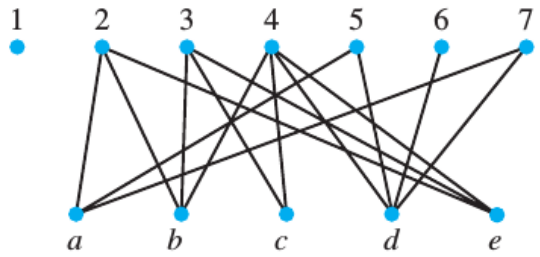
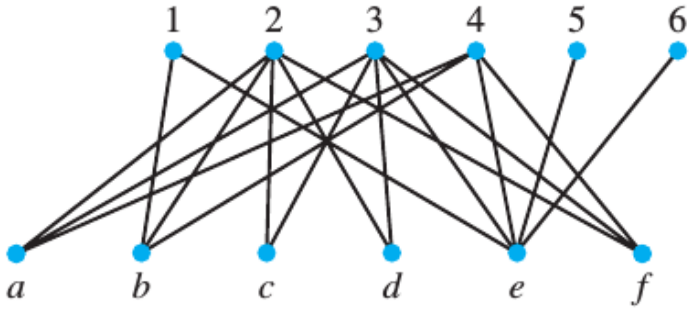
Lista de Exercícios (2)
Teoria e Aplicação de Grafos, CIC, UnB
Prof. Díbio

1. (Stein, Drysdale & Bogart, 2011). No grafo da figura a seguir, encontre um emparelhamento que sature o conjunto X={a,b,c,d,e}, ou um subconjunto S de X tal que |S| > |N(S)|.



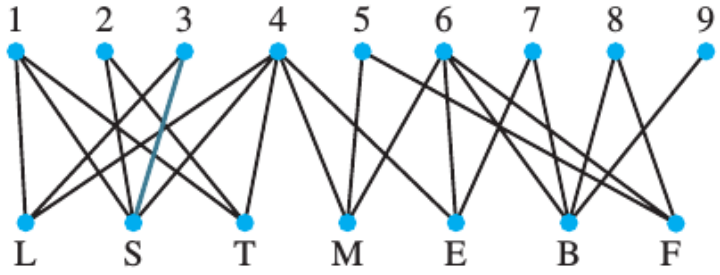
2. (Stein, Drysdale & Bogart, 2011). Encontre no grafo de 1. um emparelhamento máximo, e uma cobertura mínima de vértices.

3. (Stein, Drysdale & Bogart, 2011). No grafo da figura a seguir, encontre um emparelhamento que sature o conjunto X={a,b,c,d,e,f}, ou um subconjunto S de X tal que |S| > |N(S)|.



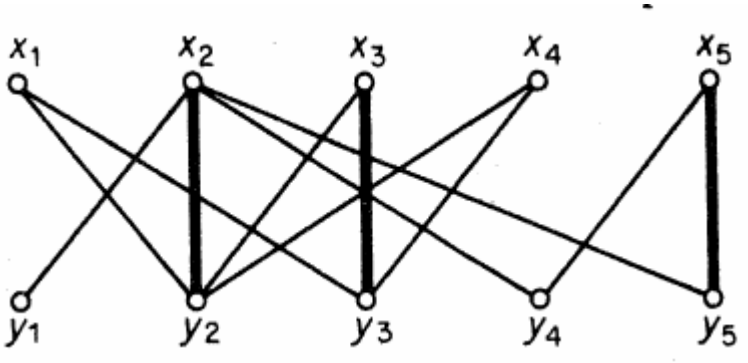
4. (Stein, Drysdale & Bogart, 2011). Encontre no grafo de 3. um emparelhamento máximo, e uma cobertura mínima de vértices.

5. (Stein, Drysdale & Bogart, 2011). Dado o emparelhamento {(S,2), (T,4), (B,7), (F,8)} no grafo a seguir, use Busca em Largura (BFS) começando no vértice 1 em uma forma alternada para encontrar um caminho aumentado iniciando em 1. Use o caminho aumentado conseguido para encontrar um emparelhamento maior do que este inicial.



6. (Stein, Drysdale & Bogart, 2011). Continue a busca como em 5 até encontrar um emparelhamento máximo.

