Signals and Systems

Lecture1: Introduction to Signals and Systems

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Partly adapted from the materials provided on the MIT OpenCourseWare

Outline

Course Introduction

This subject deals with mathematical methods used to describe signals and to analyze and synthesize systems

- Signals are variables that carry information
- Systems process input signals to produce output signals

Why is it important?

Used almost everywhere

- ISEE: communications, circuit design, video/image, sonar/radar, speech processing
- Commercial electronics
- Aeronautics and astronautics
- Seismology
- Biomedical engineering
- Energy generation and distribution
- Chemical process control
- Financial analysis

Course Introduction

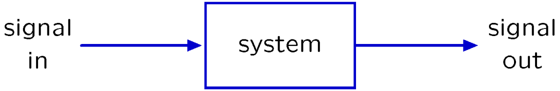
What is it all about?

- Existing system analysis
- System design
  - Signal enhancement and restoration
  - Information extraction
  - Signal design
- System control

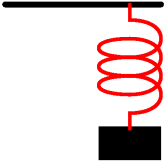
Outline

The Signals and Systems Abstraction

Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.



Example: Mass and Spring



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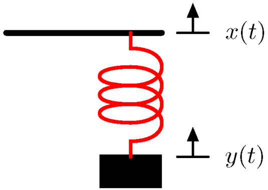

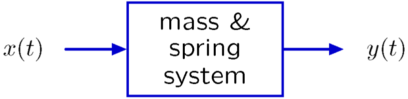

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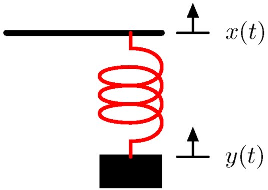
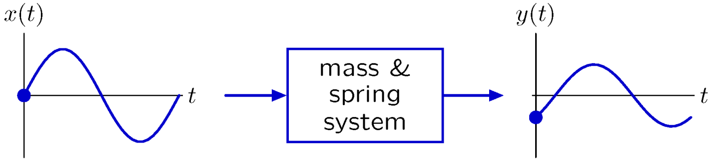
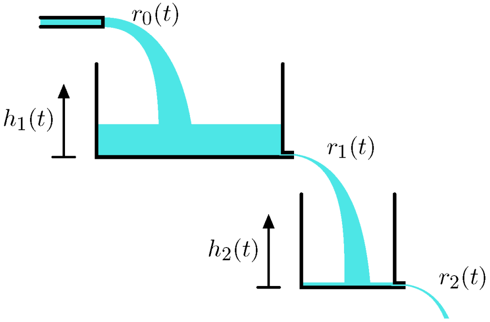
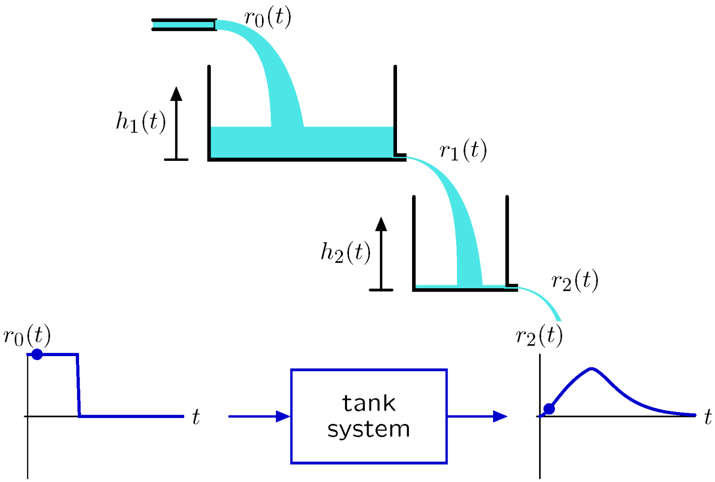




