

# **A Convolutional Fuzzy Neural Network Architecture for Object Classification with Small Training Database**

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# Outline

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Introduction

Motivation

Basic concepts

Proposed method

Results

Conclusion

# Introduction

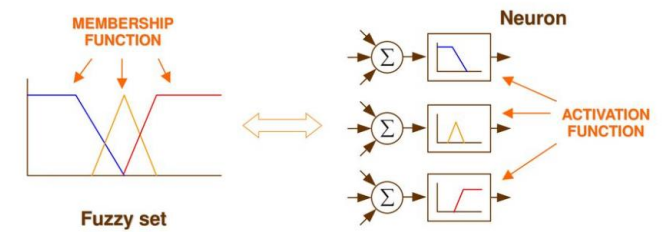
Image Classification - 90% precision rate

CNN

Fuzzy Neural Network - fuzzy values no crisp

Small data

Fully connected layers => Fuzzy neural network





# Motivation

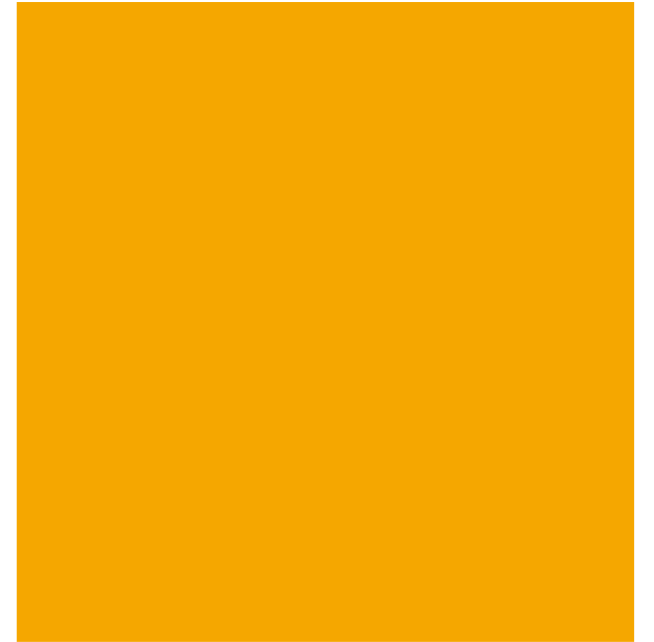


# Data!

- Sufficient training data (quality, size)
  - Not enough data – transfer learning
  - Relevant data
- 
- Crispy => fuzzy values
  - Strengthen the ability to approximate function



# Basic concepts



# CNN

- Powerful technique
  - Some correlation between input data
- Weight sharing
  - Gradient vanishing, overfitting
- Extract features from input
- Backpropagation

$$a_j^l = f\left(\sum_{i=0}^2 (x_i * w_{ij}^l) + b_j^l\right), j = 1, 2, \dots, n^l$$

$$\partial \text{loss} / \partial w_{ij}^l = a_j^l * \delta_j^{l+1}$$

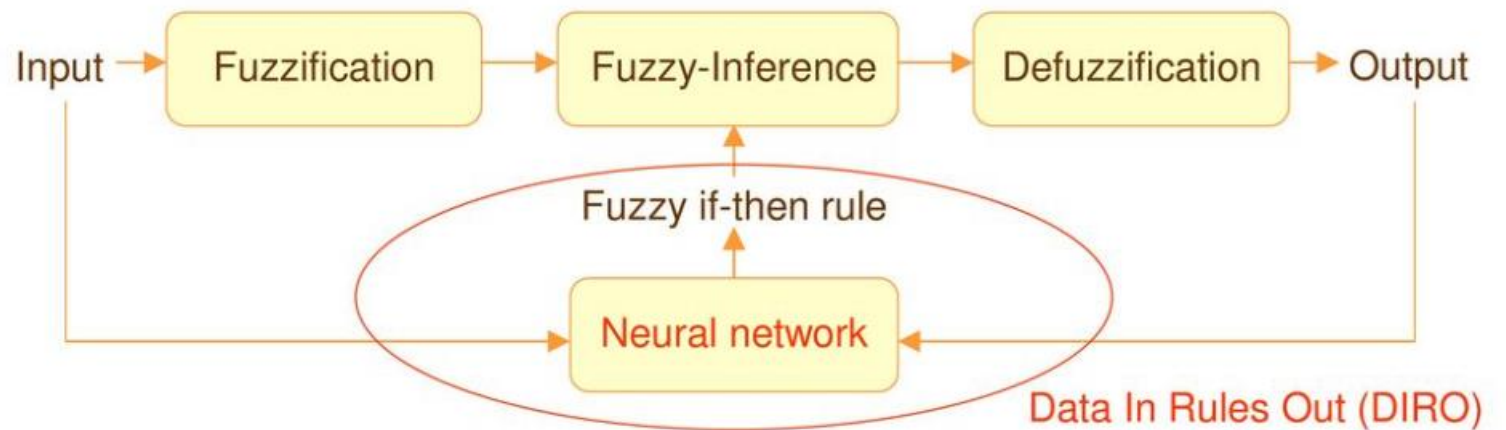
$$\delta_j^l = \begin{cases} f'(z_j^l) \otimes \text{upsample}(\delta_j^{l+1} * w_j^l), & \text{if } (l+1) \text{ layer is subsampling layer} \\ f'(z_j^l) \otimes (\delta_j^{l+1} * w_j^l), & \text{otherwise} \end{cases}$$

