PROPORTIONALITY AND SUPERPOSITION THEOREM

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INTRODUCTION

This project focusses on the verifying the proportionality and superposition theorem both experimentally and computationally.

Proportionality is a way to relate two quantities together. This means that when more input is supplied, you get more output which is proportional to the input. The Proportionality Theorem states that the response in a circuit is proportional to the source acting in the circuit. This is also known as Linearity. The proportionality constant (K) relates the input voltage to the output voltages as:

$$Vin = K.\ Vout$$

Superposition is another way to solve a linear electrical circuit. The Superposition Theorem states that in any linear electrical circuit, any voltage or current value can be obtained by taking the individual contributions to that voltage or current as a result of each source taken alone and summing them together.

TITLE OF THE EXPERIMENT

Proportionality and Superposition theorems.

PREREQUISITE

We use these principles because it will help us mathematically help in reducing the math during solving or observing the circuit.

Ohm’s Law

Ohm’s law states the relation between electric current and potential difference. The current flowing through the conductor is proportional to the voltage applied to it. One of the basic electric circuits law is ohms law.

Ohms law states that, "the voltage across a conductor is directly proportional to the current flowing through it, provided all physical conditions and temperature remains constant”.

Mathematical representation for ohms law is:

$$\boxed{V=I*R}$$

In the equation, the proportionality constant is “R”, “R” is resistance and its units is ohms. The symbol used to represent ohm is Ω. The physical factors like temperature should be constant.in some cases as the current increases, the temperature of the conductor also increases, which may lead to violation of the law. In order to find the current in the circuit we can rewrite the formula as;

$$\boxed{I=V/R}$$

If we to find the value of the resistance then we can rewrite the formula as:

$$\boxed{R=V/I}$$

Applications of ohms law:

- To determine the voltage, resistor or current of an electric circuit.
- Ohm's law is used to maintain the desired voltage drop across the electronic components.
- It is also used in DC ammeter and other dc shunts to divert the current.

Limitations of ohms law:

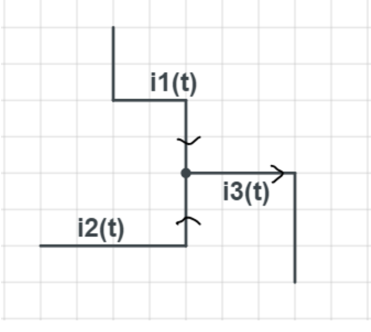
- This law cannot be applied to components with different current voltage relation for both directions I.e., cannot be applied to unilateral elements like diode, transistors etc.
- Also, not applicable to non-linear elements I.e., Element which don’t have current and voltage proportional to each other.
- Ohm’s is also not followed by semiconductors their V-I graph has a steep rising at a particular voltage, which indicates that the material begins to conduct properly only after a certain voltage.

Kirchhoff’s Current Law

Kirchhoff's junction rule also known as Kirchhoff's current law (KCL), Kirchhoff's junction rule, Kirchhoff's point rule, Kirchhoff's nodal rule. Its works on the application of the principle of conservation of electric charge. Kirchhoff's junction rule states that at any circuit junction, the sum of the currents flowing into and out of that junction are equal. We can also tell, the sum of all currents entering a node is equal to the sum of all currents leaving the node.

Solving circuits using KCL

Let us consider a network of assumed directions of the current as shown below.



Let us choose a sign convention such that currents entering the node are positive, and the currents leaving the node are negative.

With this convention, KCL applied at the node yields the equation:

$$I_1(t)+I_2(t)-I_3(t)=0$$
or,
$$I_1(t)+I_2(t)=I_3(t)$$

This amounts to the statement that the total current entering the node is the same as the total current leaving the node.

Kirchhoff's Voltage Law

Kirchhoff's voltage law is also known as Kirchhoff's second law states that the sum of all voltages around a closed loop in any circuit must be equal to zero. This follows change conservation and also conservation of energy. It states that “the net electromotive force around a closed-circuit loop is equal to the sum of potential drops around the loop”. When a charge moves around a closed loop in a circuit it must gain as much energy as it loses. The gain in charge is equal to corresponding losses in energy through resistances. Therefore, the total voltage in a closed loop of a circuit is equal to zero.

Solving circuit using Kirchhoff's second law

- The first important step is to draw a closed loop to a circuit. Once done with it draw the direction of the flow of current. Defining our sign convention is very Important.
- Draw closed loops in the circuit.
- Define the direction of flow of current in the circuit.
- Note that the direction does not have to be the Actual direction in which the current is flowing.
- Now write equations for each loop in terms of voltage drop across each component.
- Write the obtained equations in terms of Current variables obtained using Kirchhoff's current Law.
- Now we solve the problem using the obtained equations.

Effective resistance in Series

A resistor is not only a fundamental electronic component but also used to convert current to voltage and voltages to current. All the resistors follow the ohms law and Kirchhoff's law. Resistors are said to be connected in series when they are chained together in a single line resulting in a same current flowing through them. As the current enters the first resistor is same as the current in the second resistor and so on. So, the resistor in series have a common current flowing through them as current through one resistor must be also same in other resistor in series. As we know the current through all the resistor is the same,

$$I_{R1}=I_{R2}=I_{R3}=\dots=I_{RN}$$

As the resistors are connected in series the current is same through each resistor and the total resistance is given as R_T. R_T of the circuit must be equal to the sum of all the individual resistors resistance. That is:

$$R_{Total}=R_1+R_2+R_3+...R_N$$

So, we can see that we can replace all the three individual resistors with a single “equivalent” resistor which is R_T. The total resistance is generally known as the equivalent resistance and can be defined as, "a single value of resistance that can replace any number of resistors in series without altering the values of the current or the voltage in the circuit”

Series resistor current:
One important thing to note is, every time when resistors are connected in series the total resistance should be greater than the individual resistance.

Series resistor voltage:
The voltage across each resistor connected in series must be different. We know that the total voltage in the circuit is equal to the sum of the potential difference across them

$$V_T=V_1+V_2+V_3+\ldots+V_N$$

If we want to find unknown current or resistance in the circuit, we can rewrite the formula according to our requirements.

Effective resistance in Parallel

Resistors are said to connected in parallel when both of their terminals are connected respectively to each terminal of the other resistor or resistors. When the resistors are in parallel, they have same voltage across them. The current through all the resistors may not be the same. However, the voltage across all the resistors connected in parallel have the same voltage across them. So, we can define the parallel resistive circuit as a circuit where the resistors are connected to the same points or nodes and is identified by the fact that it has more than one current path connected to a common voltage source. Now in a parallel resistive circuit we know that the voltage across all the resistors will be same, hence,

$$V_{R1}=V_{R2}=V_{R3}...=V_{RN}$$

When a resistor is connected in series the total resistance is calculated as the sum of all the resistors connected in series. But when resistors are connected in parallel it is calculated in a different manner. For finding the total resistance in the circuit connected in parallel is to add the reciprocal of the resistance (1/R). Therefore, the inverse of the equivalent resistance is equal to the algebraic sum of the inverse of the individual resistances.

$$\frac{1}{R_{Total}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots+\frac{1}{R_N}$$

When two resistors of the same resistance value are connected in parallel the total resistance is equal to the half of the individual resistance. Important point to note here is the equivalent resistance is always less than the smallest resistor in the parallel network, so the total resistance, RT will always decrease. conductance is the reciprocal of the resistor.

Mesh Current Analysis

Mesh current analysis is used to find the current circulating around a mesh or loop in a closed circuit. The Kirchhoff's voltage law and current law are mainly used while solving a circuit using mesh current analysis. This method is also known as the Maxwell's circulating currents method. On using this method, the final mathematical calculations turn out to be very simple. Up to now we have seen about Mesh Current Analysis, but now we will see how to solve using this method. Here, we are using matrix concepts like multiplication, inverse matrix, etc. The general equation which is used in mesh current analysis is given as,

$$[V]=[R].\ [I]$$

As stated earlier mesh current analysis is used to calculate the current circulating around a mesh. In order to get the current matrix, we need to calculate the inverse or pseudo inverse of the resistance matrix and multiply it with the voltage matrix.

$$[I]=\text{inverse}([R]).\ [V]$$

where: [V] gives the total voltage across all the loops.
[I] gives the current in the loop.
[R] is the resistance matrix.

Limitations of Mesh Current Analysis:

- Mesh Current Analysis is limited to only a planar circuit (2D) i.e., in a given circuit no branch should cross over another branch.
- Mesh Current Analysis can be used only when the circuit is planar, otherwise we can't use this method. When we consider a large circuit then the number of meshes will be large and the total number of equations will be more. So, in that case it will be inconvenient to use this method.

Though this method has some disadvantages, it is actually a very powerful tool for circuit analysis. This method is used to solve small circuits with small numbers of meshes.

Nodal Voltage Analysis

Nodal voltage analysis is used to find the unknown voltage drops between different nodes which provides a common connection between two or more circuit components. Nodal voltage analysis uses the Kirchhoff's first law to find the voltage potentials around the circuit. Kirchhoff's first law states that "the currents entering a node are exactly equal in value to the currents leaving the node". In order to use the Kirchhoff's voltage law, the current circulating in a circuit must be converted to the voltage across the branch of the circuit

For analysing a circuit using the nodal voltage analysis, firstly, we need calculate the current at each node using Kirchhoff's Law and represent it in V/R form using Ohms Law. And now we can calculate the values of unknown voltages and current in the circuit using the nodal equations. If there are "n" nodes in the circuit then there will be an "n-1" independent nodal equation. Here one node is used as a reference node and all the other voltage will be measured with respect to this common node. Nodal voltage analysis is more appropriate when we consider a large current source.

Advantages of nodal analysis:

- It works on any circuit.
- Works with high precision and accuracy for circuits with few nodes.

Thevenin Theorem

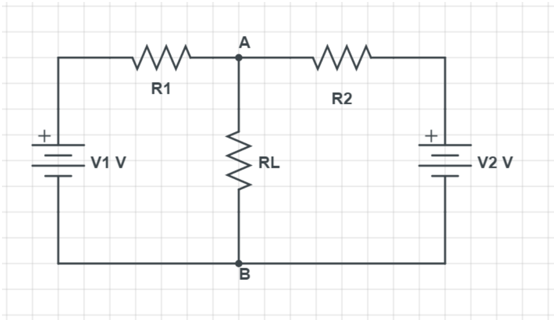
It states that any linear circuit with multiple source and multiple resistors can be simplified into an equivalent circuit with a single source and a single resistor connected in series across the load. The voltage source present in the Thevenin's equivalent circuit is called as Thevenin's equivalent voltage or simply Thevenin's voltage, V_{th}. And the resistor present in the Thevenin's equivalent circuit is called as Thevenin's equivalent resistor or simply Thevenin's resistor, R_{th}.

General Procedure for using the theorem to solve the circuit

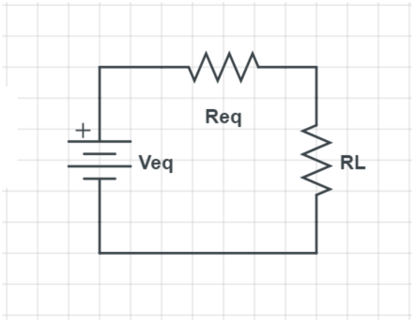
Step 1: Remove the load resistance and all power sources and solve for the effective resistance (R_{eff}) in the circuit and the obtained R_{eff} is the Thevenin's resistance R_{th}.

Step 2: Remove the load resistance and Write the equations for Kirchhoff's Current Law or Ohms law to obtain the current across the circuit.

Step 3: Using the value obtained for the current solve for the voltage across the two nodes using Kirchhoff's Voltage Law. The obtained voltage is Thevenin's voltage V_{th}.



$\Rightarrow$



$$\boxed{I = V_{\text{th}}/(R_{\text{th}} + R_L) \quad \text{or} \quad I = V_{\text{oc}}/(R_i + R_j)}$$

It can also be applied to AC circuits with components like resistors, inductors, capacitors. And also, like Thevenin's voltage and resistance, Thevenin's impedance can also be obtained by replacing all voltage sources by their internal impedances.

Advantages of Thevenin's Theorem

- It reduces a complex circuit to a simple circuit with a single source V_{th} in series with a single resistance R_{th}.
- It simplifies the part of the circuit with lesser importance and helps us to view the action of the output part directly.
- It is very useful to determine the current in a particular branch of a network as the resistance of that branch is varied while all other resistances and source remain constant.

Limitations of Thevenin's Theorem

- The Thevenin's resistance and voltage has I-V characteristic and follows Ohm's law only w.r.t to the load.
- The Thevenin's theorem can be only applied to a linear circuit and hence becomes a drawback since it can't be extended over a range.

As a whole this theorem has proved to be an easy way to analyse the circuits mainly in the circuit with the load that changes value during a process. And it also provides an efficient way to calculate voltage and current across the load without recalculating the whole circuit.

Norton's theorem

Norton's theorem states that any linear circuit containing multiple current sources and resistors can be expressed by an equivalent circuit with just a single current source and parallel resistance connected to a load.

General Procedure in using the theorem to solve it

Step 1: Remove the load resistance and all power sources, solve for the effective resistance (Reff) in the circuit.

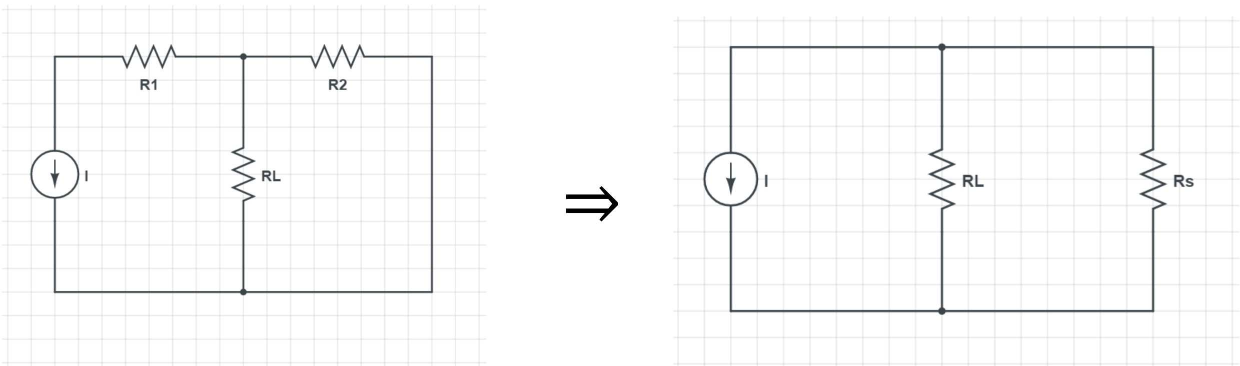
Step 2: Now the R_{eff} is in parallel with the load and the current source, now calculate the net resistance for the whole circuit i.e., for Rth and RL, using

$$R_{net} = 1/R_{eff} + 1/R_L$$

Step 3: Now calculate the voltage across the load with ohm's law,

$$V_{ab} = I * R$$

Step 4: Now calculate the Current across the load using I_{th}=V/R



The equivalent resistance of the network,

$$R= (R_1 + R_1)+R_2||\ R_3 = (R_1 + R_1) + \frac{R_2.R_3}{R_2 + R_3}$$

Current supply by the battery $I_1 = \frac{E}{R}$

Norton equivalent current I_N= current flowing through R_3,

$$I_2 = I_1 \times \frac{R_2}{R_2 + R_3} = \frac{E}{R_1 + R_i + R_2||R_3} \times \frac{R_2}{R_2 + R_3}$$

$$\boxed{I_2 = \frac{E.R_2}{(R_1 + R_i).(R_2 + R_3) + (R_2.R_3)}}$$

Advantages of Norton's Theorem

- It is useful for simplify any linear circuits, no matter how complex they are, then it will be equivalent to just a single current source and parallel resistance connected to a load.
- It's used to solve problems on parallel generators with unequal emf's and unequal impedances.
- Norton's theorem can be interchangeably by Thevenin's theorem through proper source transformations.