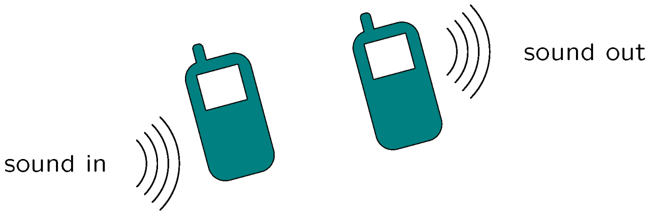
Example: Tanks

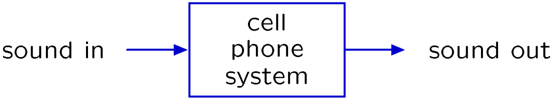


Example: Tanks

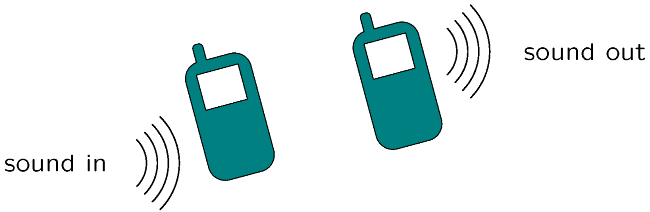


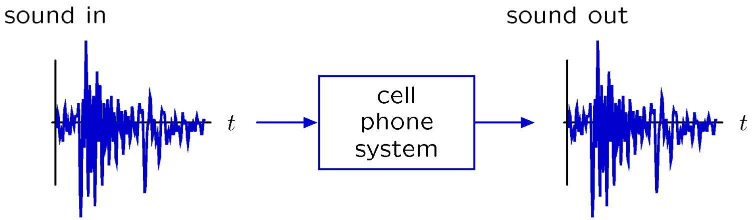
Example: Cell Phone System





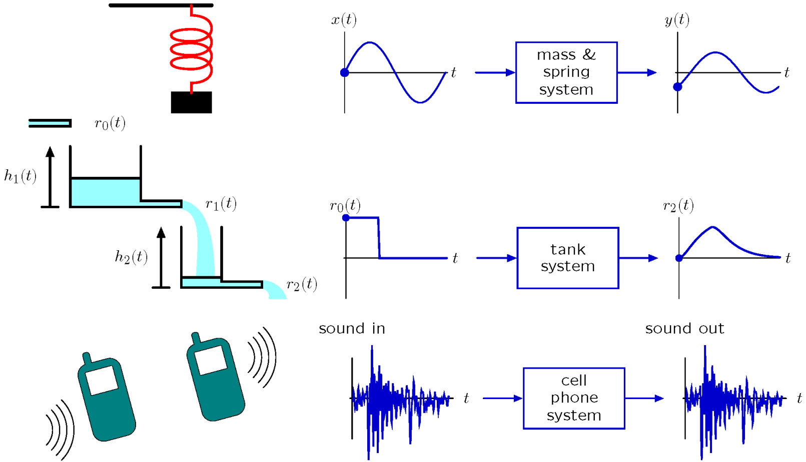
Example: Cell Phone System





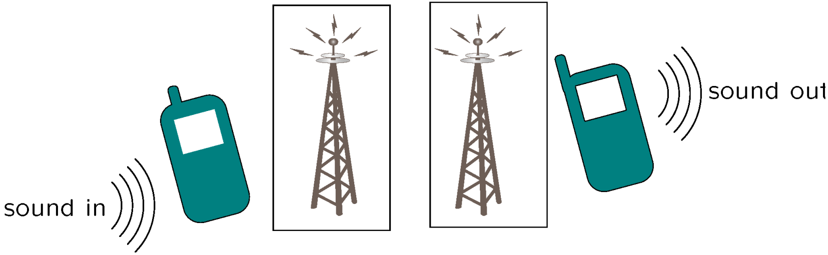
Signals and Systems: Widely Applicable

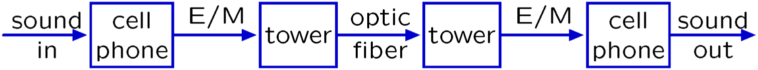
The Signals and Systems approach has broad application: electrical, mechanical, optical, acoustic, biological, financial, ...



Signals and Systems: Modular

The representation does not depend upon the physical substrate.



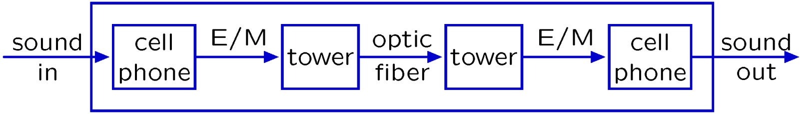


focuses on the flow of **information**, abstracts away everything else

Signals and Systems: Hierarchical

Representations of component systems are easily combined.

