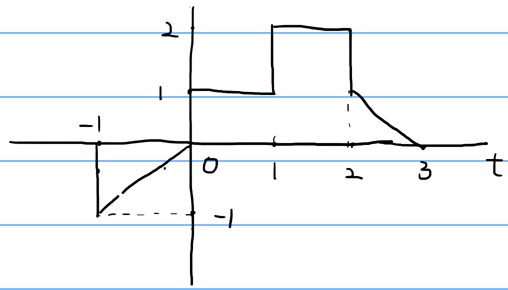
Problem 1 OWN, Problem 1.21 (a)(c)(d)(f)

Problem 2 OWN, Problem 1.22(a)(c)(e)(f)

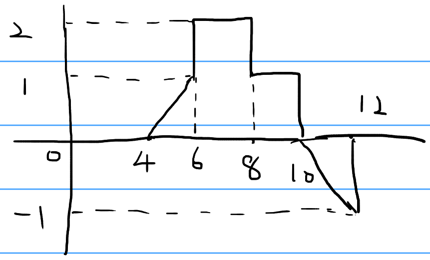
Problem 3 OWN, Problem 1.28(a)(c)(d)(e)

Problem 4 OWN, Problem 1.31

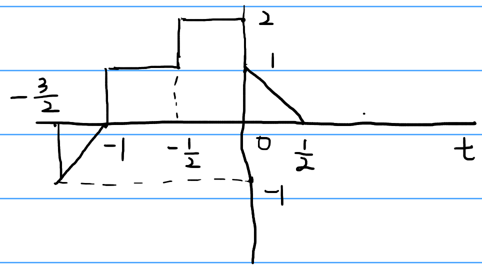
1.21 (a)



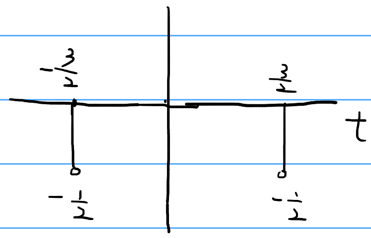
(d) $x(-\frac{1}{2}(t-8))$



(c)

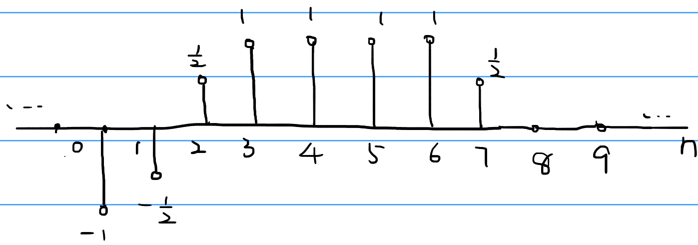


(f)

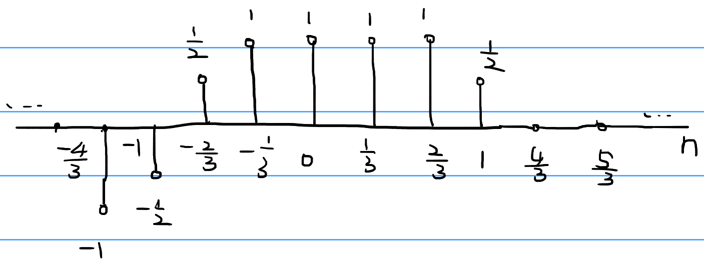


1.22 (a) (c) (e) (f)

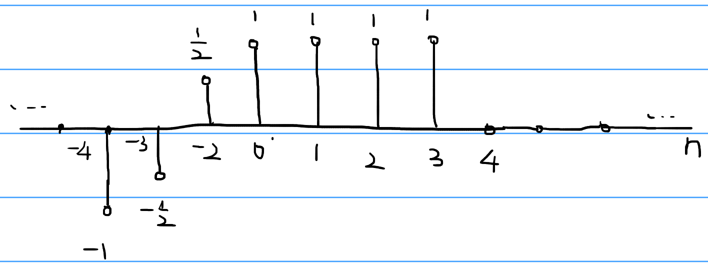
(a)



