Problem 1.

(a) False

$H(s) = \frac{1}{s-1} \Rightarrow h(t) = e^{-t} \Rightarrow \int_0^{\infty} h(t) < \infty \Rightarrow$ the system is stable

but the pole is at the right side

(b) True

Suppose that $H(s) = \frac{(s-a_1)\cdots(s-a_k)}{(s-b_1)\cdots(s-b_l)}$, $l > k$

$\Rightarrow$ $h(t)$ doesn't contain the pulse function and the derivate of it

$Y(s) = H(s) \cdot \frac{1}{s}$

$\Rightarrow y(t) = \int_{-\infty}^{t} h(t)\,dt \Rightarrow h(t) = \frac{d}{dt} y(t)$

$\Rightarrow$ $y(t)$ is continuous in $t=0$

(3) False

In (2) we haven't used that the system is causal

(4) False

$H(s) = \frac{s-2}{(s+1)(s+2)}$, $Re(s) > -1$

$\Rightarrow h(t) = (4e^{-2t} - 3e^{-t})u(t)$

The system is causal and stable but the zero is in the right side

Problem 2

$z[n] - \frac{k}{4} z[n-1] = y[n]$

$z[n] = x[n] - \frac{k}{3} z[n-1]$

$\Rightarrow z[n] = \frac{4}{7} y[n] + \frac{3}{7} x[n]$ $\Rightarrow$ $\frac{4}{7} y[n-1] + \frac{3}{7} x[n-1]$

$z[n-1] = \frac{12}{7} \cdot \frac{x[n] - y[n]}{k}$ $= \frac{12}{7} \frac{x[n] - y[n]}{k}$

$\Rightarrow y[n] + \frac{k}{3} y[n-1] = x[n] - \frac{k}{4} x[n-1]$

$$y[n] + \frac{k}{3} y[n-1] = x[n] - \frac{k}{4} x[n-1]$$

(a) $Y(z) + \frac{k}{3} z^{-1} Y(z) = X(z) - \frac{k}{4} z^{-1} X(z)$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{k}{4} z^{-1}}{1 + \frac{k}{3} z^{-1}} \qquad |z| > \left|\frac{k}{3}\right|$$

(b) When $|k| < 3$, the ROC contains the unit circle

$\Rightarrow$ When $|k| < 3$, the system is stable

(c) $k = 1$ $\quad H(z) = \dfrac{1 - \frac{1}{4} z^{-1}}{1 + \frac{1}{3} z^{-1}}$

$$\Rightarrow h[n] = \left(-\frac{1}{3}\right)^n u[n] - \frac{1}{4}\left(-\frac{1}{3}\right)^{n-1} u[n-1]$$

$$\Rightarrow y[n] = h[n] \times [n] = \left(-\frac{2}{9}\right)^n u[n] + \frac{3}{4}\left(-\frac{2}{9}\right)^n u[n-1]$$

Problem 2

(a) $V_o(s) = -K V_1(s) = -K \left(\frac{Z_2(s)}{Z_1(s) + Z_2(s)} V_i(s) + \frac{Z_1(s)}{Z_1(s) + Z_2(s)} V_o(s) \right)$

$$\Rightarrow H(s) = -\frac{K \frac{Z_2(s)}{Z_1(s) + Z_2(s)}}{1 + K \frac{Z_1(s)}{Z_1(s) + Z_2(s)}} = -\frac{K Z_2(s)}{(K+1) Z_1(s) + Z_2(s)}$$

In 11.50(c)

