Problem1

$$\frac{dy(t)}{dt} + 5y(t) = 2x(t) \iff sY(s) + 5Y(s) = 2X(s)$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s+5}$$

$$\Rightarrow S(s) = H(s) \cdot \mathcal{L}(u(t)) = \frac{2}{s+5} \cdot \frac{1}{s} = \frac{2}{s(s+5)}$$

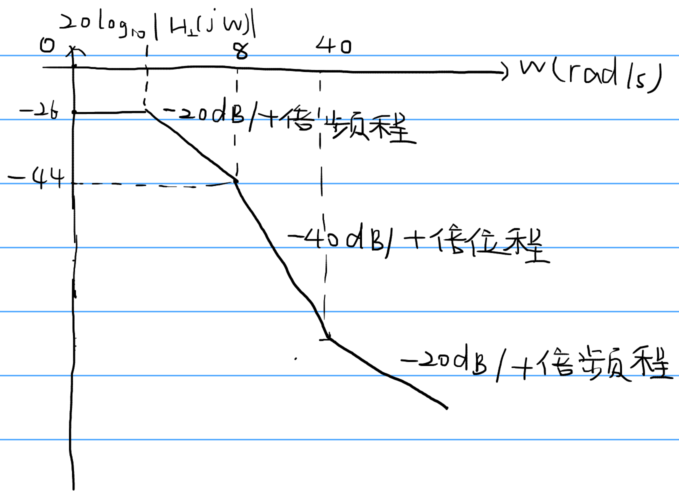
$$\Rightarrow S(\infty) = \lim_{s \to 0} sS(s) = \lim_{s \to 0} \frac{2}{s+5} = \frac{2}{5}$$

$$S(s) = \frac{2}{s(s+5)} = \frac{2}{5}\left(\frac{1}{s} - \frac{1}{s+5}\right)$$

$$\Rightarrow s(t) = \frac{2}{5} u(t) (1 - e^{-5t}) = s(\infty) u(t) (1 - e^{-5t})$$

$$S(t_0) = s(\infty)\left[1 - \frac{1}{e^2}\right] \Rightarrow t_0 = \frac{2}{5}$$