7. (Bondy & Murty, 1982). Aplique o algoritmo húngaro no grafo a seguir, com emparelhamento inicial em negrito e encontre um emparelhamento maior.



8. (Bondy & Murty, 1982). Seja uma diagonal de uma matriz nxn onde do conjunto de n valores, dos quais não há dois que possam pertencer também a uma mesma linha ou mesma coluna. O peso de uma diagonal é a soma de seus elementos. Encontre uma diagonal de menor valor possível na matriz seguinte.

$$\begin{bmatrix} 4 & 5 & 8 & 10 & 11 \\ 7 & 6 & 5 & 7 & 4 \\ 8 & 5 & 12 & 9 & 6 \\ 6 & 6 & 13 & 10 & 7 \\ 4 & 5 & 7 & 9 & 8 \end{bmatrix}$$

9. (Bondy & Murty, 1982)

3.2.8 Describe a good algorithm for finding the blocks of a graph.

10. Sejam dois conjuntos de vértices, 8 homens, 8 mulheres, em um grafo e suas preferências de casamento. Desenhe o grafo e encontre um emparelhamento estável, simulando os passos do algoritmo Gale-Shapley.

1	5	7	1	2	6	8	4	3
2	2	3	7	5	4	1	8	6
3	8	5	1	4	6	2	3	7
4	3	2	7	4	1	6	8	5
5	7	2	5	1	3	6	8	4
6	1	6	7	5	8	4	2	3
7	2	5	7	6	3	4	8	1
8	3	8	4	5	7	2	6	1

Men's Preferences

1	5	3	7	6	1	2	8	4
2	8	6	3	5	7	2	1	4
3	1	5	6	2	4	8	7	3
4	8	7	3	2	4	1	5	6
5	6	4	7	3	8	1	2	5
6	2	8	5	3	4	6	7	1
7	7	5	2	1	8	6	4	3
8	7	4	1	5	2	3	6	8

Women's Preferences

11. (Wilson, 1996)

25.2 A building contractor advertises for a bricklayer, a carpenter, a plumber and a toolmaker, and receives five applicants – one for the job of bricklayer, one for carpenter, one for bricklayer and plumber, and two for plumber and toolmaker.

(i) Draw the corresponding bipartite graph.

(ii) Check whether the marriage condition holds for this problem.

Can all of the jobs be filled by qualified people?

12. (Wilson, 1996)

25.4 (The 'harem problem') Let B be a set of boys, and suppose that each boy in B wishes to marry more than one of his girl friends. Find a necessary and sufficient condition for the harem problem to have a solution. (Hint: replace each boy by several identical copies of himself, and then use Hall's theorem.)

13. (D. Easley & J. Kleinberg, 2010)

4. Suppose we have a set of 3 sellers labeled a, b, and c, and a set of 3 buyers labeled x, y, and z. Each seller is offering a distinct house for sale, and the valuations of the buyers for the houses are as follows.

Buyer	Value for a's house	Value for b's house	Value for c's house
x	12	9	8
y	10	3	6
z	8	6	5

Suppose that a charges a price of 3 for his house, b charges a price of 1 for his house, and c charges a price of 0. Is this set of prices market-clearing? If so, explain which buyer you would expect to get which house; if not, say which seller or sellers should raise their price(s) in the next round of the bipartite-graph auction procedure from Chapter 10.

14. (D. Easley & J. Kleinberg, 2010)

13. Suppose you want to design an auction for the following type of situation: you have two identical copies of a valuable object, and there are four potential buyers for the object. Each potential buyer i wants at most one copy, and has a value v_i for either copy.

You decide to design the auction by analogy with the way in which we derived the single-item ascending-bid (English) auction from the general procedure for matching markets. In the present case, as there, you want to create a bipartite graph that encodes the situation, and then see what prices the bipartite graph auction procedure comes up with.

(a) Describe how this construction would work using an example with four potential buyers. In creating your example, first choose specific valuations for the potential buyers, and then show how the auction proceeds and what the market-clearing prices are.

(b) In the case of the single-item auction, the bipartite graph procedure yielded the simple rule from the ascending-bid (English) auction: sell to the highest bidder at the second-highest price. Describe in comparably simple terms what the rule is for the current case of two identical items (i.e. your description should not involve the terms "bipartite", "graph," or "matching").