
Efficient Reward Poisoning Attacks on Online Deep Reinforcement Learning

Yinglun Xu, Qi Zeng, Gagandeep Singh

University of Illinois Urbana-Champaign

{yinglun6, qizeng2, ggnnd}@illinois.edu

Abstract

We study data poisoning attacks on online deep reinforcement learning (DRL) where the attacker is oblivious to the learning algorithm used by the agent and does not necessarily have full knowledge of the environment. We demonstrate the intrinsic vulnerability of state-of-the-art DRL algorithms by designing a general reward poisoning framework called adversarial MDP attacks. We instantiate our framework to construct several new attacks which only corrupt the rewards for a small fraction of the total training timesteps and make the agent learn a low-performing policy. Our key insight is that the state-of-the-art DRL algorithms strategically explore the environment to find a high-performing policy. Our attacks leverage this insight to construct a corrupted environment for misleading the agent towards learning low-performing policies with a limited attack budget. We provide a theoretical analysis of the efficiency of our attack and perform an extensive evaluation. Our results show that our attacks efficiently poison agents learning with a variety of state-of-the-art DRL algorithms, such as DQN, PPO, SAC, etc. under several popular classical control and MuJoCo environments.¹

1 Introduction

In several important applications such as robot control Christiano et al. [2017] and recommendation systems Afsar et al. [2021], Zheng et al. [2018], state-of-the-art online deep reinforcement learning (DRL) algorithms rely on human feedback in terms of rewards, for learning high-performing policies. This dependency raises the threat of reward-based data poisoning attacks during training: a user can deliberately provide malicious rewards to make the DRL agent learn low-performing policies. Data poisoning has already been identified as the most critical security concern when employing learned models in the industry Kumar et al. [2020]. Thus, it is essential to study whether state-of-the-art DRL algorithms are vulnerable against reward poisoning attacks to discover potential security vulnerabilities and motivate the development of more robust training algorithms.

Challenges in poisoning DRL agents: To uncover practical vulnerabilities, it is critical that the attack does not rely on unrealistic assumptions about the attacker’s capabilities. Therefore for ensuring a practically feasible attack, we require that: (i) the attacker has no knowledge of the exact DRL algorithm used by the agent as well as the parameters of the neural network used for training, (ii) the attacker does not have detailed knowledge about the agent’s environment, and (iii) to ensure undetectability, the amount of reward corruption that the attacker can apply is limited (see Sec. 3). As we show in appendix C, these restrictions make finding an efficient attack very challenging.

This work: efficient poisoning attacks on DRL: To the best of our knowledge, no prior work studies the vulnerability of the DRL algorithms to reward poisoning attacks under the practical restrictions

¹Our code is released at https://github.com/YinglunXu/reward_poisoning_attack_drl.

mentioned above. To overcome the challenges in designing efficient attacks and demonstrate the vulnerability of the state-of-the-art DRL algorithms, we make the following contributions:

1. We propose a general, parametric reward poisoning framework for DRL algorithms, which we call adversarial MDP attack, and instantiate it to generate several attack methods.
2. We provide a theoretical analysis of our attack methods based on certain assumptions on the efficiency of the DRL algorithms which yields several insightful implications.
3. We provide an extensive evaluation of our attack methods for poisoning the training with several state-of-the-art DRL algorithms, such as DQN, PPO, SAC, etc., in the classical control and MuJoCo environments, commonly used for developing and testing DRL algorithms. Our results show that our attack methods significantly reduce the performance of the policy learned by the agent in the majority of the cases and are considerably more efficient than baseline attacks. We further validate the implications suggested by our theoretical analysis by observing the corresponding phenomena in experiments.

2 Related Works

Testing time attack on RL Testing time attack (evasion attack) in deep RL setting is popular in literature Huang et al. [2017], Kos and Song [2017], Lin et al. [2017]. For an already trained policy, testing time attack find adversarial examples where the policy have undesired behavior. In contrast, our training time attack corrupts reward to make the agent learn low-performing policies.

Data poisoning attack on bandit and tabular RL settings Jun et al. [2018], Liu and Shroff [2019], Xu et al. [2021b] study data poisoning attack against bandit algorithms. Ma et al. [2019] studies the attack in the offline tabular RL setting. Rakhsha et al. [2020], Zhang et al. [2020] study the online tabular RL setting relying on full or partial knowledge on the environment and the learning algorithm. Liu and Lai [2021], Xu et al. [2021a] discuss the attack that can work with no knowledge or weak assumptions on the learning algorithm or the environment.

Data poisoning attack on DRL There are a number of works study data poisoning attack on DRL through injecting adversarial examples during training time. Behzadan and Munir [2017], Kiourtzi et al. [2019]. In practice there is no theoretical analysis on the attack, and the attack is applied at every round. The attack method that is most related to our attack on DRL is Sun et al. [2020]. This work proposes a method to identify and apply corruption at timesteps which have more influence on learning. The main limitation in their attack is that (1) it only works for on-policy learning algorithms, and (2) the attack relies on strong assumption on learning algorithms that the behaviors of different learning algorithms trained on the same data are similar. Our attacks can work on model-free DRL algorithms of all popular learning paradigms, and only assume the learning algorithm to be efficient.

3 Background

Reinforcement learning. We consider a standard RL setting where an agent trains by interacting with an environment. The interaction involves the agent observing a state representation of the environment, taking an action to change this state, and receiving a reward for its actions. Formally, an environment is represented by a Markov decision process (MDP), $\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \mu\}$, where \mathcal{S} is the state space, \mathcal{A} is the action space, \mathcal{P} is the state transition function, \mathcal{R} is the reward function, and μ is the distribution of the initial states. The training process consists of multiple episodes where each episode is initialized with a state sampled from μ . The agent interacts with the environment at each timestep until the episode ends. A policy $\pi : \mathcal{S} \rightarrow \mathcal{A}$ is a mapping from the state space to the action space. A value function $V_{\mathcal{M}}^{\pi}(s)$ is the expected reward an agent obtains by following the policy π starting at state s in the environment \mathcal{M} . We denote $\mathcal{V}_{\mathcal{M}}^{\pi} := \mathbb{E}_{s_0 \sim \mu} V_{\mathcal{M}}^{\pi}(s_0)$ as the policy value for a policy π in the environment \mathcal{M} , which we use to measure the performance of π . The goal of the RL agent is to find and report the optimal policy with the highest policy value $\pi^* = \text{argmax}_{\pi} \mathcal{V}_{\mathcal{M}}^{\pi}$.

Reward poisoning attack on deep RL. In this work we consider a standard data poisoning attack setting Jun et al. [2018], Rakhsha et al. [2020] where a malicious adversary tries to manipulate the agent by poisoning the reward received by the agent from the environment during training. The attacker observes the state-action pair (s^t, a^t) generated during training at each timestep t and injects a corruption Δ^t on the true reward r^t . As a result, the environment returns the agent with the

corrupted observation $(s^t, a^t, s^{t+1}, r^t + \Delta^t)$ where s^{t+1} is the next state. Next, we describe the restrictions on the attacker's capabilities as mentioned in the introduction:

1. **Limited budget.** The attacker can only corrupt a small number of timesteps C , i.e., $\sum_{t=0}^T \mathbb{1}\{\Delta^t \neq 0\} \leq C$ and $C \ll T$ where T is the total number of training steps.
2. **Limited per-step corruption.** The corruption at each timestep is limited by $|\Delta^t| \leq B, \forall t \in [T]$.
3. **Limited per-episode corruption:** The total corruption across an episode is limited by $\sum_{t \in t^e} |\Delta^t| \leq E$ where t^e is the set of all timesteps in an episode e .
4. **Oblivious of the DRL algorithm.** The attacker has no knowledge of the training algorithm or any parameters in the network used by the agent while training.
5. **Oblivious of the environment.** The attacker has no knowledge about the MDP \mathcal{M} except for the number of dimensions and range of each dimension in the state and action space \mathcal{S}, \mathcal{A} . We also discuss cases where the attack uses additional information on \mathcal{M} to improve its efficiency.

Under the above constraints, the goal of the attacker is to minimize the performance of the learned policy reported by the DRL algorithm that it believes to be the best by the end of the training. More specifically, let π_0 be the policy learned by the DRL algorithm, the goal of the attacker is to minimize $\mathcal{V}_{\mathcal{M}}^{\pi_0}$. We note that $\pi^* \neq \pi_0$ in general. In practice, for experiments in Sec. 6, we represent $\mathcal{V}_{\mathcal{M}}^{\pi_0}$ by the highest policy value of all policies learned by the algorithm at the end of each epoch during training, where an epoch is a fixed number of timesteps. Next we will formulate the problem of finding an efficient attack, provide new attack methods and analyze their efficiency. To simplify the theoretical analysis, we do not consider the constraint on per-episode corruption E in Sec. 4 and Sec. 5. We will consider the full constraints in experiments in Sec. 6.

4 Formulating Reward Poisoning Attack

A reward poisoning attack algorithm can be represented by its attack strategy A^t at each timestep during training. An attack strategy A^t can depend on the full observation before that attack, that is, all the states $s^{1:t}$, actions $a^{1:t}$, rewards $r^{1:t-1}$, and corruptions $\Delta^{1:t-1}$. Then the output of A^t , i.e, the corruption on reward, satisfies $\Delta^t = A^t(s^{1:t}, a^{1:t}, r^{1:t-1}, \Delta^{1:t-1})$. In appendix C we will show that the space for possible attack algorithms is exponentially large in T . Searching for the optimal attack is computationally hard in the DRL setting, and requires full knowledge of the learning algorithm and environment. Thus the goal of our work is not to find the optimal attacks, but finding feasible attacks that can significantly influence learning algorithms under practical constraints.