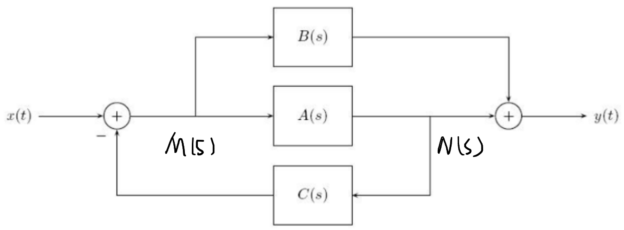
$$-K \left(V_1(s) \cdot \frac{Z_2(s)}{Z_1(s) + Z_2(s)} + \frac{Z_1(s)}{Z_1(s) + Z_2(s)} \cdot V_o(s) \right) = V_o S$$

$$\Rightarrow H(s) = -\frac{K Z_2(s)}{(K+1) Z_1(s) + Z_2(s)}$$

$\Rightarrow$ The system function is the same

(b) $K \gg 1$. $\quad H(s) = -\dfrac{K Z_2(s)}{(K+1) Z_1(s) + Z_2(s)} \approx -\dfrac{Z_2(s)}{Z_1(s)}$

Problem 4 Consider the following block diagram:



Find the transfer function $H(s)=Y(s)/X(s)$ of the overall system in terms of $A(s)$, $B(s)$ and $C(s)$. Note that the adder on the left side has one minus sign.

$$M(s) = X(s) - C(s)N(s)$$

$$Y(s) = N(s) + B(s)M(s)$$

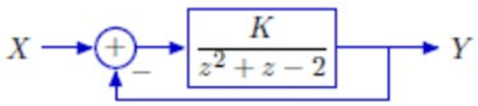
$$N(s) = M(s)A(s)$$

$$\Rightarrow M(s) = \frac{1}{1+C(s)A(s)}X(s) \qquad N(s) = \frac{A(s)}{1+C(s)A(s)}X(s)$$

$$\Rightarrow Y(s) = \frac{A(s)+B(s)}{1+C(s)A(s)} \cdot X(s) \quad \Rightarrow \quad H(s) = \frac{A(s)+B(s)}{1+C(s)A(s)}$$

Problem 5

Consider the following feedback system in which the box represents a causal LTI DT system that is represented by its system function.



a. Determine the range of K for which this feedback system is stable.

b. Determine the range of K for which this feedback system has real-valued poles.

$$\left(X(z) - Y(z)\right)\frac{k}{z^2+z-2} = Y(z)$$

$$\Rightarrow H(z) = \frac{\frac{k}{z^2+z-2}}{1+\frac{k}{z^2+z-2}} = \frac{k}{z^2+z+k-2}$$

the pole is at $z = -\frac{1}{2} \pm \sqrt{\frac{9}{4} - k}$

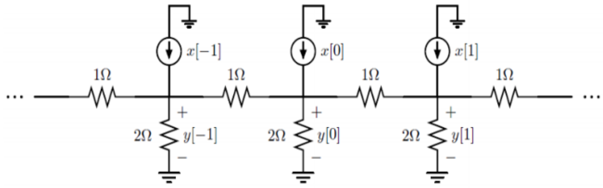
a. the system is stable when the poles are at the inside of unit circle

$\Rightarrow \quad 2 < k < 3$

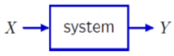
b. $\quad k < \frac{9}{4}$

Problem 6

An infinite network of resistors is excited by an infinite network of current sources as shown below.



We can consider the transformation from x to y as a DT system.



a. Show that this system is linear and "time"-invariant.

b. Determine the unit-sample response $h[n]$.

c. Determine the system function $H(z)$ and region of convergence.

d. Determine the system's pole(s) and zero(s).

$$x[n] = \frac{y[n]}{2} + \frac{y[n]-y[n+1]}{1} + \frac{y[n]-y[n-1]}{1} = \frac{5}{2}y[n] - y[n+1] - y[n-1]$$

a. $x_1[n] = \frac{5}{2}y_1[n] - y_1[n+1] - y_1[n-1]$

$x_2[n] = \frac{5}{2}y_2[n] - y_2[n+1] - y_2[n-1]$

suppose that $x_3[n] = a x_1[n] + b x_2[n]$

$$\Rightarrow x_3[n] = a\left(\frac{5}{2}y_1[n] - y_1[n+1] - y_1[n-1]\right) + b\left(\frac{5}{2}y_2[n] - y_2[n+1] - y_2[n-1]\right)$$

$$= \frac{5}{2}(a y_1[n] + b y_2[n]) - (a y_1[n+1] + b y_2[n+1]) - (a y_1[n-1] + b y_2[n-1])$$

