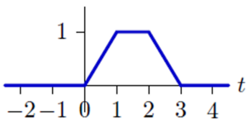
Problem 1

Determine the Laplace transforms (including the regions of convergence) of each of the following signals:

a. $x_1(t) = e^{-2(t-3)}u(t-3)$

b. $x_2(t) = |t|e^{-|t|}$

c. $x_3(t)$



a. $X(s) = \int_3^{\infty} e^{-2(t-3)} e^{-st}\,dt = \frac{1}{s+2} e^{-3s}$, $Re(s) > -2$

b. $X(s) = \int_{-\infty}^{0} -te^{t} e^{-st}\,dt + \int_0^{\infty} te^{-t} e^{-st}\,dt$

$L(e^{t}) = \frac{1}{1-s}$, $Re(s) < 1$ $\quad L(e^{-t}) = \frac{1}{s+1}$, $Re(s) > -1$

$\Rightarrow X(s) = \frac{1}{(1-s)^2} - \frac{1}{(s+1)^2}$, $-1 < Re(s) < 1$

c. $X(s) = \int_0^1 t e^{-st}\,dt + \int_1^2 e^{-st}\,dt + \int_2^3 (-t+3)e^{-st}\,dt$

$= \left(-\frac{1}{s} - \frac{1}{s^2}\right)e^{-s} + \frac{1}{s^2} - \frac{1}{s}e^{-2s} + \frac{1}{s}e^{-s} + \left(\frac{3}{s} + \frac{1-3s}{s^2}\right)e^{-3s} - \left(\frac{2}{s} + \frac{1-3s}{s^2}\right)e^{-2s}$

$= -\frac{1}{s^2}e^{-s} - \frac{1}{s^2}e^{-2s} + \frac{1}{s^2}e^{-3s} + \frac{1}{s^2}$

$s \in \mathbb{C}$

Determine and sketch all possible signals with Laplace transforms of the following forms. For each signal, indicate the associated region of convergence.

a. $X_1(s) = \dfrac{s+2}{(s+1)^2}$

b. $X_2(s) = \left(\dfrac{1-e^{-s}}{s}\right)^2$

a. $X_1(s) = \frac{1}{s+1} + \frac{1}{(s+1)^2} = \frac{1}{s+1} - \frac{d}{ds}\frac{1}{s+1}$

pole is -1

$\Rightarrow x(t) = (e^{-t} + te^{-t})u(t)$, $Re(s) > -1$

or $x(t) = (-e^{-t} - te^{-t})u(-t)$, $Re(s) < -1$

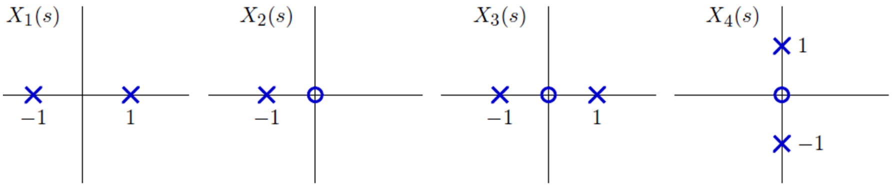
b. $X_2(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$

$L\left(\frac{1}{s^2}\right) = \int_{-\infty}^{t} u(\tau)\, d(\tau) = t\,u(t)$

$\Rightarrow X_2(s) = t\,u(t) - 2(t-1)u(t-1) + (t-2)u(t-2)$, $\operatorname{Re}(s) > 0$

Problem 3

Determine which of the following pole-zero diagrams could represent Laplace transforms of even functions of time. Determine expressions for the time functions of those that can represent even functions of time. For those that cannot, explain why they cannot.



How can you determine if a signal is even or not by looking at its Laplace transform and region of convergence?

