$y[n]+2y[n-1] = x[n]+2x[n-2]$

2.31 $\quad n\le -3,\ x[n]=0 \Rightarrow n\le -3,\ y[n]=0$

$y[n] = -2y[n-1]+x[n]+2x[n-2]$

$\Rightarrow \quad y[-2] = -2y[-3]+x[-2]+2x[-4] = 1$

$y[-1] = -2y[-2]+x[-1]+2x[-3] = 0$

$y[0] = -2y[-1]+x[0]+2x[-2] = 5$

$y[1] = -2y[0]+x[1]+2x[-1] = -4$

$y[2] = -2y[1]+x[2]+2x[0] = 16$

$y[3] = -2y[2]+x[3]+2x[1] = -27$

$y[4] = -2y[3]+x[4]+2x[2] = 58$

$y[5] = -2y[4]+x[5]+2x[3] = -114$

$y[6] = -2y[5]+x[6]+2x[4] = 228$

$y[7] = -2y[6]+x[7]+2x[5] = (-2)^2(-114)$

$\vdots$

$y[n] = -114(-2)^{n-5},\ n\ge 5$

2.33 (a) (i) $\quad \dfrac{dy(t)}{dt}+2y(t) = x(t)$

$\Rightarrow e^{2t}\left(\dfrac{dy(t)}{dt}+2y(t)\right) = e^{2t}x(t)$

$\Rightarrow \dfrac{d}{dt}e^{2t}y(t) = e^{5t}u(t) = \dfrac{d}{dt}\dfrac{1}{5}e^{5t},\quad t>0$

$\Rightarrow e^{2t}y_1(t) = \frac{1}{5}e^{5t}+c \Rightarrow y_1(t) = \frac{1}{5}e^{3t}+ce^{-2t}$

$SR \Rightarrow y_1(0)=0 \Rightarrow y_1(t) = \frac{1}{5}e^{3t}-\frac{1}{5}e^{-2t},\ t>0$

(ii) $\quad \dfrac{d}{dt}e^{2t}y(t) = e^{4t}u(t) = \dfrac{d}{dt}\dfrac{1}{4}e^{4t},\ t>0$

$\Rightarrow y(t) = \frac{1}{4}e^{2t}+ce^{-2t},\ t>0$

$SR \Rightarrow y(0)=0 \Rightarrow y_2(t) = \frac{1}{4}e^{2t}-\frac{1}{4}e^{-2t}$

(iii) $\quad \dfrac{d}{dt}e^{2t}y(t) = e^{2t}(\alpha e^{3t}+\beta e^{2t})u(t) = \dfrac{d}{dt}\left(\frac{1}{5}\alpha e^{5t}+\frac{1}{4}\beta e^{4t}\right),\ t>0$

$\Rightarrow y(t) = \frac{1}{5}\alpha e^{3t}+\frac{1}{4}\beta e^{2t}+ce^{-2t}$

$SR \Rightarrow y(0)=0 \Rightarrow y_3(t) = \frac{1}{5}\alpha e^{3t}+\frac{1}{4}\beta e^{2t}+\left(-\frac{1}{5}\alpha-\frac{1}{4}\beta\right)e^{-2t}$

$= \alpha y_1(t)+\beta y_2(t)$

(iv) $\frac{dy_1(t)}{dt}+2y_1(t)=x_1(t)$, $x_1(t)=0$, $t<t_1$

$\frac{dy_2(t)}{dt}+2y_2(t)=x_2(t)$, $x_2(t)=0$, $t<t_2$

$\Rightarrow \frac{d}{dt}(\alpha y_1(t)+\beta y_2(t)) + 2(\alpha y_1(t)+\beta y_2(t)) = \alpha x_1(t)+\beta x_2(t)$

$\alpha x_1(t)+\beta x_2(t)=0$, $t<\min\{t_1,t_2\}$

$\Rightarrow$ 输入 $\alpha x_1(t)+\beta x_2(t)$, 输出 $\alpha y_1(t)+\beta y_2(t)$

(b) (i) $\frac{d}{dt}e^{2t}y(t)=ke^{4t}u(t) = \frac{d}{dt}\frac{1}{4}ke^{4t}$, $t>0$

$\Rightarrow y_1(t)=\frac{1}{4}ke^{2t}+ce^{-2t}$, $t>0$

SR $\Rightarrow y(0)=0 \Rightarrow y_1(t)=\frac{1}{4}ke^{2t}-\frac{1}{4}ke^{-2t}$, $t>0$

(ii) $\frac{d}{dt}e^{2t}y(t)=ke^{4t-2T}u(t-T)=\frac{d}{dt}\frac{1}{4}ke^{4t-2T}$, $t>T$