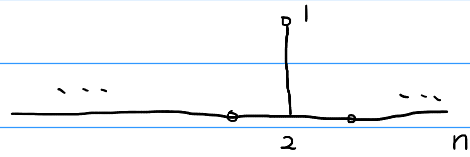
(c)



(e)



(f)



1.28 (a) (1) $y[1] = x[-1]$ $\Rightarrow$ with memory

(2) 设 $x_2[n] = x_1[n+T]$ $x_2[n] \to y_2[n] = x_2[-n] = x_1[-n+T] = y_1[n-T]$

$\Rightarrow$ time-invarient

(3) $x_1[n] \to y_1[n] = x_1[-n]$, $x_2[n] \to y_2[n] = x_2[-n]$

设 $x_3[n] = a x_1[n] + b x_2[n]$

$a x_1[n] + b x_2[n] = x_3[n] \to y_3[n] = x_3[-n] = a x_1[-n] + b x_2[-n] = a y_1[n] + b y_2[n]$

$\Rightarrow$ linear

(4) $y[-1] = x[1]$ $\Rightarrow$ Noncasual

(5) if $x[n]$ is bounded , $y[n]$ is bounded

$\Rightarrow$ stable

(c) $y[n] = n x[n]$

(1) (4) without memory and casual

(2) $x_1[n+T] = x_2[n] \to y_2[n] = n x_2[n] = n x_1[n+T] \neq y_1[n+T]$

$\Rightarrow$ time-varient

(3) $x_1[n] \to y_1[n] = n x_1[n]$, $x_2[n] \to y_2[n] = n x_2[n]$

设 $x_3[n] = a x_1[n] + b x_2[n]$

$a x_1[n] + b x_2[n] = x_3[n] \to y_3[n] = n x_3[n] = a n x_1[n] + b n x_2[n] = a y_1[n] + b y_2[n]$

$\Rightarrow$ linear

(5) when $x[n]$ is bounded

$\lim_{n\to\infty} y[n] = \lim_{n\to\infty} n x[n] = \infty$

$\Rightarrow$ non-stable

(d) $y[n] = \mathcal{E}v\{x[n-1]\} = \frac{1}{2}\{x[n-1] + x[-n-1]\}$

(1) $y[1] = \frac{1}{2}\{x[0] + x[-2]\}$ $\Rightarrow$ with memory

(2) $x_1[n+T] = x_2[n] \rightarrow y_2[n] = \frac{1}{2}\{x_2[n-1] + x_2[-n-1]\} = \frac{1}{2}\{x_1[n-1+T] + x_1[-n-1+T]\}$
$\neq y[n+T]$

$\Rightarrow$ time-varient

(3) $x_1[n] \rightarrow y_1[n] = \frac{1}{2}\{x_1[n-1] + x_1[-n-1]\}$

$x_2[n] \rightarrow y_2[n] = \frac{1}{2}\{x_2[n-1] + x_2[-n-1]\}$

设 $x_3[n] = a x_1[n] + b x_2[n]$

$a x_1[n] + b x_2[n] = x_3[n] \rightarrow y_3[n] = \frac{1}{2}\{x_3[n-1] + x_3[-n-1]\}$

$= \frac{1}{2}\{a x_1[n-1] + b x_2[n-1]$

$+ a x_1[-n-1] + b x_2[-n-1]\}$

$= a y_1[n] + b y_2[n]$

$\Rightarrow$ linear

(4) $y[-1] = \frac{1}{2}\{x[-2] + x[0]\}$ $\Rightarrow$ non-casual

(5) $y[n]$ is bounded if $x[n]$ is bounded

$\Rightarrow$ stable

(e) (1)(4) with memory and non-casual

(2) $x_1[n+T] = x_2[n] \rightarrow y_2[n] = \begin{cases} x_2[n], & n \geq 1 \\ 0, & n = 0 \\ x_2[n+1], & n \leq -1 \end{cases} = \begin{cases} x_1[n+T], & n \geq 1 \\ 0, & n = 0 \\ x_1[n+T+1], & n \leq -1 \end{cases}$