Limitations of Norton's Theorem

- The formula is appropriate for linear modules like resistors.
- It does not work for modules which are not linear like diodes, the transistor.
- It's also not useful for magnetic locking circuitries.
- It also does not work for such circuitries which have been loaded in parallel with dependent supplies.

DESCRIPTION/FUNCTIONALITY OF THE COMPONENTS

Electric Battery

It is an electric device with a positive and a negative terminal that supplies power. The positive terminal is called anode and the negative is called cathode. The electrons flow from the negative terminal of the battery to the positive terminal through an external circuit completing and closing the circuit. The principle of operation of the battery converts the chemical energy stored to electrical energy directly. And also, the batteries are designed in such a way that it is energetically favourable for a redox reaction to take place.

Voltage source

It's a device that supplies constant voltage or maintains a fixed voltage drop across its terminals. It can be either an independent source or a dependent source (controlled voltage source). The dependent source either relays on another voltage source or another current source. It is represented in terms of Volts (SI unit -V)
An ideal voltage source has a zero internal resistance so it is able to supply and absorb any amount of current through it. The current through the circuit is completely determined by an external circuit if we consider the case of an ideal voltage source. When the load resistance is connected to the voltage source, the current through the source approaches infinity as the load resistance approaches zero i.e., a short circuit, therefore the voltage source can supply unlimited power. However, the voltage source cannot be ideal due to the presence of non-zero effective of internal resistance.

Current Source

It's a device that generates a current that is independent of the voltage changes across it. And the voltage across the current source is determined by the external circuit i.e., it does not depend on current source.in the current source the current flows from positive(+ve) terminal to the negative (-ve) terminal and is indicated by the arrow. The current can either be dependent or independent, if it is a dependent source then it directly depends upon another current source or a voltage source.
For an ideal current source, the internal resistance is infinite, but it is not practically possible so the load resistance is used to achieve the infinite condition. When the load resistance is connected the voltage, source approaches infinity and so does the effective resistance in the circuit. Therefore, for effective usage of the current it is always connected in parallel with load resistance.

Wire

It is a component used to connect two electrical components together in a circuit and which allows the flow of a charge through it with some resistance i.e., it conducts current through it.

Resistor

It is an electronic component that resists the flow of the current through it and also helps in dividing the voltages and terminating transmission lines. The resistor can either be fixed or variable resistor depending on the usage of the resistor. Resistor is always present in all electrical components as an internal resistor. A resistor can either be connected in series or parallel with other resistors and components and the effective resistance in the circuit can be calculated using simple formulae. In high resistance resistors there is heat dissipation due to heavy resistance to flow of current due to which the electrical energy is converted in heat and dissipated out.

The resistor works on the principle of the ohms law i.e., R = V/ I i.e., it is directly proportional to the voltage and inversely proportional to current in the circuit. And the power dissipation is calculated using, P=I*V which indicates that the amount of heat released is directly proportional the current and voltage across the resistor.

Load Resistor

Load resistor is when the resistor is said to be loaded, i.e., a loaded component can dissipate power from the source to get a desired output. The power consumed is in the ranges from few watts to kilowatts. The nature of the can either be passive or active and can either be linear or nonlinear in nature. Also, the properties of a load resistor are of a pure resistor i.e., there is no reactance.

The heavy or light load resistor is used based on the desired output, here the heavy load consumes more current whereas the light load consumes less. And in case of a non-circuit load it will consume a minimum current. The load resistance is used to check the performance of the power with variable load. And for the ac sources circuit inductance and capacitance of the load resistor must be taken into consideration for numerical problems.

Capacitor

It is an electric device which has the ability to store electrical energy in the electrical field by producing potential difference across its plates. The dielectric present in between the plates is used to increase the capacitors charge capacity. The capacitor does not dissipate energy like a resistor.

The capacitor is charged when there is voltage difference applied across it due to which there is an electric field developed across it causing the positive charge to collect on a plate and the negative charge on another. When the capacitor is charged it allows no flow of current through it. And only when an alternating or accelerating voltage is applied, the capacitor allows the flow of a displacement current.

It is mostly used in circuits for blocking the direct current and while allowing alternating current to pass. It is also used to tune radio frequencies and also used to stabilize voltage and power flow in power transmission. And its I-V relation is q=V*C.

Inductor

An inductor is a passive two-terminal electrical component which stores magnetic field when electric current flows through it.an inductor is wounded by wire into a coil which is insulated. Inductor does not allow change in current through it. According to Lenz’s law, the induced voltage has a polarity which opposes the change in current that created it. The SI unit of inductor is Henry(H). inductors are widely used in alternating current (AC) electronic equipment which is particularly used in radio equipment. They are also used to in electronic filters to separate signals of different frequencies. The inductance of the inductor is defined as the ratio of the voltage to the rate of change of current. The voltage of the inductor is directly proportional to change in current in the circuit. We can also tell the induced emf across a coil is directly proportional to the rate of change of current through it.

A current through any wire create a magnetic field across it. The inductor is a wire shaped so the magnetic field will be much stronger. Then inductor acts in that way is because of the magnetic field. Inductor can store energy for a small period of time because the energy is stored as magnetic field which will be gone when the applied source is removed. The function of an inductor is to control signals and store energy.

1.PROPORTINALITY THEOREM

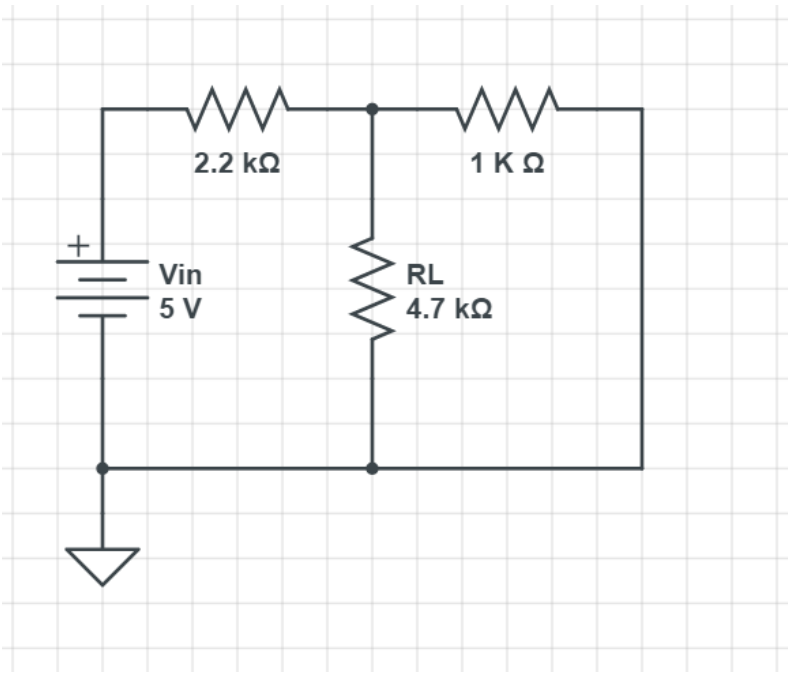
Theorem Statement

The Proportionality Theorem states that the response of a circuit is proportional to the source acting on the circuit. This is also known as linearity. The proportionality constant A relates the input voltage to the output voltage as,

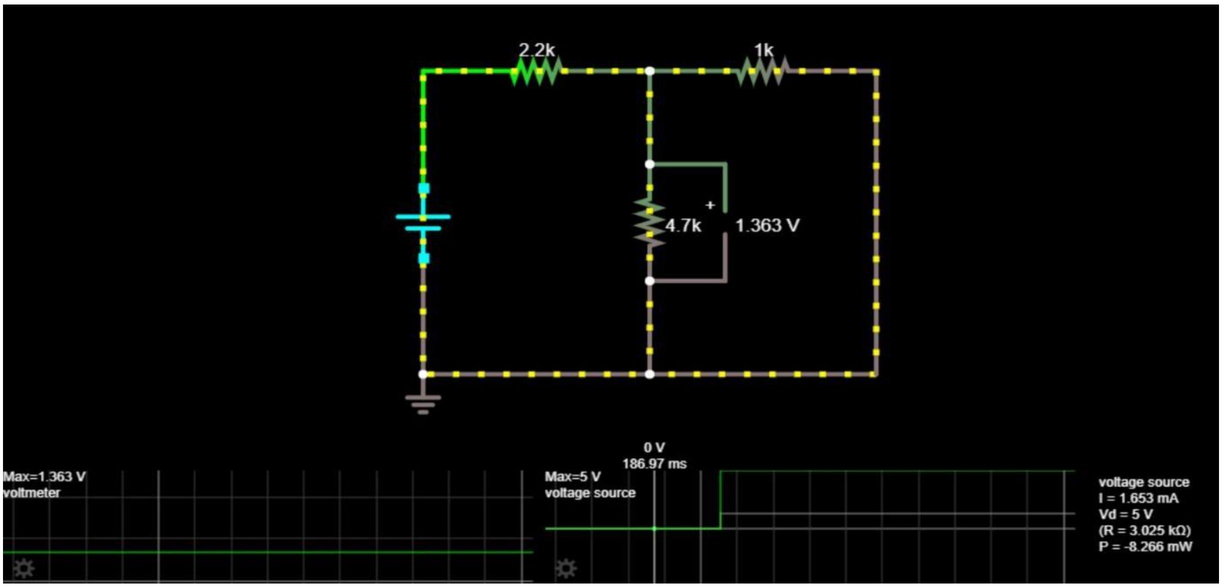
$$\boxed{V_{out}=A\cdot V_{in}}$$

Problem statement

The proportionality factor A is sometimes referred to as the gain of a circuit. For the circuit in the given diagram, the source voltage is Vin. The response Vout is across the 4.7kΩ resistor.



Simulation of circuit using Falstad
Circuit Diagram



Link: https://tinyurl.com/ycqfcg4v

Observation

Link for excel with observations (the first sheet):
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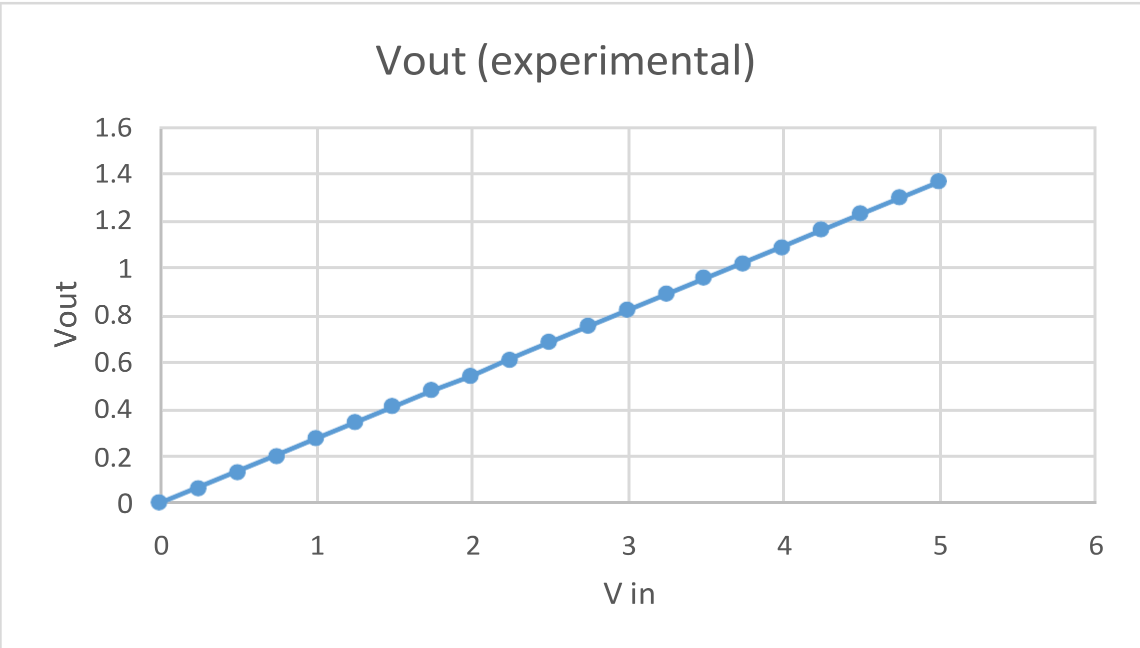
Proportionality Theorem- V_{out} vs V_{in} Graph



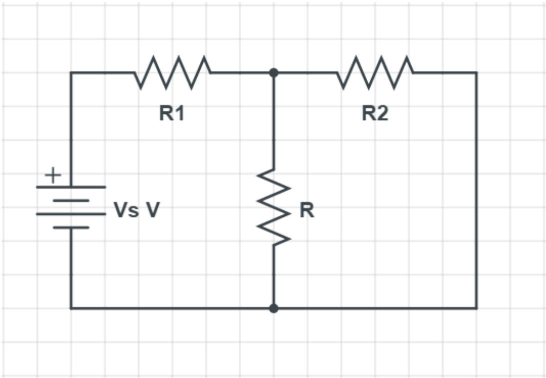
The graph we get on plotting the experimantal values in excel

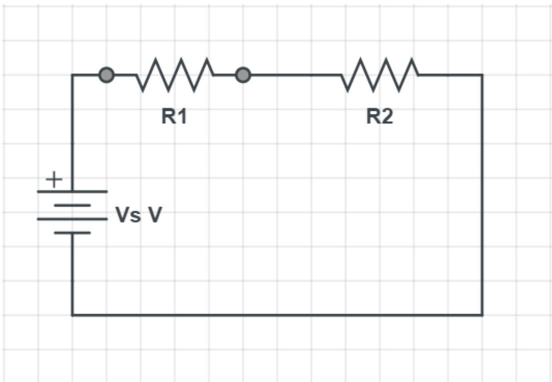
Computation

Derivation

Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load.

