Problem 1

(a) $x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau = \int_{-T_1}^{T_1} x(\tau) h(t-\tau) d\tau$

$|t-\tau| > T_2 \Leftrightarrow t > \tau + T_2$ or $t < -T_2 + \tau$

$\Rightarrow x(t) * h(t) = 0$ if $t > T_1 + T_2$ or $t < -T_2 - T_1$

$\Rightarrow T_3 = T_1 + T_2$

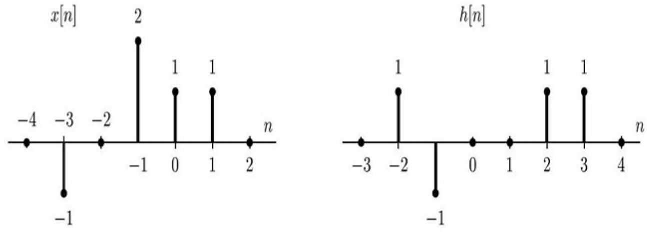
(d) $y(0) = \int_{-\infty}^{+\infty} h(\tau) x(-\tau) d\tau$

$\Rightarrow$ We need to know $x(t)$ in $[1,2]$ and $t=-6$

Problem 2

Compute the convolution $y[n] = x[n] * h[n]$ of each of the two following pairs of signals:

(a). $x[n]$ and $h[n]$ are depicted below



(b). $x[n] = u[n+4] - u[n-1]$, $h[n] = 2^n u[2-n]$.

(a) $y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$

$y[-6] = 0$

$y[-5] = x[-3]\cdot h[-2] = -1$

$y[-4] = x[-3]\cdot h[-1] = 1$

$y[-3] = x[-1]\cdot h[-2] = 2$

$y[-2] = x[-1]\cdot h[-1] + x[0]\, h[-2] = -1$

$y[-1] = x[-3]\, h[2] + x[0]\, h[-1] + x[1]\, h[-2] = -1$

$y[0] = x[-3]\, h[3] + x[1]\, h[-1] = -2$

$y[1] = x[-1]\, h[2] = 2$

$y[2] = x[-1]\, h[3] + x[0]\, h[2] = 3$

$y[3] = x[0]\, h[3] + x[1]\, h[2] = 2$

$y[4] = x[1]\, h[3] = 1$ $\quad y[5] = 0$

$y[n] = 0,\ n \le -6$ 或 $n \ge 5$

(b) $y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] = \sum_{k=-\infty}^{+\infty} \left(u[k+4] - u[k-1]\right)\cdot 2^{n-k}\, u[2+k-n]$

$= \sum_{k=-4}^{0} 2^{n-k}\, u[2+k-n]$

$= 2^{n+4}\, u[-2-n] + 2^{n+3}\, u[-1-n] + 2^{n+2}\, u[-n]$

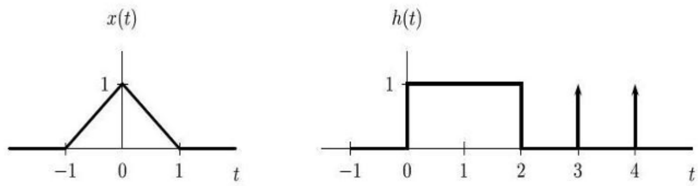
$+ 2^{n+1}\, u[1-n] + 2^{n}\, u[2-n]$

Problem 3

Compute the convolution $y(t) = x(t) * h(t)$ for each of the following pairs of signals:

(a). $x(t) = e^{-t}u(t+1)$, $h(t) = e^{2t}u(-t)$

(b). $x(t)$ and $h(t)$ are depicted below:



(a) $y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$

$= \int_{-\infty}^{+\infty} e^{-\tau} u(\tau+1)\, e^{2(t-\tau)}\, u(-t+\tau)\, d\tau$

$t \ge -1$ 时

$y(t) = \int_{t}^{+\infty} e^{2t-3\tau}\, d\tau = \frac{1}{3} e^{-t}$

$t < -1$ 时

$y(t) = \int_{-1}^{+\infty} e^{2t-3\tau}\, d\tau = \frac{1}{3} e^{2t+3}$

(b) $h(t) = u(t) - u(t-2) + \delta(t-3) + \delta(t-4)$

$y(t) = x(t) * h(t) = \int_{-\infty}^{t} x(\tau)\, d\tau - \int_{-\infty}^{t-2} x(\tau)\, d\tau + x(t-3) + x(t-4)$

$$= \begin{cases} 0 & t \le -1 \\ \frac{1}{2}t^2 + t + \frac{1}{2} & -1 < t \le 0 \\ -\frac{1}{2}t^2 + t + \frac{1}{2} & 0 < t \le 1 \\ \frac{1}{2}(t-2)^2 + (t-2) + \frac{3}{2} & 1 < t \le 2 \\ -\frac{1}{2}(t-2)^2 + (t-2) + \frac{3}{2} + x(t-3) & 2 < t \le 3 \\ 2 + x(t-3) + x(t-4) & t > 3 \end{cases}$$

Problem 4

The following are impulse responses of either discrete-time or continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answer:

(a). $h[n] = 2^n u[3-n]$

(b). $h(t) = u(1-t) - \frac{1}{2}e^{-t}u(t)$

(c). $h[n] = [1-(0.99)^n]u[n]$

(d). $h(t) = e^{15t}[u(t-1) - u(t-100)]$

(a) $h[-1] = \frac{1}{2} \Rightarrow$ System is not causal

$\sum_{k=-\infty}^{+\infty} |h[k]| = \sum_{k=-\infty}^{3} 2^n < \infty \Rightarrow$ System is stable

