

Uncover viral sharing through global structure of host-virus meta-network

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Purpose: This template provides a series of scripts to render a markdown document into an interactive website and a series of PDFs.

Internals: GitHub actions and a series of python scripts. The markdown is handled with pandoc.

Motivation: It makes collaborating on text with GitHub easier, and means that we never need to think about the output.

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Last revision: *July 21, 2023*

1 Introduction

2 Methods

3 Communicability quantify how well information transit between two nodes by considering all possible path in a
4 network and penalizing longer ones. It is compute with the exponent of the adjacency matrix of the network.

$$G = \sum_{k=0}^{\infty} \frac{(A^k)}{k!} = e^A$$

5 where G is the communicability matrix, A the adjacency matrix and k is used as a penalizing term. It is possible
6 to compute the exponential of a matrix with the graph spectrum :

$$G = \sum_{j=1}^n \varphi_j \varphi_j^T e^{\lambda_j}$$

7 where φ_j and λ_j are respectively the j^{th} eigenvectors and eigenvalues of the matrix A . The spectral form of G
8 can be decompose by the following way :

$$G = \varphi_1 \varphi_1^T e^{\lambda_1} + \sum_{j=2}^n \varphi_j^+ \varphi_j^{+T} e^{\lambda_j} + \sum_{j=2}^n \varphi_j^- \varphi_j^{-T} e^{\lambda_j} + \sum_{j=2}^n \varphi_j^- \varphi_j^{+T} e^{\lambda_j} \quad (1)$$

9

10

11

12 where φ^+ or φ^- indicate respectively all the positives or negatives values of the j^{th} eigenvector. The first is not
13 include because all the values the eigenvector the same sign. Estrada & Hatano (2008) explain that “two nodes
14 have the same sign in an eigenvector if they can be considered as being in the same partition of the network,
15 while those pairs having different signs correspond to nodes in different partitions.” So to make partition we are
16 mostly interested by the sign of the sums in eq. 1 :

$$\sum_{j=2}^{intracluster} \varphi_j \varphi_j^T e^{\lambda_j} = \sum_{j=2}^n \varphi_j^+ \varphi_j^{+T} e^{\lambda_j} + \sum_{j=2}^n \varphi_j^- \varphi_j^{-T} e^{\lambda_j}$$

17 and

$$\sum_{j=2}^{intercluster} \varphi_j \varphi_j^T e^{\lambda_j} = \sum_{j=2}^n \varphi_j^- \varphi_j^{+T} e^{\lambda_j}$$

18 so the clustering matrix is obtain with

$$\Delta G = \sum_{j=2}^{intracluster} \varphi_j \varphi_j^T e^{\lambda_j} - \left| \sum_{j=2}^{intercluster} \varphi_j \varphi_j^T e^{\lambda_j} \right|$$

19 in short it is in fact

$$\Delta G = G - \varphi_1 \varphi_1^T e^{\lambda_1}$$

```
library(tidyverse)
library(lattice)
library(igraph)
library(colorRamps)
A = matrix(c(0,1,0,1,1,0,0,0,0,0,0,
            1,0,1,1,1,0,0,0,0,0,0,
            0,1,0,1,1,0,0,0,0,0,0,
            1,1,1,0,1,0,0,0,0,0,0,
            1,1,1,1,0,1,0,0,0,0,0,
            0,0,0,0,1,0,1,0,0,0,0,
            0,0,0,0,0,1,0,1,1,0,1,
            0,0,0,0,0,0,1,0,1,1,1,
            0,0,0,0,0,0,1,1,0,1,1,
            0,0,0,0,0,0,0,1,1,0,1,
            0,0,0,0,0,0,1,1,1,1,0), nrow =11, ncol =11)
grap = graph_from_adjacency_matrix(A, mode = "undirected")
plot(grap)
```

20 **Example** A graph with 11 nodes and 2 distinct group. First we need to compute the graph spectrum

```
spectra = eigen(A)
levelplot(spectra$vectors, ylab = "eigenvectors", xlab = "j th position")
```

21 Now let's take the 2nd dimension as an example.

```
##
G_dim2 = spectra$vectors[,2]%*%t(spectra$vectors[,2])*exp(spectra$values[2])
intra = G_dim2[G_dim2]
levelplot(G_dim2, ylab = "node", xlab = "node", col.regions = rev(matlab.like(16)))
```

22 And that it ! The second dimension of the graph communicability identify 2 cluster (blue).

23 We can compute for the third dimension

```
G_dim3 = spectra$vectors[,3]%*%t(spectra$vectors[,3])*exp(spectra$values[3])
levelplot(G_dim3, ylab = "node", xlab = "node", col.regions = rev(matlab.like(16)))
```

24 Which identify clusters between 5:6 and 6:7. The cluster of the third dimension are less “obvious” than those
25 from the second dimension

26 Now if we want to use other communicability dimension we just have to add

```
levelplot(G_dim2+G_dim3, ylab = "node", xlab = "node", col.regions = rev(matlab.like(16)))
```

27 We could continue like that till the last dimension (11th), but it was for the explanation. So now we can compute
28 directly ΔG by adding all the 10 dimension

```
delta_G = matrix(0, nrow =11, ncol =11)
for(dim in 2:11){
  delta_G = delta_G + spectra$vectors[,dim]%*%t(spectra$vectors[,dim])*exp(spectra$values[dim])
}
levelplot(delta_G, ylab = "node", xlab = "node", col.regions = rev(matlab.like(16)))
```

29 Results

30 ![Global matrix of clustering communicability. Positive values indicate that species are in the same cluster]

³¹ **Conclusion**

³² Estrada, E. & Hatano, N. (2008). [Communicability in complex networks](#). *Physical Review E*, 77, 036111.