Using the above stated Thevenin's theorem we can simplify the given circuit as follows,





By observation we can clearly conclude that the two resistance R1 and R2 are in parallel, hence the net resistance of the circuit comes out to be,

$$\boxed{R_{\text{eq}} = \frac{R_1 . R_2}{R_1 + R_2}}$$

To obtain the net voltage across the wire AB we need to calculate the current flowing through the circuit. For calculating it we can use OHM's law. OHM's law states that the current through a conductor between two points is directly proportional to the voltage across the two points. From OHM's law,

$$\boxed{I = \frac{V}{R}}$$

Here, V is the net voltage of the circuit and R is the net resistance of the circuit after removing the joint across AB, hence in the given circuit we can write the current as,

$$\boxed{I = \frac{V_S}{R_1 + R_2}}$$

Now for finding the voltage across AB we can use the Kirchhoff's loop rule. Kirchhoff's loop rule states that the sum of all the electric potential differences around a loop is zero. Now on applying Kirchhoff's loop rule in the above circuit we get the following,

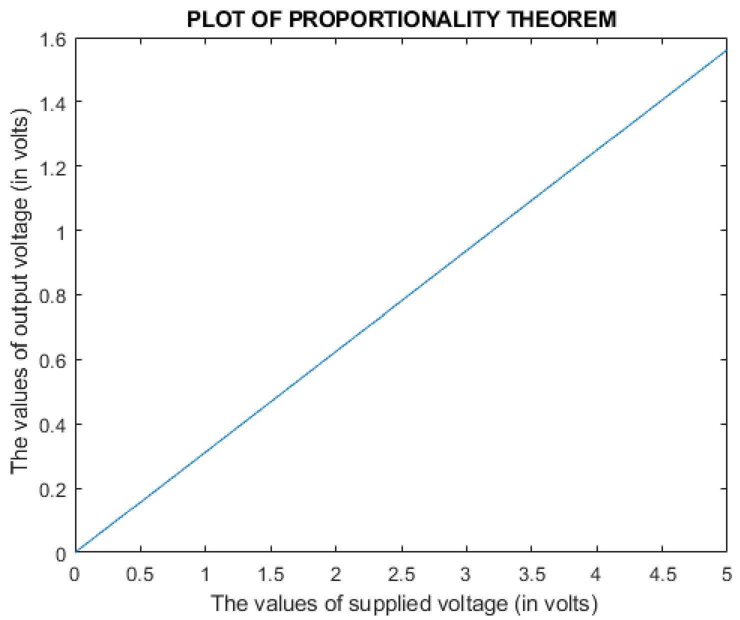
$$V_{\text{out}} = V_s - \left(\frac{V_S}{R_1 + R_2}\right) . R_1$$

And on further simplification we get the following

$$\boxed{V_{\text{out}} = V_{\text{in}} \left(\frac{R_2}{R_1 + R_2}\right)}$$

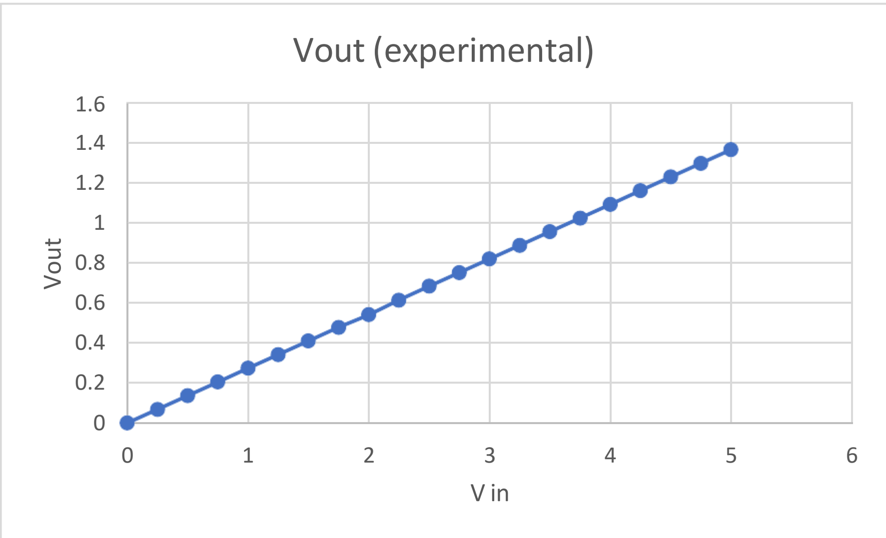
MATLAB code

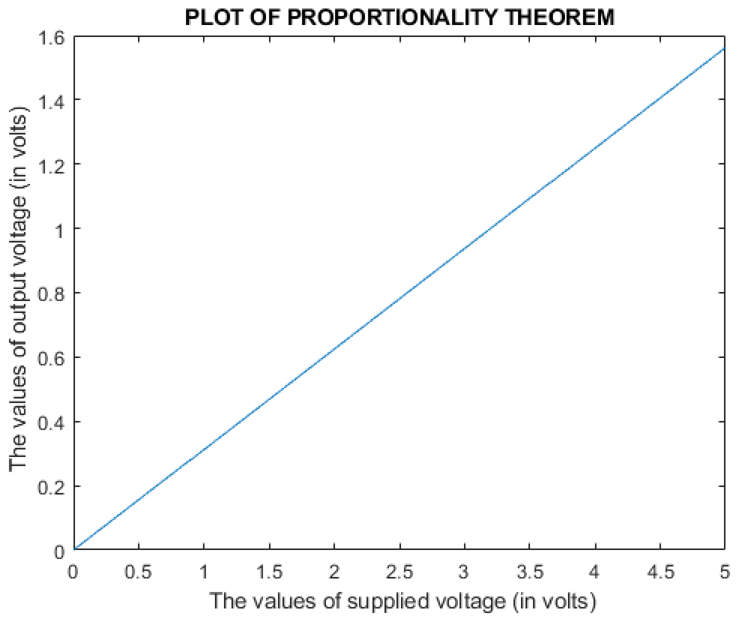
```
Vin = 0:0.25:5;
R1=2.2;
R2=1;
R=R2/(R1+R2);
Vout = Vin*R;
plot(Vin,Vout)
title("PLOT OF PROPORTIONALITY THEOREM")
xlabel(" The values of supplied voltage (in volts)")
ylabel(" The values of output voltage (in volts)")
```



RESULT

Now on plotting a graph of the given values with Vin along x-axis and the experimental and computational values of Vout along y-axis we get the following graphs,





INFERENCE

The value A is calculated by taking the ratio of Vout to Vin or finding the slope of the above graphs i.e.,

$$\boxed{A = Vout/Vin}$$

After calculating the value of A, we can observe that it is constant for any value of input voltage Vin. This constant A is called the voltage gain of the circuit. Hence, the Proportionality theorem is verified for this circuit.

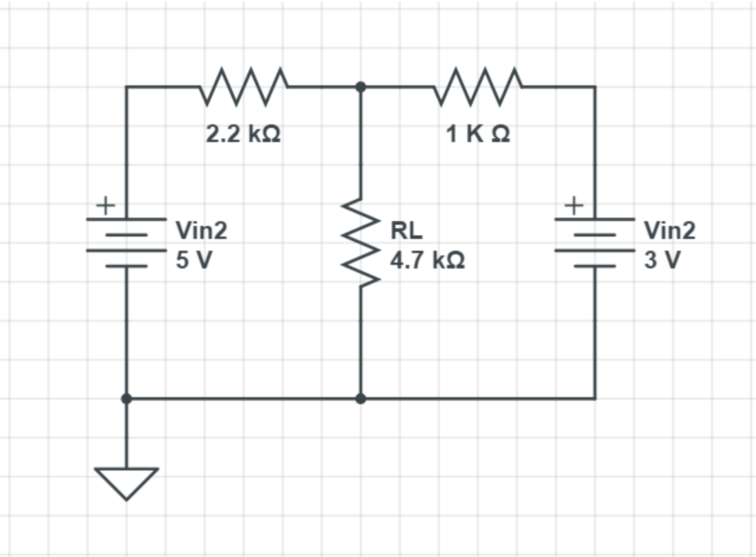
2. Superposition Theorem

Theorem Statement

The Superposition Theorem states that the response of a linear circuit with multiple independent sources, such as in the given figure, can be obtained by adding the individual responses caused by the individual sources acting alone. For an independent source acting alone, all other independent voltage sources in the circuit are replaced by short circuits and all other independent current sources are replaced by open circuits.

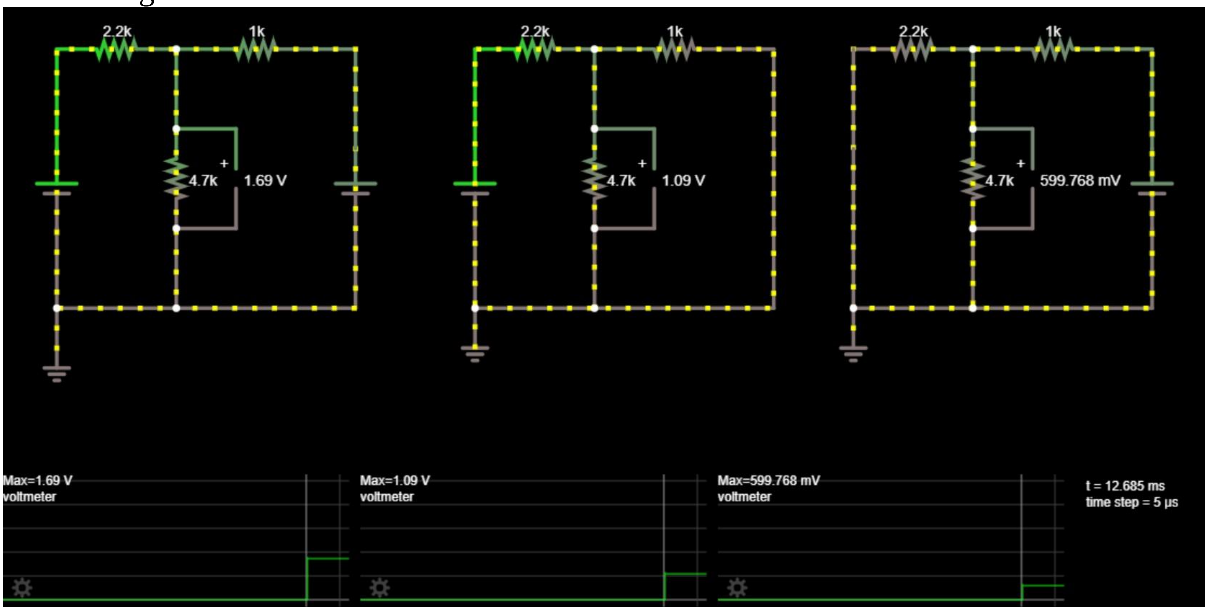
Problem statement

Check whether the given circuit follows the Superposition



Circuit Stimulation

Circuit diagram



Link: https://tinyurl.com/ybah4tgt

Observation:

Link: (the second sheet)
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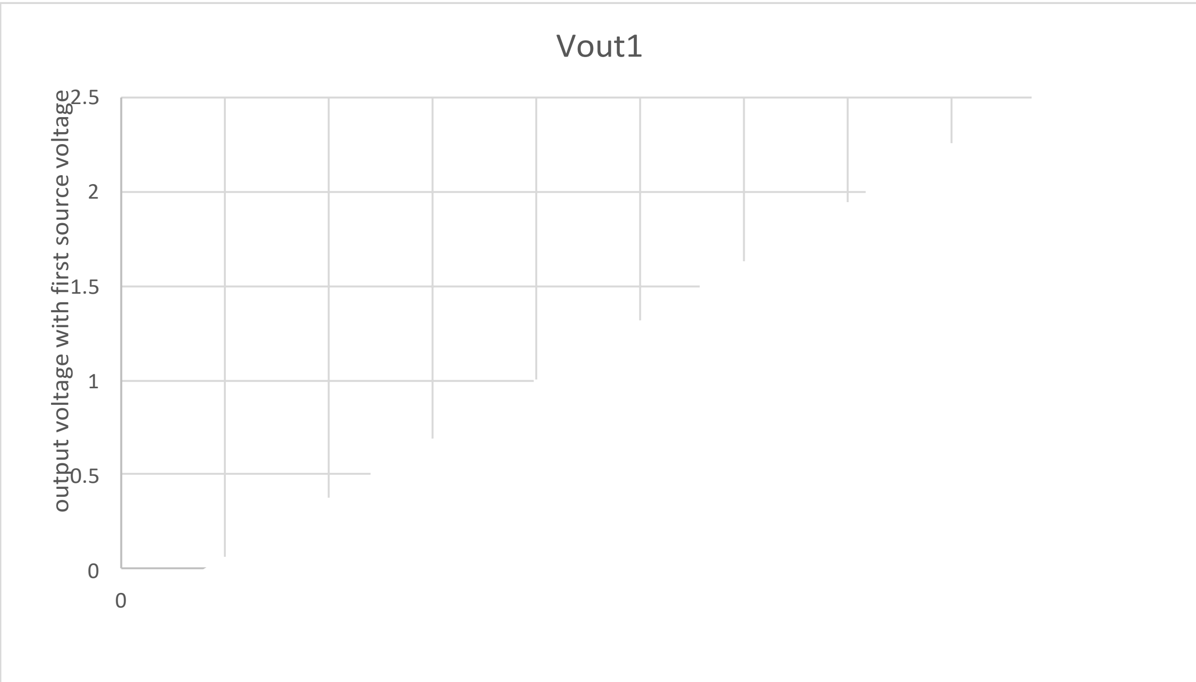
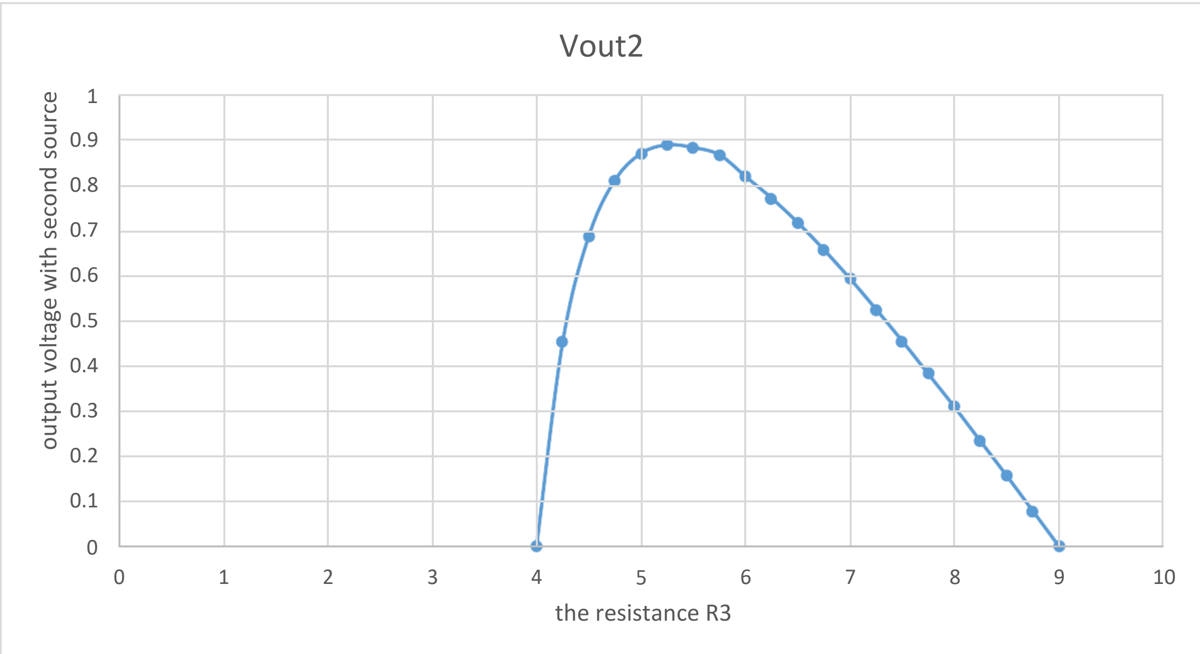
RESULT

