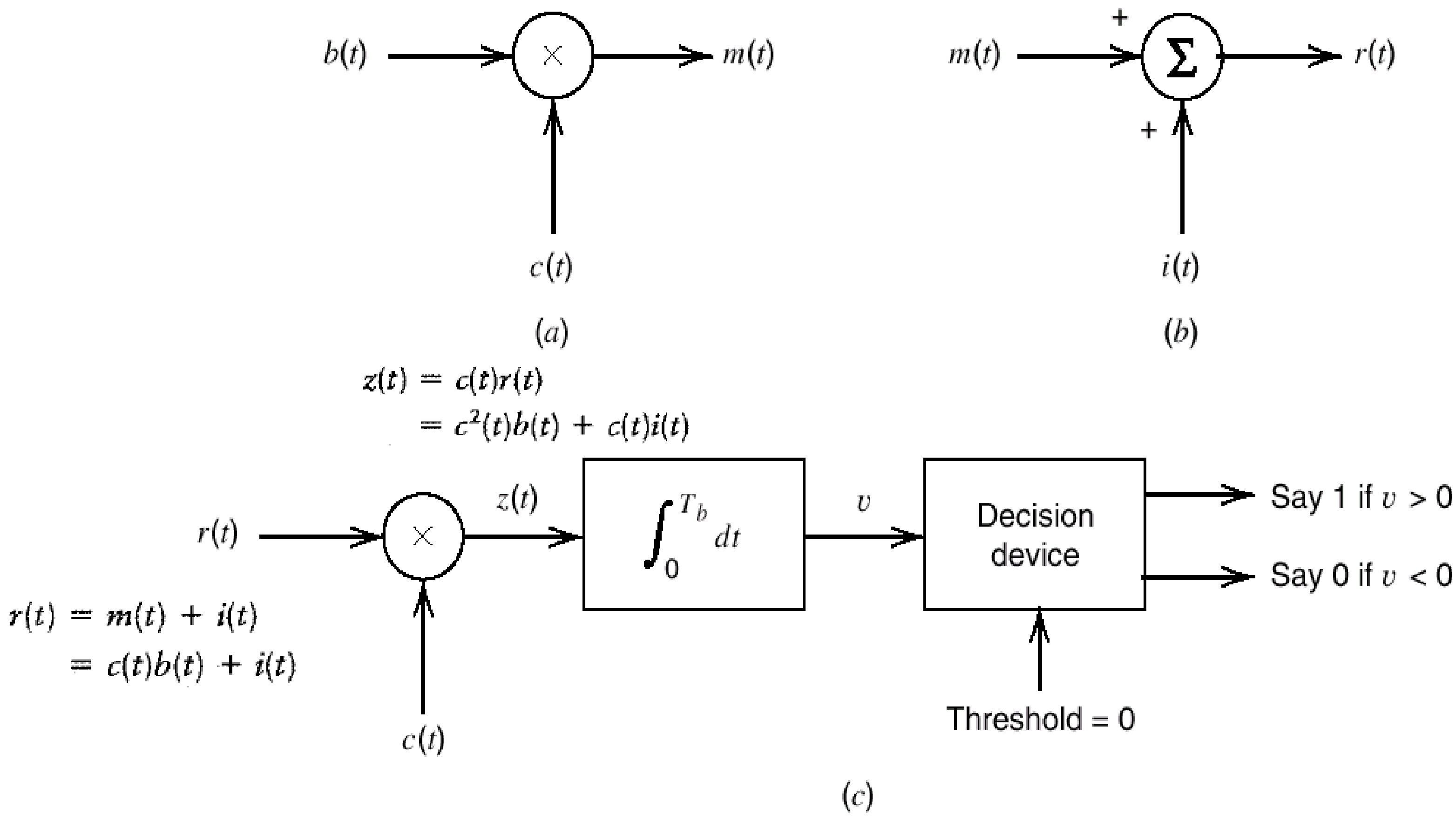
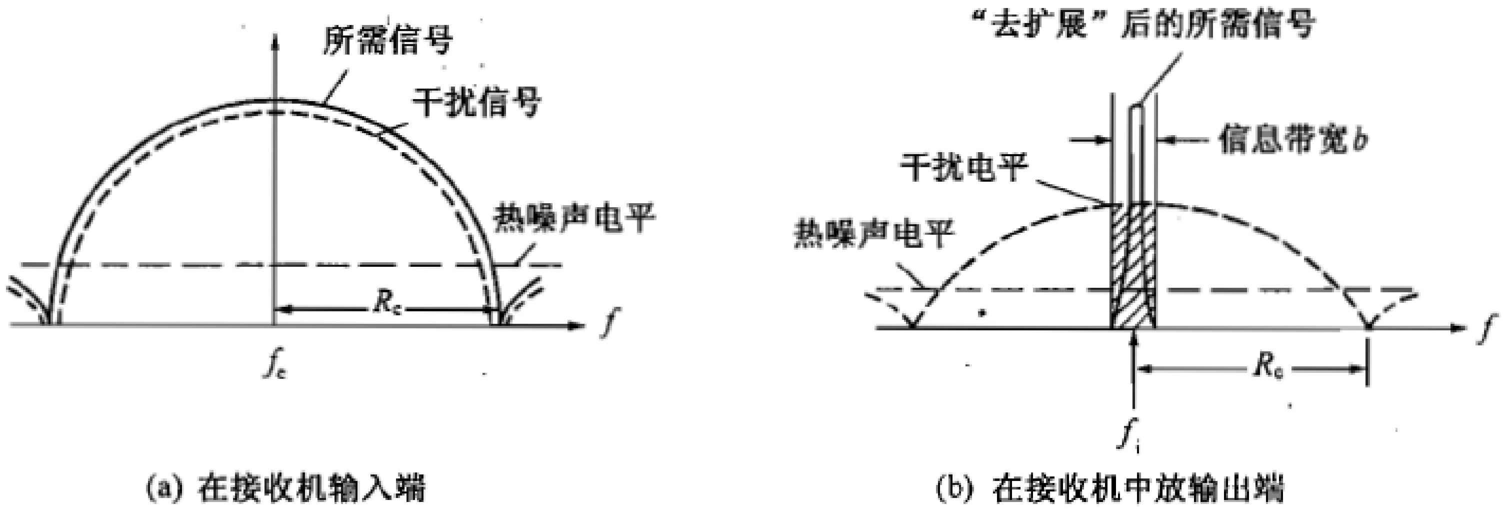
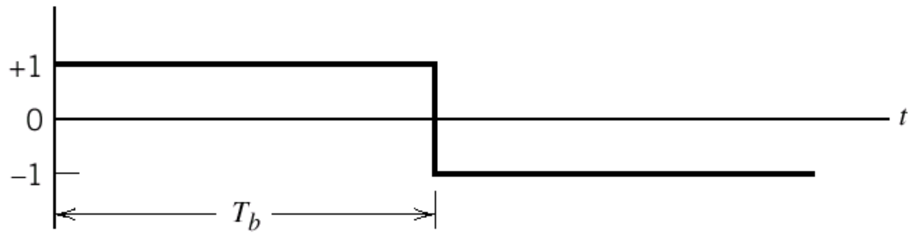