# Fuzzy Neural Network (FNN)

- Input unit is graded - membership to fuzzy set
- Fuzzy value

$R^l$  : IF  $x_1$  is  $A_1^l$  and  $\dots x_n$  is  $A_n^l$   
THEN  $y_1$  is  $w_1^l$  and  $\dots y_m$  is  $w_m^l$

$$y_j = \frac{\sum_{l=1}^h w_j^l \left( \prod_{i=1}^n \mu_{A_i^l}(x_i) \right)}{\sum_{l=1}^h \left( \prod_{i=1}^n \mu_{A_i^l}(x_i) \right)}$$





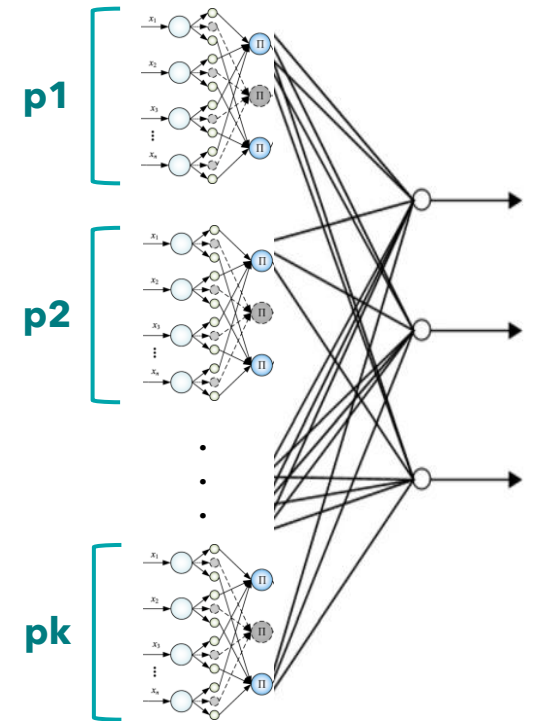
# Fuzzy Neural Network (FNN)

## When input number is too large!

- K independent inference engine

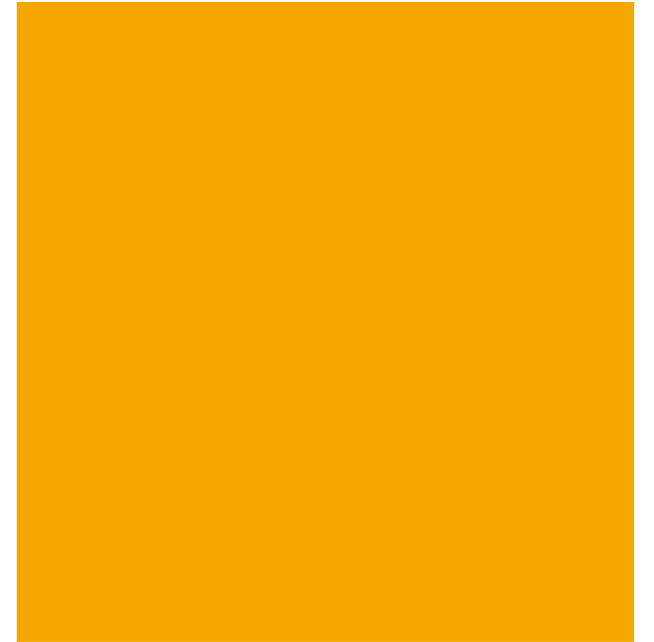
$$\hat{x}_i^p, p = 1, 2, \dots, k, i = 1, 2, \dots, (n/k)$$

$$R^{p,l} : \text{IF } x_1^p \text{ is } A_1^l \text{ and } \dots x_n^p \text{ is } A_n^l \\ \text{THEN } y_1 \text{ is } w_1^{p,l} \text{ and } \dots y_m \text{ is } w_m^{p,l}$$