Problem formulation: find a feasible poisoning attack We consider a reward poisoning attack as feasible if it can make a learning algorithm learn a policy with low policy value under the attack. Formally, the attacker's objective can be stated using the following constraints:

$$\mathcal{V}_{\mathcal{M}}^{\pi_0} \leq V; \Delta^t = A^t(s^{1:t}, a^{1:t}, \hat{r}^{1:t-1}); \sum_{t=1}^T \mathbb{1}[\Delta^t \neq 0] \leq C; |\Delta^t| \leq B, \forall t \in [T]. \quad (1)$$

An attack algorithm is efficient if it satisfies (1) with low values for B, C, V with $V < \mathcal{V}_{\mathcal{M}}^{\pi^*}$ otherwise the attack is trivial as $\mathcal{V}_{\mathcal{M}}^{\pi_0} \leq \mathcal{V}_{\mathcal{M}}^{\pi^*}$ holds without any attack. As confirmed by our theoretical analysis in Sec. 5 and experiments in Sec. 6, finding an efficient attack based on (1) is non-trivial.

Adversarial MDP attack To find attack algorithms for solving (1), we introduce a general parametric attack framework called "adversarial MDP attack" for poisoning the training of deep RL agents. The high level idea behind our attack is to construct a fixed adversarial environment to train the agent. This idea has been applied in designing attack in the simpler bandit Liu and Shroff [2019] and tabular setting Rakhsa et al. [2020] where they formulate the problem of finding the best adversarial environment for their attack goal as an optimization problem and solve it directly. Solving such an optimization problem is computational infeasible in the deep RL setting due to the complexity of both the environment and learning algorithm. Therefore we design new efficient algorithms that are suited to the deep RL setting and our attack scenario. In our attack, the attacker constructs an adversarial MDP $\widehat{\mathcal{M}} = \{\mathcal{S}, \mathcal{A}, \mathcal{P}, \widehat{\mathcal{R}}, \gamma, \mu\}$ for the agent to train on by injecting the corrupted reward $\widehat{\mathcal{R}}$ to the environment during training. More specifically, for an adversarial MDP attack with $\widehat{\mathcal{M}}$, its attack strategy at round t only depends on the current state and action taken by the learning algorithm:

$$\Delta^t = A^t(s^t, a^t) = \hat{R}(s^t, a^t) - R(s^t, a^t) \quad (2)$$

Next, we compute bounds on V , B , and C such that $A^{1:T}$ constructed using (2) from a given $\widehat{\mathcal{M}}$ in our framework is a feasible solution to (1). We make two simplifying assumptions: (i) the learning algorithm can always find and report the optimal policy from a fixed environment, i.e., $\pi_0 = \pi^*$ always holds. We note that our experimental results show that our attack succeeds with low values of V , B and C on state-of-the-art deep RL algorithms that do not always learn an optimal policy, and (ii) the learning algorithm explores strategically, that is, instead of uniformly exploring all state action pairs, the algorithm does not waste many rounds to explore the state action pairs that have little value. This assumption is satisfied by RL algorithms Dong et al. [2019], Jin et al. [2018], Agarwal et al. [2019] and also validated in our experiments.

Lower bounds on V , B , and C The bound on V relates to the optimal policy $\hat{\pi}^* := \operatorname{argmax}_{\pi} \mathcal{V}_{\mathcal{M}}^{\pi}$ that the algorithm will learn under $\widehat{\mathcal{M}}$ based on our first assumption, i.e., $\pi_0 = \hat{\pi}^*$. Then we can directly bound V by the policy value of $\hat{\pi}^*$ under \mathcal{M} : $V \geq \mathcal{V}_{\mathcal{M}}^{\hat{\pi}^*}$. It is straightforward to give the bound on B as $B \geq \|\hat{\mathcal{R}} - \mathcal{R}\|_{\infty}$. For bounding C , our attack applies corruption whenever the learning algorithm chooses an action a^t at a state s^t such that the reward function at this state action pair are different for the real and adversarial MDP, i.e., $\hat{\mathcal{R}}(s^t, a^t) \neq \mathcal{R}(s^t, a^t)$. Then C can be bound as $C \geq \sum_{t=1}^T \mathbb{1}[\hat{\mathcal{R}}(s^t, a^t) \neq \mathcal{R}(s^t, a^t)]$. Under our second assumption, the bound on C will be low if $\widehat{\mathcal{M}}$ satisfies $\hat{\mathcal{R}}(s, \hat{\pi}^*(s)) = \mathcal{R}(s, \hat{\pi}^*(s))$ for all states s , as for most of the time $a^t = \hat{\pi}^*(s)$, resulting in $\mathbb{1}[\hat{\mathcal{R}}(s^t, a^t) \neq \mathcal{R}(s^t, a^t)] = 0$. Intuitively, raising the value of B or C increases the number of candidate adversarial MDP's that satisfy (1) with low value of V . Next we will instantiate our framework to construct specific $\widehat{\mathcal{M}}$ that result in efficient attacks which significantly reduce the performance of the learned policy under low budget.

5 Poisoning Attack Methods

In this section we design new reward poisoning attacks for deep RL by instantiating our adversarial MDP attack framework. Each instantiation provides a parameterized way to construct an adversarial MDP $\widehat{\mathcal{M}}$ via a parameter Δ , corresponding to the amount of reward corruption for poisoning applied by the attacker at a timestep. The definition of Δ yields $|\Delta| = \|\hat{\mathcal{R}} - \mathcal{R}\|_{\infty}$, resulting in a lower bound on requirement $B \geq |\Delta|$. For a given value of V , let $\Delta(V)$ be the minimum absolute value of the parameter Δ for the instantiation to satisfy the requirement on V in (1). We note that $\Delta(V)$ corresponds to the minimum requirement on B for the attack to satisfy V in (1). We provide symbolic expressions for $\Delta(V)$ corresponding to each instantiation and then use it to construct an upper bound on $\Delta(V)$, which is easier to reason about than exact expressions. We define $G_V^{\mathcal{M}}$ and $B_V^{\mathcal{M}}$ with respect to value V to be the set of all policies with policy value $> V$ and $\leq V$ respectively under \mathcal{M} . The upper bound on $\Delta(V)$ will be constructed using $G_V^{\mathcal{M}}$ and $B_V^{\mathcal{M}}$. We will empirically examine the requirement on C for each instantiation in Sec. 6. An attack is efficient if to satisfy a certain value of V in (1), it requires low value of C and $B = \Delta(V)$. All the attack methods we are going to propose do not require any knowledge about both the learning algorithm and the environment, though one method gain benefits from full or partial knowledge of the environment as we find in practice. For simplicity, we start from attack on environments with discrete action space and then show how to transfer the attack and the corresponding analysis to continuous action space. Note that our analysis in this section is based on the two assumptions we made in Sec. 4 that the learning algorithms can always learn the optimal policy and explore strategically. All the proofs for the theorems and lemmas can be found in the appendix B.

Uniformly at random (UR) attack. We first introduce a trivial instantiation of adversarial MDP attack framework as a baseline, which we call the UR attack. Here, the attacker randomly decides whether to corrupt the reward at a timestep with a fixed probability p and amount Δ regardless of the current state and action. Formally, the attack strategy of the UR attacker at time t is: $A^t(s^t, a^t) = \Delta$ with probability p , otherwise $A^t(s^t, a^t) = 0$. Let $V_{\max} = \max_{\pi} \mathcal{V}_{\mathcal{M}}^{\pi}$ and $V_{\min} = \min_{\pi} \mathcal{V}_{\mathcal{M}}^{\pi}$ be the maximum and minimum policy value computed over all policies under \mathcal{M} . We provide the following bounds on $\Delta(V)$ for the UR attack:

Theorem 5.1. For the UR attack parameterized with probability p , the exact expression and an upper bound on $\Delta(V)$ are:

$$\Delta(V) = \min_{\pi_1 \in B_V^{\mathcal{M}}} \max_{\pi_2 \in G_V^{\mathcal{M}}} \frac{\mathcal{V}_M^{\pi_2} - \mathcal{V}_M^{\pi_1}}{p \cdot |L^{\pi_1} - L^{\pi_2}|} \leq \frac{V_{\max} - V_{\min}}{p \cdot \min_{\pi_1 \in G_V^{\mathcal{M}}} \max_{\pi_2 \in B_V^{\mathcal{M}}} |L^{\pi_1} - L^{\pi_2}|}$$

where $L^\pi := \sum_s \mu^\pi(s)$ is the expected length of an episode for an agent following the policy π . Moreover, the expression only holds for Δ with the same sign as $L^{\pi_1} - L^{\pi_2}$ for the π_1, π_2 that realize the min-max condition.

Theorem 5.1 shows that the value of $\Delta(V)$ depends on the difference in episode length from policies in $G_V^{\mathcal{M}}$ and $B_V^{\mathcal{M}}$. A low value of p also makes high value of $\Delta(V)$. In addition, the sign of Δ needs to be chosen correctly to make $\mathcal{V}_M^{\hat{\pi}^*} < \mathcal{V}_M^{\pi^*}$, otherwise it can make the optimal policies look even better under $\widehat{\mathcal{M}}$, which yields the following implications:

Implication 1. If the UR attack applies corruption in the wrong direction, then compared to the attack in the right direction, it requires higher value of $|\Delta|$ to make the learning agent learn policy of same performance, or it can never make it regardless of the value of $|\Delta|$.

Learned policy evasion (LPE) attack The high-level idea behind the LPE attack is to make all policies of good performance appear bad to the learning agent. Intuitively, policies of good performance should share similar behavior as there is usually certain general strategy to behave well under the environment. Therefore if the attacker can make the actions correspond to such behavior look bad, then all the good policies will appear bad to the agent. Formally, the LPE attack is characterized by the policy π^\dagger available to the attacker, and it penalizes the learning algorithm whenever it chooses an action that corresponds to π^\dagger . Correspondingly, the attack strategy is: $A^t(s^t, a^t) = \Delta \cdot \mathbb{1}\{a^t = \pi^\dagger(s^t)\}$, where $\Delta < 0$ is a fixed value. π^\dagger is learned offline by the attacker before the agent starts learning. We will show different ways the attacker can generate π^\dagger in Sec. 6. Next we analyze the efficiency of the attack. To help our analysis, we introduce the following definition to measure the similarity $D(\pi_1, \pi_2)$ between two policies π_1 and π_2 :

Definition 5.2. (Similarity between policies) The similarity of a policy π_1 to a policy π_2 is: $D(\pi_1, \pi_2) = \sum_{s \in \mathcal{S}} \mu^{\pi_1}(s) \mathbb{1}[\pi_1(s) = \pi_2(s)]$.