$\Rightarrow y_3[n] = a y_1[n] + b y_2[n] \Rightarrow$ the system is linear

suppose that $x_4[n] = x_1[n+T]$

$\Rightarrow x_4[n] = \frac{5}{2}y_1[n+T] - y_1[n+T+1] - y_1[n+T-1]$

$\Rightarrow y_4[n] = y_1[n+T]$

$\Rightarrow$ the system is time-invarient

(b)(c) $X(z) = \frac{5}{2}Y(z) - zY(z) - \frac{1}{z}Y(z)$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{z}{-z^2+\frac{5}{2}z-1} = -\frac{z}{(z-2)(z-\frac{1}{2})}$$

$$= \frac{2}{3}\left(\frac{z}{z-\frac{1}{2}} - \frac{z}{z-2}\right)$$

$$\Rightarrow h(z) = H(z)\cdot X(z) = \frac{2}{3}\left(\frac{z}{z-\frac{1}{2}} - \frac{z}{z-2}\right)$$

$h(z)$	$h[n]$	ROC
$\frac{z}{z-\frac{1}{2}}$	$(\frac{1}{2})^n u[n]$	$\lvert z\rvert > \frac{1}{2}$
	$-(\frac{1}{2})^n u[-1-n]$	$\lvert z\rvert < \frac{1}{2}$
$\frac{z}{z-2}$	$2^n u[n]$	$\lvert z\rvert > 2$
	$-2^n u[-1-n]$	$\lvert z\rvert < 2$

$$\Rightarrow \quad h[n] = (\tfrac{1}{2})^n u[n] - 2^n u[n] \qquad |z| > 2$$

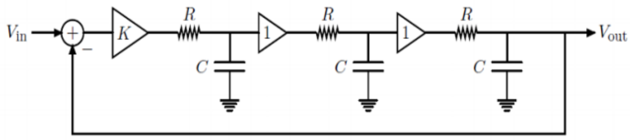
$$(\tfrac{1}{2})^n u[n] + 2^n u[-1-n] \qquad \tfrac{1}{2} < |z| < 2$$

$$-(\tfrac{1}{2})^n u[-1-n] + 2^n u[-1-n] \qquad |z| < \tfrac{1}{2}$$

(d) poles, $z = 2$, $z = \frac{1}{2}$ zeros: $z = 0$

Problem 7

The following feedback circuit was the basis of Hewlett and Packard's founding patent.

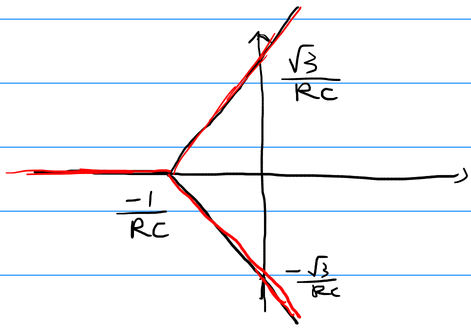


a. With $R = 1\,\mathrm{k}\Omega$ and $C = 1\mu\mathrm{F}$, sketch the pole locations as the gain K varies from 0 to ∞, showing the scale for the real and imaginary axes. Find the K for which the system is barely stable and label your sketch with that information. What is the system's oscillation period for this K?

b. How do your results change if R is increased to $10\,\mathrm{k}\Omega$?

a. $$H(s) = \frac{\frac{k}{(1+sRC)^3}}{1+\frac{k}{(1+sRC)^3}} = \frac{k}{(1+sRC)^3+k}$$

$\Rightarrow$ the poles are $\frac{-(k)^{\frac{1}{3}}-1}{RC}$, $\frac{e^{\frac{j\pi}{3}}(k)^{\frac{1}{3}}-1}{RC}$, $\frac{e^{\frac{-j\pi}{3}}(k)^{\frac{1}{3}}-1}{RC}$



the system is barely stable when the pole is at the $j\omega$ axis $\Rightarrow \sqrt[3]{k} = 2 \Rightarrow k = 8$