# Proposed method



# Proposed method

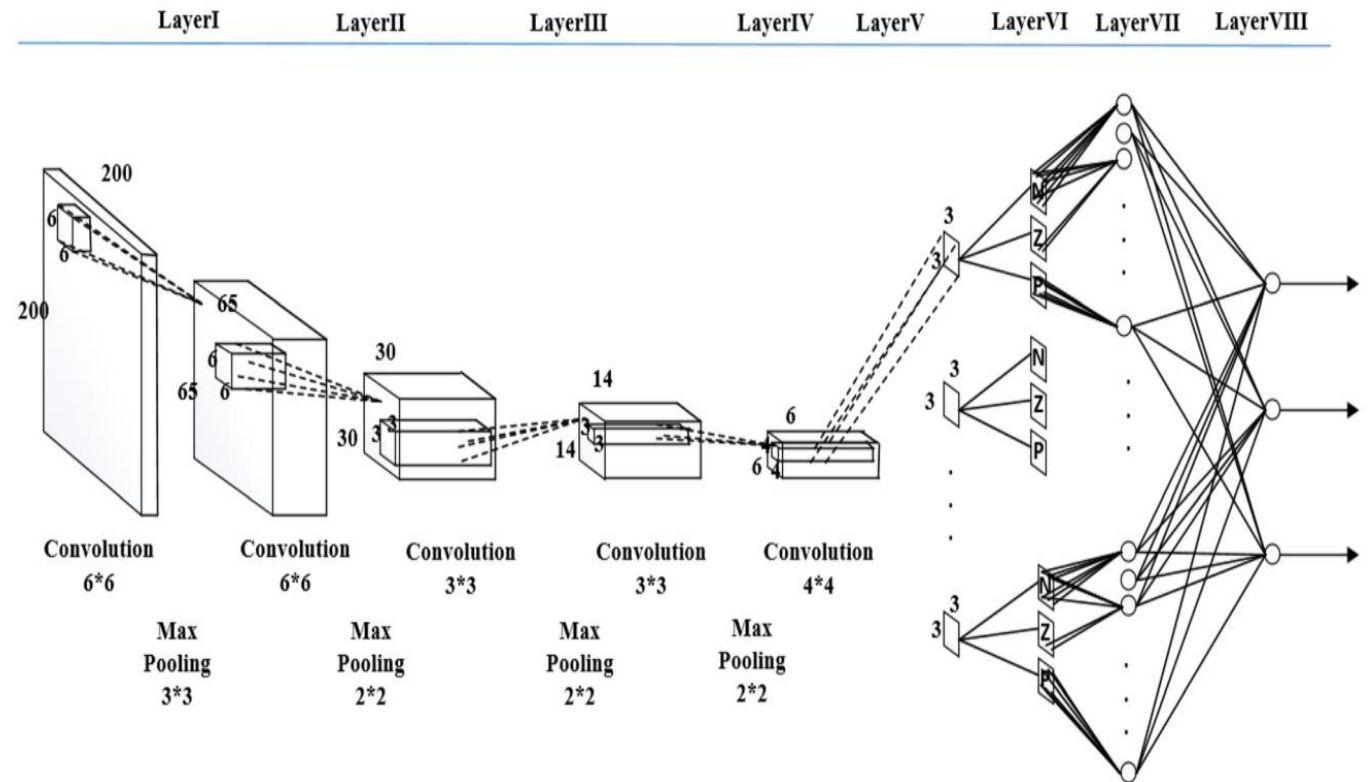
- Dropout - overfitting

$$\phi_q^p = \prod_{i=1}^{h \times w} \mu_{F_i^q}(x_i)$$

$$N = e^{-\frac{(x+m)^2}{2\sigma^2}}, Z = e^{-\frac{x^2}{2\sigma^2}}, P = e^{-\frac{(x-m)^2}{2\sigma^2}}$$

Fuzzy inference units:

$$3^{3 \times 3 \times 80} \rightarrow 3^{3 \times 3}$$



# Proposed method

- $\phi_q^p = \prod_{i=1}^9 \mu_{F_i^q}^p(x_i)$

$$\mu_{F_i^q}^p(\zeta_{p,i}^5) = \begin{bmatrix} N(\zeta_{p,1}^5) \times \dots \times N(\zeta_{p,i}^5) \times \dots \times N(\zeta_{p,9}^5) \\ Z(\zeta_{p,1}^5) \times N(\zeta_{p,i}^5) \times \dots \times N(\zeta_{p,9}^5) \\ P(\zeta_{p,1}^5) \times \dots \times P(\zeta_{p,i}^5) \times \dots \times P(\zeta_{p,9}^5) \end{bmatrix}$$

$$\psi = \begin{bmatrix} \phi_1^1 \\ \phi_2^1 \\ \vdots \\ \phi_{19683}^{80} \end{bmatrix}, \quad z^8 = \begin{bmatrix} z_1^8 \\ z_2^8 \\ z_3^8 \end{bmatrix}, \quad w^8 = \begin{bmatrix} w_{1,1}^8 & \dots & w_{1,19683}^8 \\ w_{2,1}^8 & \dots & w_{2,19683}^8 \\ w_{3,1}^8 & \dots & w_{3,19683}^8 \end{bmatrix}$$

$$z^8 = w^8 \times \psi, \quad y = a^8 = \frac{1}{e^{z_1^8} + e^{z_2^8} + e^{z_3^8}} \begin{bmatrix} e^{z_1^8} \\ e^{z_2^8} \\ e^{z_3^8} \end{bmatrix}$$