(b) $h(-1) = u(2) = 1 \Rightarrow$ System is not causal

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{-\infty}^{+\infty} |u(1-t)|\,dt - \int_{-\infty}^{+\infty} \frac{1}{2}e^{-t}u(t)\,dt$$

$$= \int_{-\infty}^{1} dt - \int_{0}^{\infty} \frac{1}{2}e^{-t}\,dt = \Big|_{-\infty}^{1} - \frac{1}{2} = \infty$$

$\Rightarrow$ System is not stable

(c) $h[n] = (1-0.99^n)u[n]$

$h[n] = 0,\ n<0 \Rightarrow$ System is causal

$$\sum_{n=-\infty}^{+\infty} (1-0.99^n)u[n] = \sum_{n=0}^{\infty} 1-0.99^n = \left(\sum_{n=0}^{\infty} 1\right) - 100 = \infty$$

$\Rightarrow$ System is not stable

(d) $h(t) = e^{15t}[u(t-1) - u(t-100)]$

when $t<0$, $u(t-1) = u(t-100) = 0 \Rightarrow h(t) = 0$

$\Rightarrow$ System is causal

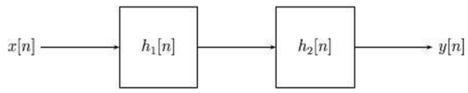
$$\int_{-\infty}^{+\infty} dt\, h(t) = \int_{-\infty}^{+\infty} e^{15t}[u(t-1) - u(t-100)]\,dt$$

$$= \int_{1}^{100} e^{15t}\,dt = \frac{1}{15}(e^{1500} - e)$$

$\Rightarrow$ System is stable

Problem 5

Consider the cascade of LTI systems with unit sample responses $h_1[n]$ and $h_2[n]$ depicted below:

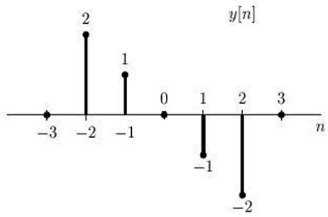


Suppose we are given the following information:

- $h_2[n] = \delta[n] - \delta[n-1]$
- If the input is

$$x[n] = u[n] - u[n-2]$$

then the output is as depicted below



Find $h_1[n]$.

$$y[n] = 2\delta[n+2] + \delta[n+1] - \delta[n-1] - 2\delta[n-2]$$

Suppose that the output of $h_1[n]$ is $y_1[n]$

$$\Rightarrow y[n] = y_1[n] - y_1[n-1]$$

$$\Rightarrow y_1[n] = 2\delta[n+2] + 3\delta[n+1] + 3\delta[n] + 2\delta[n-1]$$

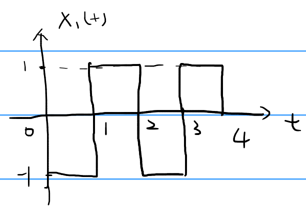
$$x[n] = u[n] - u[n-2] = \delta[n] + \delta[n-1]$$

$$\Rightarrow y_1[n] = 2x[n+2] + x[n+1] + 2x[n]$$

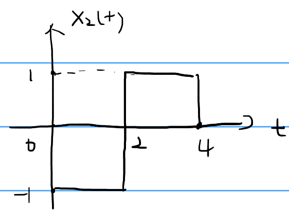
$$\Rightarrow h_1[n] = 2\delta[n+2] + \delta[n+1] + 2\delta[n]$$

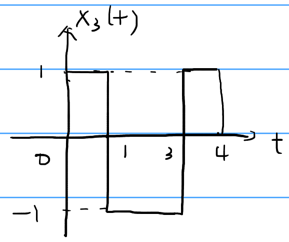
Problem 6.

(a)



(b)

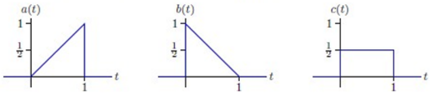




(c) $y_{ij}(t) = 0$, $t = 4$. $i, j = 1, 2, 3$, $i \neq j$

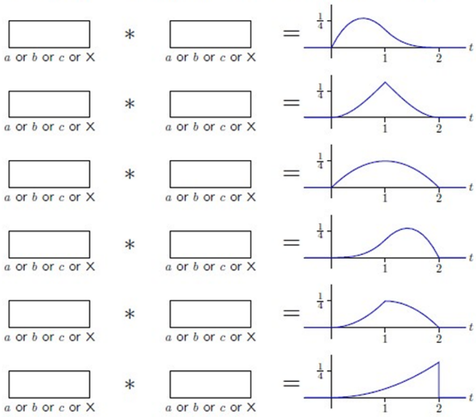
Problem 7

Consider the convolution of two of the following signals.



Determine if each of the following signals can be constructed by convolving (a or b or c) with (a or b or c). If it can, indicate which signals should be convolved. If it cannot, put an X in both boxes.

Notice that there are ten possible answers: $(a * a)$, $(a * b)$, $(a * c)$, $(b * a)$, $(b * b)$, $(b * c)$, $(c * a)$, $(c * b)$, $(c * c)$, or (X,X). Notice also that the answer may not be unique.



(1) $b * b$

(2) $a * b$ $b * a$

(3) $X * X$

(4) $a * a$

(5) $a * c$ $c * a$

(6) $X * X$