Figure 7.5

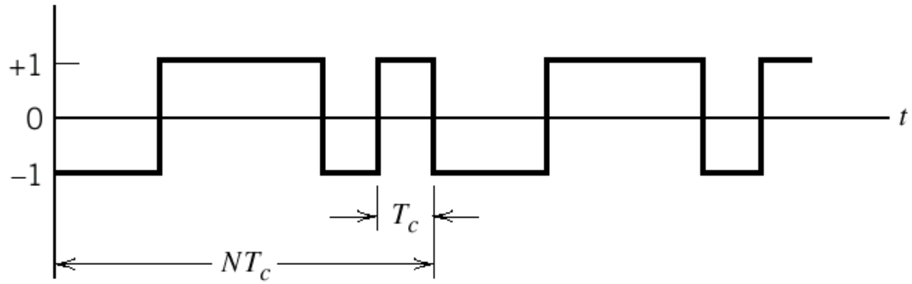
Idealized model of *baseband* spread-spectrum system. (*a*) Transmitter. (*b*) Channel. (*c*) Receiver.



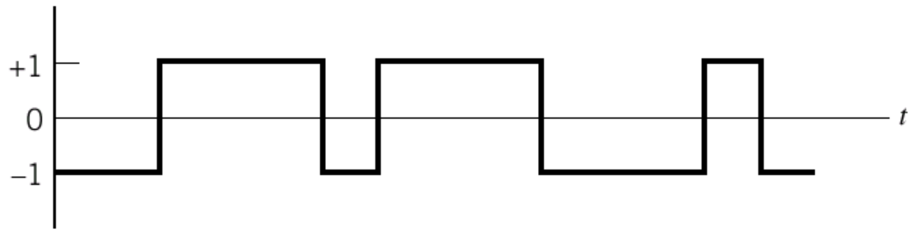
信号频谱在接收机中的变化



(a) Data signal $b(t)$



(b) Spreading code $c(t)$



(c) Product signal $m(t)$

Figure 7.6 Illustrating the waveforms in the transmitter of Figure 7.5*a*.

7.3 A Notion of Spread Spectrum

- **Advantage of the use of a spreading code:** wideband transmitted signal --- noiselike to a receiver having no knowledge of the spreading code
- **Price paid:**
  - increased transmission bandwidth
  - system complexity
  - and processing delay

7.4 Direct-Sequence Spread Spectrum with Coherent Binary Phase-shift Keying

- DS/SS in the baseband transmission discussed in section 7.3

- DS/SS in the passband transmission using coherent PSK discussed in this section

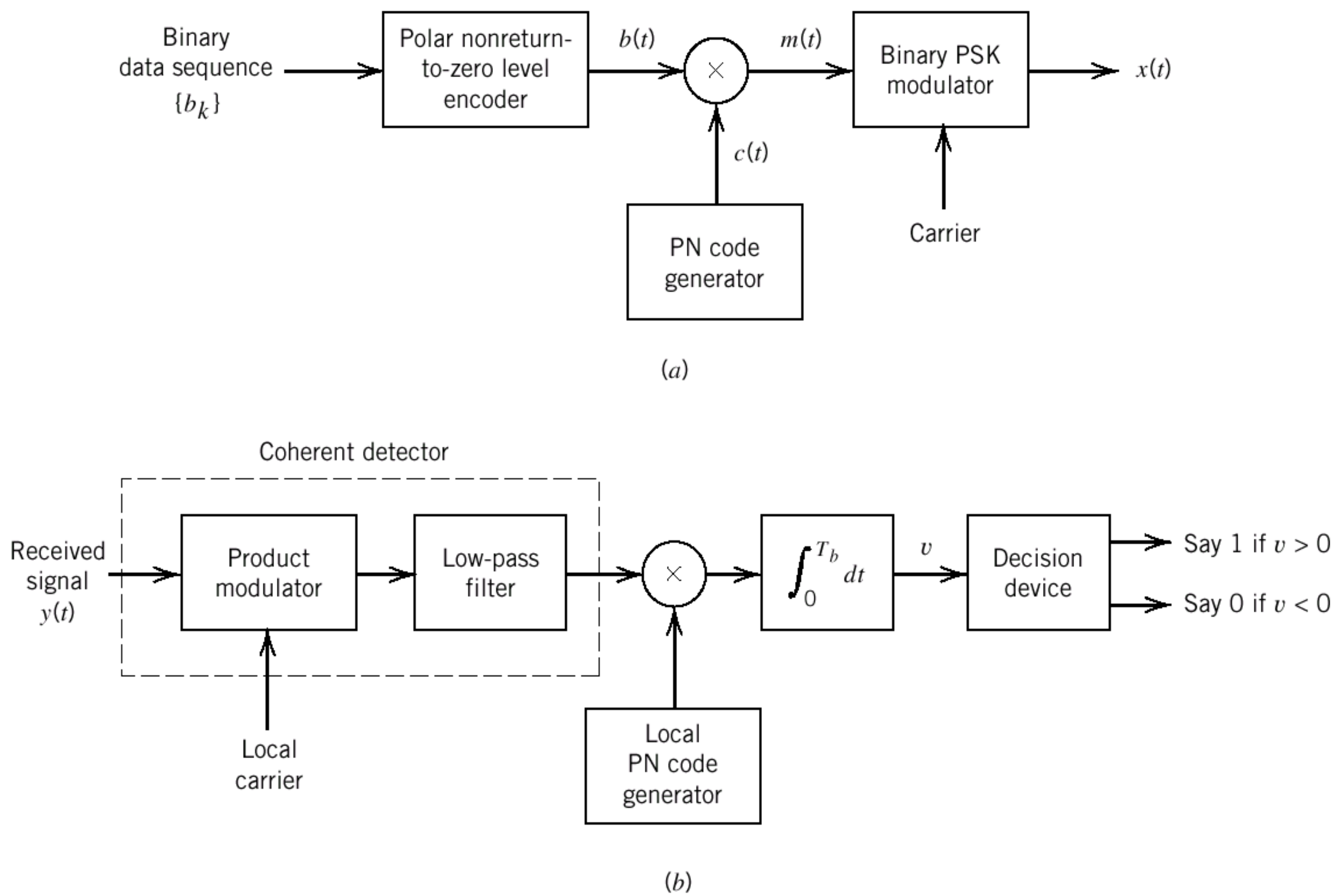


Figure 7.7 Direct-sequence spread coherent phase-shift keying.
(*a*) Transmitter. (*b*) Receiver.