# Proposed method

CFNN	Feature numbers	Weight numbers
Layer I	$65 \times 65 \times 20$	$6 \times 6 \times 3 \times 20$
Convolution $6 \times 6$		
Max pooling $3 \times 3$		
Layer II	$30 \times 30 \times 40$	$6 \times 6 \times 20 \times 40$
Convolution $6 \times 6$		
Max pooling $2 \times 2$		
Layer III	$14 \times 14 \times 40$	$3 \times 3 \times 40 \times 40$
Convolution $3 \times 3$		
Max pooling $2 \times 2$		
Layer IV	$6 \times 6 \times 40$	$3 \times 3 \times 40 \times 40$
Convolution $3 \times 3$		
Max pooling $2 \times 2$		
Layer V	$3 \times 3 \times 80$	$4 \times 4 \times 40 \times 80$
Convolution $3 \times 3$		
Layer VI	2160	N/A
Fuzzifier		
Layer VII	1,574,640	N/A
Inference Layer		
Layer VIII	3	$1,574,640 \times 3$
Defuzzifier		

# Proposed method

- Cross entropy

$$L = - \sum_{i=1}^3 d_i \ln(y_i)$$

$$\partial L / \partial z^8 = \partial y / \partial z^8 \partial L / \partial y$$

$$\frac{\partial y_i}{\partial z^8} = \begin{bmatrix} \frac{\partial y_1}{\partial z_1^8} & \frac{\partial y_2}{\partial z_1^8} & \frac{\partial y_3}{\partial z_1^8} \\ \frac{\partial y_1}{\partial z_2^8} & \frac{\partial y_2}{\partial z_2^8} & \frac{\partial y_3}{\partial z_2^8} \\ \frac{\partial y_1}{\partial z_3^8} & \frac{\partial y_2}{\partial z_3^8} & \frac{\partial y_3}{\partial z_3^8} \end{bmatrix} \text{ and } \frac{\partial L}{\partial y} = \begin{bmatrix} \frac{\partial L}{\partial y_1} \\ \frac{\partial L}{\partial y_2} \\ \frac{\partial L}{\partial y_3} \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\partial y_i / \partial z_j^8,$$

$$\frac{\partial y_i}{\partial z_j^8} = \begin{cases} \frac{e^{z_j^8} \sum_{k=1}^3 e^{z_k^8} - e^{z_j^8} e^{z_j^8}}{\left( \sum_{k=1}^3 e^{z_k^8} \right)^2} = \frac{e^{z_j^8}}{\sum_{k=1}^3 e^{z_k^8}} \left( 1 - \frac{e^{z_j^8}}{\sum_{k=1}^3 e^{z_k^8}} \right) = y_j(1 - y_j), & \text{if } i = j \\ \frac{0 - e^{z_i^8} e^{z_j^8}}{\left( \sum_{k=1}^3 e^{z_k^8} \right)^2} = \frac{-e^{z_i^8}}{\sum_{k=1}^3 e^{z_k^8}} \frac{e^{z_j^8}}{\sum_{k=1}^3 e^{z_k^8}} = -y_i y_j, & \text{if } i \neq j \end{cases}$$

$$\begin{aligned} \frac{\partial L}{\partial z_j^8} &= - \sum_{i=1, i \neq j}^3 \frac{d_i}{y_i} (-y_i y_j) - \frac{d_j}{y_j} (y_j(1 - y_j)) \\ &= \sum_{i=1, i \neq j}^3 d_i y_j - d_j + d_j y_j = -d_j + y_j \sum_{i=1}^3 d_i. \end{aligned}$$

# Proposed method

- Cross entropy

$$\sum_{i=1}^3 d_i = 1,$$

$$\partial L / \partial z_j^8 = -d_j + y_j.$$

$$\partial L / \partial w^8 = \partial L / \partial z^8 \psi^T = (a^8 - d) \psi^T.$$

$$\partial L / \partial \phi^p = (w^8)^T \partial L / \partial z^8 = (w^8)^T (a^8 - d)$$

$$\begin{aligned} \frac{\partial L}{\partial z_p^5} &= \left( \frac{\partial(\phi_1^p, \dots, \phi_{19688}^p)}{\partial(z_{p,1}^5, \dots, z_{p,9}^5)} \right)^T \times \frac{\partial L}{\partial \phi^p} \\ &= \begin{bmatrix} \frac{\partial \phi_1^p}{\partial z_{p,1}^5} & \frac{\partial \phi_2^p}{\partial z_{p,1}^5} & \dots & \frac{\partial \phi_{19688}^p}{\partial z_{p,1}^5} \\ \frac{\partial \phi_1^p}{\partial z_{p,2}^5} & \dots & \dots & \cdot \\ \vdots & \dots & \dots & \cdot \\ \frac{\partial \phi_1^p}{\partial z_{p,9}^5} & \dots & \dots & \frac{\partial \phi_{19688}^p}{\partial z_{p,9}^5} \end{bmatrix} \times \begin{bmatrix} \frac{\partial L}{\partial \phi_1^p} \\ \frac{\partial L}{\partial \phi_2^p} \\ \vdots \\ \frac{\partial L}{\partial \phi_{19688}^p} \end{bmatrix} \end{aligned}$$