The similarity of π_1 to π_2 increases with the frequency with which π_2 takes the same action as π_1 in the same state s . Note that $D(\pi_1, \pi_2) \neq D(\pi_2, \pi_1)$, and $D(\pi_1, \pi_2) \leq L^{\pi_1}$.

Theorem 5.3. For LPE attack with π^\dagger , the expression and an upper bound on $\Delta(V)$ are:

$$\Delta(V) = \min_{\pi_1 \in B_V^{\mathcal{M}}} \max_{\pi_2 \in G_V^{\mathcal{M}}} \frac{\mathcal{V}_M^{\pi_2} - \mathcal{V}_M^{\pi_1}}{D(\pi_2, \pi^\dagger) - D(\pi_1, \pi^\dagger)} \leq \frac{V_{\max} - V_{\min}}{\min_{\pi \in G_V^{\mathcal{M}}} D(\pi, \pi^\dagger) - \min_{\pi \in B_V^{\mathcal{M}}} D(\pi, \pi^\dagger)}.$$

In appendix B we will show that $\min_{\pi \in B_V^{\mathcal{M}}} D(\pi, \pi^\dagger)$ is likely to be 0 in general cases. Theorem 5.3 shows that the requirement on Δ for the LPE attack is inversely proportional to the minimum similarity between π^\dagger and a policy from $G_V^{\mathcal{M}}$. In practice we observe that in most cases there are usually certain behaviors shared in common by the non-trivial policies that have better performance than random ones. This yields the following implications:

Implication 2. With the same per-step reward corruption Δ , the LPE attack can make the agent learn policies of worse performance with high performing π^\dagger compared to the LPE attack with random policies.

The LPE attack can generate a random policy as π^\dagger . For experiments in Sec. 6, to simplify implementation, we generate the random policy through random initialization of a learning algorithm different to that used by the agent. When the attacker has access to the environment, it can learn a high performing policy as π^\dagger which may provide better attack performance. To estimate the requirement on C , first we give the following lemma:

Lemma 5.4. For the LPE attack, $\widehat{\mathcal{R}}(s, \hat{\pi}^*(s)) = \mathcal{R}(s, \hat{\pi}^*(s)) \iff D(\pi^\dagger, \hat{\pi}^*) = 0$. Given sufficient Δ , the optimal policy under $\widehat{\mathcal{M}}$ constructed by the LPE attack with π^\dagger satisfies $D(\pi^\dagger, \hat{\pi}^*) = 0$.

Recall that in Sec. 4, we estimate that the requirement on C will be low if the adversarial and real reward are the same at all $(s, \hat{\pi}^*(s))$, i.e., $\widehat{\mathcal{R}}(s, \hat{\pi}^*(s)) = \mathcal{R}(s, \hat{\pi}^*(s))$. Lemma 5.4 suggests that this condition will hold for the LPE attack given sufficient Δ , as for the LPE attack $D(\pi^\dagger, \hat{\pi}^*) = 0 \iff \widehat{\mathcal{R}}(s, \hat{\pi}^*(s)) = \mathcal{R}(s, \hat{\pi}^*(s))$. This yields

Implication 3: Under the LPE attack with sufficient $|\Delta|$, the learning agent will gradually converge to $\hat{\pi}^*$ under $\widehat{\mathcal{M}}$ eventually and no corruptions will be applied afterwards, resulting in a decrease in attack frequency as the training goes on.

Random policy inducing (RPI) attack The intuitive idea behind the RPI attack is to make the agent believes that a random policy is an optimal one. To achieve this, the attacker can make all the actions that are different to the ones given by the random policy look bad. The RPI attack is characterized by a randomly generated policy π^\dagger , and it penalizes the learning algorithm whenever it doesn't follow the action that corresponds to π^\dagger . Formally, the attack strategy of the RPI attack with a policy π^\dagger is: $A^t(s^t, a^t) = \Delta \cdot \mathbb{1}\{a^t \neq \pi^\dagger(s^t)\}$, where $\Delta < 0$ is a fixed value. We have the following expression and upper bound on $\Delta(V)$:

Theorem 5.5. *For RPI attack with policy π^\dagger , the expression and an upper bound on $\Delta(V)$ are:*

$$\Delta(V) = \min_{\pi_1 \in B_V} \max_{\pi_2 \in G_V^\mathcal{M}} \frac{\mathcal{V}_\mathcal{M}^{\pi_2} - \mathcal{V}_\mathcal{M}^{\pi_1}}{(L^{\pi_2} - L^{\pi_1}) - (D(\pi_2, \pi^\dagger) - D(\pi_1, \pi^\dagger))} \leq \frac{V_{\max} - V_{\min}}{\min_{\pi \in G_V^\mathcal{M}} (L^\pi - D(\pi, \pi^\dagger))}$$

Theorem 5.5 suggests that the requirement on B will be low if all policies in $G_V^\mathcal{M}$ have low similarity to the policy π^\dagger . With the same observation we give about similarity between policies better than the random ones, it suggests that $\Delta(V)$ will be less when π^\dagger is a random policy. Then theorem 5.5 gives

Implication 4: With the same per-step reward corruption Δ , the RPI attack with random π^\dagger can make the learning algorithm learn policies of worse performance than the RPI attack with high performing π^\dagger .

To estimate the requirement on C , first we give the following lemma:

Lemma 5.6. *For the RPI attack, $\widehat{\mathcal{R}}(s, \hat{\pi}^*(s)) = \mathcal{R}(s, \hat{\pi}^*(s)) \iff \hat{\pi}^* = \pi^\dagger$. Given sufficient Δ , the optimal policy under $\widehat{\mathcal{M}}$ constructed by the RPI attack with π^\dagger is $\hat{\pi}^* = \pi^\dagger$.*

Lemma 5.6 suggests that with sufficient value of $|\Delta|$, the RPI attack can make $\hat{\pi}^* = \pi^\dagger$, then no corruption will be applied when the learning agent converges to $\hat{\pi}^*$, resulting in less requirement on C . This implies that

Implication 5: *For the RPI attack with sufficient $|\Delta|$, the learning agent will gradually converge to π^\dagger , resulting in a decrease in the frequency of corruption as the training goes on.*

Our two main attack methods LPE and RPI proposed so far work with negative corruption on reward and avoid attacking the action corresponding to $\hat{\pi}^*$ in every state. The methods will work well under our assumptions on learning algorithms. An alternative is to attack with positive corruption on reward. Under the framework of adversarial MDP attack, such attack can result in less requirement on C when the learning agent does the opposite to our second assumption, i.e., it explores the sub-optimal actions more often. To compare the difference in performance between the two types of attack, we propose and analyze a variant of RPI attack called random policy promoting (RPP) attack.

Random policy promoting (RPP) attack The RPP attack shares the same intuition about highlighting a random policy, but instead it positively rewards the actions corresponding to the random policy. Formally, the attack strategy of a RPP attack with a policy π^\dagger is given as $A^t(s^t, a^t) = \Delta \cdot \mathbb{1}\{a^t = \pi^\dagger(s^t)\}$, where $\Delta > 0$ is a fixed value. The expression and an upper bound on $\Delta(V)$ are:

Theorem 5.7. *Under RPP attack with policy π^\dagger , the expression and an upper bound for $\Delta(V)$ are:*

$$\Delta(V) = \min_{\pi_1 \in B_V} \max_{\pi_2 \in G_V^\mathcal{M}} \frac{\mathcal{V}_\mathcal{M}^{\pi_2} - \mathcal{V}_\mathcal{M}^{\pi_1}}{D(\pi_2, \pi^\dagger) - D(\pi_1, \pi^\dagger)} \leq \frac{V_{\max} - V_{\min}}{\max\{L^{\pi^\dagger} - \max_{\pi \in G_V^\mathcal{M}} D(\pi, \pi^\dagger), 0\}}$$

We note that compared to the RPI attack, the RPP attack requires more B if $L^{\pi^\dagger} < \min_{\pi \in G_V^M} L^\pi$, as it results in $L^{\pi^\dagger} - \max_{\pi \in G_V^M} D(\pi, \pi^\dagger) < \min_{\pi \in G_V^M} L^\pi - \max_{\pi \in G_V^M} D(\pi, \pi^\dagger) \leq \min_{\pi \in G_V^M} (L^\pi - D(\pi, \pi^\dagger))$. Further if $L^{\pi^\dagger} - \max_{\pi \in G_V^M} D(\pi, \pi^\dagger)$ becomes less than 0, then (1) can never be satisfied with any value of B . This happens when a policy in G_V^M benefits more from the positive corruption than policies from B_V^M . This implies that

Implication 6: For environments where policies of high values are associated with long episodes, the RPI attack can make the learning agent learn worse policy than the RPP attack, and the RPP attack may even be ineffective regardless of Δ .

As we mentioned before, the requirement on C for the RPP and RPI attack depends on how much the learning algorithm deviates from our second assumption in Sec. 4. From experiments in Sec. 6, under the same constraints on attack, we observe that for most learning algorithms, the RPI attack usually achieves better attack results. Our results suggest that although the RPP attack can be more efficient for certain learning algorithms and environments, the RPI attack is in general more reliable and efficient than the RPP attack.

Finally, note that the upper bound we provide for $\Delta(V)$ of all attacks here satisfies $\Delta(V) \geq (V_{\max} - V_{\min})/L_{\max}$, where $L_{\max} := \max_\pi L^\pi$ is the maximum episode length. Therefore for experiments in Sec. 6, we always let $|\Delta| > (V_{\max} - V_{\min})/L_{\max}$.