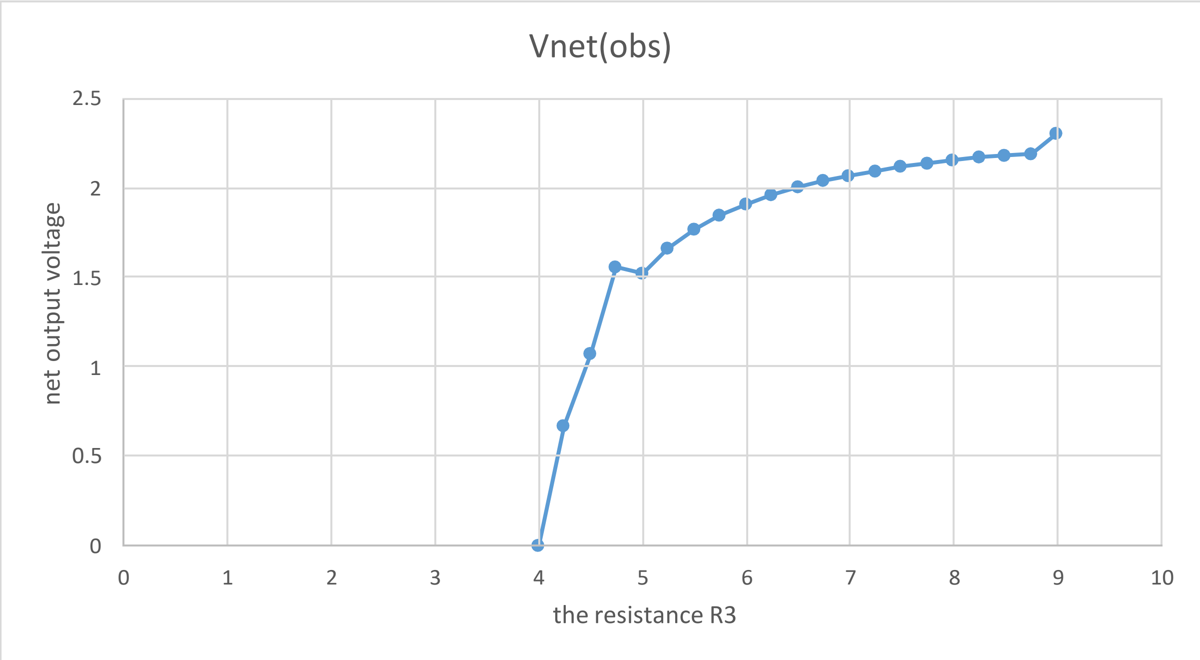
Now on plotting a graph of the given values with R_3 along x-axis and the experimental values of Vr1, Vr2, Vr along y-axis we get the following graphs.

Variation of Output voltage with R3(with one source voltage as V1 and V2 respectively)





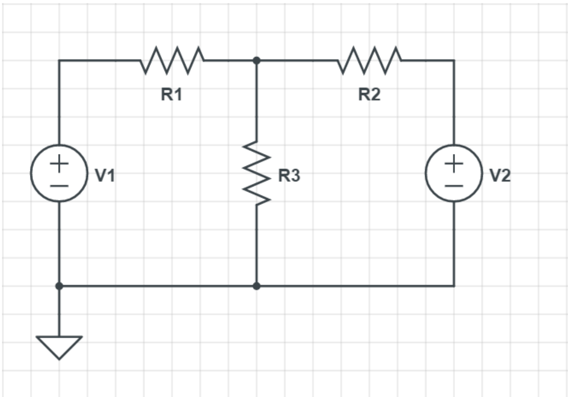
Variation of Output voltage with R3(with two source voltages – V1 & V2)



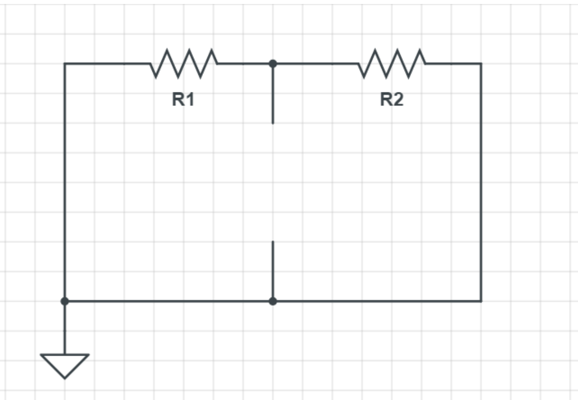
Computation

Let us assume V_1 , V_2 as source voltage. let $V_1 > V_2$.

Case 1 [with 2 source voltage]:

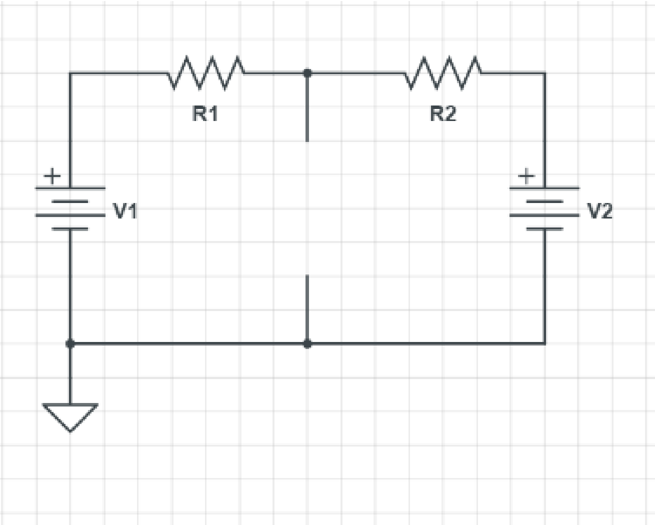


Finding the equivalent resistance, by removing the 'R_3' resistor. Also, by removing the voltage sources (short circuiting).



$$\boxed{R_{\mathrm{TH}} = \frac{R_1 . R_2}{R_1 + R_2}}$$

Finding equivalent voltage [without 'R_3' resistor]



By KVL,

V_1 - IR_1 - IR_2 - V_2 = 0

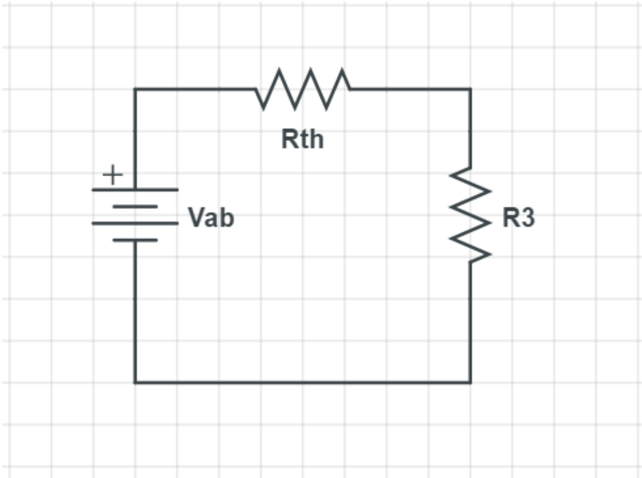
$I_1 = \dfrac{V_1 - V_2}{R_1 + R_2}$,

I$\longrightarrow$ current flowing in the circuit.

Let V_{AB} is the equivalent voltage across A&B,

$V_{\mathrm{AB}} = V_1 - IR_1 = V_2 + IR_2$

Simplified version of circuit [as per Thevenin's rule]



By applying KVL,

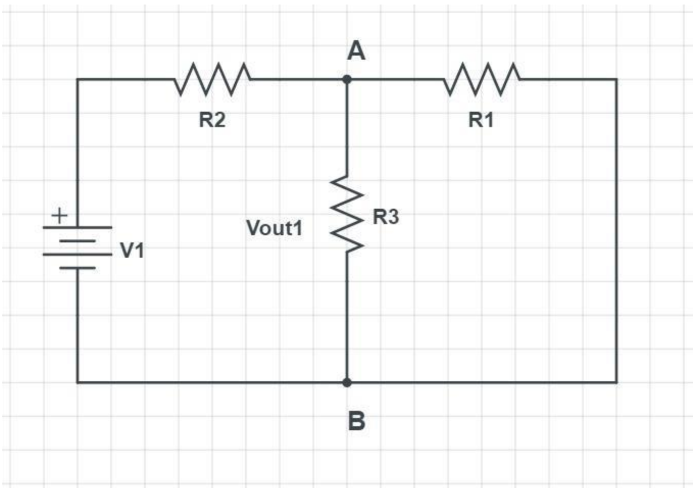
$V_{\mathrm{AB}} - I_1' R_{\mathrm{TH}} - I_1' R_3 = 0$

$I_1' = \dfrac{V_{\mathrm{AB}}}{R_{\mathrm{TH}} + R_3}$

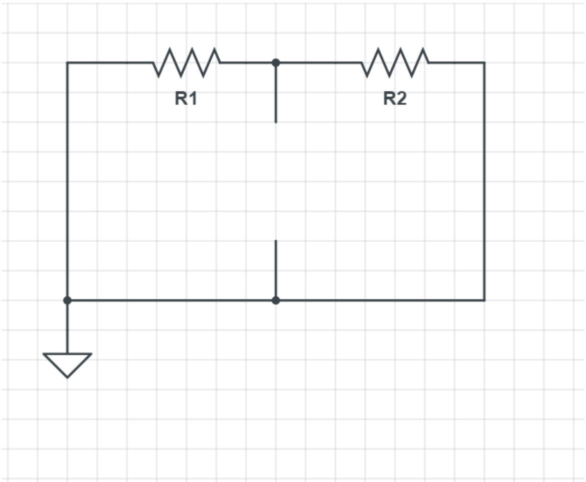
by ohm's law,

$V_{\mathrm{out}} = I_1' \times R_3$ where V_{out} is voltage acrossR_3

Case 2(with source voltage V_1 and where V_2 is the short circuited)

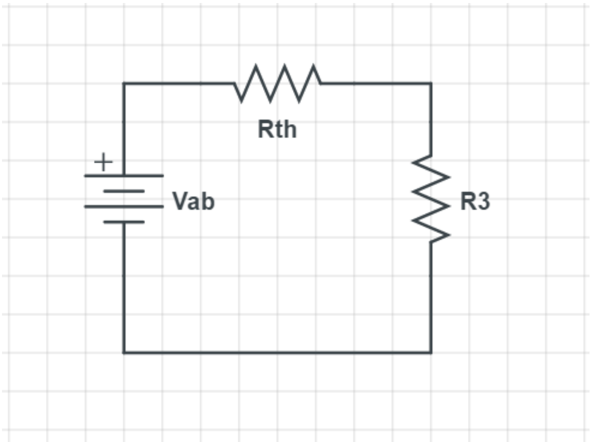


Finding the equivalent resistance across A&B by removing 'R_3' resistor, [also by short circuiting source voltage]



$$R_{\text{TH}} = \frac{R_1 . R_2}{R_1 + R_2}$$

Simplified version of the circuit, [using Thevenin's theorem]



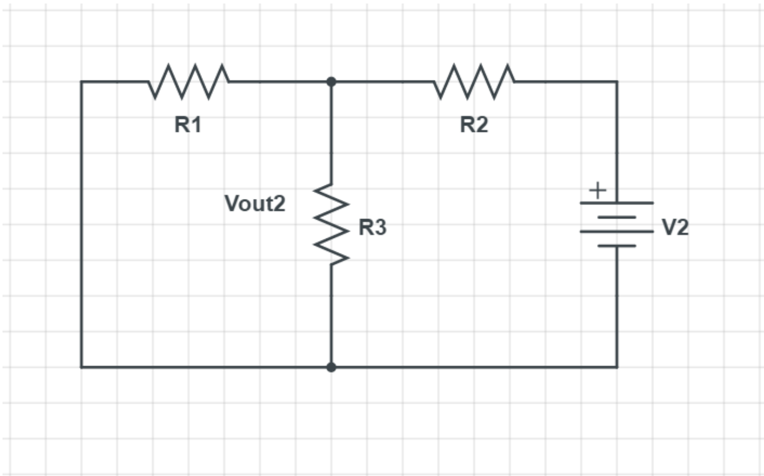
by KVL,

$$V_{\text{AB}} - I_2' R_{\text{TH}} - I_2' R_3 = 0$$

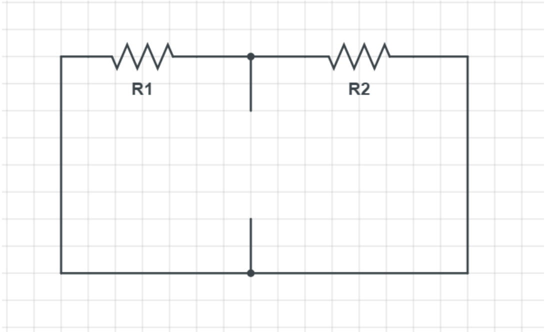
$$I_2' = \frac{V_{\text{AB}}}{R_{\text{TH}} + R_3}$$

HENCE, $V_{\text{out 1}} = I_2' R_3 \longrightarrow$ by ohm's law

Case 3(with source voltage V_2 with V_1 is short circuited)

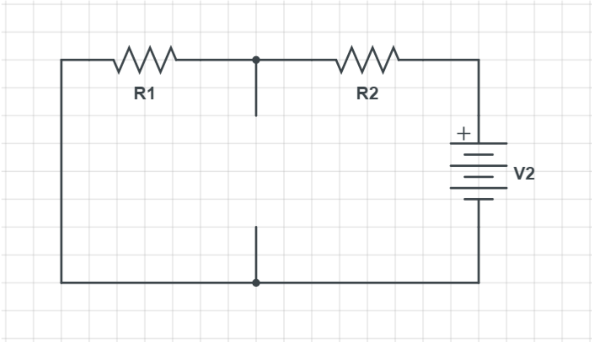


Finding the equivalent resistance across A&B by removing 'R_3' resistor (also by short circuiting source voltage)



$$R_{\mathrm{TH}} = \frac{R_1 R_2}{R_1 + R_2}$$

Finding equivalent voltage across A&B



By KVL,

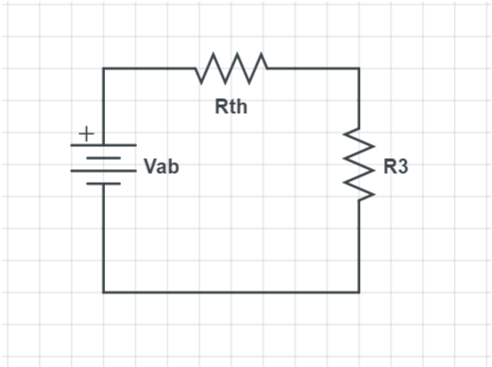
$$V_2 - I_3 R_2 - I_3 R_1 = 0$$

$$I_3 = \frac{V_2}{R_1 + R_2}$$

$$V_{\mathrm{AB}} = V_2 - I_3 R_2$$

where V_{AB} is voltage across A&B.