$\Rightarrow y_2(t)=\frac{1}{4}ke^{2(t-T)}+ce^{-2t}$, $t>T$

SR $\Rightarrow y_2(T)=0 \Rightarrow y_2(t)=\frac{1}{4}ke^{2(t-T)}-\frac{1}{4}ke^{-2(t-T)}=y_1(t-T)$

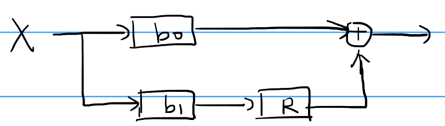
(iii) $\frac{d}{dt}y_1(t)+2y_1(t)=x_1(t) \Rightarrow \frac{d}{dt}y_1(t-T)+2y_1(t-T)=x_1(t-T)=x_2(t)$

输入 $x_2(t)$, 输出 $y_1(t-T)$ $\quad = \frac{d}{dt}y_2(t)+2y_2(t)$

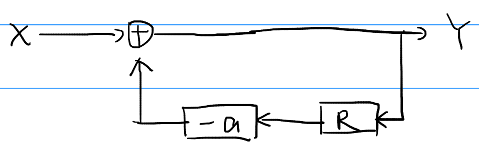
$\Rightarrow y_2(t)=y_1(t-T)$

2.57 (a) $x[n] \xrightarrow{S_1} b_0x[n]+b_1x[n-1] \xrightarrow{S_2} y[n]=-ay[n]+b_0x[n]+b_1x[n-1]$

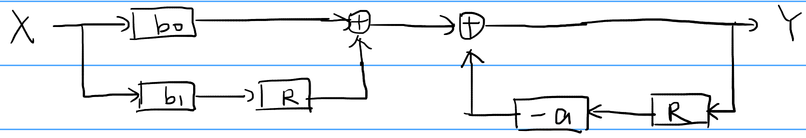
(b) $Y=b_0X+b_1RX$



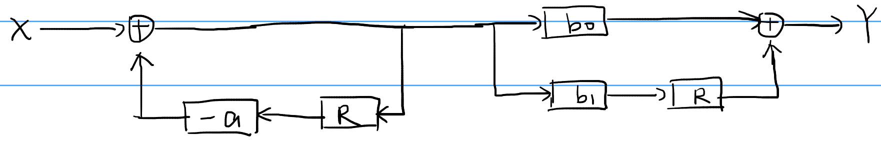
(c) $Y=-aRY+X$



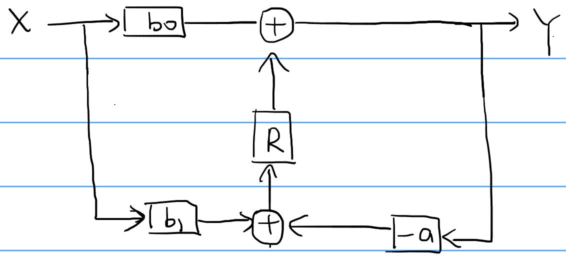
(d)



(f)



(e)



2.60 (a) $a_2 Y + a_1 A Y + a_0 A^2 Y = b_0 A^2 X + b_1 A X + b_2 X$

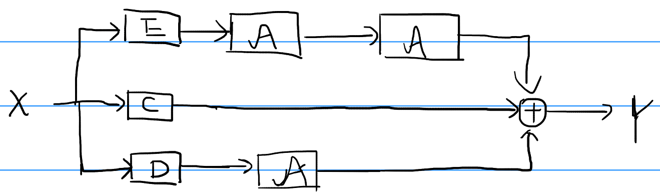
(A is integration signal)

$$\Rightarrow y(t) = -\frac{a_1}{a_2}\int_{-\infty}^{t} y(\tau)\,d\tau - \frac{a_0}{a_2}\int_{-\infty}^{t}\left(\int_{-\infty}^{\tau} y(\sigma)\,d\sigma\right)d\tau$$
$$+ \frac{b_2}{a_2}x(t) + \frac{b_1}{a_2}\int_{-\infty}^{t} x(\tau)\,d\tau + \frac{b_0}{a_2}\int_{-\infty}^{t}\left(\int_{-\infty}^{\tau} x(\sigma)\,d\sigma\right)d\tau$$

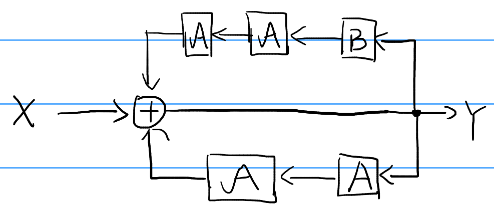
$A = -\frac{a_1}{a_2}$ $\quad B = -\frac{a_0}{a_2}$ $\quad C = \frac{b_2}{a_2}$ $\quad D = \frac{b_1}{a_2}$ $\quad E = \frac{b_0}{a_2}$