$y_1[n+T] = \begin{cases} x_1[n+T], & n \geq 1-T \\ 0, & n = -T \\ x_1[n+T+1], & n \leq -1-T \end{cases}$ $x_1[n+T] \not\rightarrow y_1[n+T]$

$\Rightarrow$ time-varient

(3) $x_1[n] \rightarrow y_1[n] = \begin{cases} x_1[n], & n \geq 1 \\ 0, & n = 1 \\ x_1[n+1], & n \leq -1 \end{cases}$

$x_2[n] \rightarrow y_2[n] = \begin{cases} x_2[n], & n \geq 1 \\ 0, & n = 1 \\ x_2[n+1], & n \leq -1 \end{cases}$

设 $x_3[n] = a x_1[n] + b x_2[n]$

$a x_1[n] + b x_2[n] = x_3[n] \rightarrow y_3[n] = \begin{cases} x_3[n], & n \geq 1 \\ 0, & n = 1 \\ x_3[n+1], & n \leq -1 \end{cases} = \begin{cases} a x_1[n] + b x_2[n], & n \geq 1 \\ 0, & n = 1 \\ a x_1[n+1] + b x_2[n+1], & n \leq -1 \end{cases}$

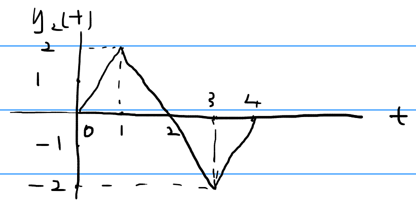
$= a y_1[n] + b y_2[n]$

$\Rightarrow$ linear

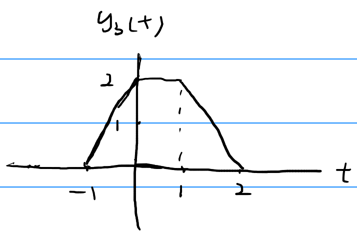
(5) $y[n]$ is bounded if $x[n]$ is bounded

$\Rightarrow$ stable

1.31 (a) $x_2(t) = x_1(t) - x_1(t-2) \longrightarrow y_1(t) - y_1(t-2)$



(b) $x_3(t) = x_1(t) + x_1(t+1) \rightarrow y_1(t) + y_1(t+1)$



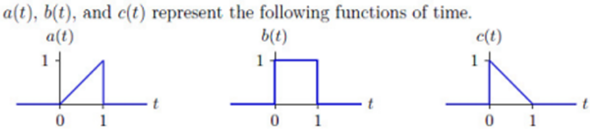
Problem 5

a. $\frac{1}{1-a} = \sum_{n=0}^{\infty} a^n$, $|a|<1$

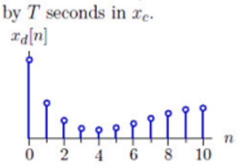
b. $\sum_{n=0}^{N-1} a^n = \begin{cases} \frac{1-a^N}{1-a} & a \neq 1 \\ N-1 , & a=1 \end{cases}$

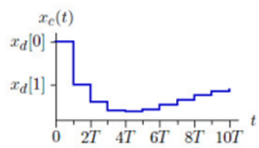
Problem 6

Let $a(t)$, $b(t)$, and $c(t)$ represent the following functions of time.



Let $x_c(t)$ represent a continuous-time signal derived from the discrete-time signal $x_d[n]$ using a zero-order hold, as illustrated below, where consecutive samples of x_d are separated by T seconds in x_c.



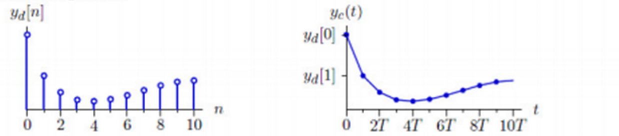


a. Determine an expression for $x_c(t)$ in terms of the samples $x_d[n]$ and the functions $a(t)$, $b(t)$, and $c(t)$.

(a)

$x_c(t) = \sum_{n=1}^{9} x_d(n) \cdot b(\frac{1}{T}t - n)$, $0 \leq t \leq 10T$

Let $y_c(t)$ represent a continuous-time signal derived from the discrete-time signal $y_d[n]$ using a piecewise linear interpolator, so that sucessive samples of y_d are connected by straight line segments.



b. Determine an expression for $y_c(t)$ in terms of the samples $y_d[n]$ and the functions $a(t)$, $b(t)$, and $c(t)$.