Problem 2

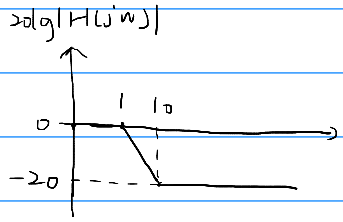


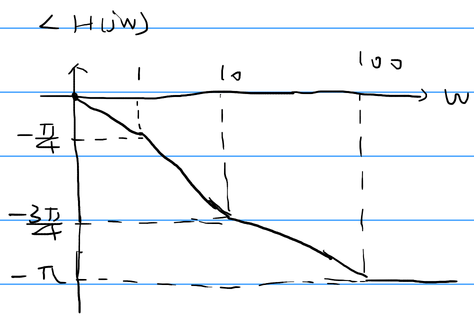

Problem3

$$x(t) = e^{jt} \Rightarrow y(t) = H(j)\, x(t) = -2j e^{jt}$$

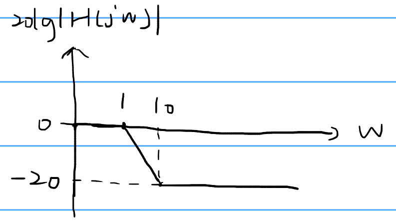
Problem4

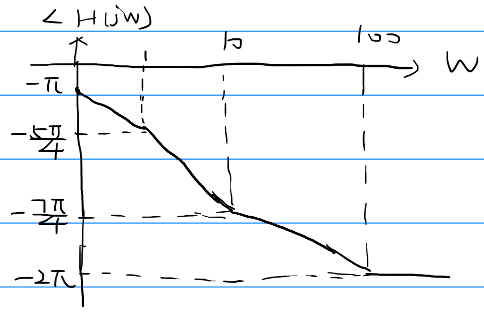
$$\frac{1 - \frac{jw}{10}}{1 + jw}$$



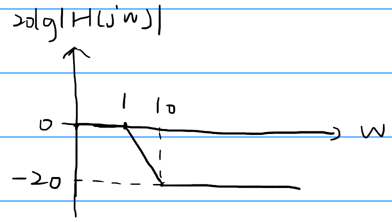



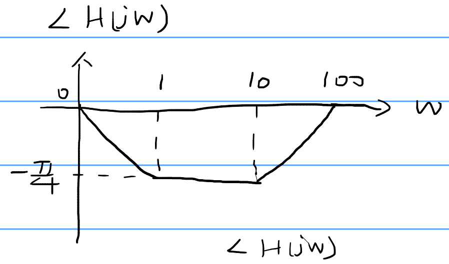
$$\frac{\frac{jw}{10}-1}{1+jw}$$



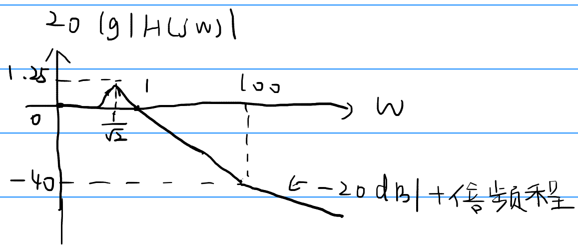

$$\frac{1+\frac{jw}{10}}{1+jw}$$

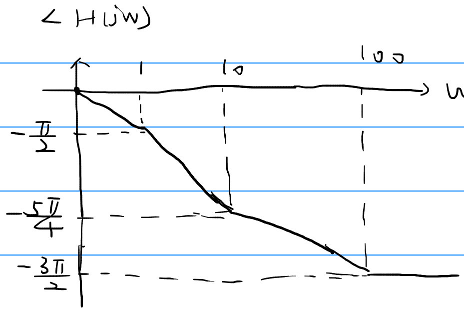



$$\frac{1-\frac{jw}{10}}{(jw)^2+(jw)+1}$$

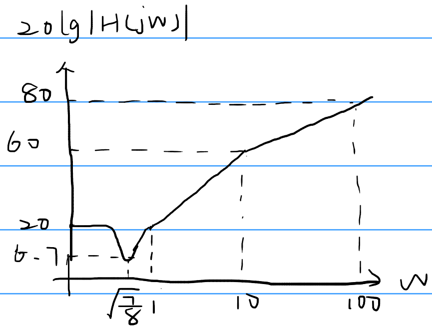
$$= \frac{1-\frac{jw}{10}}{(1-w^2)+jw}$$

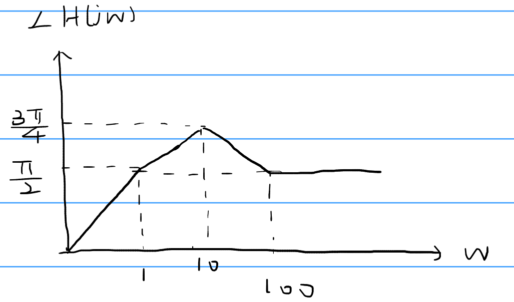



$$\frac{10+5jw+10(jw)^2}{1+\frac{jw}{10}}$$

$$= \frac{(10-10w^2)+5jw}{1+\frac{jw}{10}}$$



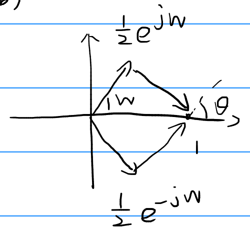

problem 5

(a) $H(e^{jw}) = e^{-jw}\dfrac{1-\frac{1}{2}e^{jw}}{1-\frac{1}{2}e^{-jw}}$

$$\Rightarrow |H(e^{jw})| = \frac{|1-\frac{1}{2}e^{jw}|}{|1-\frac{1}{2}e^{-jw}|} = \frac{|1-\frac{1}{2}\cos w - \frac{1}{2}j\sin w|}{|1-\frac{1}{2}\cos w + \frac{1}{2}j\sin w|} = 1$$