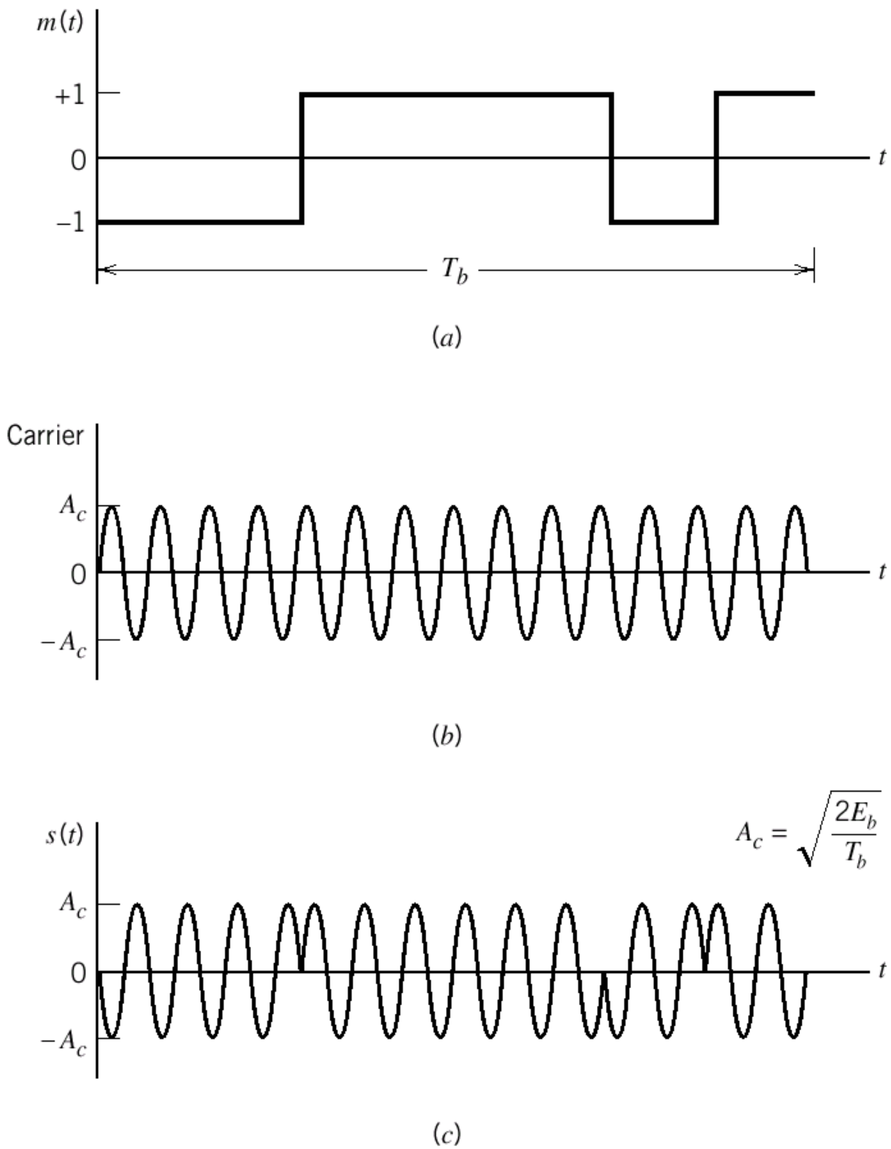


Figure 7.8

(*a*) Product signal $m(t) = c(t)b(t)$.
(*b*) Sinusoidal carrier. (*c*) DS/BPSK signal.