Extension to environments with continuous action space To extend the attacks above from discrete to continuous action space, we adaptively discretize the continuous action space with respect to the action from the learning agent. Formally, we consider two actions as the same if the distance between them in the action space is less than a threshold r . Then the aforementioned attack methods can decide whether to apply corruption based on whether two actions are considered as the same given r . For example, the attack strategy for the LPE attack with π^\dagger in continuous action space is $A^t = \Delta \cdot \mathbb{1}[||a^t - \pi^\dagger(s^t)||_2 \leq r]$. Accordingly, we define the similarity between policies for continuous action space parameterized by r :

Definition 5.8. (Similarity between policies under distance r) The similarity of a policy π_1 to a policy π_2 in continuous action space parameterized by r is defined as $D_c(\pi_1, \pi_2, r) = \sum_{s \in \mathcal{S}} \mu^{\pi_1}(s) \mathbb{1}[||\pi_1(s) - \pi_2(s)||_2 \leq r]$.

By replacing $D(\pi_1, \pi_2)$ with $D_c(\pi_1, \pi_2, r)$, we can transfer the analysis for the attack from discrete action space to continuous action space. Note that while we measure the distance in L2-norm, but any other norm will also work. The extension adds an additional parameter r for the attack methods to decide. In appendix A we show how the choice on r influence the attacks.

6 Experiments

We evaluate our attacks methods from Sec. 5 for poisoning training with state-of-the-art DRL algorithms in both the discrete and continuous setting. We consider learning in environments typically used for assessing the performance of the DRL algorithms in the literature. We also experimentally show that the implications in Sec. 5 hold when attacking practical DRL algorithms even though they do not necessarily satisfy our assumptions from Sec. 3.

Learning algorithms and environments We consider 4 common Gym environments Brockman et al. [2016] in the discrete case: CartPole, LunarLander, MountainCar, and Acrobot, and the continuous case: HalfCheetah, Hopper, Walker2d, and Swimmer. The DRL algorithms in the discrete setting are: dueling deep Q learning (Duel) Wang et al. [2016] and double dueling deep Q learning (Double) Van Hasselt et al. [2016] while for the continuous case we choose: deep deterministic policy gradient (DDPG), twin delayed DDPG (TD3), soft actor critic (SAC), and proximal policy optimization (PPO). Overall, the 6 algorithms we consider cover the popular paradigms in model-free learning algorithms: policy gradient, Q-learning, and their combination. The implementation of the algorithms are based on the spinningup project Achiam [2018].

Experimental setup We consider more strict and practical constraints (as described in Sec. 3) on the attacker than in our theoretical analysis. To work with extra constraints, we modify our adversarial MDP attack framework: if applying corruption as per the attack strategy given by the framework in

Sec. 4 will break the constraints on E or C at a time step, then the attacker applies no corruption at that time step. We choose T to ensure that the learning algorithm can converge within T time steps without the attack. We evaluate the effectiveness of our attacks with different values of C such that the ratio C/T is low. Since the attacker is unaware of the learning algorithm in all of our attacks, for each environment, whenever the attacker needs to learn π^\dagger offline, it does so with an algorithm different from the agent's learning algorithm. More specifically, for each environment we select a pair of learning algorithms that are most efficient in learning from the environment (without attack), then while we use one of them for the learning agent to train, the other will be used by the attacker to learn a high performing or generate a random π^\dagger . Our criteria yields cases where the learning algorithms in a pair belong to different learning paradigms and has different architectures of neural networks. Our results demonstrate that the efficiency of our attack methods does not depend upon similarity between the learning algorithms.

To determine E and B , we note that $V_{\max} - V_{\min}$ represents the maximum environment-specific net reward an agent can get during an episode, and $\frac{(V_{\max} - V_{\min})}{L_{\max}}$ represents the range of average reward at a time step for an agent. We set $E = \lambda_E \cdot (V_{\max} - V_{\min})$, and $B = \lambda_B \cdot \frac{(V_{\max} - V_{\min})}{L_{\max}}$ where $\lambda_E \leq 1$, $\lambda_B > 1$ are normalization constants to ensure that the values of E and B represent similar attack power across different experiments. We choose $\lambda_E \leq 1$ to ensure that the corruption in an episode is \leq the maximum net reward a policy can achieve during an episode. We choose $\lambda_B > 1$ that ensures $B > \frac{(V_{\max} - V_{\min})}{L_{\max}}$. This is because the upper bounds on $\Delta(V)$ according to our theorems in Sec. 5 should be $> \frac{(V_{\max} - V_{\min})}{L_{\max}}$ for all attack algorithms. The exact values of T , B , and E for different environments are in the appendix A.

Main results A subset of our main results are shown in Fig. 1. The remaining set can be found in

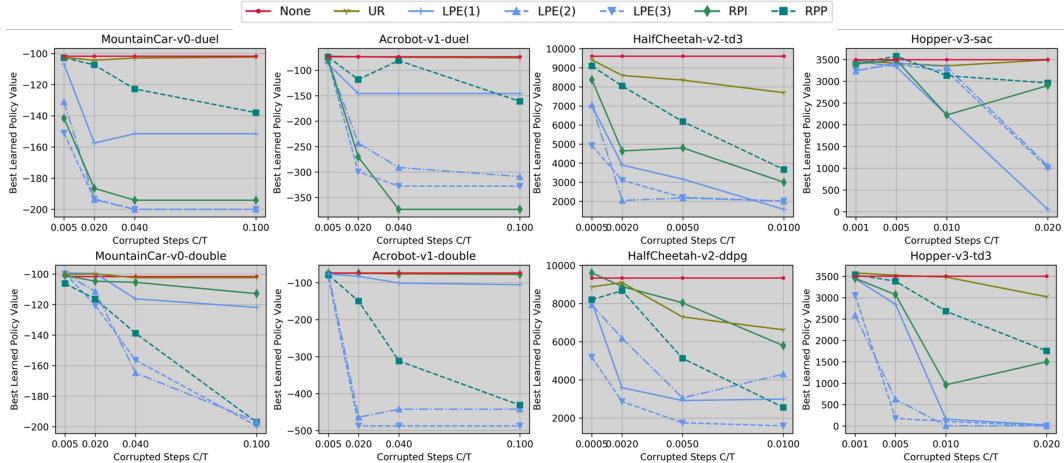


Figure 1: Highest policy value of learned policies by algorithms under no or different attacks.

appendix A. The x axis is C/T ; the y axis is the policy value of the best policy the learning algorithm learned after each epoch across the whole training process. The y value at each data point is averaged over 10 runs under the same setting. The lower the value on the y axis under an attack, the more powerful the attack is. We consider an attack to be successful if the resulting policy value is lower than the one from the two baselines: the learning agent under no attack and the UR attack. For the UR attack, we set $p = C/T$ so that the corrupted rounds are randomly distributed in the whole training process. We always choose the sign of Δ that gives the best attack result. In Fig. 1, the best UR attack has only a small influence on the learning algorithm.

For the LPE attack, we consider three variants based on π^\dagger : (1) the attacker does not have any knowledge about the environment and uses a random policy as π^\dagger , (2) the attacker trains in the environment for T steps and chooses the best policy as π^\dagger , and (3) Same as (2) except that the attacker selects a policy as π^\dagger which has a policy value that is the closest to the mean of the policy values in the first two cases. This variant is used to check the effect of learning suboptimal π^\dagger on the attack performance. We observe in Fig. 1 that the variants (2) and (3) always succeed while (1) fails

in 1 case (LunarLander learned by Double). Comparing the three variants, in general the variants (2), (3) usually achieve better attack results than the variant (1). Especially, we notice that LPE (2) achieves the worst learning result for training with the Duel algorithm in MountainCar and with the Double algorithm in Acrobot (-200 and -500 are the minimum rewards from an episode in the two environments respectively) with corruption budget $C/T = 0.04$ and 0.02 respectively, suggesting that the learning algorithms do not learn policies better than random ones.

For the RPI and RPP attacks, they are effective in most cases except that RPI fails in 2 cases (Swimmer learned by PPO, and Acrobot learned by Double), and RPP fails in 1 case (CartPole learned by Double). Across all cases, the RPI attack usually has better performance than the RPP attack. Especially, we notice that in Acrobot and MountainCar environments, RPI attack achieves better attack result when the learning algorithm is Duel, and the opposite is true when the learning algorithm is Double. This is probably because the Double algorithm does more exploration in suboptimal state action pairs than the Duel algorithm.

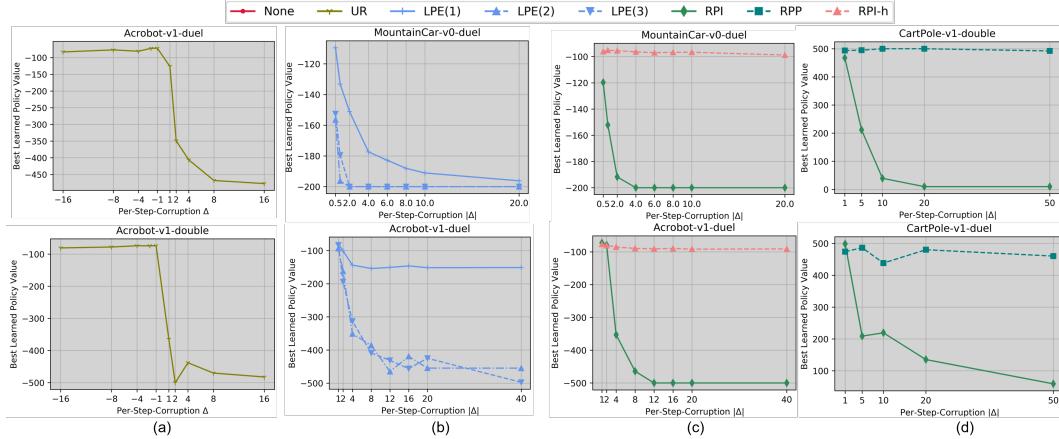


Figure 2: Experimental observations validating implications 1,2,4, and 6 from Sec. 5.