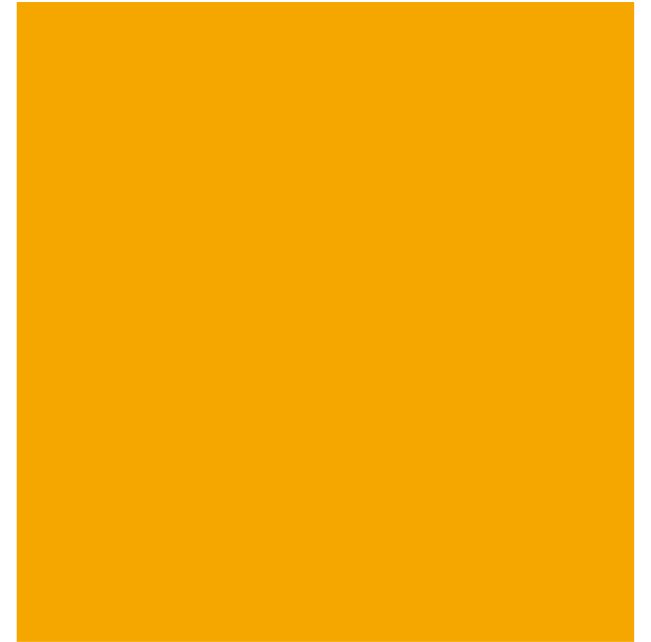
$$\partial \phi_q^p / \partial z_{p,i}^5 = (\partial \phi_q^p / \partial \mu_{F_i^q}) \times (\partial \mu_{F_i^q} / \partial \zeta_{p,i}^5) \times (\partial \zeta_{p,i}^5 / \partial z_{p,i}^5),$$

$$\frac{\partial \mu_{F_i^q}}{\partial \zeta_{p,i}^5} = \frac{\partial e^{-\frac{(\zeta_{p,i}^5 + m_q)^2}{2\sigma^2}}}{\partial \zeta_{p,i}^5} = -e^{-\frac{(\zeta_{p,i}^5 + m_q)^2}{2\sigma^2}} \times \frac{\zeta_{p,i}^5}{\sigma^2} = \mu_{F_i^q} \times \frac{\zeta_{p,i}^5}{\sigma^2},$$

$$\frac{\partial \phi_q^p}{\partial \mu_{F_i^q}} = \frac{\prod_{j=1}^9 \left( \mu_{F_j^q} \left( \zeta_{p,j}^5 \right) \right)}{\mu_{F_i^q} \left( \zeta_{p,i}^5 \right)} \quad \frac{\partial \zeta_{p,i}^5}{\partial z_{p,i}^5} = \begin{cases} 0, & \text{if dropped out} \\ 1, & \text{if not dropped out} \end{cases}$$



# Results

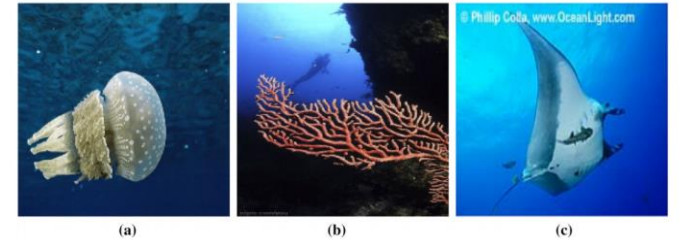




# Case 1

TWGC (Taiwan GPU Cloud) - Tesla V100 GPUs - 16 GB RAM cluster

- 3 classes
- 700 image per class
- ImageNet
- 200x200 RGB
- K-fold validation –  $k^* / k-1$
- Dropout