(b) $x(t) \xrightarrow{S_1} C x(t) + D\int_{-\infty}^{t} x(\tau)\,dt + E\int_{-\infty}^{t} d\tau \int_{-\infty}^{\tau} x(\sigma)\,d\sigma \xrightarrow{S_2} y(t)$

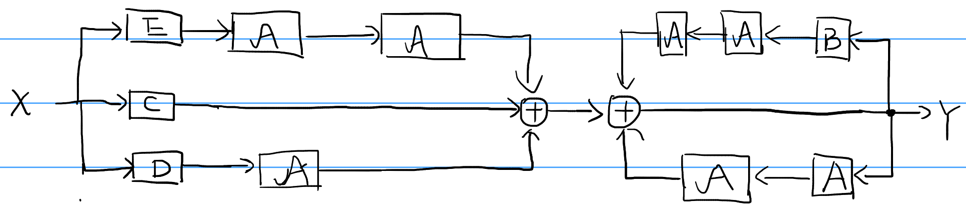
S_1

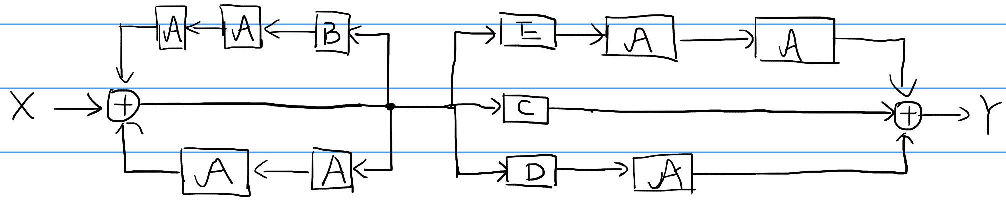


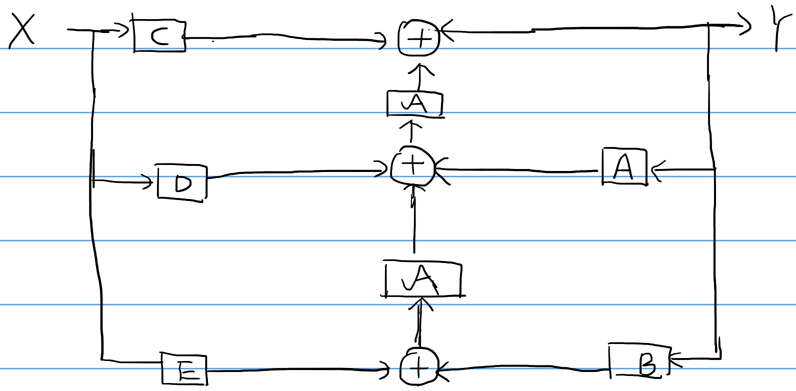
S_2



S.

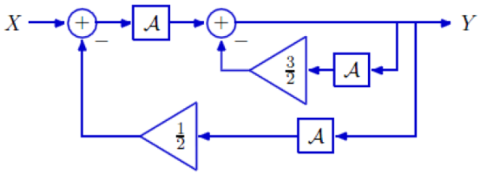






Problem 5

Consider the system defined by the following block diagram:



a. Determine the system functional $H = \frac{Y}{X}$.

b. Determine the poles of the system.

c. Determine the impulse response of the system.

a. $Y = A(X - \frac{1}{2}AY) - \frac{3}{2}AY = AX - \frac{1}{2}A^2Y - \frac{3}{2}AY$

$\Rightarrow H = \frac{Y}{X} = \frac{A}{\frac{1}{2}A^2 + \frac{3}{2}A + 1}$

b. $H = \frac{A}{\frac{1}{2}A^2 + \frac{3}{2}A + 1} = \frac{2A}{A^2 + 3A + 2} = \frac{2A}{A+1} - \frac{2A}{A+2} = \frac{2}{s+1} - \frac{2}{2s+1}$

$\Rightarrow$ poles are $s_1 = -1$, $s_2 = -\frac{1}{2}$

c. $H = \frac{2A}{A+1} - \frac{2A}{A+2} = 2A\left(1 + (-A) + (-A)^2 + \cdots\right) - A\left(1 + (-\frac{A}{2}) + (-\frac{A}{2})^2 + \cdots\right)$

$\Rightarrow Y = 2\left(1 + (-A) + (-A)^2 + \cdots\right)u(t) - \left(1 + (-\frac{A}{2}) + (-\frac{A}{2})^2 + \cdots\right)u(t)$

$= 2e^{-t} - e^{-\frac{1}{2}t}$

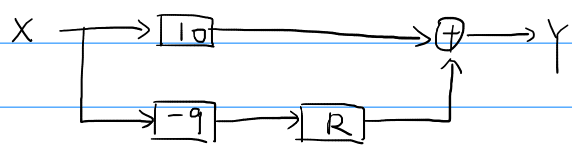
Problem 6 Finding a system

a. Determine the difference equation and block diagram representations for a system whose output is 10, 1, 1, 1, 1, … when the input is 1, 1, 1, 1, 1, … .