Simplified version of the circuit, (using Thevenin's theorem)



by KVL,

$$V_{\mathrm{AB}} - I'_2 R_{\mathrm{TH}} - I'_3 R_3 = 0$$

$$I'_3 = \frac{V_{\mathrm{AB}}}{R_1 + R_2}$$

hence, $V_{\mathrm{out}\,2} = I'_3 R_3$

Now, we can see that, $\boxed{V_{\mathrm{out}} = V_{\mathrm{out}\,1} + V_{\mathrm{out}\,2}}$

Hence, super-position theorem is proved.

Also, $\boxed{I'_1 = I'_2 + I'_3}$

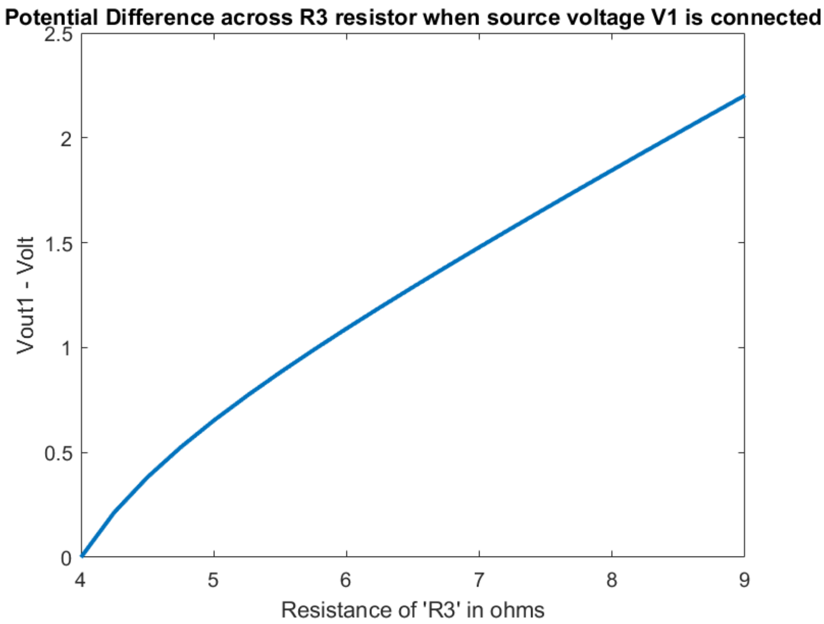
$I'_1 \longrightarrow$ current flowing through R_3 when sourcevoltages are there

$I'_2 \longrightarrow$ current flowing through R_3 when 'V_1' is there.

$I'_3 \longrightarrow$ current flowing through R_3 when 'V_2' is there.

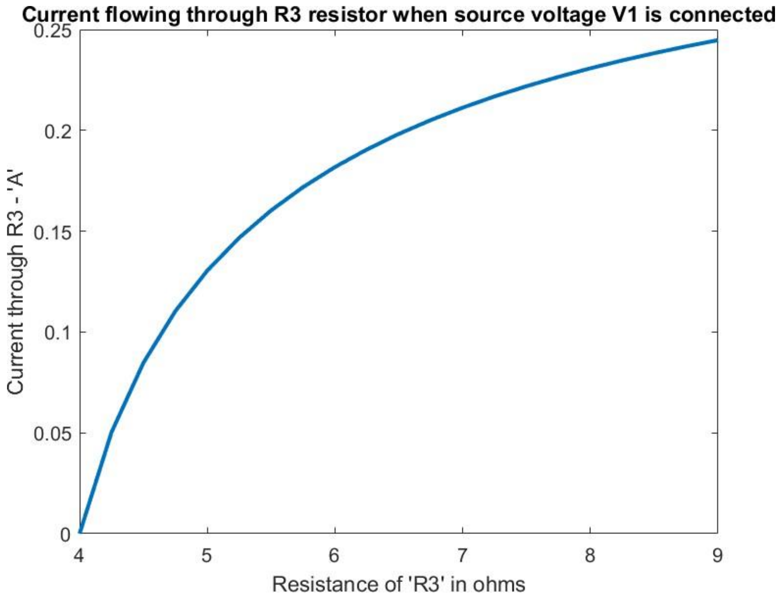
MATLAB code

```
clc;
clear all;
close all;

% Resistance Values
R1 = 0:0.25:5;
R2 = 2:0.25:7;
R3 = 4:0.25:9;

% Source Voltages
V1 = 0:0.25:5;
V2 = 5:-0.25:0;

% With source voltage V1
R_TH = R1.*R2 ./ (R1 + R2);
I2 = (V1) ./ (R1 + R2);
V_ab_1 =  V1 - I2.*R1;
IR3_1 = V_ab_1 ./ (R3 + R_TH);
Vout1 = IR3_1 .* R3;
figure
plot(R3,Vout1,"LineWidth",2)
title("Potential Difference across R3 resistor when source voltage V1 is
connected")
xlabel("Resistance of 'R3' in ohms")
ylabel("Vout1 - Volt")
```

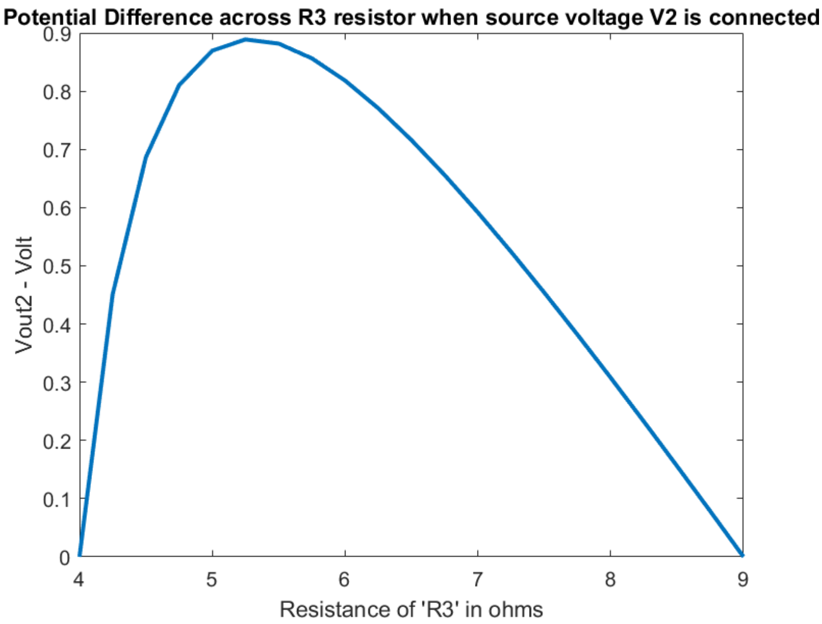



```
plot(R3,IR3_1,"LineWidth",2)
title("Current flowing through R3 resistor when source voltage V1 is
connected")
xlabel("Resistance of 'R3' in ohms")
```

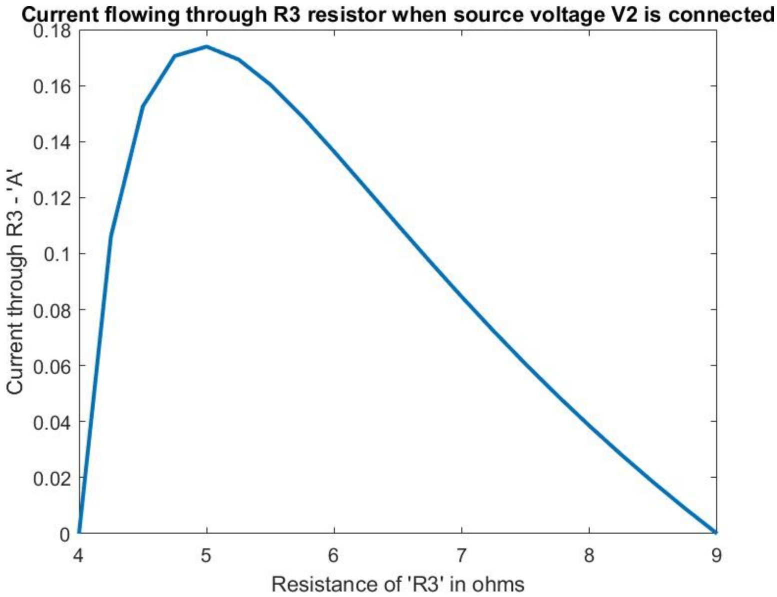
```
ylabel("Current through R3 - 'A'")
```



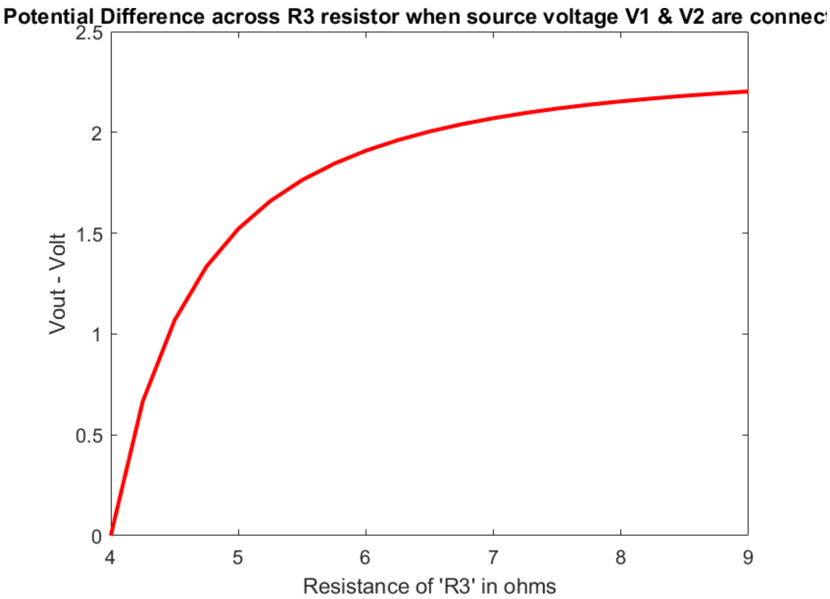

```
% With source voltage V2
R_TH = R1.*R2 ./ (R1 + R2);
I3 = (V2) ./ (R1 + R2);
V_ab_2 =  V2 - I3.*R2;
IR3_2 = V_ab_2 ./ (R3 + R_TH);
Vout2 = IR3_2 .* R3;
figure
plot(R3,Vout2,"LineWidth",2)
title("Potential Difference across R3 resistor when source voltage V2 is
connected")
xlabel("Resistance of 'R3' in ohms")
ylabel("Vout2 - Volt")
```



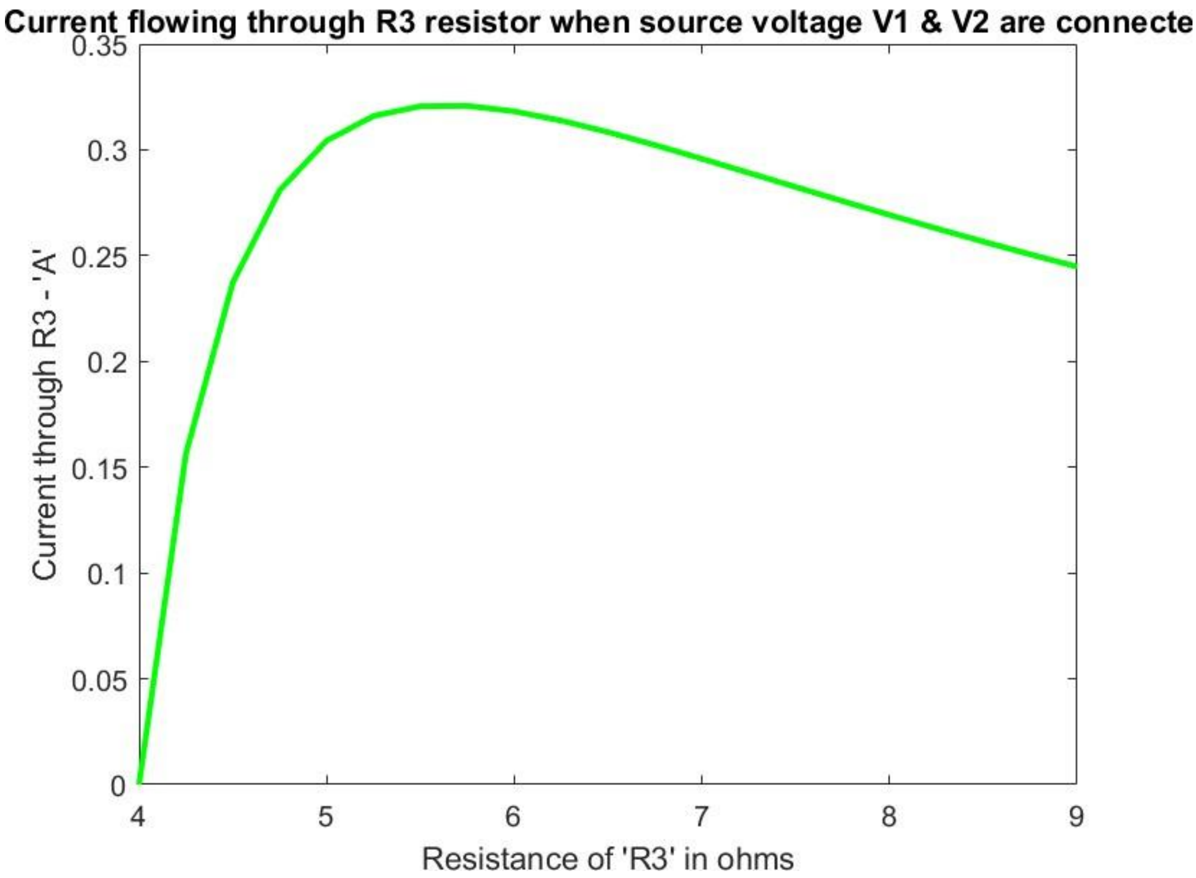
```
plot(R3,IR3_2,"LineWidth",2)
title("Current flowing through R3 resistor when source voltage V2 is
connected")
xlabel("Resistance of 'R3' in ohms")
ylabel("Current through R3 - 'A'")
```



