Signals and Systems – Spring 2025

Problem Set 2

Issued: Mar. 6, 2025 Due: Mar. 11, 2025

Reading Assignments:

Signals and Systems (OWN), Chapter 1.4, 2.4, 2.5; Supplementary notes, Chapter 3-4

Problem 1 OWN, Problem 2.31 (using the step-by-step method)

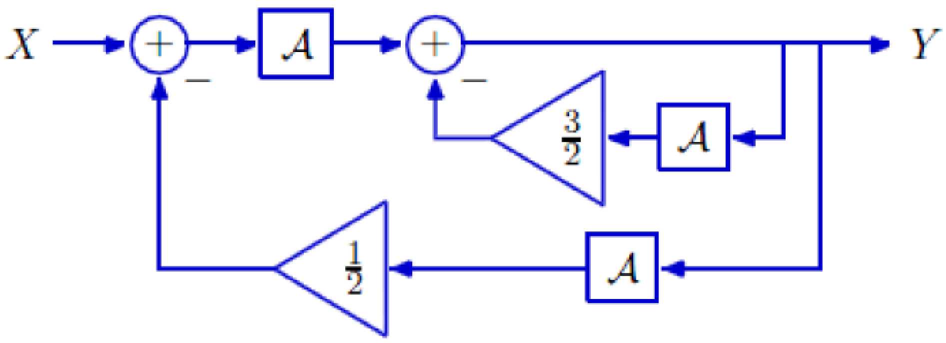
Problem 2 OWN, Problem 2.33

Problem 3 OWN, Problem 2.57

Problem 4 OWN, Problem 2.60

Problem 5

Consider the system defined by the following block diagram:



a. Determine the system functional $H = \frac{Y}{X}$.

b. Determine the poles of the system.

c. Determine the impulse response of the system.

Problem 6 Finding a system

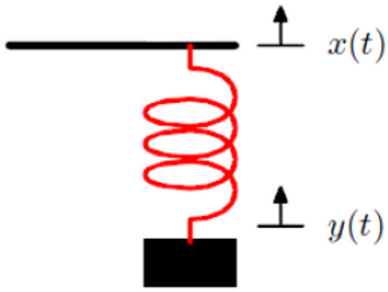
a. Determine the difference equation and block diagram representations for a system whose output is 10, 1, 1, 1, 1, … when the input is 1, 1, 1, 1, 1, … .

b. Determine the difference equation and block diagram representations for a system whose output is 1, 1, 1, 1, 1, . . . when the input is 10, 1, 1, 1, 1, ...

c. Compare the difference equations in parts a and b. Compare the block diagrams in parts a and b.

Problem 7

The following figure illustrates a mass and spring system. The input $x(t)$ represents the position of the top of the spring. The output $y(t)$ represents the position of the mass.



The mass is $M = 1$ kg and the spring constant is $K = 1$ N/m. Assume that the spring obeys Hooke's law and that the reference positions are defined so that if the input x(t) is equal to zero, then the resting position of y(t) is also zero.

a. Determine a differential equation that relates the input $x(t)$ and output $y(t)$.

b. Calculate the step response of the system.

c. The differential equation in part a contains a second derivative (if you did part a correctly). We wish to develop a forward-Euler approximation for this derivative. One method is to write the second-order differential equation in part a as a part of first order differential equations. Then apply the forward-Euler approximation to the first order derivatives:

$$\left.\frac{dy(t)}{dt}\right|_{t=nT} \approx \frac{y[n+1]-y[n]}{T}.$$

Use this approach to find a difference equation to approximate the behavior of the mass and spring system. Determine the step response of the system and compare your results to those in part b.

d. An alternative to the forward-Euler approximation is the backward-Euler approximation:

$$\left.\frac{dy(t)}{dt}\right|_{t=nT} \approx \frac{y[n]-y[n-1]}{T}.$$

Repeat the exercise in the previous part, but using the backward-Euler approximation instead of the forward-Euler approximation.

e. The forward-Euler method approximates the second derivative at $t = nT$ as

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=nT} = \frac{y[n+2]-2y[n+1]+y[n]}{T^2}.$$

The backward-Euler method approximates the second derivative at $t = nT$ as

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=nT} = \frac{y[n]-2y[n-1]+y[n-2]}{T^2}.$$

Consider a compromise based on a centered approximation:

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=nT} = \frac{y[n+1]-2y[n]+y[n-1]}{T^2}\,.$$

Use this approximation to determine the step response of the system. Compare your results to those in the two previous parts of this problem.