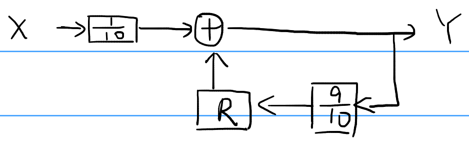
b. Determine the difference equation and block diagram representations for a system whose output is 1, 1, 1, 1, 1, . . . when the input is 10, 1, 1, 1, 1, ...

c. Compare the difference equations in parts a and b. Compare the block diagrams in parts a and b.

a. $y[n] = 10x[n] - 9x[n-1]$; $n \leq 0$ 时 $x[n] = y[n] = 0$



b. $x[n] = 10y[n] - 9y[n-1]$; $n \leq 0$ 时 $x[n] = y[n] = 0$



equation,

c. a does difference equation for $x[n]$

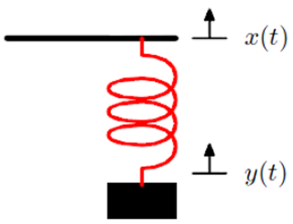
b does difference equation for $y[n]$

block:

b has the feedback and a doesn't have

Problem 7

The following figure illustrates a mass and spring system. The input *x(t)* represents the position of the top of the spring. The output *y(t)* represents the position of the mass.



The mass is $M = 1$ *kg* and the spring constant is $K = 1$ *N/m*. Assume that the spring obeys Hooke's law and that the reference positions are defined so that if the input x(t) is equal to zero, then the resting position of y(t) is also zero.

(a) $K\left(x(t) - y(t)\right) = M\ddot{y}(t)$

(b) $A^2(X - Y) = Y \quad \Rightarrow \quad \frac{Y}{X} = \frac{A^2}{1+A^2} = \frac{1}{2}A^2\left(\frac{1}{1-iA} + \frac{1}{1+iA}\right)$

$\Rightarrow Y = \frac{A^2}{2}\left(1 + iA + (iA)^2 + \cdots\right)X + \frac{A^2}{2}\left(1 + (-iA) + (-iA)^2 + \cdots\right)X$

$= \frac{1}{2}A^2 e^{it} + \frac{1}{2}A^2 e^{-it} = A^2 \cos t$

$= \int_0^t d\tau \int_0^\tau \cos\sigma \, d\sigma = \int_0^t \sin\tau \, d\tau = -\cos t + 1, \quad t > 0$

(c) $\frac{dy(t)}{dt} = v(t) \quad \Rightarrow \quad \frac{y[n+1] - y[n]}{T} = v[n]$

$x(t) - y(t) = \frac{dv(t)}{dt} \qquad x[n] - y[n] = \frac{v[n+1] - v[n]}{T}$

set $T = 0.1$, $y[0] = 0$, $v[0] = 0$, $x[n] = 1$, $n \geq 0$

$\Rightarrow v[1] = 0.1 \quad y[1] = 0$

$v[2] = 0.2 \quad y[2] = 0.01$

$v[3] = 0.299 \quad y[3] = 0.03$

$v[4] = 0.396 \quad y[4] = 0.0599$

$(1 - \cos t)\big|_{t=0.4} = 0.0789$

the difference can't be ignored!

(c) $\frac{dy(t)}{dt} = v(t)$ $\Rightarrow$ $\frac{y[n]-y[n-1]}{T} = v[n]$

$x(t) - y(t) = \frac{dv(t)}{dt}$ $\quad$ $x[n] - y[n] = \frac{v[n]-v[n-1]}{T}$

set $T = 0.1$, $y[0] = 0$, $v[0] = 0$, $x[n] = 1$, $n \geq 0$

$\Rightarrow$ $v[1] = 0.099$ $\quad y[1] = 0.0099$

$v[2] = 0.196$ $\quad y[2] = 0.0295$

$v[3] = 0.29$ $\quad y[3] = 0.0585$

$v[4] = 0.38$ $\quad y[4] = 0.0965$

$(1-\cos t)\big|_{t=0.4} = 0.0789$

the difference can't be ignored

(e) $x(t) - y(t) = \frac{d^2}{dt^2} y(t)$ $\Rightarrow$ $x[n] - y[n] = \frac{y[n+1] - 2y[n] + y[n-1]}{T^2}$

set $T = 0.1$, $y[0] = 0$, $x[n] = 1$, $n \geq 0$

$\Rightarrow$ $y[1] = 0.01$

$y[2] = 0.0299$

$y[3] = 0.059501$

$y[4] = 0.0985$

the result is similar to backforward-euler method