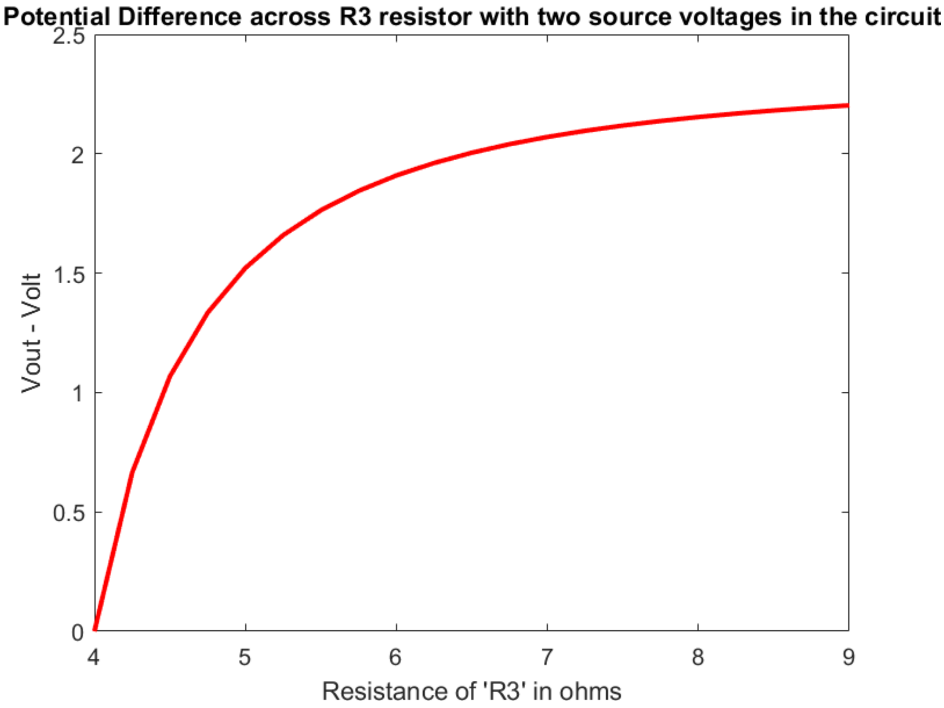

```
% Summing up both potentials Vout1 and Vout2 to get Vout
% Summing up current flowing through Resistor R3, found above, IR3_1 & IR3_2
Vout = Vout1 + Vout2;
IR3 = IR3_1 + IR3_2;
plot(R3,Vout,"LineWidth",2,"Color",'r')
title("Potential Difference across R3 resistor when source voltage V1 & V2
are connected")
xlabel("Resistance of 'R3' in ohms")
ylabel("Vout - Volt")
```



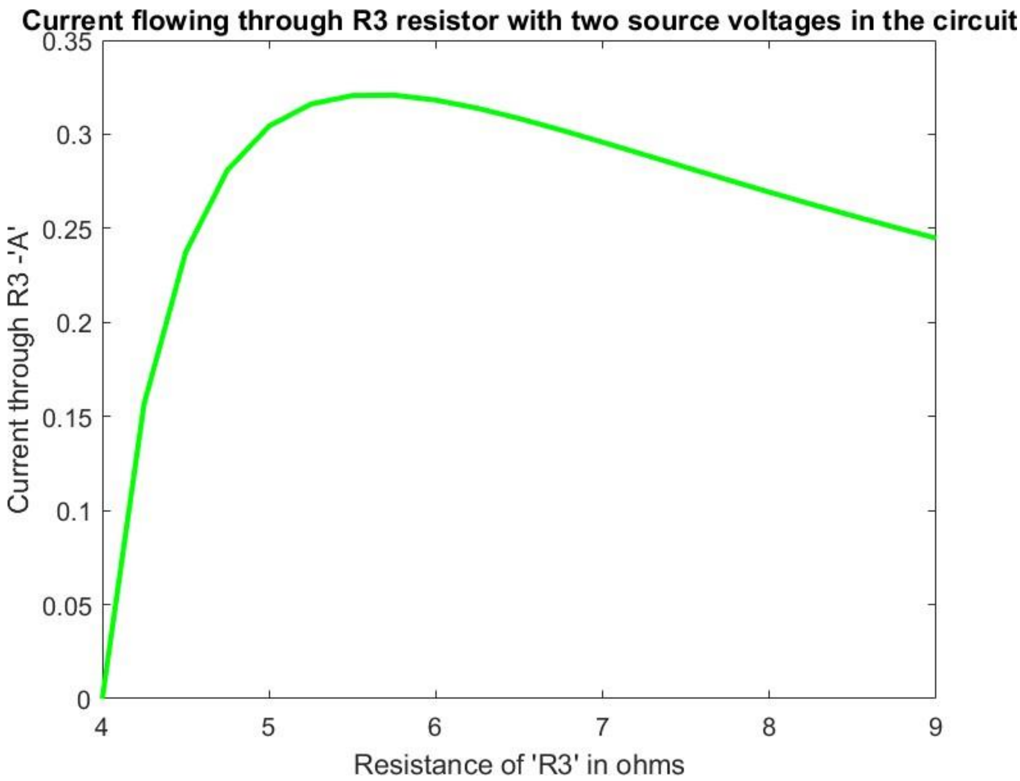
```
plot(R3,IR3,"LineWidth",2,"Color",'green')
title("Current flowing through R3 resistor when source voltage V1 & V2 are
connected")
xlabel("Resistance of 'R3' in ohms")
ylabel("Current through R3 - 'A'")
```




```
% With source voltageS V1 and V2
% Summing up both potentials Vout1 and Vout2 to get Vout
% Summing up current flowing through Resistor R3, found above, IR3_1 & IR3_2

R_TH = R1.*R2 ./ (R1 + R2);
I1 = (V1 - V2) ./ (R1 + R2);
V_ab =  V1 - I1.*R1;
IR3 = V_ab ./ (R3 +R_TH);
Vout = IR3 .* R3;
plot(R3,Vout,"LineWidth",2,"Color",'r')
title("Potential Difference across R3 resistor with two source voltages in
the circuit")
xlabel("Resistance of 'R3' in ohms")
ylabel("Vout - Volt")
```



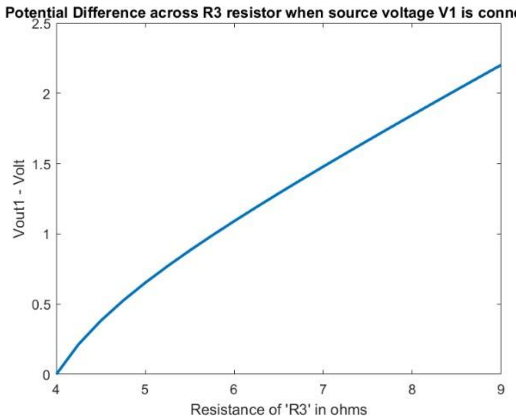

```
plot(R3,IR3,"LineWidth",2,"Color",'green')
title("Current flowing through R3 resistor with two source voltages in the circuit")
xlabel("Resistance of 'R3' in ohms")
ylabel("Current through R3 -'A'")
```

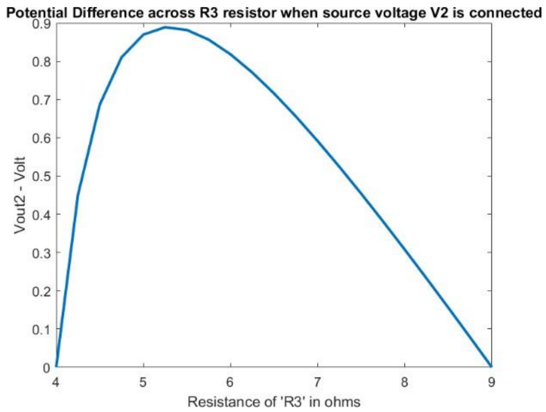


Result

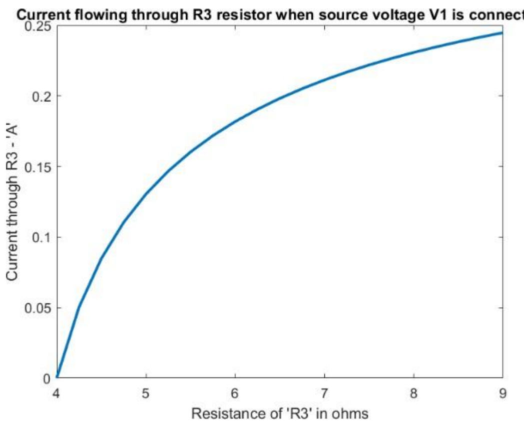
Now on plotting a graph of the given values with R_3 along x-axis and the experimental values of Vr1, Vr2, Vr along y-axis we get the following graphs.

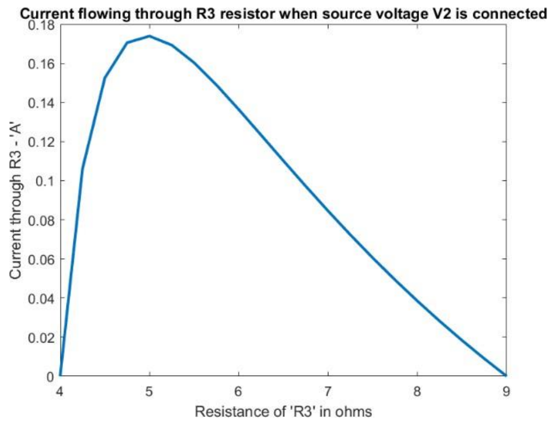
Variation of Output voltage with R3(with one source voltage as V1 and V2 respectively)



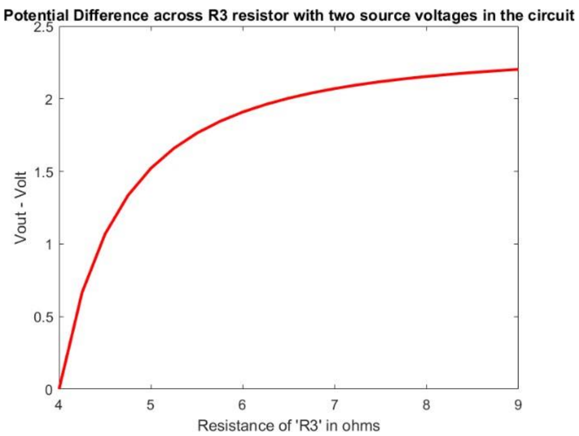


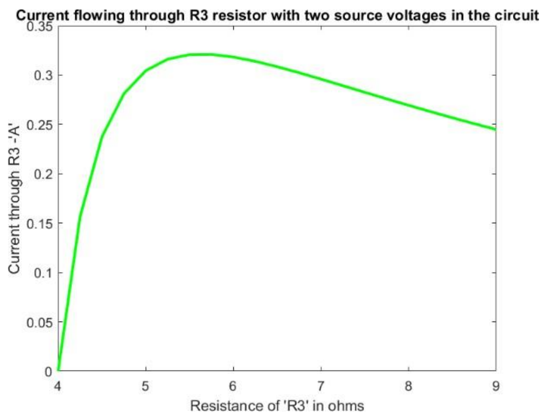
Variation of Output current with R3(with one source voltage as V1 and V2 respectively)





Variation of Output voltage and current with R3(with two source voltages – V1 & V2)





Inference

We can observe that the sum of individual output voltages is equal to the output voltage when both source voltages (V1 and V2) are connected in the circuit.

- For Vin = V1 and V2, we get output voltage across resistor Vr_out and current flowing through the circuit is I_out.
- For Vin = V1, we get output voltage across resistor Vr_out1 and current flowing through the circuit is I_out1.
- For Vin = V2, we get output voltage across resistor as Vr_out2 and current flowing through the circuit is I_out2.

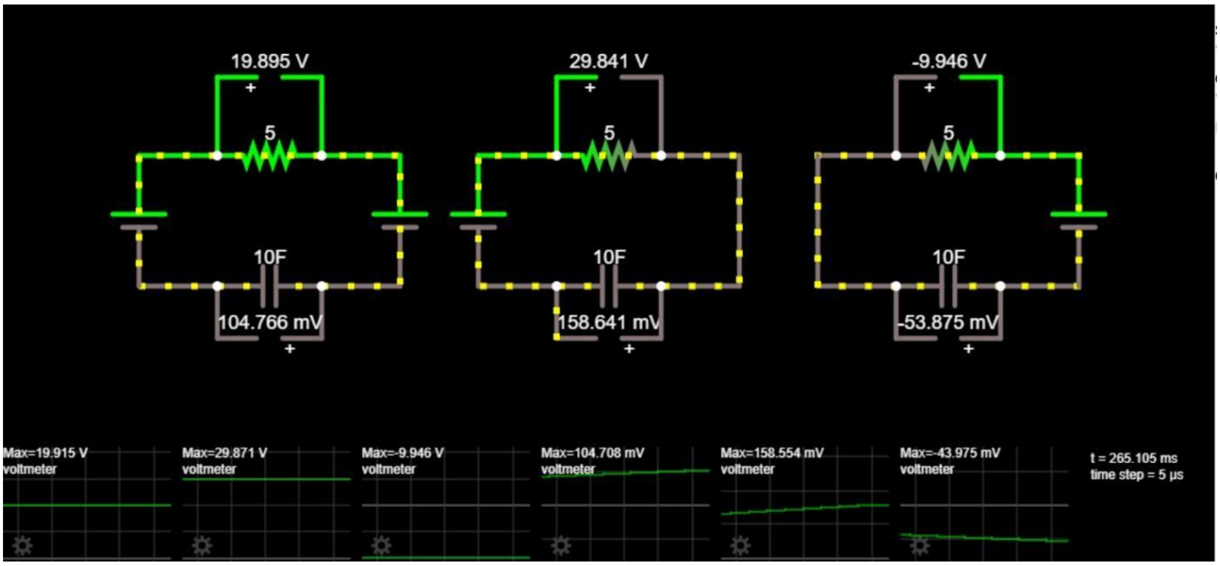
We observe that:

- Vr_out = Vr_out1 + Vr_out2
- I_out = I_out1 + I_out2

Hence, the superposition theorem is proved.