Suppose that $f(t)$ is even function

$\int_0^\infty f(t) e^{-st}\, dt$

$\int_{-\infty}^{0} f(t) e^{-st}\, dt = \int_0^\infty f(-t) e^{st}\, dt = \int_0^\infty f(t) e^{st}\, dt$

$\Rightarrow$ the ROCs are symmetrical

$\Rightarrow X_2(s)$ cannot represent even function of time

$X_1(s)$ can be Laplace transform of $f(t) = e^{|t|}$

$X_3(s)$ can be Laplace transform of $f(t) = \frac{1}{2}e^{t} + \frac{1}{2}e^{-t}$

$X_4(s)$ can be Laplace transform of $f(t) = \cos t$

Problem 4

a. Use the initial and final value theorems (where applicable) to find $x(0)$ and $x(\infty)$ for the signals with the following Laplace transforms:

1. $\dfrac{1 - e^{-sT}}{s}$
2. $\dfrac{(s+1)^2 - 1}{[(s+1)^2 + 1]^2}$

b. Find the inverse Laplace transforms for each of the previous parts and show that the time waveforms and initial and final values agree.

a.1 $x(0^+) = \lim_{s\to\infty} s X(s) = \lim_{s\to\infty} 1 - e^{-sT} = 1$

$x(\infty) = \lim_{s\to 0} s X(s) = \lim_{s\to 0} 1 - e^{-sT} = 0$

2. $x(0^+) = \lim_{s\to\infty} s X(s) = \lim_{s\to\infty} s \cdot \frac{(s+1)^2 - 1}{[(s+1)^2+1]^2} = 0$

$x(\infty) = \lim_{s\to 0} s X(s) = \lim_{s\to 0} s\, \frac{(s+1)^2 - 1}{[(s+1)^2+1]^2} = 0$

b.1 $X(s) = \frac{1-e^{-sT}}{s} = \frac{1}{s} - \frac{e^{-sT}}{s}$

$\Rightarrow x(t) = u(t) - u(t-T) \Rightarrow x(0) = 1, \; x(\infty) = 0$

2 $X(s) = \frac{(s+1)^2-1}{[(s+1)^2+1]^2} = -\frac{d}{ds}\frac{(s+1)}{(s+1)^2+1} =$

$L^{-1}\left[\frac{s+1}{(s+1)^2+1}\right] = e^{-t}\cos t$

$\Rightarrow x(t) = te^{-t}\cos t \Rightarrow x(0)=0, \; x(\infty)=0$

Problem 5

(a) $L\left(\frac{dy_1}{dt}\right) = sH_1(s) \quad L\left(\frac{d^2y_1}{dt^2}\right) = s^2H_1(s)$

$H(s) = 2s^2H_1(s) + 4sH_1(s) - 6H_1(s)$

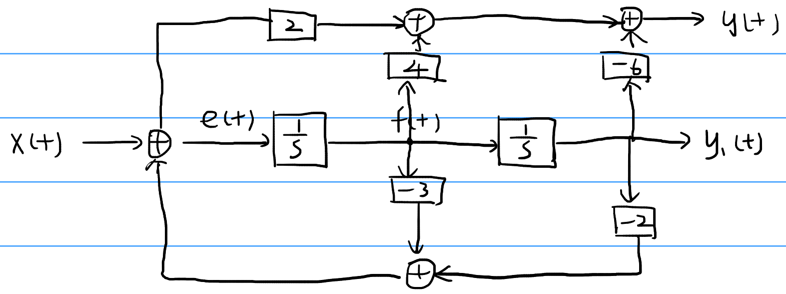
$\Rightarrow y(t) = 2\frac{d^2y_1(t)}{dt^2} + 4\frac{dy_1(t)}{dt} - 6y_1(t)$

(b)(c) $L\left(\int_{-\infty}^{\tau} x(\tau)d\tau\right) = \frac{X(s)}{s} \Rightarrow \boxed{\frac{1}{s}}$ is equal to an int device