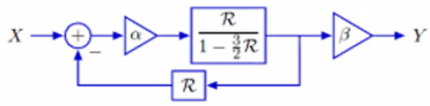
c. Determine an expression for $\frac{dy_c(t)}{dt}$ in terms of the samples $y_d[n]$ and the functions $a(t)$, $b(t)$, and $c(t)$.

b. $y_c(t) = \sum_{n=0}^{9} y_d[n+1] - y_d[n] \; a(\frac{1}{T}t - n)$, $0 \le t \le 10T$

c. $$\frac{d y_c(t)}{dt} = \sum_{n=0}^{9} y_d[n+1] - y_d[n] \cdot \frac{1}{T} b(\frac{1}{T}t - n)$$, $0 \le t \le 10T$ 且 $t \ne 0, T, 2T, \cdots 10T$

Problem 7 Missing Parameters

Consider the following system.



Assume that X is the unit-sample signal, $x[n] = \delta[n]$. Determine the values of α and β for which $y[n]$ is the following sequence (i.e., $y[0]$, $y[1]$, $y[2]$, ...):

$$0,\ 1,\ \frac{3}{2},\ \frac{7}{4},\ \frac{15}{8},\ \frac{31}{16},\ \ldots$$

$$Y[n] = \beta \cdot \alpha \cdot \frac{R}{1-\frac{3}{2}R} \cdot \left[X[n] - R \cdot \frac{Y[n]}{\beta}\right]$$

when $n=1$

$$Y[1] - \frac{3}{2}Y[0] = \alpha\beta R\left[X[n] - \frac{Y[n-1]}{\beta}\right] = \alpha\beta X[n-1]$$

$\Rightarrow 1 = \alpha\beta$

when $n \geq 2$

$$\Rightarrow Y[n] - \frac{3}{2}Y[n-1] = \alpha\beta\left\{X[n-1] - \frac{1}{\beta}Y[n-2]\right\}$$

$$\Rightarrow Y[n] - \frac{3}{2}Y[n-1] + \alpha Y[n-2] = \alpha\beta X[n-1]$$

令 $n=3$ $\frac{7}{4} - \frac{3}{2}\cdot\frac{3}{2} + \alpha = 0$ $\Rightarrow \alpha = \frac{1}{2}$

because of $\alpha\beta = 1$, $\beta = 2$

$\Rightarrow \alpha = \frac{1}{2}$, $\beta = 2$

Problem 8 Choose a bank

Consider two banks. Bank #1 offers a 3% annual interest rate, but charges a ￥1 service charge each year, including the year when the account was opened. Bank #2 offers a 2% annual interest rate, and has no annual service charge. Let $y_i[n]$ represent the balance in bank i at the beginning of year n and $x_i[n]$ represent the amount of money you deposit in bank i during year n. Assume that deposits during year n are credited to the balance at the end of that year but earn no interest until the following year.

a. Use difference equations to express the relation between deposits and balances for each bank.

b. Assume that you deposit ￥100 in each bank and make no further deposits. Solve your difference equations in part a numerically to determine your balance in each bank for the next 5 years. Which account has the larger balance 5 years after the initial investment (one year without interest and 4 years with interest)?

a. $y_1[n] = 1.03\, y_1[n-1] - 1 + x_1[n-1]$

$y_2[n] = 1.02\, y_2[n-1] + x_2[n-1]$

b. $y_1[0] = y_2[0] = 0, \quad x_1[0] = x_2[0] = 100. \quad x_i[n] = 0, \; n \geq 1$

$\Rightarrow \quad y_1[n] = 1.03^{n-1} \left\{ x_1[0] - \frac{100}{3} \right\} + \frac{100}{3}$

$y_2[n] = 1.02^{n-1} \cdot x_2[0].$

令 $n = 5$, $\quad y_1[5] = 108.367$

$y_2[5] = 108.243$

$\Rightarrow$ bank1 has the larger balance