Experimental validation of implications 1, 2, 4, and 6 We empirically examine the best value of V learned by the agent under our attack methods parameterized with different values of Δ . Note that the value of $|\Delta|$ is also the requirement on B for the attack. To remove the dependency of V on C , we set $C = T$, $E = \infty$ so that the attacker is never out of budget due to C and E . We choose representative environments to validate our implications in Sec. 5, and the results are shown in Fig. 2. The x axis is the value of Δ in (a) the $|\Delta|$ for the rest used by the attacks. The y axis has the same meaning as in Fig. 1. In Fig. 2 (a) we run the UR attack with both signs of Δ on environment Acrobot. We observe that when corrupting in the wrong direction $\Delta < 0$, the Duel algorithm can still learn the policies of optimal performance. When corrupting in the right direction $\Delta > 0$, the Duel algorithm learns much worse policies as Δ increases. This observation agrees with implication 1. In Fig. 2 (b) we run the three variants of the LPE attack on environment MountainCar and Acrobot. We observe that for the same value of Δ , LPE (2) (3) attack can always result in lower policy value than LPE (1) attack, which agrees with implication 2. In Fig. 2 (c) we run the RPI attack together with its special variant (RPI-h) which uses high performing π^\dagger . We observe that the RPI attack with a random policy π^\dagger always leads to a lower value of the best learned policy compared to the RPI-h with high performing π^\dagger , which agrees with implication 4. In Fig. 2 (d) we run the RPI and RPP attack with random π^\dagger on CartPole. Since in CartPole the policy value is the same as the episode length of a policy, it is an environment that satisfies the condition in implication 6. We observe that RPI attack leads to very low value of the best learned policy while the RPP attack does not influence the learned policy value no matter how large $|\Delta|$ it uses, which agrees with implication 6.

We also experimentally validate the implications 3, 5 given by our lemmas in Sec. 5. The details are given in appendix A.

7 Conclusion

In this work, we study the security vulnerability of DRL algorithms against training time attacks. We designed a general, parametric framework for reward poisoning attacks and instantiated it to create several efficient attacks. We provide theoretical analysis yielding insightful implications validated by our experiments. Our detailed evaluation confirms the efficiency of our attack methods pointing to the practical vulnerability of popular DRL algorithms.

References

- Joshua Achiam. Spinning Up in Deep Reinforcement Learning. 2018.
- M Mehdi Afsar, Trafford Crump, and Behrouz Far. Reinforcement learning based recommender systems: A survey. *arXiv preprint arXiv:2101.06286*, 2021.
- Alekh Agarwal, Nan Jiang, Sham M Kakade, and Wen Sun. Reinforcement learning: Theory and algorithms. *CS Dept., UW Seattle, Seattle, WA, USA, Tech. Rep*, 2019.
- Vahid Behzadan and Arslan Munir. Vulnerability of deep reinforcement learning to policy induction attacks. In *International Conference on Machine Learning and Data Mining in Pattern Recognition*, pages 262–275. Springer, 2017.
- Greg Brockman, Vicki Cheung, Ludwig Pettersson, Jonas Schneider, John Schulman, Jie Tang, and Wojciech Zaremba. Openai gym. *arXiv preprint arXiv:1606.01540*, 2016.
- Paul F. Christiano, Jan Leike, Tom B. Brown, Miljan Martic, Shane Legg, and Dario Amodei. Deep reinforcement learning from human preferences. In *Proc. Neural Information Processing Systems (NeurIPS)*, pages 4299–4307, 2017.
- Kefan Dong, Yuanhao Wang, Xiaoyu Chen, and Liwei Wang. Q-learning with ucb exploration is sample efficient for infinite-horizon mdp. *arXiv preprint arXiv:1901.09311*, 2019.
- Sandy Huang, Nicolas Papernot, Ian Goodfellow, Yan Duan, and Pieter Abbeel. Adversarial attacks on neural network policies. *arXiv preprint arXiv:1702.02284*, 2017.
- Chi Jin, Zeyuan Allen-Zhu, Sébastien Bubeck, and Michael I Jordan. Is q-learning provably efficient? *Advances in neural information processing systems*, 31, 2018.
- Kwang-Sung Jun, Lihong Li, Yuzhe Ma, and Xiaojin Zhu. Adversarial attacks on stochastic bandits. *arXiv preprint arXiv:1810.12188*, 2018.
- Panagiota Kiourti, Kacper Wardega, Susmit Jha, and Wenchao Li. Trojdril: Trojan attacks on deep reinforcement learning agents. *arXiv preprint arXiv:1903.06638*, 2019.
- Jernej Kos and Dawn Song. Delving into adversarial attacks on deep policies. *arXiv preprint arXiv:1705.06452*, 2017.
- Ram Shankar Siva Kumar, Magnus Nyström, John Lambert, Andrew Marshall, Mario Goertzel, Andi Comissoneru, Matt Swann, and Sharon Xia. Adversarial machine learning-industry perspectives. In *2020 IEEE Security and Privacy Workshops (SPW)*, pages 69–75. IEEE, 2020.
- Yen-Chen Lin, Zhang-Wei Hong, Yuan-Hong Liao, Meng-Li Shih, Ming-Yu Liu, and Min Sun. Tactics of adversarial attack on deep reinforcement learning agents. *arXiv preprint arXiv:1703.06748*, 2017.
- Fang Liu and Ness Shroff. Data poisoning attacks on stochastic bandits. In *International Conference on Machine Learning*, pages 4042–4050. PMLR, 2019.
- Guanlin Liu and Lifeng Lai. Provably efficient black-box action poisoning attacks against reinforcement learning. *Advances in Neural Information Processing Systems*, 34, 2021.
- Yuzhe Ma, Xuezhou Zhang, Wen Sun, and Xiaojin Zhu. Policy poisoning in batch reinforcement learning and control. *arXiv preprint arXiv:1910.05821*, 2019.
- Amin Rakhsa, Goran Radanovic, Rati Devidze, Xiaojin Zhu, and Adish Singla. Policy teaching via environment poisoning: Training-time adversarial attacks against reinforcement learning. In *International Conference on Machine Learning*, pages 7974–7984. PMLR, 2020.
- Yanchao Sun, Da Huo, and Furong Huang. Vulnerability-aware poisoning mechanism for online rl with unknown dynamics. *arXiv preprint arXiv:2009.00774*, 2020.
- Hado Van Hasselt, Arthur Guez, and David Silver. Deep reinforcement learning with double q-learning. In *Proceedings of the AAAI conference on artificial intelligence*, volume 30, 2016.

- Ziyu Wang, Tom Schaul, Matteo Hessel, Hado Hasselt, Marc Lanctot, and Nando Freitas. Dueling network architectures for deep reinforcement learning. In *International conference on machine learning*, pages 1995–2003. PMLR, 2016.
- Hang Xu, Rundong Wang, Lev Raizman, and Zinovi Rabinovich. Transferable environment poisoning: Training-time attack on reinforcement learning. In *Proceedings of the 20th International Conference on Autonomous Agents and MultiAgent Systems*, pages 1398–1406, 2021a.
- Yinglun Xu, Bhuvesh Kumar, and Jacob D Abernethy. Observation-free attacks on stochastic bandits. *Advances in Neural Information Processing Systems*, 34, 2021b.
- Xuezhou Zhang, Yuzhe Ma, Adish Singla, and Xiaojin Zhu. Adaptive reward-poisoning attacks against reinforcement learning. In *International Conference on Machine Learning*, pages 11225–11234. PMLR, 2020.
- Guanjie Zheng, Fuzheng Zhang, Zihan Zheng, Yang Xiang, Nicholas Jing Yuan, Xing Xie, and Zhenhui Li. DRN: A deep reinforcement learning framework for news recommendation. In *Proc. World Wide Web Conference on World Wide Web, WWW*, pages 167–176. ACM, 2018.

A Experiments details and additional experiments

The hyper parameters for the learning algorithms can be found in the codes. The parameters for the setup of the experiments are given in Table 1. Here the parameter r is the additional parameter for attack against environment with continuous action space as discussed in Sec. 5. The choice on r for the LPE attack and RPI/RPP attack are different. In practice we choose the parameters that significantly reduce the performance of the best learned policy by the learning algorithms. In Table 2 We provide the value of V_{\min} , V_{\max} , and L_{\max} for each environment we use to determine the constraints on the attack. These value are given by either the setup of the environment or empirically estimation. For example, in MountainCar-v0, $L_{\max} = 200$ and $V_{\min} = -200$ are given by the set up directly, as an episode will be terminated after 200 steps, and the reward is -1 for each step. $V_{\max} = -100$ is empirically estimated by the highest reward given by the best policy learned by the most efficient learning algorithm. In Table 3 we provide the policy values (expected total reward from an episode) of π^\dagger used by LPE attack (2) and (3). Recall that for LPE attack (2), π^\dagger represents an expert policy that have very high performance, and for LPE attack (3), π^\dagger represents a median expert policy that also have high performance but less than that for LPE attack (2). The whole set of the main results for our attack methods against learning algorithms under full constraints are shown in Fig. 3.