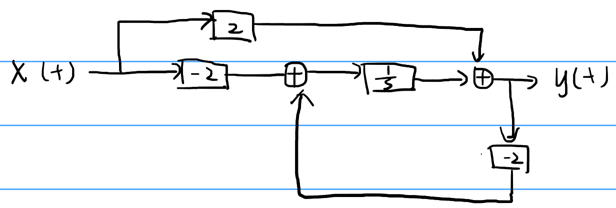
$\Rightarrow f(t) = \frac{dy_1(t)}{dt}, \quad e(t) = \frac{d^2y_1(t)}{dt^2}$

(d) $y(t) = 2e(t) + 4f(t) - 6y_1(t)$

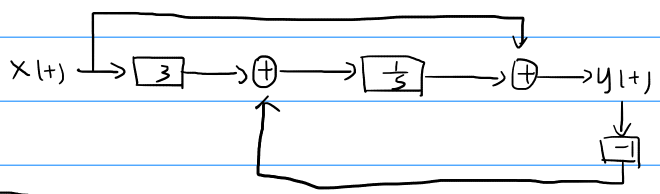
(e)



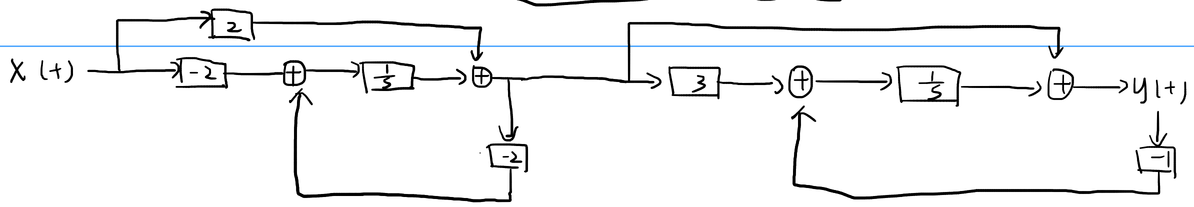
(f) $\frac{2(s+1)}{s+2}$



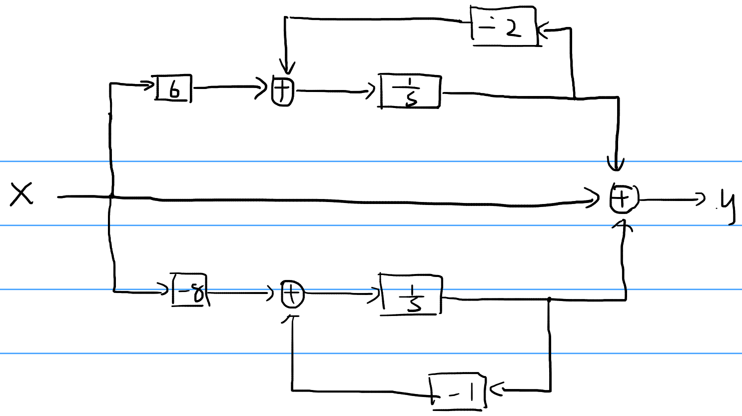
$\frac{s+3}{s+1} = \frac{Y}{X}$



H(s)



(g)



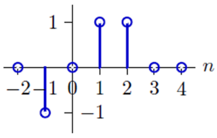
Problem 6

Determine the Z transform (including the region of convergence) for each of the following signals:

a. $x_1[n] = \left(\frac{1}{2}\right)^n u[n-3]$

b. $x_2[n] = (1+n)\left(\frac{1}{3}\right)^n u[n]$

c. $x_3[n]$



a. $H_1(z) = \sum_{n=3}^{\infty} \left(\frac{1}{2}\right)^n \cdot z^{-n} = \frac{\left(\frac{1}{2z}\right)^3}{1-\frac{1}{2z}} = \frac{1}{8z^3-4z^2}$, $|z| > \frac{1}{2}$

b. $H_2(z) = \sum_{n=0}^{\infty} (n+1)\left(\frac{1}{3}\right)^n z^{-n} = \frac{9z^2}{(3z-1)^2}$, $|z| > \frac{1}{3}$

c. $H_3(z) = (-1)z + z^{-1} + z^{-2} = -z + \frac{1}{z} + \frac{1}{z^2}$, $z \in C$