Example: cascade of component systems



Composite system

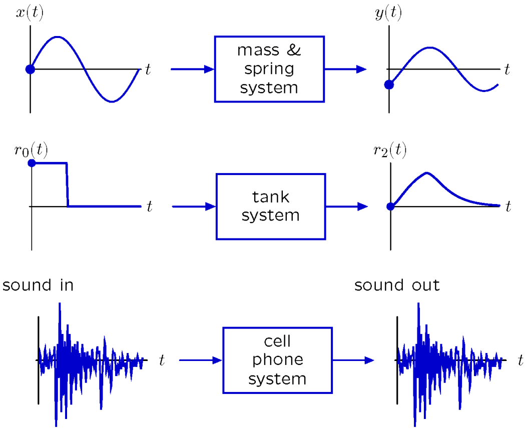
Component and composite systems have the same form, and are analyzed with same methods.

Outline

Signals

Signals are mathematical functions.

- independent variable = time
- dependent variable = voltage, flow rate, sound pressure



Signals

Generic time

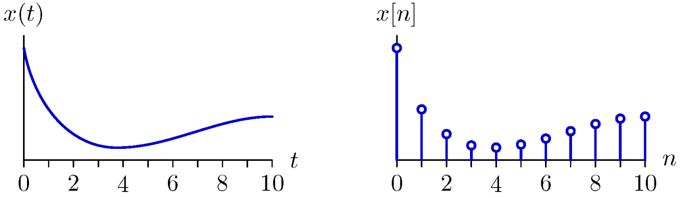
The term time is often used generically to represent the independent variable of a signal. The independent variable may be

- continuous or discrete
- 1-D, 2-D, $\cdots$ N-D

For this course: Focus on a single (1-D) independent variable which we call time.

Signals: Continuous Time and Discrete Time

continuous "time" (CT) and discrete "time" (DT)



Many physical systems operate in continuous time.

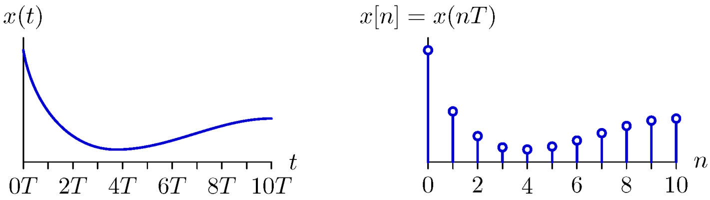
- mass and spring
- leaky tank

Digital computations are done in discrete time.

- state machines: given the current input and current state, what is the next output and next state.

Signals: Continuous Time and Discrete Time

Sampling: converting CT signals to DT



$T =$ sampling interval

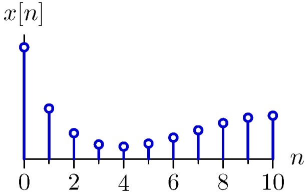
Important for computational manipulation of physical data.

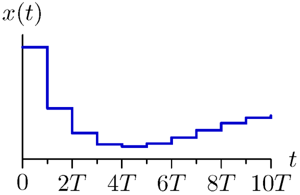
- digital representations of audio signals (e.g., MP3)
- digital representations of pictures (e.g., JPEG)

Signals: Continuous Time and Discrete Time

Reconstruction: converting DT signals to CT

zero-order hold





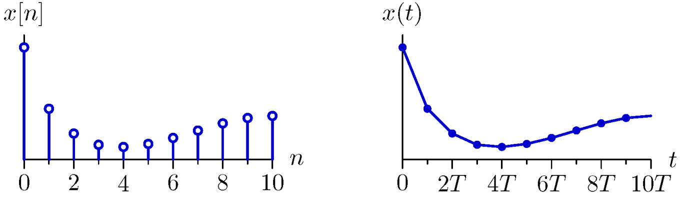
$T =$ sampling interval

commonly used in audio output devices such as CD players

Signals: Continuous Time and Discrete Time

Reconstruction: converting DT signals to CT

piecewise linear



$T =$ sampling interval

commonly used in rendering images

Signals: Real and Complex

Signals can be real, imaginary, or complex. An important class of signals are the complex exponentials:

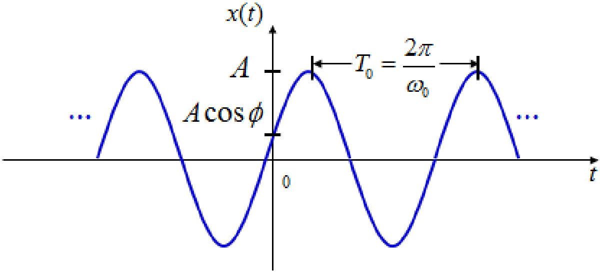
- the CT signal $x(t) = e^{st}$ where s is a complex number,
- the DT signal $x[n] = z^n$ where z is a complex number.

Q. Why do we deal with complex signals?

A. They are often analytically simpler to deal with than real signals.

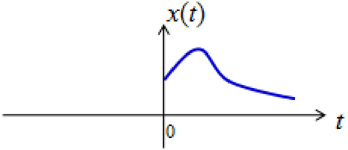
Signals: Periodic and Aperiodic

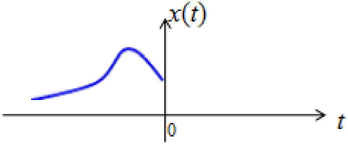
Periodic signals have the property that $x(t + T) = x(t)$ for all t. The smallest value of T that satisfies the definition is called the period.



Signals: Causal and Anti-causal

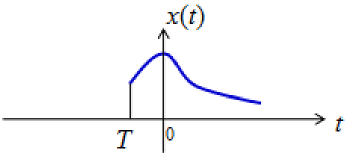
A causal signal is zero for $t < 0$ and anti-causal signal is zero for $t > 0$

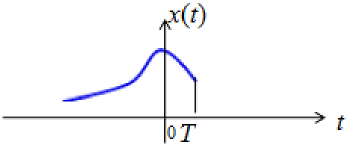




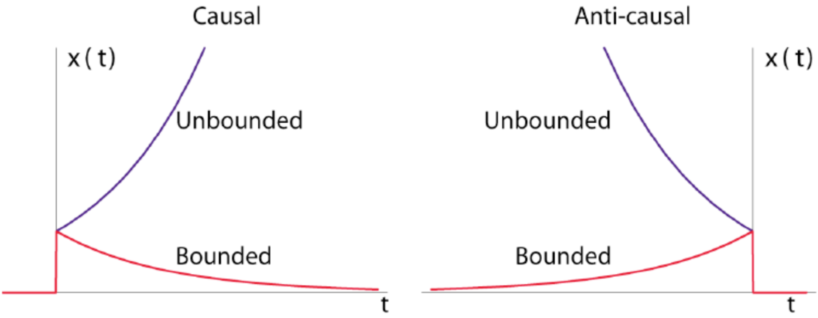
Signals: Right- and Left-sided

A right-sided signal is zero for $t < T$ and left-sided signal is zero for $t > T$ where T can be positive or negative.





Signals: Bounded and Unbounded



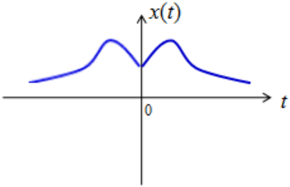
Whether the output signal of a system is bounded or unbounded determines the stability of the system.

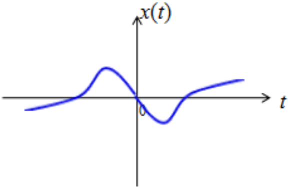
Signals: Even and Odd

$$x_e(t) = x_e(-t) \qquad x_o(t) = -x_o(-t)$$

- Any signal can be broken into a sum of even and odd signals

$$Ev\{x(t)\} = \frac{x(t) + x(-t)}{2} \qquad Od\{x(t)\} = \frac{x(t) - x(-t)}{2}$$