$\Rightarrow w = \frac{\sqrt{3}}{RC} = \sqrt{3} \times 10^3$

$\Rightarrow T = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{3}} \times 10^{-3}$ s

b. When R is increase to $10\,\mathrm{k}\Omega$, $k = 8$, $T = \frac{2\pi}{\sqrt{3}} \times 10^{-2}$ s

Problem 8

(a) $Y(s) = (X(s)-Y(s))\, H_c(s)\, H_p(s) = \frac{k\partial}{s+\partial}(X(s)-Y(s))$

$$\Rightarrow Y(s) = \frac{\frac{k\partial}{s+\partial}}{1+\frac{k\partial}{s+\partial}} X(s) = \frac{k\partial}{s+(k+1)\partial} X(s)$$

$$\Rightarrow e(s) = X(s) - Y(s) = \frac{s+\partial}{s+(k+1)\partial} X(s)$$

when $x(t) = u(t) \Rightarrow X(s) = \frac{1}{s}$

$$\Rightarrow e(s) = \frac{s+\partial}{s+(k+1)\partial} \cdot \frac{1}{s} = \frac{1}{k+1} \cdot \frac{1}{s} + \frac{k}{k+1} \frac{1}{s+(k+1)\partial}$$

$$\Rightarrow e(t) = \frac{1}{k+1} u(t) + \frac{k}{k+1} e^{-(k+1)\partial t} u(t)$$

$\Rightarrow$ when $x(t) = u(t)$, $e(t) \not\to 0$

(b) $$Y(s) = \frac{\frac{\partial}{s+\partial}(k_1 + \frac{k_2}{s})}{1+\frac{\partial}{s+\partial}(k_1+\frac{k_2}{s})} X(s)$$

$$\Rightarrow X(s) - Y(s) = \frac{1}{1+\frac{\partial}{s+\partial}(k_1+\frac{k_2}{s})} X(s)$$

$x(t) \neq u(t)$, let $k_2 = 0$, we can find k_1

$x(t) = u(t)$. $X(s) = \frac{1}{s}$

$$e(s) = X(s) - Y(s) = \frac{1}{s+\frac{\partial}{s+\partial}(k_1 s + k_2)}$$

$$= \frac{s+\partial}{s^2 + (\partial k_1 + \partial)s + \partial k_2}$$

$\Rightarrow k_1 > -1$, $k_2 > 0$, the system is stable

$$\Rightarrow \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\, e(s) = \lim_{s\to 0} \frac{s(s+\partial)}{s^2+(\partial k_1+\partial)s+\partial k_2} = 0$$

use PID

(3) $$e(s) = \frac{1}{1+H_c(s)H_p(s)} X(s) = \frac{1}{1+(k_1+\frac{k_2}{s}+k_3 s)\cdot\frac{1}{(s-1)^2}} \cdot X(s)$$

$$= \frac{(s-1)^2}{(s-1)^2+(k_1+\frac{k_2}{s}+k_3 s)} X(s)$$

$$= \frac{s(s-1)^2}{s^3+(k_3-2)s^2+(k_1+1)s+k_2} X(s)$$

$k_3>2$. $k_1>-1$, $k_2>0$, $(k_3-2)(k_1+1)>k_2$, the system is stable

when $x(t)=u(t) \Rightarrow X(s)=\frac{1}{s}$

$$\Rightarrow \lim_{t\to\infty} e(t) = \lim_{s\to 0} s e(s) = \lim_{s\to 0} \frac{s(s-1)^2}{s^3+(k_3-2)s^2+(k_1+1)s+k_2} = 0$$

use PI

$$e(s) = \frac{1}{1+(k_1+\frac{k_2}{s})\cdot\frac{1}{(s-1)^2}} X(s)$$

$$= \frac{s(s-1)^2}{s^3-2s^2+(k_1+1)s+k_2}$$

the system is stable when $-2>0$

$\Rightarrow$ the controler can't make the system stable