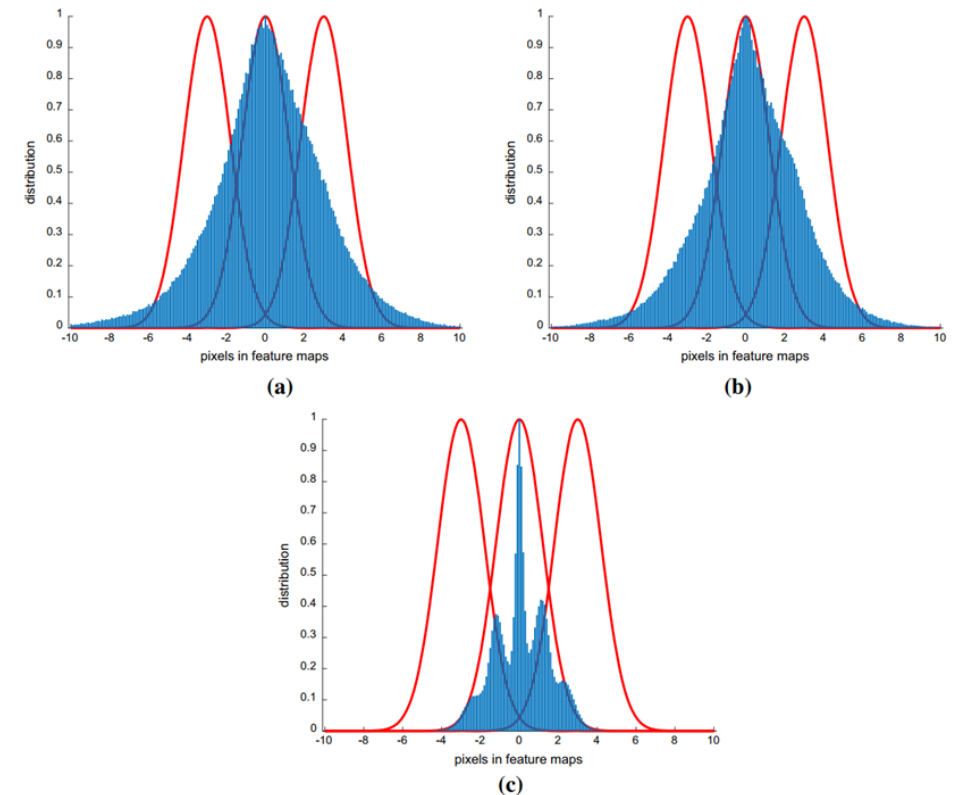


# Case 1

- Expert rules

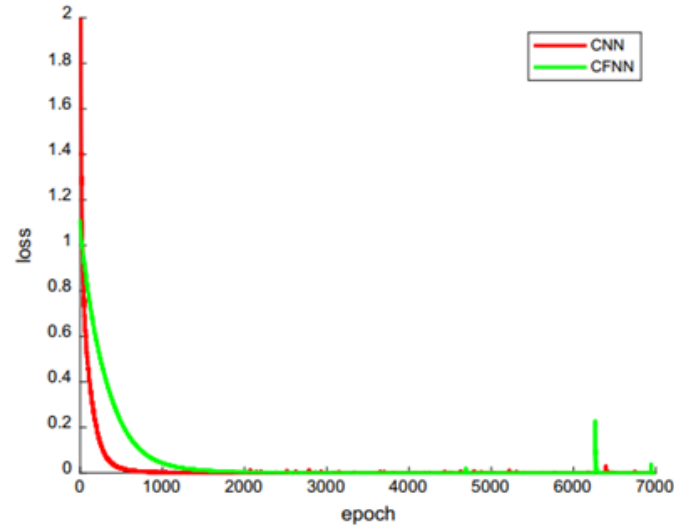
$$m = 3.0, \sigma = 1.2.$$

- Gaussian distribution randomly

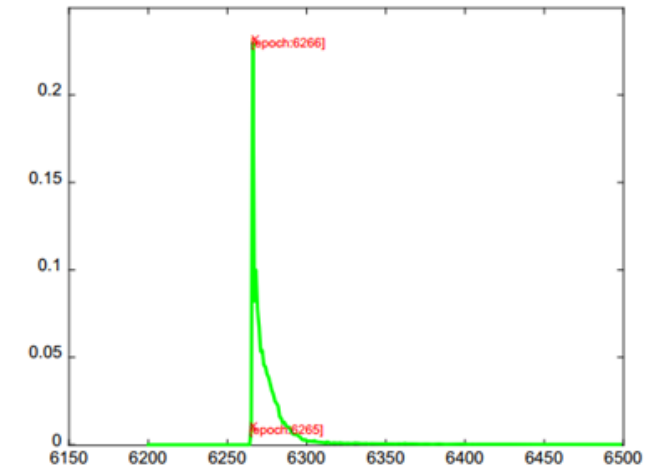


# Case 1

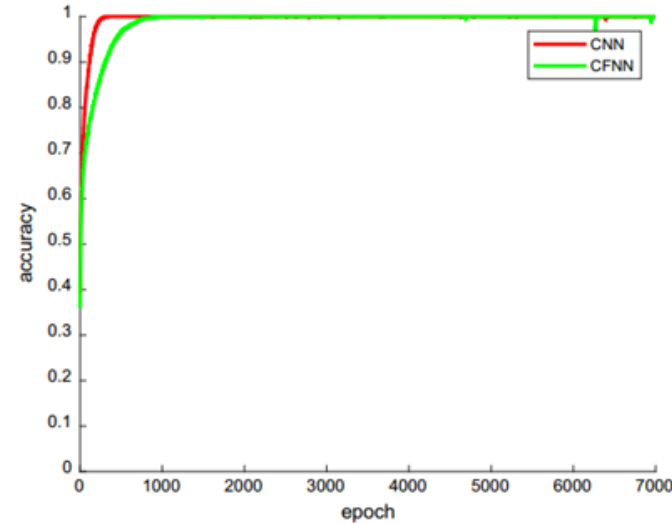
- Dropout !



(a)

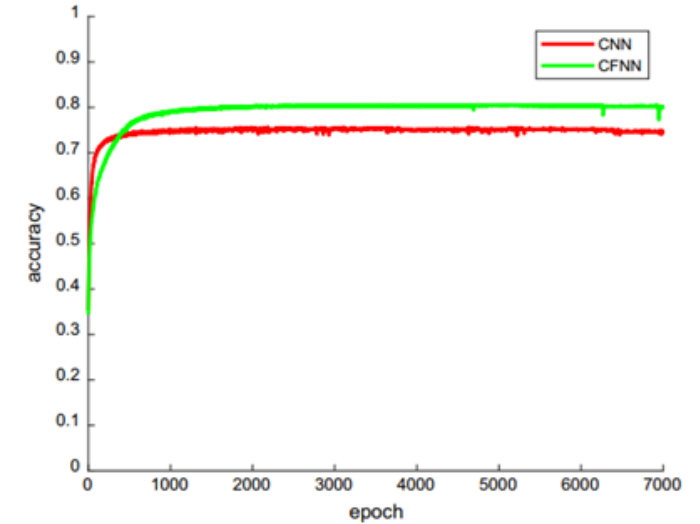


(b)