Model for Analysis

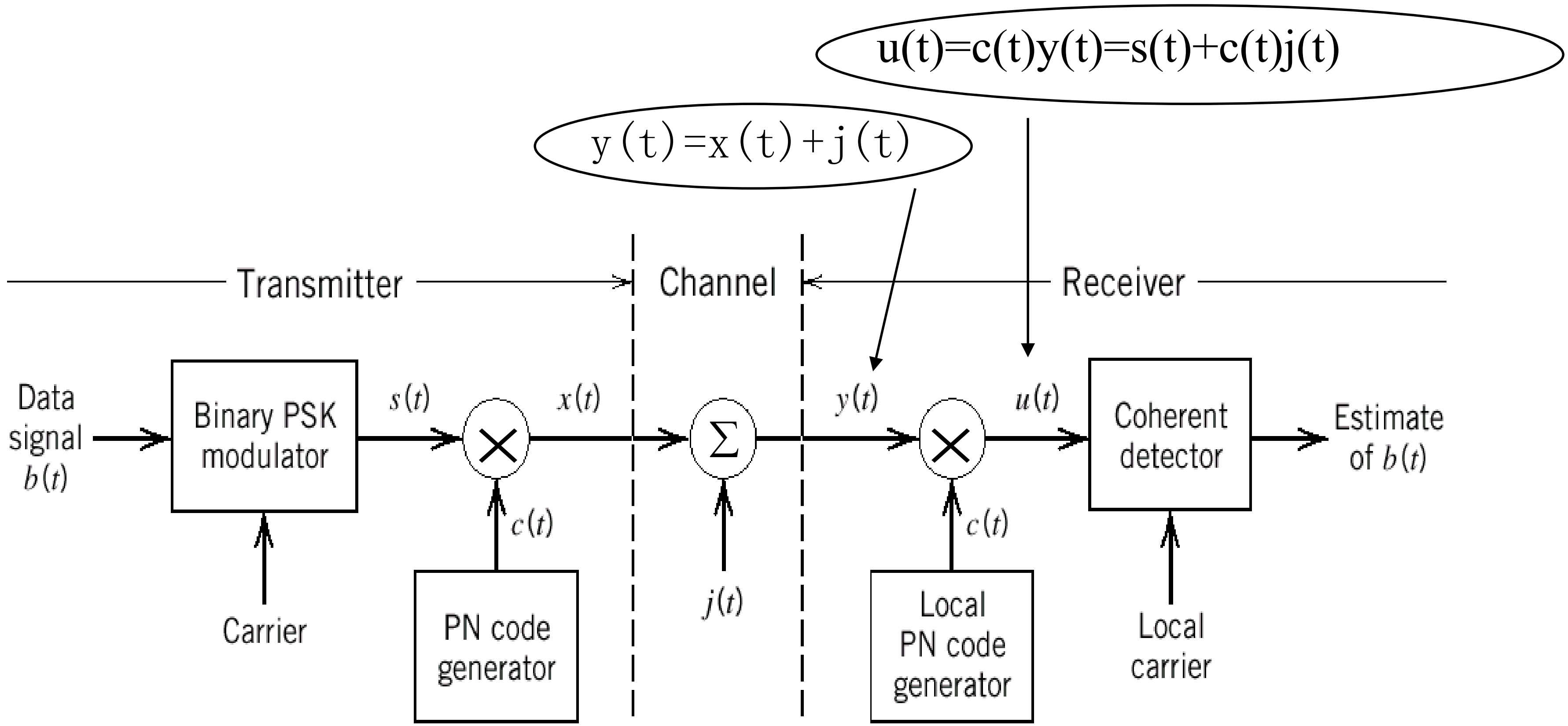


Figure 7.9 Model of direct-sequence spread binary PSK system.

7.4 Direct-Sequence Spread Spectrum with Coherent Binary Phase-shift Keying

- Synchronization of PN sequences
  - *Acquisition*(coarse synchronization): two PN codes are aligned to within a fraction of the chip in as short a time as possible
    - measure of correlation
    - synchronism decision
  - *Tracking*(fine synchronization): phase-lock techniques used

7.5 Signal-Space Dimensionality and Processing Gain

- **Topic:** mathematical analysis of spread-spectrum modulation(DS/BPSK in figure 7.9)

- **Method:** signal-space theoretic ideas in chapter 5

7.5 Signal-Space Dimensionality and Processing Gain

- Orthonormal basis functions:

$$\phi_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}}\cos(2\pi f_c t), & kT_c \le t \le (k+1)T_c \\ 0, & \text{otherwise} \end{cases} \qquad (7.14)$$

$$\tilde{\phi}_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}}\sin(2\pi f_c t), & kT_c \le t \le (k+1)T_c \\ 0, & \text{otherwise} \end{cases} \qquad (7.15)$$

7.5 Signal-Space Dimensionality and Processing Gain

- **Transmitted signal:** N-dimensional

$$x(t) = c(t)s(t)$$

$$= \pm\sqrt{\frac{2E_b}{T_b}}c(t)\cos(2\pi f_c t)$$

$$= \pm\sqrt{\frac{E_b}{N}}\sum_{k=0}^{N-1} c_\kappa \phi_\kappa(t), \qquad 0 \le t \le T_b$$

Average input signal power: E_b/T_b

7.5 Signal-Space Dimensionality and Processing Gain

- **Interfering signal:** 2N-dimensional

$$j(t)=\sum_{k=0}^{N-1} j_k\varphi_k(t)+\sum_{k=0}^{N-1}\tilde{j}_k\tilde{\phi}_k(t),\qquad 0\le t\le T_b \qquad (7.17)$$

Where

$$j_k=\int_0^{T_b} j(t)\phi_k(t)dt,\quad k=0,\ 1,\ldots,\ N-1 \quad (7.18)$$

and

$$\tilde{j}_k=\int_0^{T_b} j(t)\tilde{\phi}_k(t)dt,\qquad k=0,\ 1,\ldots,\ N-1 \quad (7.19)$$

7.5 Signal-Space Dimensionality and Processing Gain

- *The average power of the interference j(t)*

$$J = \frac{1}{T_b}\int_0^{T_b} j^2(t)\,dt$$

$$= \frac{1}{T_b}\sum_{k=0}^{N-1} j_k^2 + \frac{1}{T_b}\sum_{k=0}^{N-1} \tilde{j}_k^2 \qquad (7.20)$$

assume

$$\sum_{k=0}^{N-1} j_k^2 = \sum_{k=0}^{N-1} \tilde{j}_k^2 \qquad (7.21)$$

Average input interference power

$$J = \frac{2}{T_b}\sum_{k=0}^{N-1} j_k^2 \qquad (7.22)$$

7.5 Signal-Space Dimensionality and Processing Gain

- The coherent detector output

$$v = \sqrt{\frac{2}{T_b}} \int_0^{T_b} u(t)\cos(2\pi f_c t)dt$$

$$= v_s + v_{cj} \qquad (7.23)$$

where

$$v_s = \sqrt{\frac{2}{T_b}} \int_0^{T_b} s(t)\cos(2\pi f_c t)dt \qquad (7.24)$$

$$v_{cj} = \sqrt{\frac{2}{T_b}} \int_0^{T_b} c(t)j(t)\cos(2\pi f_c t)dt \qquad (7.25)$$

7.5 Signal-Space Dimensionality and Processing Gain

- The despread BPSK signal *s(t)*

$$s(t) = \pm\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t), \quad 0 \le t \le T_b \tag{7.26}$$

Assuming that the carrier frequency f_c is an integer multiple of $1/T_b$, we have

$$\Longrightarrow \quad v_s = \pm\sqrt{E_b} \tag{7.27}$$

Average output signal power: $\boldsymbol{E_b}$

7.5 Signal-Space Dimensionality and Processing Gain

- Component v_{cj} **due to interference**

$$v_{cj} = \sqrt{\frac{2}{T_b}} \sum_{k=0}^{N-1} c_k \int_{kT_c}^{(k+1)T_c} j(t) \cos(2\pi f_c t) dt \qquad (7.28)$$

Using Equ. (7.17), we get

$$v_{cj} = \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} c_k j_k \qquad (7.29)$$

7.5 Signal-Space Dimensionality and Processing Gain

Assume: PN sequence -- independent and identically distributed(i.i.d) binary sequence

$$\Rightarrow \quad V_{cj} = \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} C_k j_k \qquad (7.30)$$

The mean of V_{cj}

$$E[C_k j_k | j_k] = 0 \qquad (7.32)$$

The variance of V_{cj} (*average output interference power*)

$$\mathrm{var}[V_{cj} | j] = \frac{1}{N} \sum_{k=0}^{N-1} j_k^2 = \frac{JT_c}{2} \qquad (7.34)$$

7.5 Signal-Space Dimensionality and Processing Gain

- The output signal-to noise ratio

$$(SNR)_O = \frac{2E_b}{JT_c}$$

- The input signal-to noise ratio

$$(SNR)_I = \frac{E_b/T_b}{J}$$

- Processing gain PG = Spread factor N = T_b/T_c

7.6 Probability of Error

- **Model:** DS/SS BPSK system(Fig.7.9)

  coherent output – random variable

$$V = \pm\sqrt{E_b} + V_{cj}$$

- ***Probability of error P_e*** (with large spread factor N)

$$P_e \cong \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{JT_c}}\right) \qquad (7.42)$$