Table 1: Parameters for experiments

ENVIRONMENT	T	B	E	$r(\text{LPE})$	$r(\text{RPI/RPP})$
CARTPOLE	80000	5	500	/	/
LUNARLANDER	120000	4	800	/	/
MOUNTAINCAR	80000	2.5	200	/	/
ACROBOT	80000	4	500	/	/
HALFCHEETAH	600000	42	6300	2	1.5
HOPPER	600000	25	2500	2	1.1
WALKER2D	600000	25	2500	2.2	1.5
SWIMMER	600000	0.8	80	2.2	1.0

Table 2: Values used to determine the constraints on attack

ENVIRONMENT	L_{\max}	V_{\max}	V_{\min}	$V_{\max} - V_{\min}$	$\frac{V_{\max} - V_{\min}}{L_{\max}}$
CARTPOLE	500	500	0	500	1
LUNARLANDER	1000	200	-1000	1200	1.2
MOUNTAINCAR	200	-100	-200	100	0.5
ACROBOT	500	-100	-500	400	0.8
HALFCHEETAH	1000	12000	0	12000	12
HOPPER	1000	4000	0	4000	4
WALKER2D	1000	5000	0	5000	5
SWIMMER	1000	120	0	120	0.12

To validate implication 3 and 5, we experimentally examine the requirement on attack budget C for our attack methods with sufficient value of $|\Delta|$. For a fixed value of Δ , we remove the constraints on C and E so that the corruption can always be applied following A^t defined by different attack methods, and then we measure how many steps are corrupted in each epoch under the attack. The environment we choose is MountainCar, and the value of $|\Delta|$ for both LPE (all the three variants as described in Sec. 6) and RPI attack is 10. The results are shown in Fig. 4. The x axis is the index of epochs during training, and the y axis is the number of time steps that are corrupted in the epoch. For both LPE and RPI attack, we observe that the number of corrupted steps decrease with time and eventually approaches 0. This suggests that for both attacks, the agent gradually never take actions a^t at the states s^t that correspond to the ones where no corruption will be applied under the attack, i.e., $A^t(s^t, a^t) \neq 0$. This observation agrees with implication 3 and 5.

By the definition 5.8, higher value of r results in higher similarity between policies. By theorem 5.3, the LPE attack should have lower value of $\Delta(V)$ given higher value of r ; by theorem 5.5 and 5.7, the

Table 3: Policy value of π^\dagger used by LPE attack (2) and (3). Here ALG1 and ALG2 are the pair of learning algorithms we use in each environment. $\mathcal{V}_M^{\pi^\dagger}-(2)-1$ is the policy value of π^\dagger for LPE attack (2) when the learning algorithm for the agent is ALG1 (implying that the learning algorithm used by the attacker to learn π^\dagger is ALG2). The meanings for the last three columns are similar.

ENVIRONMENT	ALG1	ALG2	$\mathcal{V}_M^{\pi^\dagger}-(2)-1$	$\mathcal{V}_M^{\pi^\dagger}-(3)-1$	$\mathcal{V}_M^{\pi^\dagger}-(2)-2$	$\mathcal{V}_M^{\pi^\dagger}-(3)-2$
CARTPOLE	DUEL	DOUBLE	500	220	500	199
LUNARLANDER	DUEL	DOUBLE	154	2	202	10
MOUNTAINCAR	DUEL	DOUBLE	-108	-158	-101	-156
ACROBOT	DUEL	DOUBLE	-101	-200	-100	-199
HALFCHEETAH	DDPG	SAC	12374	6007	12766	5974
HOPPER	TD3	SAC	3619	1828	3562	1801
WALKER2D	TD3	SAC	5172	2552	4622	2426
SWIMMER	DDPG	PPO	120	61	120	61

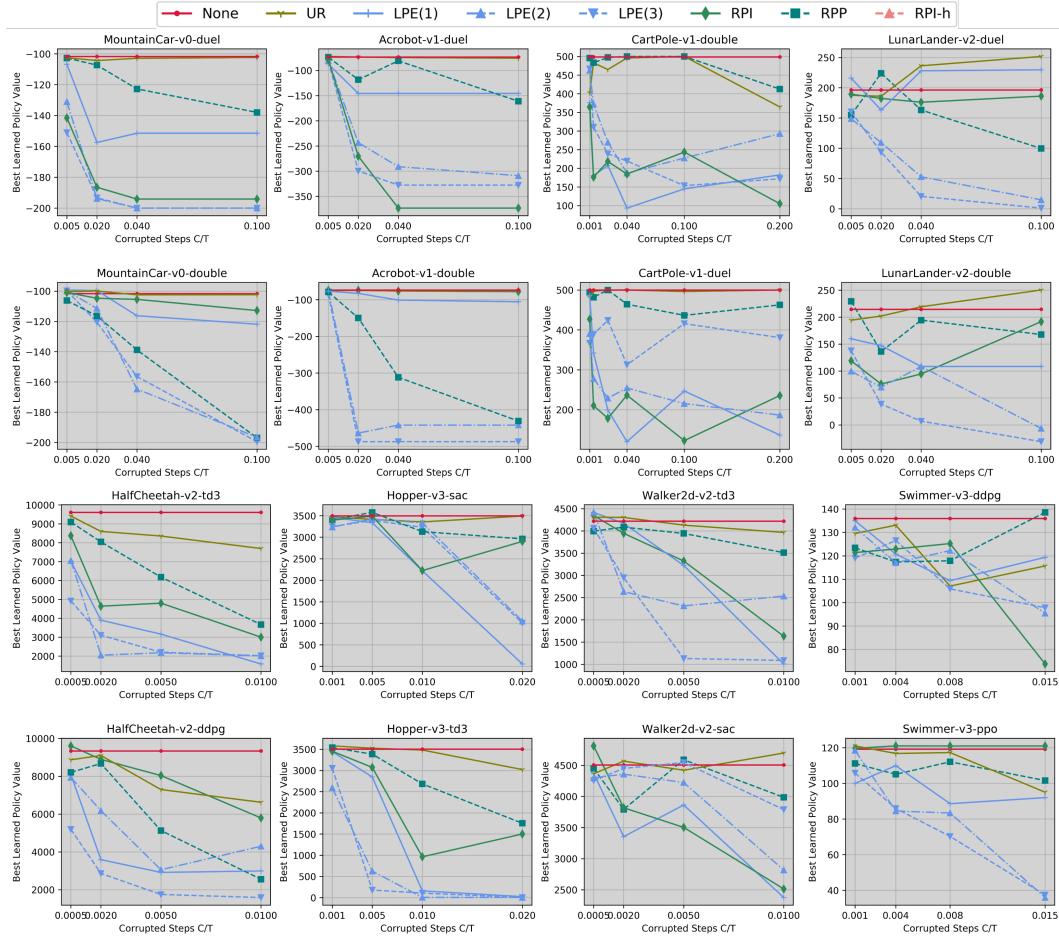


Figure 3: The complete main results for our attack methods against different learning algorithms in different environment

RPI and RPP attack should have lower value of $\Delta(V)$ given lower value of r . This gives the follow implications:

Implication 8: Given unlimited buget on C and E , with the same value of $|\Delta|$, the LPE attack can make the learning algorithm learn worse policy with higher value of r , and the opposite is true for the RPI and RPP attack.

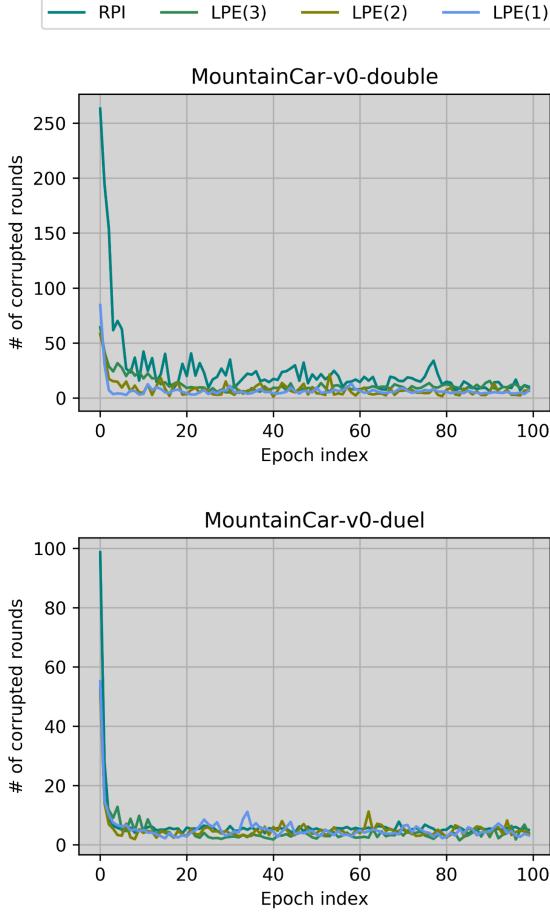


Figure 4: Experimental observations validating implications 3 and 5 from Sec. 5

To experimentally validate this implication, we run experiments on Hopper environment and TD3 learning algorithm as an example. We set $C = T$, $E = \infty$, and $|\Delta| = 25$. The results are shown in Fig. 5. The observation validates implication 8.