Problem 7

Determine and sketch all possible signals with Z transforms of the following forms. For each signal, indicate the associated region of convergence.

a. $X_1(z) = \dfrac{1}{z(z-1)^2}$

b. $X_2(z) = \left(\dfrac{1-z^2}{z}\right)^2$

$-n-1$

$n-1$

a. $X_1(z) = z^{-2}\cdot(-z)\frac{d}{dz}\frac{1}{z-1}$

$\frac{1}{z-1} = z^{-1}\cdot\frac{z}{z-1} \Rightarrow z^{-1}\left[\frac{z}{z-1}\right] = u[n-1]$, $|z|>1$

或 $= -u[-n]$, $|z|<1$

$\Rightarrow x(z) = (n-2)u[n-3]$, $|z|>1$

或 $= -(n-2)u[-n+2]$, $|z|<1$

b. $X_2(z) = \frac{1}{z^2} + z^2 - 2 = \delta[n-2] + \delta[n+2] - 2\delta[n]$, $|z|>0$

Problem 8

Let $X(z)$ represent the Z transform of $x[n]$, and let $r_0 < |z| < r_1$ represent its region of convergence (ROC).

Let $x[n]$ be represented as the sum of even and odd parts

$$x[n] = x_e[n] + x_o[n]$$

where $x_e[n] = x_e[-n]$ and $x_o[n] = -x_o[-n]$.

a. Under what conditions does the Z transform of $x_e[n]$ exist?

b. Assuming the conditions given in part a, find an expression for the Z transform of $x_e[n]$, including its region of convergence.

(a) when $r_0 < |z| < r_1$, $\sum_{n=-\infty}^{+\infty} x_e[n] z^{-n} < \infty$

(b) $X_e(z) = \sum_{-\infty}^{+\infty} x_e[n] z^{-n} = \sum_{n=-\infty}^{-1} x_e[n] z^{-n} + \sum_{n=0}^{\infty} x_e[n] z^{-n}$

$= \sum_{n=1}^{\infty} x_e[n] z^{n} + \sum_{n=0}^{\infty} x_e[n] z^{-n}$

$= x_e[0] + \sum_{n=1}^{\infty} x_e[n] (z^{n} + z^{-n})$

the convergence is relate to $x_e[n]$

Problem 9

$$H_1(z) = k_1 \frac{(z - \frac{3}{4}e^{j\frac{\pi}{4}})(z - \frac{3}{4}e^{j(-\frac{\pi}{4})})}{(z - \frac{3}{4}e^{j\frac{3\pi}{4}})(z - \frac{3}{4}e^{j(-\frac{3\pi}{4})})}$$

$$H_2(z) = k_2 \frac{(z - \frac{1}{2}e^{j\frac{3\pi}{4}})(z - \frac{1}{2}e^{j(-\frac{3\pi}{4})})}{(z - \frac{1}{2}e^{j\frac{\pi}{4}})(z - \frac{1}{2}z^{j(-\frac{\pi}{4})})}$$

$H_2(z) = \frac{k_2}{k_1} H_1(\frac{3}{2}e^{j\pi} z) = \frac{k_2}{k_1} H_1(-\frac{3}{2}z)$

$\Rightarrow h_2[n] = \frac{k_2}{k_1}(-\frac{2}{3})^n h_1[n]$

$\Rightarrow g[n] = \frac{k_2}{k_1}(-\frac{2}{3})^n$

$g[n] = 0.\quad n<0 \quad \Rightarrow g[n] = \frac{k_2}{k_1}(-\frac{2}{3})^n u[n]$

$\sum_{k=0}^{\infty} |g[k]| = 3 \quad \Rightarrow \sum_{k=0}^{\infty} \frac{k_2}{k_1}(\frac{2}{3})^k = 3 \quad \Rightarrow \frac{k_2}{k_1} = 1$

$\Rightarrow g[n] = (-\frac{2}{3})^n u[n]$