7.6 Probability of Error

- **Antijam Characteristics**
  - P_e of coherent BPSK system

$$P_e \cong \frac{1}{2} erfc(\sqrt{\frac{E_b}{N_0}}) \tag{7.43}$$

  - **Jamming margin**

$$\frac{J}{P} = \frac{PG}{E_b / N_0} \tag{7.46}$$

  - **Example 7.3** -- powerful advantage against interference

7.7 Frequency-Hop Spread Spectrum

- **Problem of DS/SS systems:** practical *limit on the attainable processing gain* because of the capabilities of physical devices to generate a PN sequence with narrow chip duration
- **Solution:** Frequency hopping

7.7 Frequency-Hop Spread Spectrum

- **FH:** randomly hopping the data-modulated carrier from one frequency to the next(2^k frequency hops) and on a single hop

  bandwidth of transmitted signal = bandwidth of a conventional MFSK

- **Common modulation format for FH systems:** MFSK (see Chapter 6) ---> FH/MFSK

7.7 Frequency-Hop Spread Spectrum

- *Difference:*
  - **DS** instantaneous spreading of the transmission bandwidth
  - **FH** sequential spreading of the transmission bandwidth
- *Advantage:* FH bandwidths >> DS bandwidths
- *Disadvantage:* most FH/SS systems use noncoherent detection

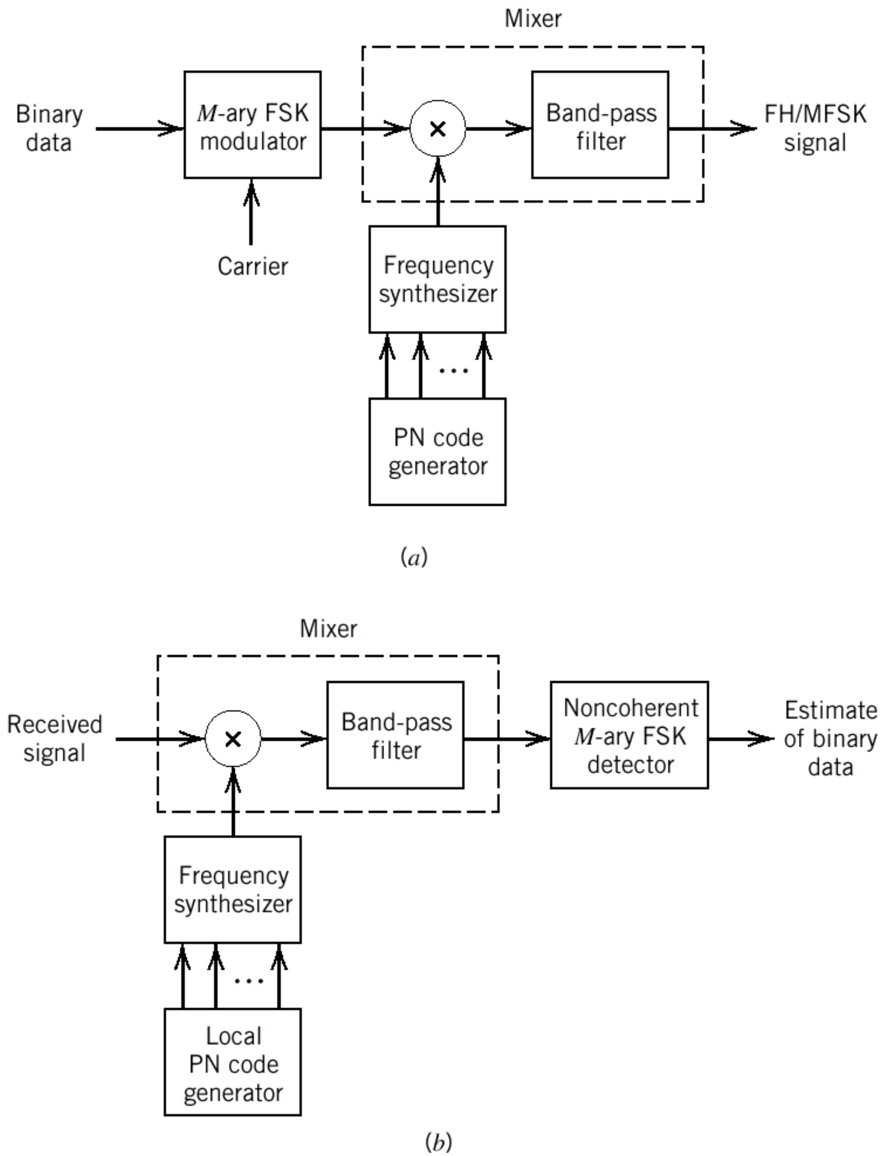


Figure 7.10 Frequency-hop spread *M*-ary frequency-shift keying. (*a*) Transmitter. (*b*) Receiver.

7.7 Frequency-Hop Spread Spectrum

- Slow-Frequency Hopping
  - (symbol rate) $R_s = L*R_h$ (hop rate)
    several symbols transmitted per hop
  - **chip rate** $R_c = max(R_s, R_h) = R_s = R_b/K$
  - **processing gain** $PG = W_c/R_s = 2^k$
  - Example 7.4