(c)

training

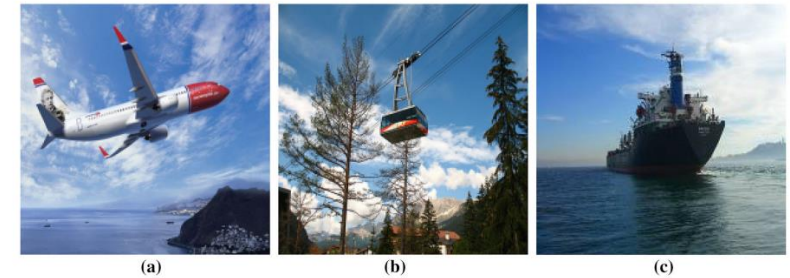


(d)

test

## Case 2

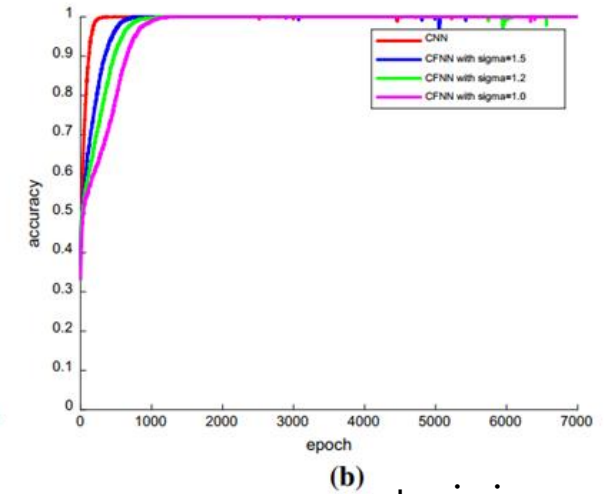
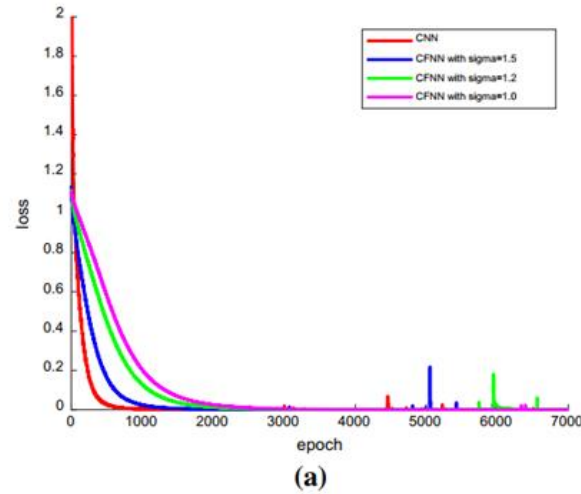
- 700 images per class - ImageNet
- Different membership function
- 10 fold - 210 training set - 1890 test set



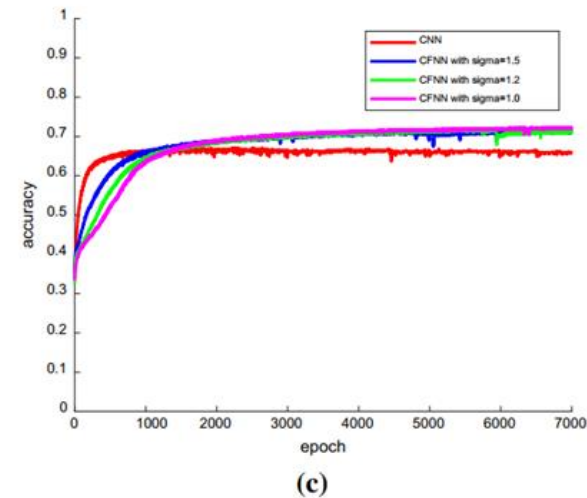
Network architecture	The testing accuracy (%)
CNN	65.83
The proposed CFNN with $\sigma = 1.5$	70.95
The proposed CFNN with $\sigma = 1.2$	71.11
The proposed CFNN with $\sigma = 1.0$	71.98

## Case 2

- CNN – fastest converge
- CNN – overfit rapidly
- CFNN
  - – better result – insufficient data