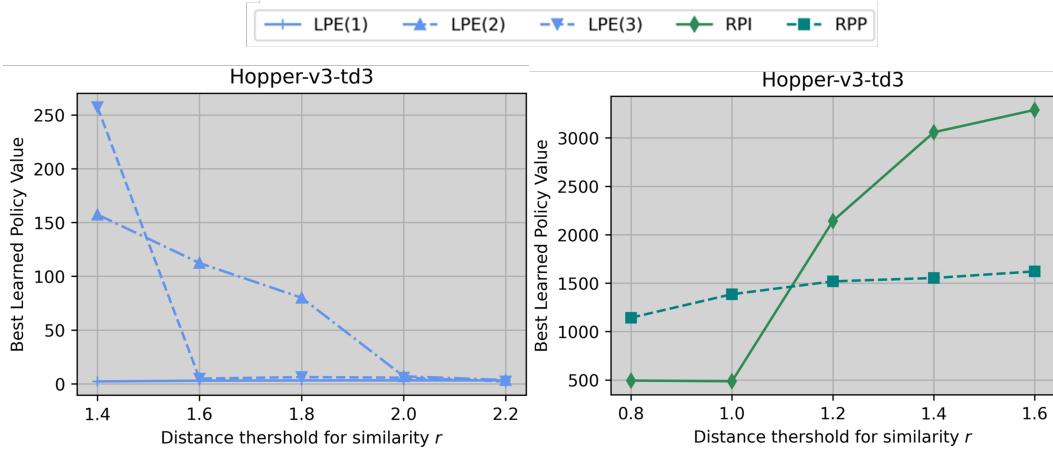


Figure 5: Influence of the distance threshold for similarity r on different attack methods

B Proof for theorems and lemmas

At the beginning we introduce a simple yet useful lemma for the purpose of simplifying the proof of our theorems and lemmas:

Lemma B.1. *A necessary and sufficient condition for the optimal policy $\hat{\pi}^*$ under $\widehat{\mathcal{M}}$ has a policy value less than V under \mathcal{M} is:*

$$\mathcal{V}_{\mathcal{M}}^{\hat{\pi}^*} \leq V \iff \max_{\pi \in B_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi} > \max_{\pi \in G_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi}.$$

Proof: Necessity: When the l.h.s $\mathcal{V}_{\mathcal{M}}^{\hat{\pi}^*} \leq V$ is true, by definition of $B_V^{\mathcal{M}}$ we have $\hat{\pi}^* \in B_V^{\mathcal{M}}$. Since $\widehat{\mathcal{M}}$ is the policy of highest policy value under $\widehat{\mathcal{M}}$, we have $\max_{\pi \in B_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi} = \max_{\pi} \mathcal{V}_{\mathcal{M}}^{\pi} > \max_{\pi \in G_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi}$. Sufficiency: When the r.h.s $\max_{\pi \in B_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi} > \max_{\pi \in G_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi}$ is true, then $\hat{\pi}^* \in B_V^{\mathcal{M}}$, and by the definition of $B_V^{\mathcal{M}}$, we have $\mathcal{V}_{\mathcal{M}}^{\hat{\pi}^*} \leq V$.

For convenience, we introduce a definition called value loss to measure the difference between the policy value of a policy under the original and adversarial MDP, :

Definition B.2. (Value loss) The value loss of a policy π given the environment \mathcal{M} and an adversarial MDP $\widehat{\mathcal{M}}$ is defined as

$$\delta\mathcal{V}_{\mathcal{M}, \widehat{\mathcal{M}}}^{\pi} := \sum_s \mu^{\pi}(s)(\mathcal{R}(s, \pi(s))) - \dot{\mathcal{R}}(s, \pi(s))$$

, where $\mu^{\pi}(s)$ is the state distribution for the policy π representing in expectation how often a state s will be visited in an episode.

The definition of value loss can help us rewriting lemma B.1 as

$$\mathcal{V}_{\mathcal{M}}^{\hat{\pi}^*} \leq V \iff \max_{\pi \in B_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi} - \delta\mathcal{V}_{\mathcal{M}, \widehat{\mathcal{M}}}^{\pi} > \max_{\pi \in G_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi} - \delta\mathcal{V}_{\mathcal{M}, \widehat{\mathcal{M}}}^{\pi}. \quad (3)$$

We will frequently using this result for the proof next.

Proof for theorem 5.1 Under the UR attack, the value loss of a policy is

$$\delta\mathcal{V}_{\mathcal{M}, \widehat{\mathcal{M}}}^{\pi} = -p \cdot \Delta \cdot \sum_s \mu^{\pi}(s) = -p \cdot \Delta \cdot L^{\pi}.$$

To make the r.h.s in Equation 3 hold, the following needs to be satisfied:

$$\max_{\pi \in B_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi} + p \cdot \Delta \cdot L^{\pi} > \max_{\pi \in B_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^{\pi} + p \cdot \Delta \cdot L^{\pi}.$$

Equivalently, it requires that there exists $\pi_1 \in B_V^{\mathcal{M}}$ such that for all $\pi_2 \in G_V^{\mathcal{M}}$, the following holds $(\mathcal{V}_{\mathcal{M}}^{\pi_1} + p \cdot \Delta \cdot L^{\pi_1}) - (\mathcal{V}_{\mathcal{M}}^{\pi_2} + p \cdot \Delta \cdot L^{\pi_2}) > 0$. This can further be formalized as $\min_{\pi_2 \in G_V^{\mathcal{M}}} \max_{\pi_1 \in B_V^{\mathcal{M}}} (\mathcal{V}_{\mathcal{M}}^{\pi_1} + p \cdot \Delta \cdot L^{\pi_1}) - (\mathcal{V}_{\mathcal{M}}^{\pi_2} + p \cdot \Delta \cdot L^{\pi_2}) > 0$, which gives

$$|\Delta| > \min_{\pi_1 \in B_V^{\mathcal{M}}} \max_{\pi_2 \in G_V^{\mathcal{M}}} \frac{\mathcal{V}_{\mathcal{M}}^{\pi_2} - \mathcal{V}_{\mathcal{M}}^{\pi_1}}{p \cdot |L^{\pi_1} - L^{\pi_2}|}.$$

Then by the definition of $\Delta(V)$, we have $\Delta(V) = \min_{\pi_1 \in B_V^{\mathcal{M}}} \max_{\pi_2 \in G_V^{\mathcal{M}}} \frac{\mathcal{V}_{\mathcal{M}}^{\pi_2} - \mathcal{V}_{\mathcal{M}}^{\pi_1}}{p \cdot |L^{\pi_1} - L^{\pi_2}|}$, and an upper bound can be directly derived as

$$\min_{\pi_1 \in B_V^{\mathcal{M}}} \max_{\pi_2 \in G_V^{\mathcal{M}}} \frac{\mathcal{V}_{\mathcal{M}}^{\pi_2} - \mathcal{V}_{\mathcal{M}}^{\pi_1}}{p \cdot |L^{\pi_1} - L^{\pi_2}|} \leq \min_{\pi_1 \in B_V^{\mathcal{M}}} \max_{\pi_2 \in G_V^{\mathcal{M}}} \frac{V_{\max} - V_{\min}}{p \cdot |L^{\pi_1} - L^{\pi_2}|} = \frac{V_{\max} - V_{\min}}{p \cdot \max_{\pi_1 \in B_V^{\mathcal{M}}} \min_{\pi_2 \in G_V^{\mathcal{M}}} |L^{\pi_1} - L^{\pi_2}|}.$$

Proof for theorem 5.3

The value loss for a policy π under the LPE attack with policy π^\dagger is:

$$\begin{aligned} \delta\mathcal{V}_{\mathcal{M}, \widehat{\mathcal{M}}}^{\pi} &= - \sum_s \mu^{\pi}(s) \cdot \Delta \cdot \mathbb{1}[\pi(s) = \pi^\dagger(s)] \\ &= -\Delta \cdot D(\pi, \pi^\dagger). \end{aligned} \quad (4)$$

Equation 4 says that the value loss for policy π is proportional to its similarity with π^\dagger . To make the r.h.s in Equation 3 hold, the following should be satisfied:

$$\max_{\pi_1 \in B_V^{\mathcal{M}}} V_{\mathcal{M}}^\pi + \Delta \cdot D(\pi_1, \pi^\dagger) > \max_{\pi_2 \in G_V^{\mathcal{M}}} V_{\mathcal{M}}^\pi + \Delta \cdot D(\pi_2, \pi^\dagger).$$

By similar analysis from proof for theorem 5.1, it can equivalently be rewritten as

$$|\Delta| > \min_{\pi_1 \in B_V^{\mathcal{M}}} \max_{\pi_2 \in G_V^{\mathcal{M}}} \frac{V_{\mathcal{M}}^{\pi_2} - V_{\mathcal{M}}^{\pi_1}}{D(\pi_2, \pi^\dagger) - D(\pi_1, \pi^\dagger)} = \Delta(V).$$

Next we give an upper bound on $\Delta(V)$. Let $\pi_0 := \operatorname{argmin}_{\pi \in B_V^{\mathcal{M}}} D(\pi, \pi^\dagger)$ then the maximum policy value of a policy from $B_V^{\mathcal{M}}$ is lower bound by

$$\max_{\pi \in B_V^{\mathcal{M}}} V_{\mathcal{M}}^\pi = \max_{\pi \in B_V^{\mathcal{M}}} (V_{\mathcal{M}}^\pi - \Delta \cdot D(\pi, \pi^\dagger)) \geq V_{\mathcal{M}}^{\pi_0} - \Delta \cdot D(\pi_0, \pi^\dagger) \geq V_{\min} - \min_{\pi \in B_V^{\mathcal{M}}} D(\pi, \pi^\dagger)$$

For the maximum policy value of a policy from $G_V^{\mathcal{M}}$, it can be directly upper bound by

$$\max_{\pi \in G_V^{\mathcal{M}}} V_{\mathcal{M}}^\pi = \max_{\pi \in G_V^{\mathcal{M}}} (V_{\mathcal{M}}^\pi - \Delta \cdot D(\pi, \pi^\dagger)) \leq \max_{\pi \in B_V^{\mathcal{M}}} V_{\mathcal{M}}^\pi - \min_{\pi \in B_V^{\mathcal{M}}} \Delta \cdot D(\pi, \pi^\dagger) = V_{\max} - \Delta \cdot \min_{\pi \in B_V^{\mathcal{M}}} D(\pi, \pi^\dagger).$$

Then the upper bound on $\Delta(V)$ can be given as

$$\Delta(V) \leq \frac{V_{\max} - V_{\min}}{\min_{\pi \in G_V^{\mathcal{M}}} D(\pi, \pi^\dagger) - \min_{\pi \in B_V^{\mathcal{M}}} D(\pi, \pi^\dagger)}$$

Note that one can always find a policy π that share no similarity to π^\dagger by always choosing a different action to π^\dagger , that is $\pi(s) \neq \pi^\dagger(s), \forall s$, then $D(\pi, \pi^\dagger) = 0$. In the situation where the number of actions is large than 2, then such policy can still have random behavior which usually corresponds to low policy value, and we can assume that a policy with no similarity to π^\dagger can always be found in $B_V^{\mathcal{M}}$, suggesting that $\min_{\pi \in B_V^{\mathcal{M}}} D(\pi, \pi^\dagger)$, then the upper bound on $\Delta(V)$ can be rewritten as $\Delta(V) \leq \frac{V_{\max} - V_{\min}}{\min_{\pi \in G_V^{\mathcal{M}}} D(\pi, \pi^\dagger)}$.