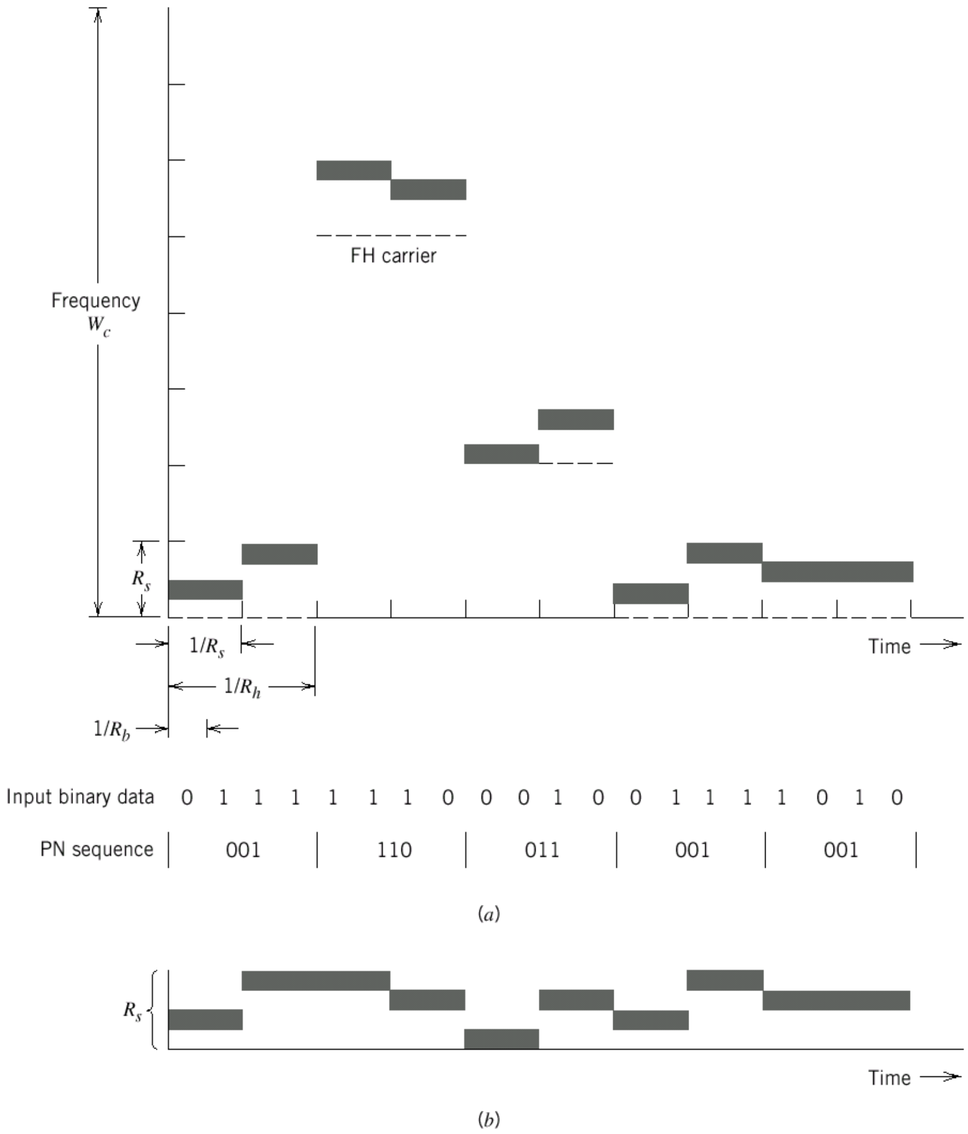


Figure 7.11 Illustrating slow-frequency hopping. (*a*) Frequency variation for one complete period of the PN sequence. (*b*) Variation of the dehopped frequency with time.

7.7 Frequency-Hop Spread Spectrum

- **Fast-Frequency Hopping**
  - (hop rate) $R_h = L * R_s$ (symbol rate)
    hop several times per data symbol
  - **chip rate** $R_c = max(R_s, R_h) = R_h$
  - **defeat smart jammer**
  - **two detection procedures**
    - separate decisions + majority vote
    - maximum likelihood detection (optimum)
  - Example 7.5

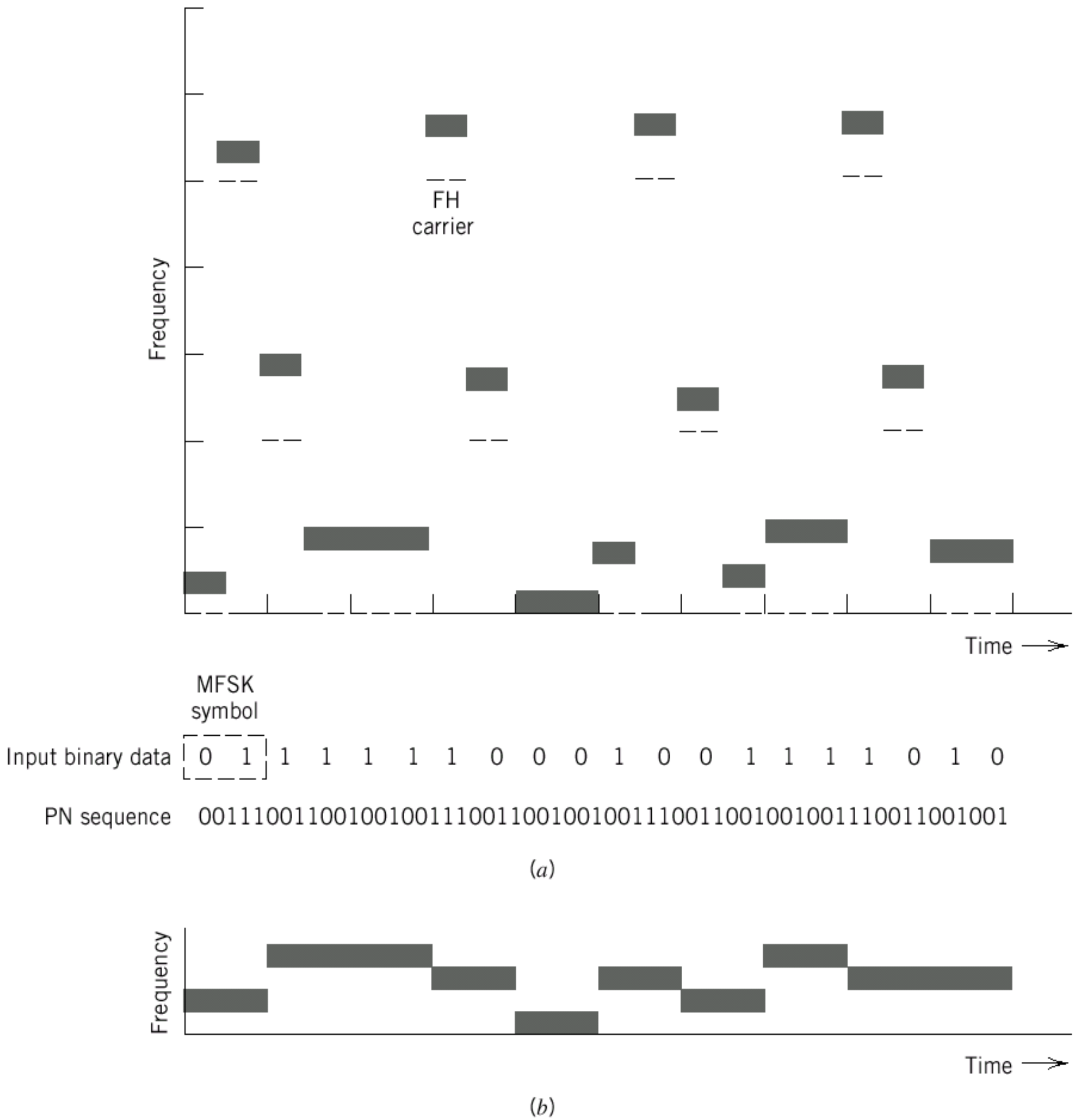


Figure 7.12 Illustrating fast-frequency hopping. (*a*) Variation of the transmitter frequency with time. (*b*) Variation of the dehopped frequency with time.