APPLICATION OF SUPER-POSITION IN RC CIRCUIT

Simulation of the circuit online



Link: https://tinyurl.com/ybm6tyey

Super-Position theorem for differential equation

Suppose y1(t) solves the differential equation, $\frac{dy}{dt}$ + f(t) = g1(t)

Suppose y2(t) solves the differential equation, $\frac{dy}{dt}$ + f(t) = g2(t)

Then y1 + y2 is a solution to $\frac{dy}{dt}$ + f(t) = g1(t) + g2(t)

Proof:

y = y1 + y2

y' = y1' + y2'

y1' + y2' + f(t) * [y1 + y2] = g1(t) + g2(t)

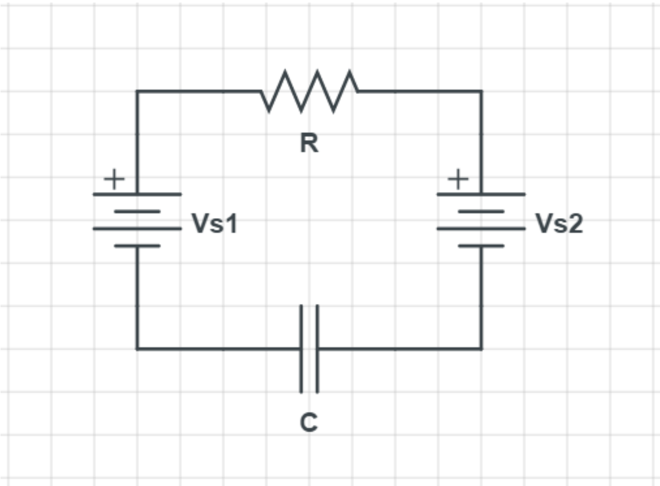
[y1' + f(t) * y1] + [y2' + f(t) * y2] = g1(t) + g2(t)

g1(t) + g2(t) = g1(t) + g2(t)

Hence Proved.

Proof for Super-Position Theorem

With 2 source voltages V_{S1} and V_{S2}



Let Vc be the voltage across the capacitor.

Let Vr be the voltage across the resistor.

Let V_{S1} be source voltage 1.

Let V_{S2} be source voltage 2.

Let R be the resistance of the resistor and C be the capacitance of the capacitor.

Let q(t) be charge on capacitor,

q(t) = C x Vc(t)

$\frac{\mathrm{dq}(t)}{\mathrm{dt}}$ = C x Vc'(t)

Vc'(t) = $\frac{I(t)}{C}$ ---- (i)

By Ohm's law, I = $\frac{V}{R}$

By, Kirchhoff’s voltage law [Sum of voltage around any closed loop is zero],

Vs1 - Vs2 - Vr - Vc(t) = 0

Vr = Vs1 - Vs2 - Vc(t) ---- (ii)

Use (ii) in (i),

Vc'(t) = $\frac{\mathrm{Vs1} - \mathrm{Vs2} - \mathrm{Vc}(t)}{R \; x \; C}$

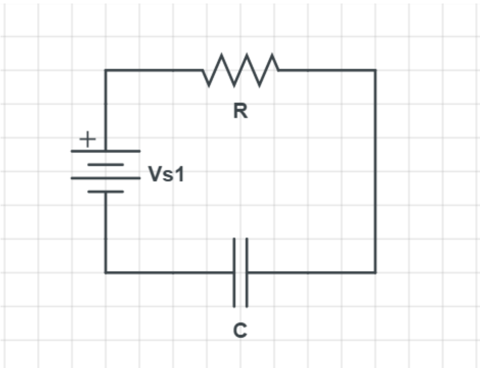
By Euler's method,

Vc(n+1) = Vc(n) + ot x $\frac{\mathbf{Vs1 - Vs2 - Vc(\boldsymbol{n})}}{\boldsymbol{R\ x\ C}}$

I = $\frac{\mathbf{Vs1 - Vs2 - Vc(t)}}{\boldsymbol{R}}$

In this case, let voltage across the resistor, capacitor be Vr_out and Vc_out. Let the total current flowing in the circuit be I_out.

With source voltage Vs1



By, Kirchhoff’s voltage law [Sum of voltage around any closed loop is zero],

Vs1 - Vr - Vc(t) = 0

Vr = Vs1 - Vc(t) → (ii)

Use (ii) in (i),

Vc'(t) = $\frac{\mathrm{Vs1 - Vc}(t)}{R\ x\ C}$

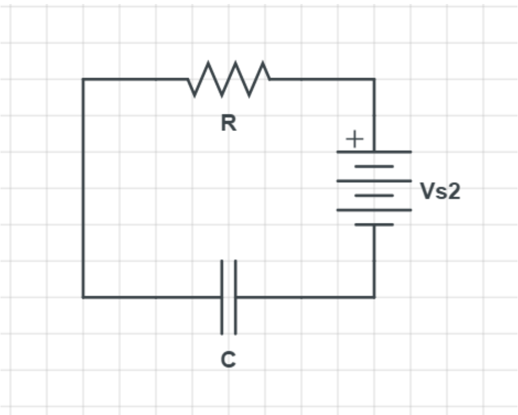
By Euler's method,

Vc(n+1) = Vc(n) + ot x $\frac{\mathbf{Vs1 - Vc(\boldsymbol{n})}}{\boldsymbol{R\ x\ C}}$

I1 = $\frac{\mathbf{Vs1 - Vc(t)}}{\boldsymbol{R}}$

In this case, let voltage across the resistor, capacitor be Vr_out1 and Vc_out1. Let the total current flowing in the circuit be I1.

With source voltage Vs2



By, Kirchhoff's voltage law (Sum of voltage around any closed loop is zero)

-Vs2 - Vr - Vc(t) = 0

Vr = -Vs2 - Vc(t) → (ii)

Use (ii) in (i),

$$Vc'(t) = \frac{-Vs2 - Vc(t)}{R\ x\ C}$$

By Euler's method,

$$\mathbf{Vc(n+1) = Vc(n) + ot\ x\ \frac{-Vs2 - Vc(n)}{R\ x\ C}}$$

$$\mathbf{I2 = \frac{-Vs2 - Vc(t)}{R}}$$

In this case, let voltage across the resistor, capacitor be Vr_out2 and Vc_out2. Let the total current flowing in the circuit be I2.

By super-position theorem,

Vc_out = Vc_out1 + Vc_out2

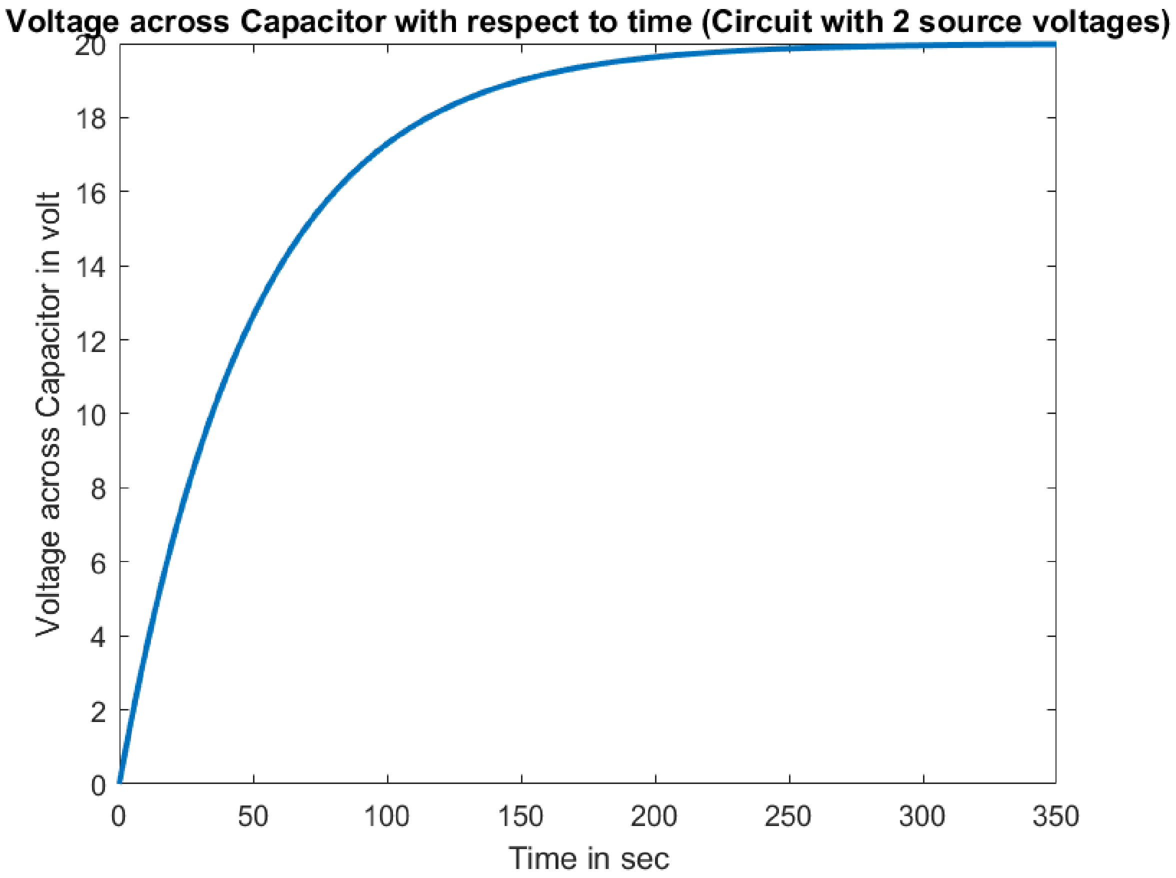
Vr_out = Vr_out1 + Vr_out2

I_out = I1 + I2

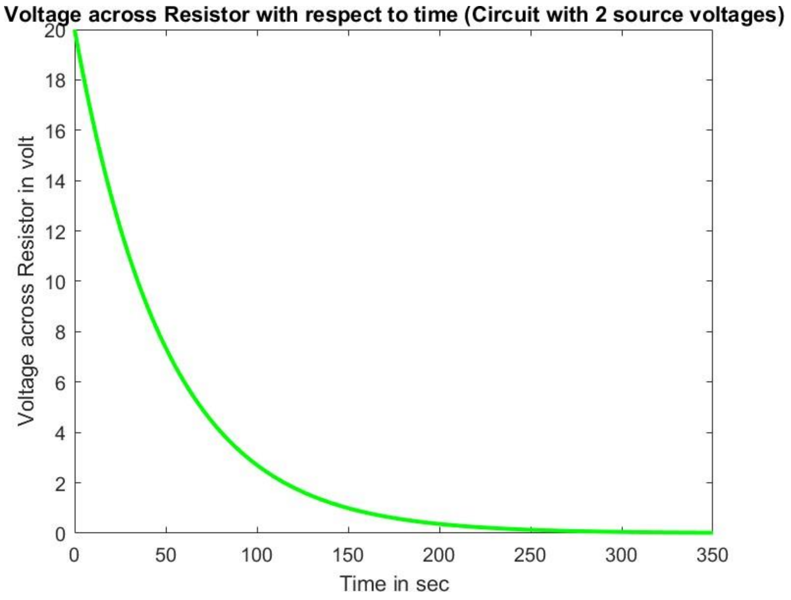
MATLAB code

```
clc;
clear all;
close all;

R = 5;
C = 10;
time_constant = R * C;
% To get a smooth graph we here take 7 times the time constant
% Actually steady state is achieved at 5 times the time constant
T = 7 * time_constant;
```

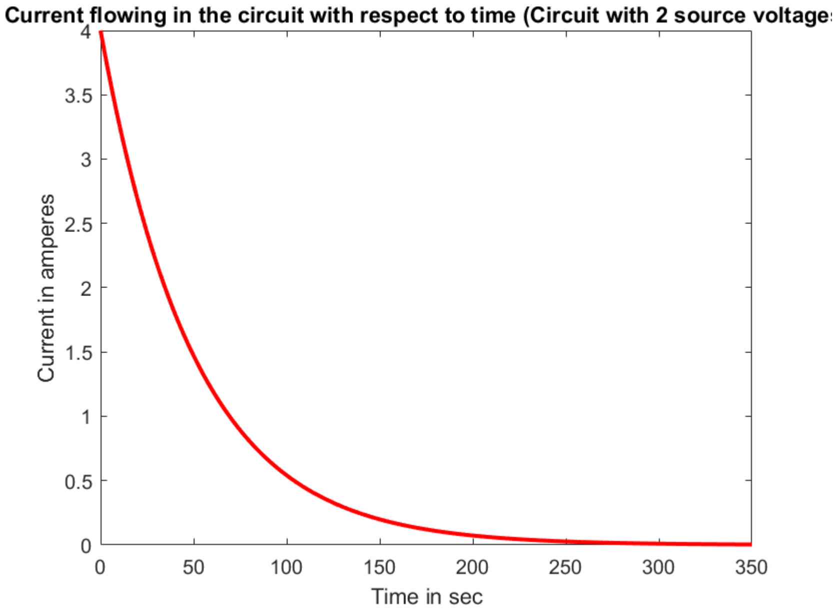
```
dt = 0.01;
t = 0:dt:T;
Vs1 = 30;
Vs2 = 10;
Vc(1) = 0;

for n = 1:(length(t)-1)
Vc(n+1) = Vc(n) + dt*((Vs1 - Vs2 - Vc(n))*(1/(R*C)));
end
Vr = Vs1 - Vs2 - Vc;
I = (Vs1 - Vs2 - Vc) / R;

% Plotting the graph
plot(t,Vc,"LineWidth",2)
title("Voltage across Capacitor with respect to time (Circuit with 2 source
voltages)")
xlabel("Time in sec")
ylabel("Voltage across Capacitor in volt")
```




```
plot(t,Vr,"LineWidth",2,"Color",'green')
title("Voltage across Resistor with respect to time (Circuit with 2 source
voltages)")
xlabel("Time in sec")
ylabel("Voltage across Resistor in volt")
```