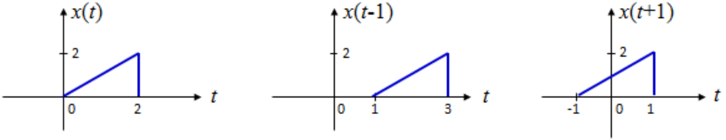


Transformations of time

- Time shift

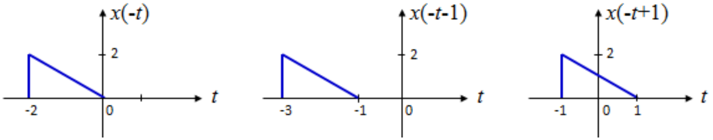
$$x(t) \Rightarrow x(t-t_0) \quad t_0 > 0 \quad \text{delayed,} \quad \text{move to right}$$

$$t_0 < 0 \quad \text{advanced,} \quad \text{move to left}$$



- Time reversal

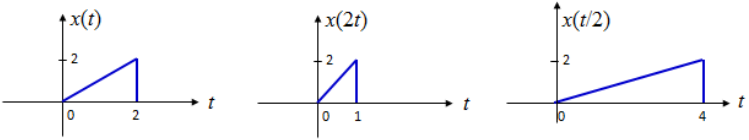
$$x(t) \Rightarrow x(-t)$$



Transformations of time

- Time scaling

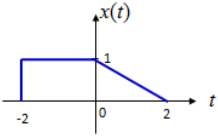
$$x(t) \Rightarrow x(\alpha t) \begin{cases} |\alpha| < 1 & \text{stretched} \\ |\alpha| > 1 & \text{compressed} \end{cases}$$



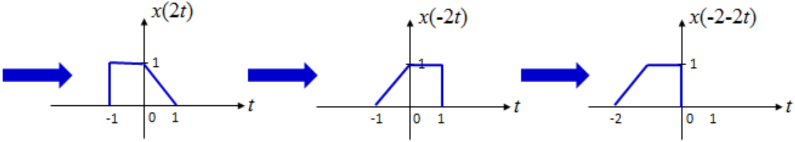
Time scaling of signals in DT?

Check Yourself

Given $x(t)$, sketch $x(-2-2t)$.



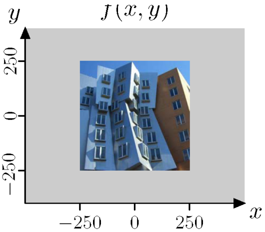
① $x(t) \Rightarrow x(2t) \Rightarrow x(-2t) \Rightarrow x(-2-2t)$



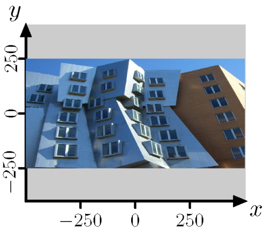
② $x(t) \Rightarrow x(2t) \Rightarrow x(2t-2) \Rightarrow x(-2t-2)$

③ ...

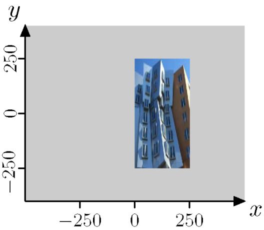
Check Yourself



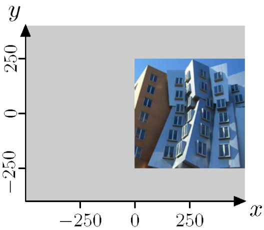
How many images match the expressions beneath them?



$f_1(x, y) = f(2x, y)$?

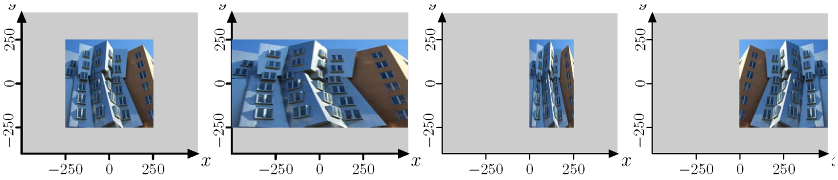


$f_2(x, y) = f(2x - 250, y)$?



$f_3(x, y) = f(-x - 250, y)$?