7.8 Computer Experiments: Maximal-Length and Gold Codes

- **Purpose:** study a certain class of spreading codes for CDM system that provide a satisfactory performance
- **CDM**
  - bandwidth allocation of FDM is not needed
  - time synchronization of TDM is not needed
  - common channel is used through the assignment of a spreading code to each user

7.8 Computer Experiments: Maximal-Length and Gold Codes

- In an ideal CDM system

  the **cross-correlation** between any two user = 0

  ⟹ the cross-correlation function between the spreading codes = 0 (for all cyclic shifts)

Unfortunately, ***ordinary PN sequences*** do not satisfy this requirement as relatively ***poor cross-correlation properties.***

- Then we use a special class of PN sequences called ***Gold sequences*** (code).

7.8 Computer Experiments: Maximal-Length and Gold Codes

- Gold sequences
  - period = 2^m-1
  - number of sequences = 2^m+1
  - generation: a preferred pair of primitive polynomials $g_1(X)$ and $g_2(X)$
- A preferred pair of primitive polynomials

whose cross-correlation function has a magnitude less than or equal to

$2^{(m+1)/2}+1$ for m odd

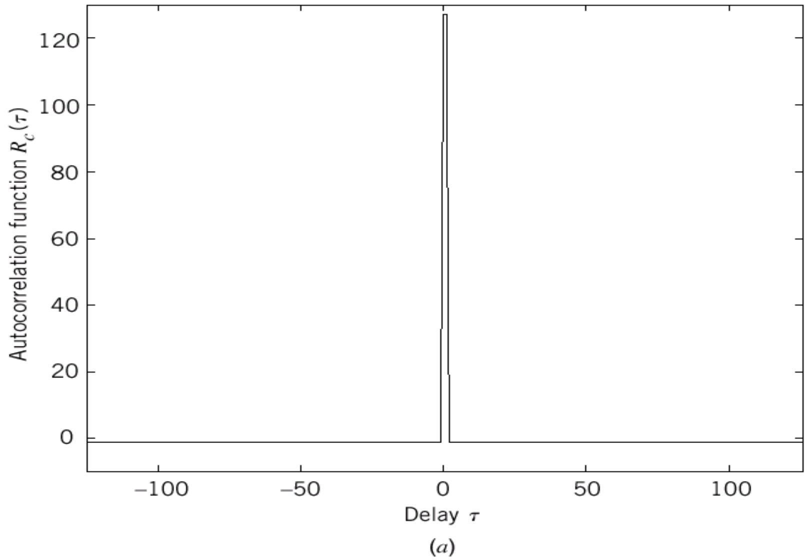
$2^{(m+2)/2}+1$ for m even and $m \neq 0 \bmod 4$

7.8 Computer Experiments: Maximal-Length and Gold Codes

- Primitive polynomials g(X)
  - g(X) is irreducible (cannot be factored)
  - polynomial g(X) divides the factor X^N+1, where $N=2^m-1$
  - polynomial g(X) cannot divide the factor X^q+1, where $q<2^m-1$

  Back to 7.2 Pseudo-Noise Sequences

- Experiments
  - Experiment 1. Correlation properties of PN sequences
  - Experiment 2. Correlation properties of Gold sequences



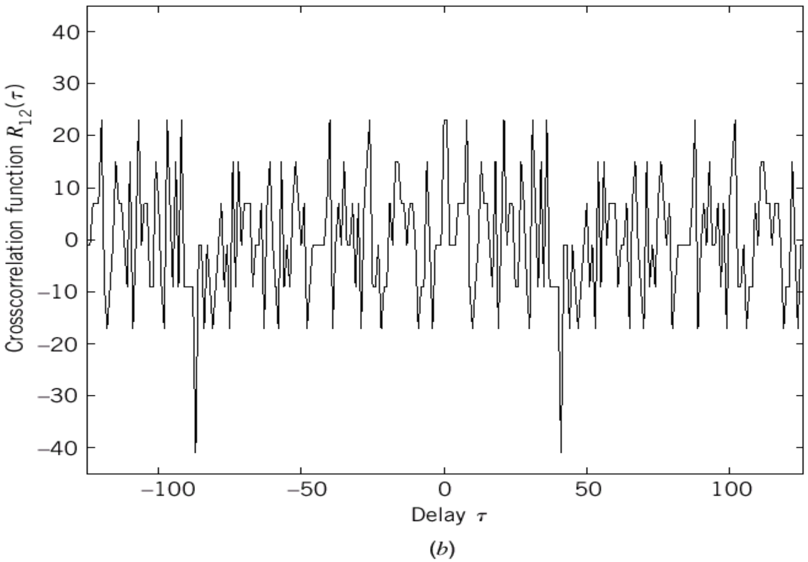


Figure 7.13

(*a*) Autocorrelation function $R_c(t)$, and (*b*) cross-correlation function $R_{12}(t)$ of the two PN sequences [7, 1] and [7, 6, 5, 4].