Proof for theorem 5.5

The value loss for a policy π under the RPI attack with policy π^\dagger is:

$$\begin{aligned} \delta V_{\mathcal{M}, \widehat{\mathcal{M}}}^\pi &= - \sum_s \mu^\pi(s) \cdot \Delta \cdot \mathbb{1}[\pi(s) \neq \pi^\dagger(s)] \\ &= - \sum_s \mu^\pi(s) \cdot \Delta \cdot (1 - \mathbb{1}[\pi(s) = \pi^\dagger(s)]) \\ &= - \sum_s \mu^\pi(s) \cdot \Delta - \sum_s \mu^\pi(s) \cdot \Delta \cdot \mathbb{1}[\pi(s) \neq \pi^\dagger(s)] \\ &= -\Delta \cdot (L^\pi - D(\pi, \pi^\dagger)) \end{aligned} \tag{5}$$

Note that for the attack policy π^\dagger itself, its value loss is 0 as $L^{\pi^\dagger} = D(\pi^\dagger, \pi^\dagger)$. To make the r.h.s in Equation 3 hold, the following needs to be satisfied

$$\max_{\pi_1 \in B_V^{\mathcal{M}}} V_{\mathcal{M}}^\pi + \Delta \cdot (L^{\pi_1} - D(\pi_1, \pi^\dagger)) > \max_{\pi_2 \in G_V^{\mathcal{M}}} V_{\mathcal{M}}^\pi + \Delta \cdot (L^{\pi_2} - D(\pi_2, \pi^\dagger)).$$

It can be equivalently rewritten as

$$|\Delta| > \min_{\pi_1 \in B_V^{\mathcal{M}}} \max_{\pi_2 \in G_V^{\mathcal{M}}} \frac{V_{\mathcal{M}}^{\pi_2} - V_{\mathcal{M}}^{\pi_1}}{(L^{\pi_2} - L^{\pi_1}) - (D(\pi_2, \pi^\dagger) - D(\pi_1, \pi^\dagger))} = \Delta(V).$$

Next we give an upper bound on $\Delta(V)$. Since π^\dagger is randomly generated, we can assume that it has random behavior in the environment resulting in poor performance, then we can lower bound the the maximum policy value of a policy from $B_V^{\mathcal{M}}$ by

$$\max_{\pi \in B_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^\pi = \max_{\pi \in B_V^{\mathcal{M}}} (\mathcal{V}_{\mathcal{M}}^\pi - \Delta \cdot D(\pi, \pi^\dagger)) \geq \mathcal{V}_{\mathcal{M}}^{\pi^\dagger} - \Delta \cdot (L^{\pi^\dagger} - D(\pi^\dagger, \pi^\dagger)) = \mathcal{V}_{\mathcal{M}}^{\pi^\dagger} \geq V_{\min}.$$

For the maximum policy value of a policy from $G_V^{\mathcal{M}}$, it can be directly upper bound as

$$\begin{aligned} \max_{\pi \in G_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^\pi &= \max_{\pi \in G_V^{\mathcal{M}}} (\mathcal{V}_{\mathcal{M}}^\pi - \Delta \cdot (L^\pi - D(\pi, \pi^\dagger))) \\ &\leq \max_{\pi \in B_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^\pi - \min_{\pi \in B_V^{\mathcal{M}}} \Delta \cdot (L^\pi - D(\pi, \pi^\dagger)) \\ &= V_{\max} - \Delta \cdot \min_{\pi \in B_V^{\mathcal{M}}} (L^\pi - D(\pi, \pi^\dagger)). \end{aligned}$$

Combing both, an upper bound on $\Delta(V)$ can be given as

$$\Delta(V) \leq \frac{V_{\max} - V_{\min}}{\min_{\pi \in G_V^{\mathcal{M}}} (L^\pi - D(\pi, \pi^\dagger))}.$$

Proof for theorem 5.7 Note that the attack strategy for RPP attack has the same form as that of the LPE attack, except that the attack applies positive corruption instead of negative. We can directly write down the policy loss for a policy and $\Delta(V)$ under the RPP attack since they share the same form as those for the LPE attack.

$$\delta \mathcal{V}_{\mathcal{M}, \widehat{\mathcal{M}}}^\pi = -\Delta \cdot D(\pi, \pi^\dagger).$$

$$|\Delta| > \min_{\pi_1 \in B_V^{\mathcal{M}}} \max_{\pi_2 \in G_V^{\mathcal{M}}} \frac{\mathcal{V}_{\mathcal{M}}^{\pi_2} - \mathcal{V}_{\mathcal{M}}^{\pi_1}}{D(\pi_2, \pi^\dagger) - D(\pi_1, \pi^\dagger)} = \Delta(V).$$

For the upper bound on $\Delta(V)$, we can lower bound the maximum policy value in $\widehat{\mathcal{M}}$ of a policy in $B_V^{\mathcal{M}}$ by that of π^\dagger , that is,

$$\max_{\pi \in B_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^\pi \geq \mathcal{V}_{\mathcal{M}}^{\pi^\dagger} = \mathcal{V}_{\mathcal{M}}^{\pi^\dagger} + \Delta \cdot D(\pi^\dagger, \pi^\dagger) \geq V_{\min} + \Delta \cdot L^{\pi^\dagger}.$$

The maximum policy value in $\widehat{\mathcal{M}}$ of a policy in $G_V^{\mathcal{M}}$ can be upper bound by

$$\max_{\pi \in G_V^{\mathcal{M}}} \mathcal{V}_{\mathcal{M}}^\pi \leq V_{\max} + \Delta \cdot \min_{\pi \in G_V^{\mathcal{M}}} D(\pi, \pi^\dagger).$$

Combing both, an upper bound on $\Delta(V)$ can be given as

$$\Delta(V) \leq \frac{V_{\max} - V_{\min}}{L^{\pi^\dagger} - \max_{\pi \in G_V^{\mathcal{M}}} D(\pi, \pi^\dagger)}.$$

Proof for lemma 5.4 Under the LPE attack, the value loss for any policy is ≥ 0 as the corruption Δ is always negative. If a policy π has no similarity to the attack policy π^\dagger , that is, $D(\pi, \pi^\dagger) = 0$, then its policy loss is 0. Let D_0 be the set of policies that has no similarity to π^\dagger . Then as the value of $|\Delta|$ increases, the value loss for all policies not in D_0 increase, then eventually the policy with the highest policy value in D_0 will have the highest policy value under the LPE attack with sufficient Δ .

Proof for lemma 5.6 Under the RPI attack, the value loss for any policy is ≥ 0 as the corruption Δ is always negative. The only policy that has 0 value loss is the attack policy π^\dagger itself. As the value of $|\Delta|$ increases, the value loss for all policies increase except for π^\dagger , then eventually π^\dagger will have the highest policy value under the RPI attack with sufficient Δ .

C Hardness of finding the optimal attack algorithm

First we show the space for all possible attack algorithms is exponentially large in T . As discussed in Sec. 3, an attack algorithms can be represented by its attack strategies A^t at each round t , and the attack strategy is a mapping $A^t : \mathcal{S}^t \times \mathcal{A}^t \times \mathcal{R}^{t-1} \times \mathcal{C}^{t-1} \rightarrow \mathcal{C}$, where \mathcal{C} is the corruption space for all possible amount of corruption, $\mathcal{S}^t = \underbrace{\mathcal{S} \times \mathcal{S} \dots \times \mathcal{S}}_{t \text{ times}}$, and the meaning of \mathcal{A}^t , \mathcal{R}^{t-1} , and \mathcal{C}^{t-1} is similar. Given the constraints on the budget of the attacker defined in Sec. 3, finding the optimal attack algorithm requires solving the following optimization problem:

$$\min_{A^{t=1,\dots,T}} \mathcal{V}_{\mathcal{M}}^{\pi_0}, \text{s.t. } \Delta^t = A^t(s^{1:t}, a^{1:t}, r^{1:t-1}, \Delta^{1:t-1}), \sum_{t=1}^T \mathbb{1}[\Delta^t \neq 0] \leq C, |\Delta^{t=1,\dots,T}| \leq B. \quad (6)$$

Even in the tabular setting where the sizes of $\mathcal{S}, \mathcal{A}, \nabla, \mathcal{C}$ are all finite, in total there are $|\mathcal{S}|^{T(T+1)/2} |\mathcal{A}|^{T(T+1)/2} |\mathcal{R}|^{T(T-1)/2} |\mathcal{C}|^{T(T-1)/2}$ many possible attack algorithms. This makes exhaustive enumeration computationally infeasible.

Next we show the hardness of searching for the optimal attack strategy. Zhang et al. [2020] show that the poisoning attack problem in the simpler tabular setting can be formulated as an RL problem which is harder than the RL problem for the learning agent. More specifically, the input state of the RL problem for the attacker needs to include the parameters of the learning algorithm, and correspondingly the transition function \mathcal{P} needs to include how such parameters are updated. In the DRL setting, the learning algorithms are more complex compared to the tabular setting. For example, if the learning algorithm is a deep Q learning algorithm, then the input space for the attacker's RL problem need to include all the parameters in the neural networks. Clearly both the input space and transition functions are more complicated in the DRL setting, making the RL problem for the attacker significantly harder to solve.

At last, note that both exhaustive enumeration and the attacker's RL formulation requires that the attacker has full knowledge of both the environment and the learning algorithm which is a strong assumption on attacker's capabilities as mentioned in Sec. 4. Considering all the strong requirements for the attacker and difficulties in finding the optimal attack, the goal in our work is to not chase optimality but find efficient attack algorithms without requiring any knowledge about the environment and the learning algorithm.