

## Signals and Systems – Spring 2025

### Problem Set 1

Issued: Feb. 18, 2025

Due: Mar. 4, 2025

Reading Assignments:

Signals and Systems (OWN), Chapter 1; Supplementary notes, Chapter 1-3

**Problem 1** OWN, Problem 1.21 (a)(c)(d)(f)

**Problem 2** OWN, Problem 1.22(a)(c)(e)(f)

**Problem 3** OWN, Problem 1.28(a)(c)(d)(e)

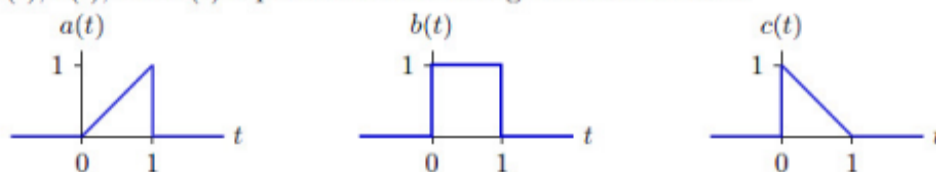
**Problem 4** OWN, Problem 1.31

#### **Problem 5** Geometric sums

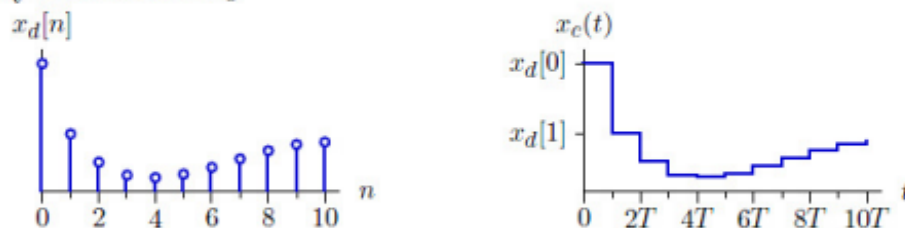
- Expand  $\frac{1}{1-a}$  in a power series. For what range of  $a$  does your answer converge?
- Find a closed-form expression for  $\sum_{n=0}^{N-1} a^n$ . For what range of  $a$  does your answer converge?

#### **Problem 6** Reconstructing CT Signals from Samples

Let  $a(t)$ ,  $b(t)$ , and  $c(t)$  represent the following functions of time.

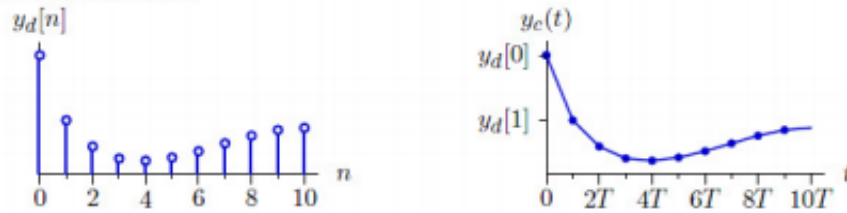


Let  $x_c(t)$  represent a continuous-time signal derived from the discrete-time signal  $x_d[n]$  using a zero-order hold, as illustrated below, where consecutive samples of  $x_d$  are separated by  $T$  seconds in  $x_c$ .



- Determine an expression for  $x_c(t)$  in terms of the samples  $x_d[n]$  and the functions  $a(t)$ ,  $b(t)$ , and  $c(t)$ .

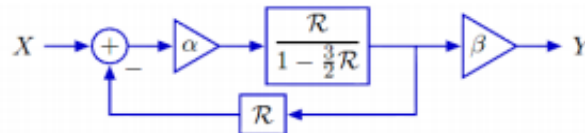
Let  $y_c(t)$  represent a continuous-time signal derived from the discrete-time signal  $y_d[n]$  using a piecewise linear interpolator, so that successive samples of  $y_d$  are connected by straight line segments.



- Determine an expression for  $y_c(t)$  in terms of the samples  $y_d[n]$  and the functions  $a(t)$ ,  $b(t)$ , and  $c(t)$ .
- Determine an expression for  $\frac{dy_c(t)}{dt}$  in terms of the samples  $y_d[n]$  and the functions  $a(t)$ ,  $b(t)$ , and  $c(t)$ .

### Problem 7 Missing Parameters

Consider the following system.



Assume that  $X$  is the unit-sample signal,  $x[n] = \delta[n]$ . Determine the values of  $\alpha$  and  $\beta$  for which  $y[n]$  is the following sequence (i.e.,  $y[0], y[1], y[2], \dots$ ):

$$0, 1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \dots$$

### Problem 8 Choose a bank

Consider two banks. Bank #1 offers a 3% annual interest rate, but charges a ¥1 service charge each year, including the year when the account was opened. Bank #2 offers a 2% annual interest rate, and has no annual service charge. Let  $y_i[n]$  represent the balance in bank  $i$  at the beginning of year  $n$  and  $x_i[n]$  represent the amount of money you deposit in bank  $i$  during year  $n$ . Assume that deposits during year  $n$  are credited to the balance at the end of that year but earn no interest until the following year.

- Use difference equations to express the relation between deposits and balances for each bank.
- Assume that you deposit ¥100 in each bank and make no further deposits. Solve your difference equations in part a numerically to determine your balance in each bank for the next 5 years. Which account has the larger balance 5 years after the initial investment (one year without interest and 4 years with interest)?