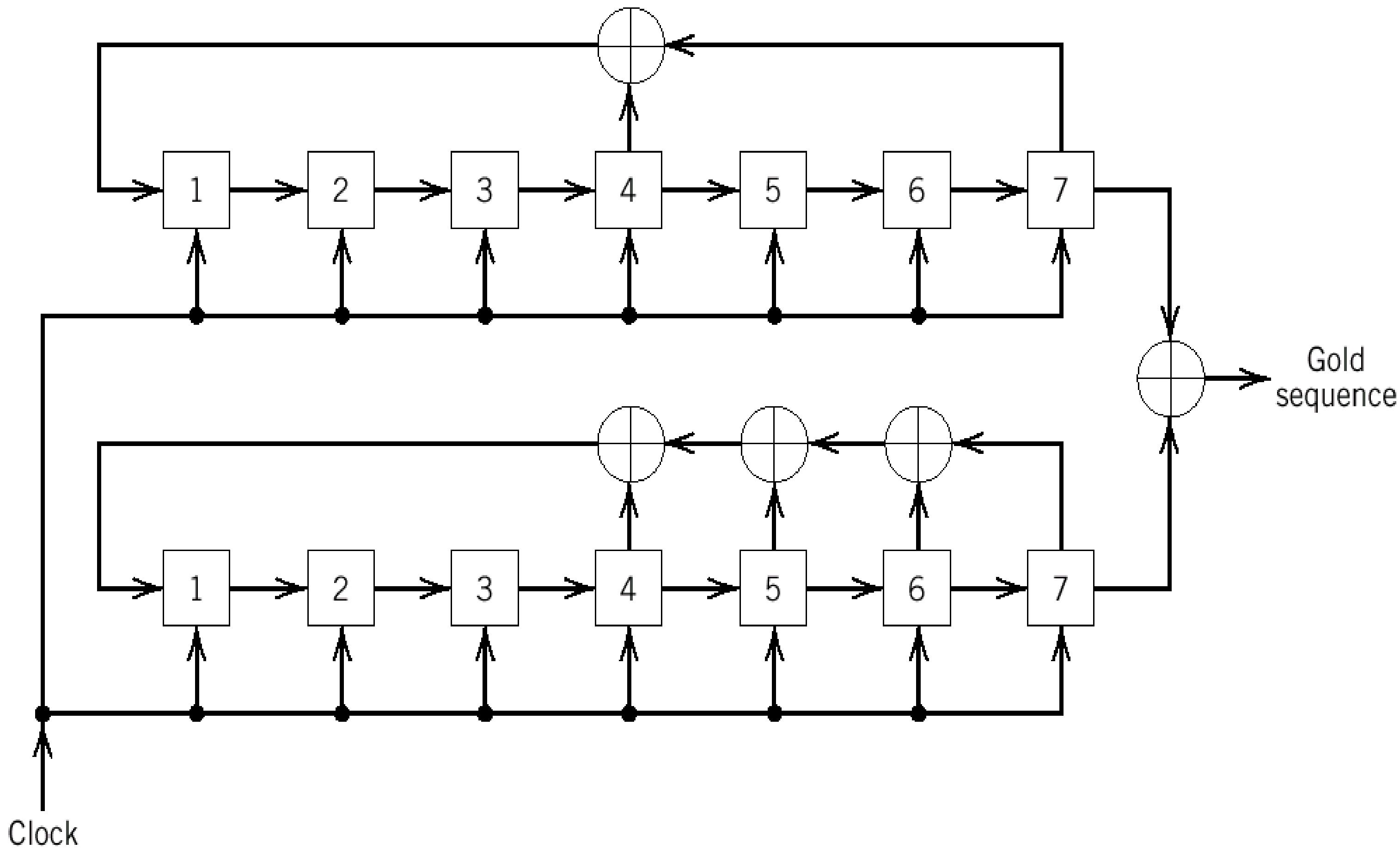


Figure 7.14

Generator for a Gold sequence of period $2^7 - 1 = 127$.

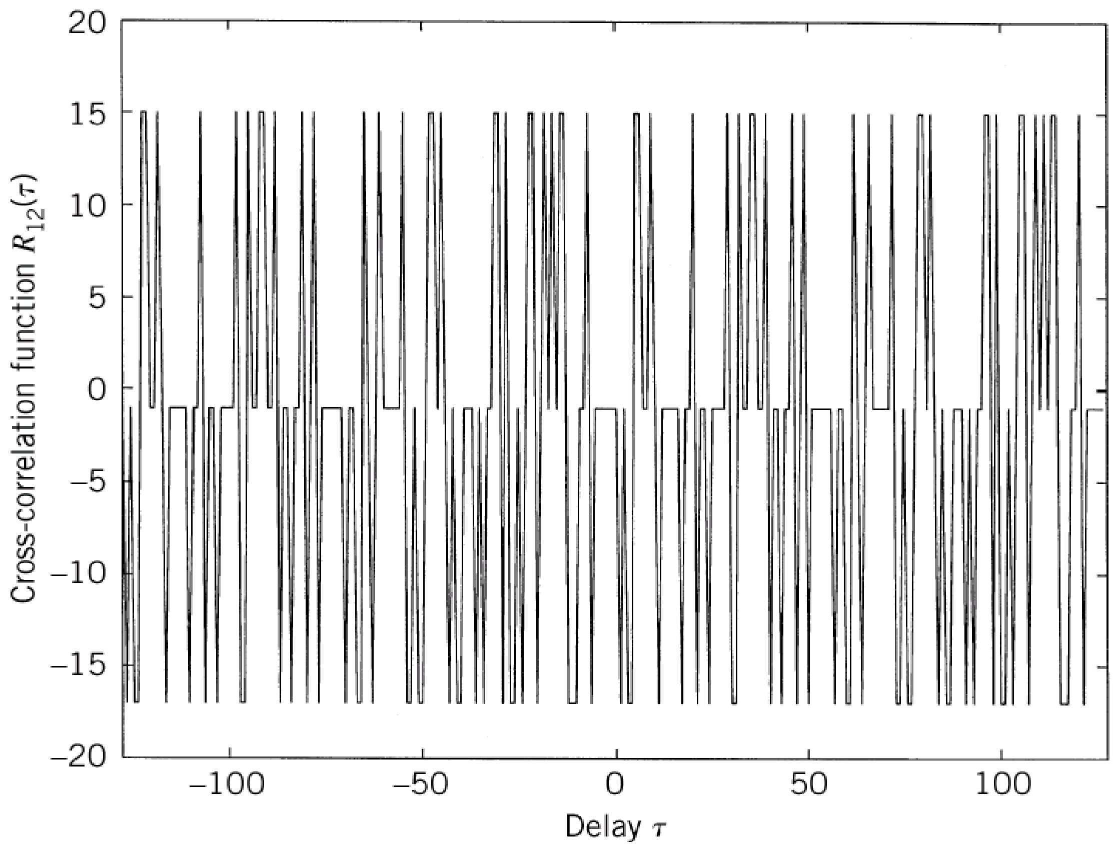


Figure 7.15 Cross-correlation function $R_{12}(t)$ of a pair of Gold sequences based on the two PN sequences [7, 4] and [7, 6, 5, 4].

7.9 Summary and Discussion

- Two categories of spread-spectrum communications
  - DS/MPSK : the PN sequence makes the transmitted signal assume a noiselike appearance by spreading its spectrum over a broad range of frequencies simultaneously
  - FH/MFSK : the PN sequence makes the carrier hop over a number of frequencies in pseudo-random manner, and the spectrum of the transmitted signal is spread in a sequential manner

7.9 Summary and Discussion

- The jammers encountered in practice include
  - the barrage noise jammer
  - the partial-band noise jammer
  - the pulsed noise jammer
  - the single-tone jammer
  - the multitone jammer