$\Rightarrow |H(jw)| = 1$ for all frequency

(b)




$$\theta = \arctan\left(\frac{\frac{1}{2}\sin w}{1-\frac{1}{2}\cos w}\right)$$

$$\Rightarrow \angle H(e^{jw}) = -w - 2\arctan\left(\frac{\frac{1}{2}\sin w}{1-\frac{1}{2}\cos w}\right)$$

(d) when $w = \frac{\pi}{3}$, $\angle H(e^{jw}) = -\frac{2\pi}{3}$

$$\Rightarrow y(t) = |H(e^{jw})| \cos\left(\frac{\pi}{3}n + \angle H(e^{jw})\right)$$

$$= \cos\left(\frac{\pi}{3}n - \frac{2\pi}{3}\right)$$

**Problem 6** For a second-order CT system with poles at $-1$ and $-4$ (and no zeros), find the frequency at which the phase is $\boxed{?}$° using any method except for the vector method. Then illustrate and confirm that result using the vector method.

The poles is $-1$ and $-4$, the $H(s) = \frac{k}{(s+1)(s+4)}$

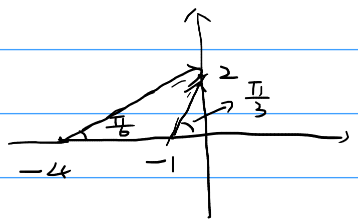
Let $s = jw$ $\Rightarrow H(jw) = \frac{1}{(jw+1)(jw+4)}$

The phase of $H(jw)$ is $-\angle(jw+1) - \angle(jw+4)$

$$= -\arctan w - \arctan\frac{w}{4}$$

$$= -\arctan\frac{5w}{4-w^2}$$

$-90° \Rightarrow w = 2$




the result is confirmed

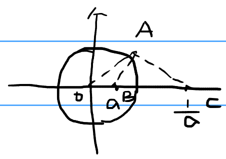
Problem 7

(a) $H(z) = k\,\frac{z-\frac{1}{a}}{z-a} \Rightarrow H(e^{jw}) = k \cdot \frac{e^{jw}-\frac{1}{a}}{e^{jw}-a}$

$$\Rightarrow |H(e^{jw})| = \left|k \cdot \frac{\cos w - \frac{1}{a} + j\sin w}{\cos w - a + j\sin w}\right|$$

$$= |k| \cdot \sqrt{\frac{(\cos w - \frac{1}{a})^2 + \sin^2 w}{(\cos w - a)^2 + \sin^2 w}} = |k|\sqrt{\frac{1-\frac{2}{a}\cos w + \frac{1}{a^2}}{1-2a\cos w + a^2}}$$

$$= \frac{|k|}{a}$$




$OA^2 = OB \cdot OC \Rightarrow \triangle OAB \sim \triangle OCA$

$$\Rightarrow \frac{AC}{AB} = \frac{OA}{OB} = \frac{1}{a}$$

$$\Rightarrow |H(e^{jw})| = |k|\frac{|e^{jw}-\frac{1}{a}|}{|e^{jw}-a|} = \frac{|k|}{a}$$

(b) $|V_1| = \sqrt{1+a^2-2a\cos w}$

(c) $|V_2| = \sqrt{1+\frac{1}{a^2}-2\cdot\frac{1}{a}\cdot\cos w} = \frac{1}{a}|V_1|$

and $\frac{1}{a}$ has nothing to do with $w$

Problem 8

pole-zero diagram 1 — impulse respose 3 — Bode magnitude 5 — Bode angle 4

pole-zero diagram 2 — impulse respose 1 — Bode magnitude 2 — Bode angle 3

pole-zero diagram 3 — impulse respose 4 — Bode magnitude 3 — Bode angle 6

pole-zero diagram 4 — impulse respose 2 — Bode magnitude 6 — Bode angle 2

pole-zero diagram 5 — impulse respose 6 — Bode magnitude 1 — Bode angle 1

pole-zero diagram 6 — impulse respose 5 — Bode magnitude 4 — Bode angle 5