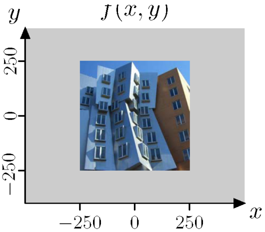
Check Yourself



$f(x,y)$ $\quad$ $f_1(x,y) = f(2x,y)$? $\quad$ $f_2(x,y) = f(2x-250,y)$? $\quad$ $f_3(x,y) = f(-x-250,y)$?

$$\begin{aligned}
x &= 0 && \rightarrow f_1(0,y) = f(0,y) && \surd \\
x &= 250 && \rightarrow f_1(250,y) = f(500,y) && \times \\
\\
x &= 0 && \rightarrow f_2(0,y) = f(-250,y) && \surd \\
x &= 250 && \rightarrow f_2(250,y) = f(250,y) && \surd \\
\\
x &= 0 && \rightarrow f_3(0,y) = f(-250,y) && \times \\
x &= 250 && \rightarrow f_3(250,y) = f(-500,y) && \times
\end{aligned}$$

Check Yourself



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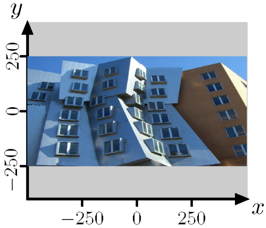
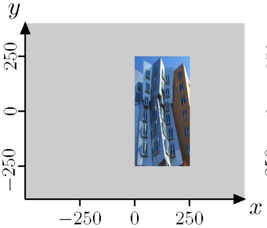
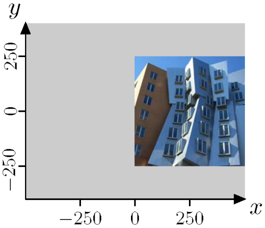


$\cancel{f_1(x, y) = f(2x, y)}$?



$f_2(x, y) = f(2x - 250, y)$?



$\cancel{f_3(x, y) = f(-x - 250, y)}$?

Outline

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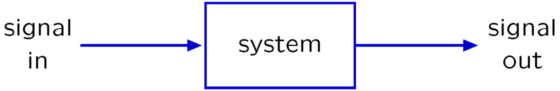
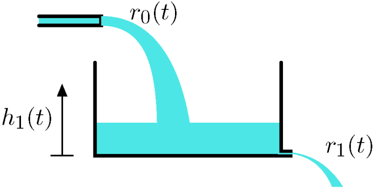
Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.



Example System: Leaky Tank

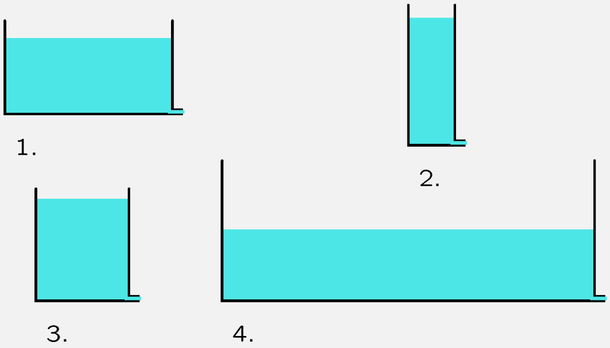
Formulate a mathematical description of this system.



What determines the leak rate?

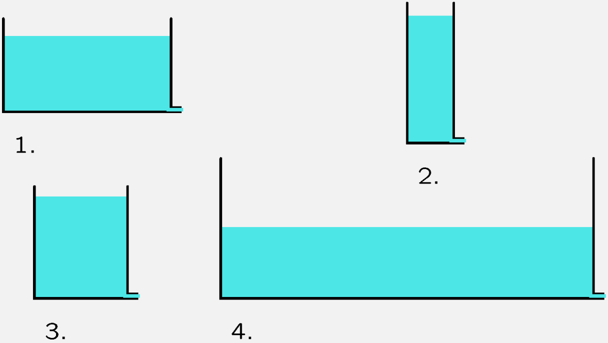
Check Yourself

The holes in each of the following tanks have equal size. Which tank has the largest leak rate $r_1(t)$?



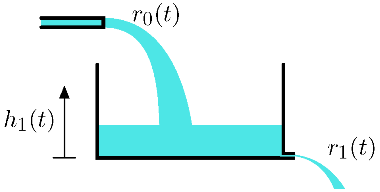
Check Yourself

The holes in each of the following tanks have equal size. Which tank has the largest leak rate $r_1(t)$? 2



Example System: Leaky Tank

Formulate a mathematical description of this system.