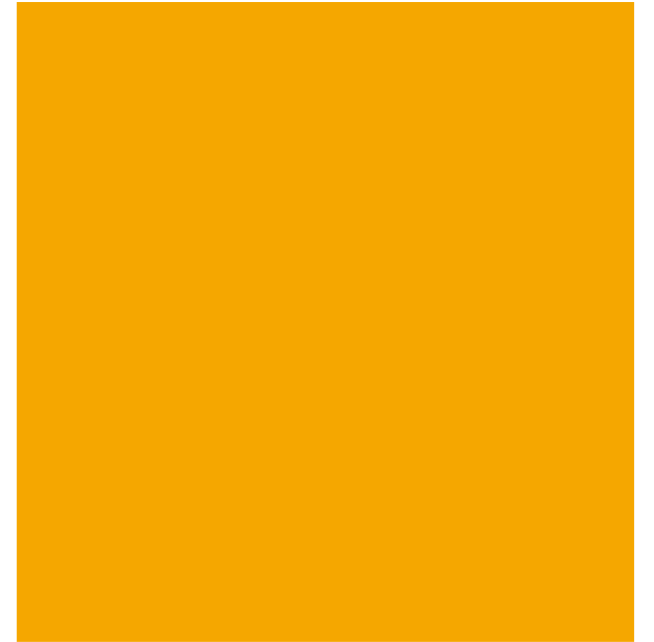
(b) training



(c) test



# Conclusion



## Conclusion and future work

Reduce overfitting

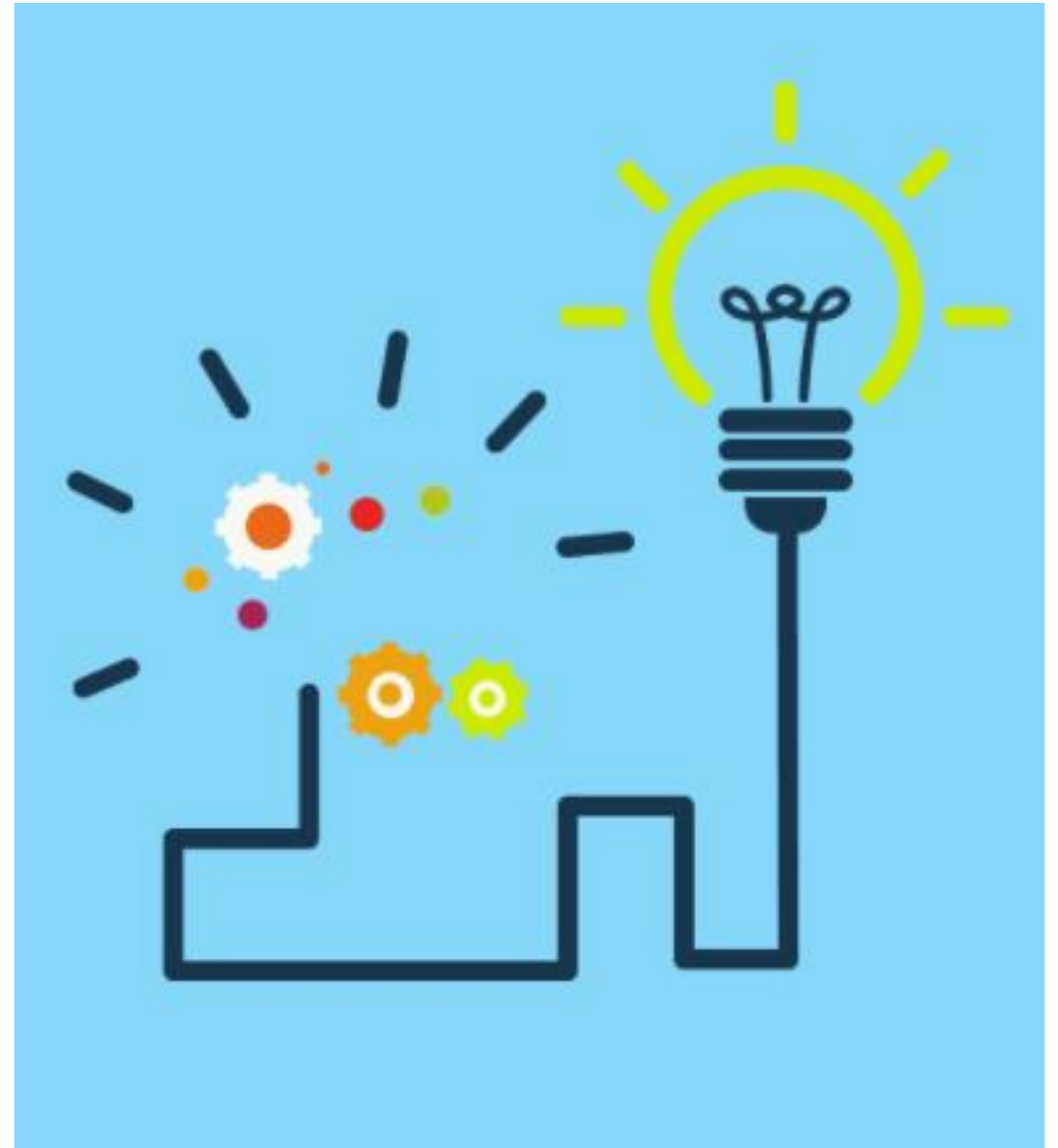
Sum-up feature information

Increase accuracy in tests

Possible to enhance testing accuracy  
by observing distribution of pixels -  
feature map - adjust membership  
function

Optimized membership function  
=>increase accuracy

Adaptive adjustment strategy



Thank you for your attention.