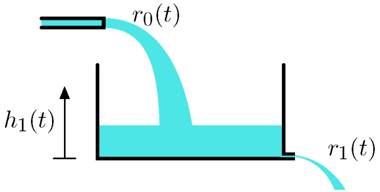


Assume linear leaking: $r_1(t) \propto h_1(t)$

What determines the height $h_1(t)$?

Example System: Leaky Tank

Formulate a mathematical description of this system.



Assume linear leaking: $r_1(t) \propto h_1(t)$

Assume water is conserved: $\dfrac{dh_1(t)}{dt} \propto r_0(t) - r_1(t)$

Solve: $\dfrac{dr_1(t)}{dt} \propto r_0(t) - r_1(t)$

Check Yourself

What are the dimensions of constant of proportionality C?

$$\frac{dr_1(t)}{dt} = C\Big(r_0(t) - r_1(t)\Big)$$

Check Yourself

What are the dimensions of constant of proportionality C?
inverse time (to match dimensions of dt)

$$\frac{dr_1(t)}{dt} = C\Big(r_0(t) - r_1(t)\Big)$$

Analysis of the Leaky Tank

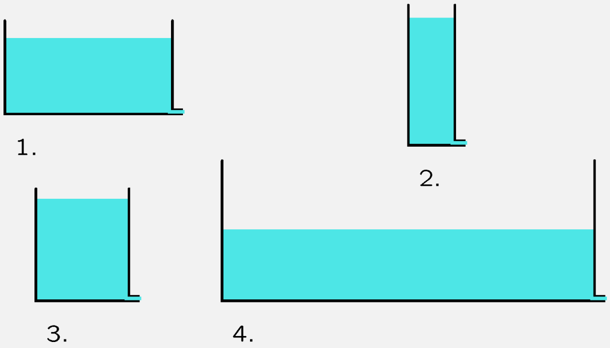
Call the constant of proportionality $1/\tau$.

Then τ is called the **time constant** of the system.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

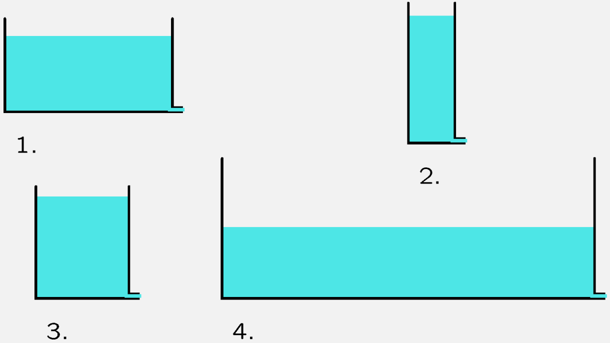
Check Yourself

Which of the following tanks has the largest time constant τ?



Check Yourself

Which of the following tanks has the largest time constant τ? 4



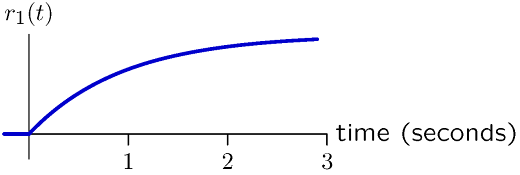
Analysis of the Leaky Tank

Call the constant of proportionality $1/\tau$.

Then τ is called the **time constant** of the system.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

Assume that the tank is initially empty, and then water enters at a constant rate $r_0(t) = 1$. Determine the output rate $r_1(t)$.



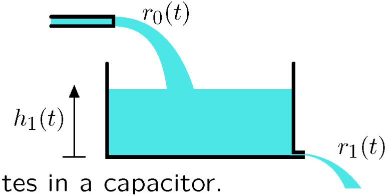
Explain the shape of this curve mathematically.

Explain the shape of this curve physically.

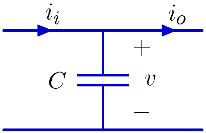
Leaky Tanks and Capacitors

Although derived for a leaky tank, this sort of model can be used to represent a variety of physical systems.

Water accumulates in a leaky tank.



Charge accumulates in a capacitor.



$$\frac{dv}{dt} = \frac{i_i - i_o}{C} \propto i_i - i_o \qquad \text{analogous to} \qquad \frac{dh}{dt} \propto r_0 - r_1$$

Outline

Assignments

- Reading Assignment: Chapter 1.0-1.3, 1.5-1.6