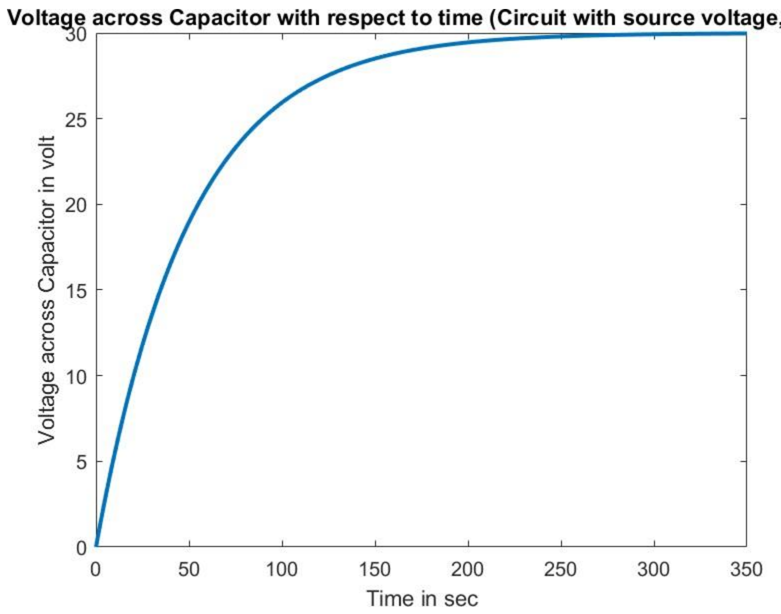

```
plot(t,I,"LineWidth",2,"Color",'r')
title("Current flowing in the circuit with respect to time (Circuit with 2 source voltages)")
xlabel("Time in sec")
ylabel("Current in amperes")
```



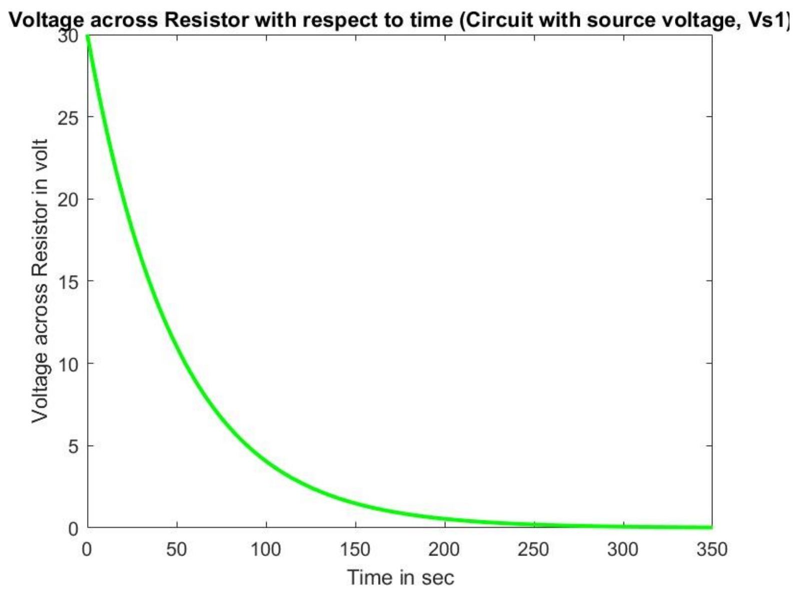
When one source voltage (Vs1) is alone connected

```
Vc_out1(1) = 0;
for i = 1:(length(t)-1)
    Vc_out1(i+1) = Vc_out1(i) + dt*((Vs1 - Vc_out1(i))*(1/(R*C)));
end
Vr_out1 = Vs1 - Vc_out1;
I1 = (Vs1 - Vc_out1) / R;
```

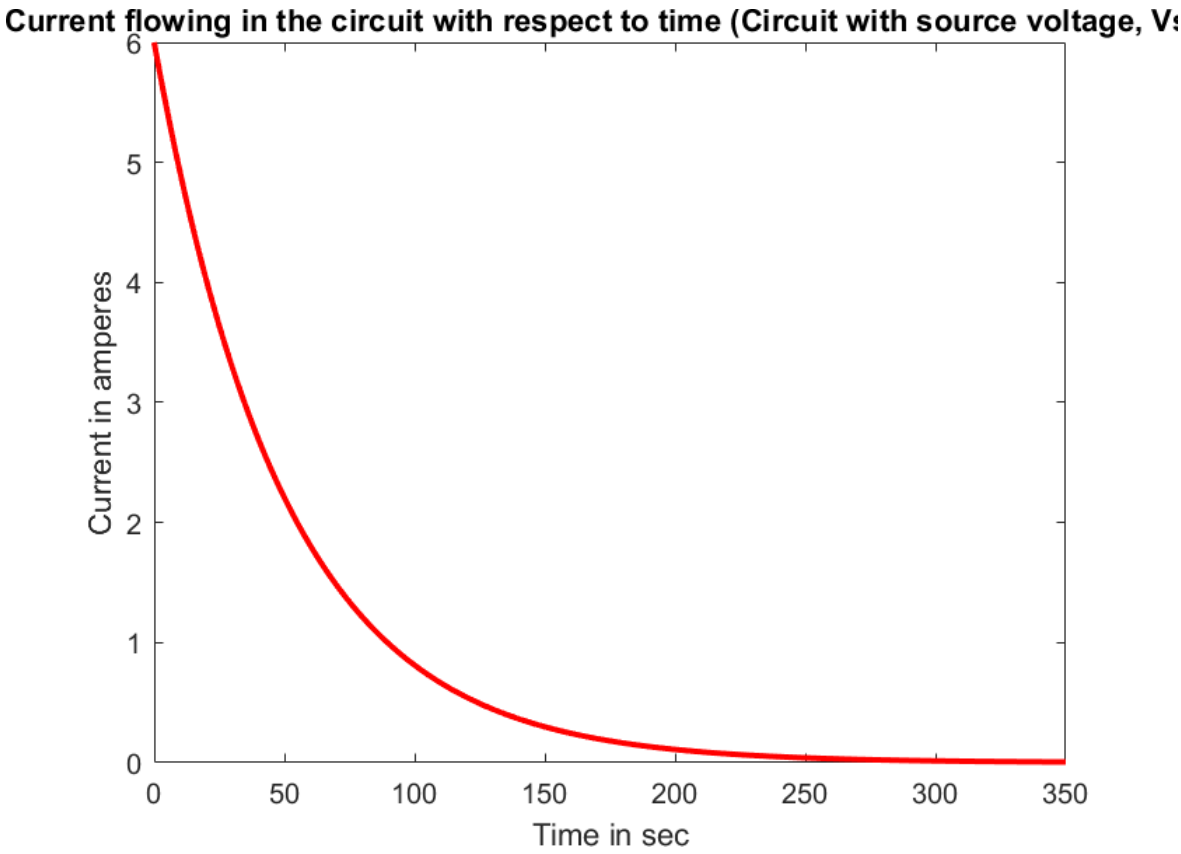
```
% Plotting the graph
plot(t,Vc_out1,"LineWidth",2)
title("Voltage across Capacitor with respect to time (Circuit with source voltage, Vs1)")
xlabel("Time in sec")
ylabel("Voltage across Capacitor in volt")
```




```
plot(t,Vr_out1,"LineWidth",2,"Color",'green')
title("Voltage across Resistor with respect to time (Circuit with source voltage, Vs1)")
xlabel("Time in sec")
ylabel("Voltage across Resistor in volt")
```

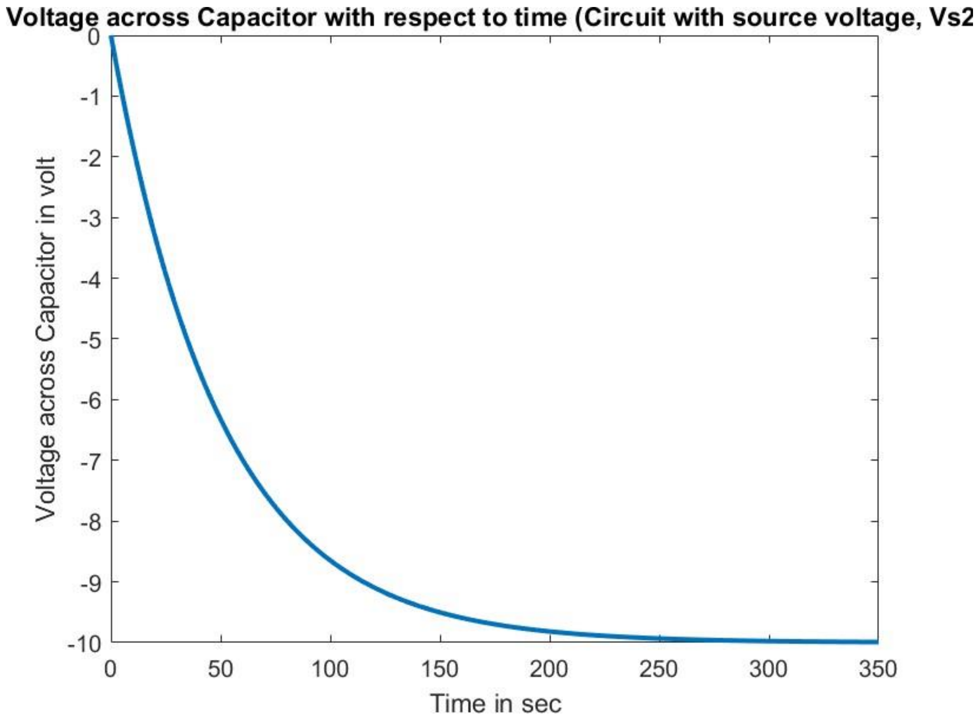


```
plot(t,I1,"LineWidth",2,"Color",'r')
title("Current flowing in the circuit with respect to time (Circuit with source voltage, Vs1)")
xlabel("Time in sec")
ylabel("Current in amperes")
```

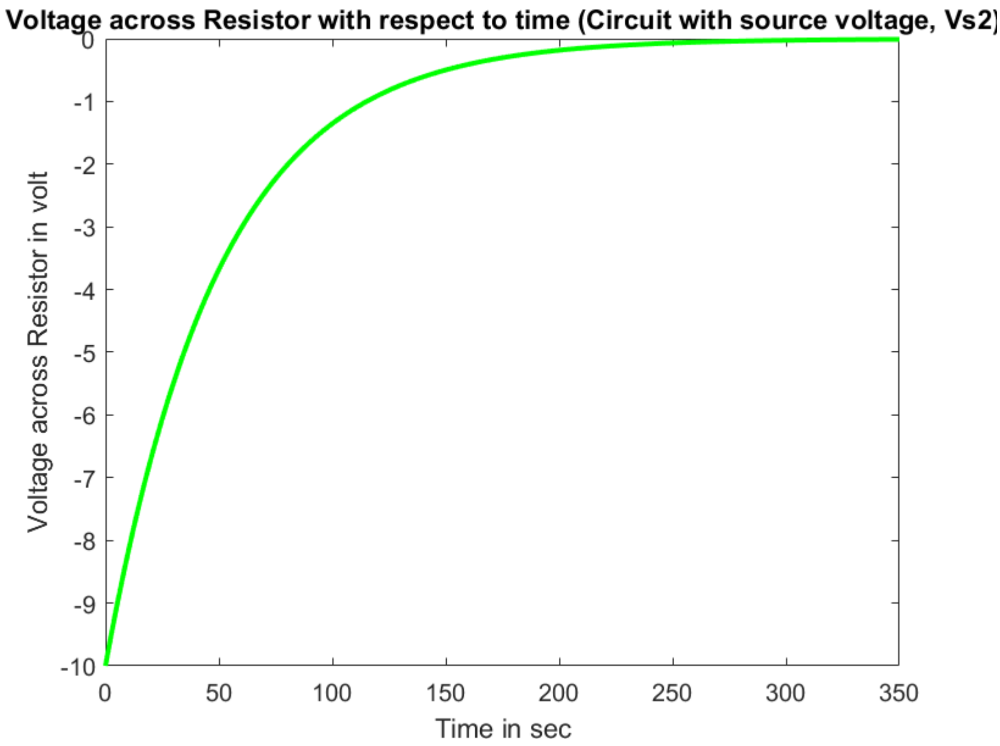


When one source voltage (Vs2) is alone connected

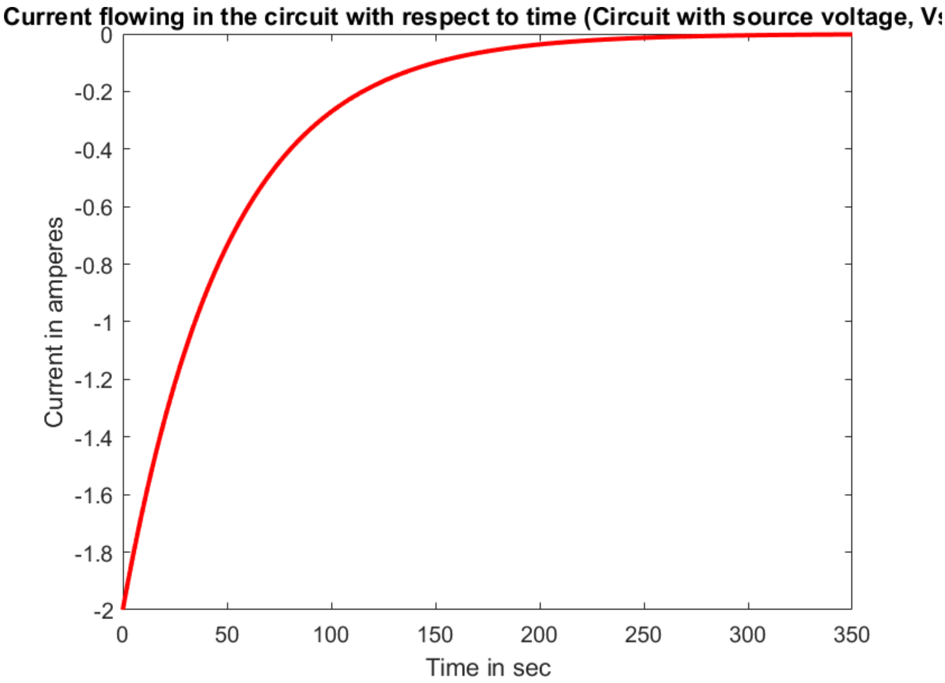
```
Vc_out2(1) = 0;
for j = 1:(length(t)-1)
    Vc_out2(j+1) = Vc_out2(j) + dt*((-Vs2 - Vc_out2(j))*(1/(R*C)));
end
Vr_out2 = -Vs2 - Vc_out2;
I2 = (-Vs2 - Vc_out2) / R;

% Plotting the graph
plot(t,Vc_out2,"LineWidth",2)
title("Voltage across Capacitor with respect to time (Circuit with source voltage,Vs2)")
xlabel("Time in sec")
ylabel("Voltage across Capacitor in volt")
```




```
plot(t,Vr_out2,"LineWidth",2,"Color",'green')
title("Voltage across Resistor with respect to time (Circuit with source
voltage,Vs2)")
xlabel("Time in sec")
ylabel("Voltage across Resistor in volt")
```



```
plot(t,I2,"LineWidth",2,"Color",'r')
title("Current flowing in the circuit with respect to time (Circuit with source voltage,Vs2)")
xlabel("Time in sec")
ylabel("Current in amperes")
```



Negative voltage in a circuit is voltage that is more negative in polarity than the ground of the circuit.

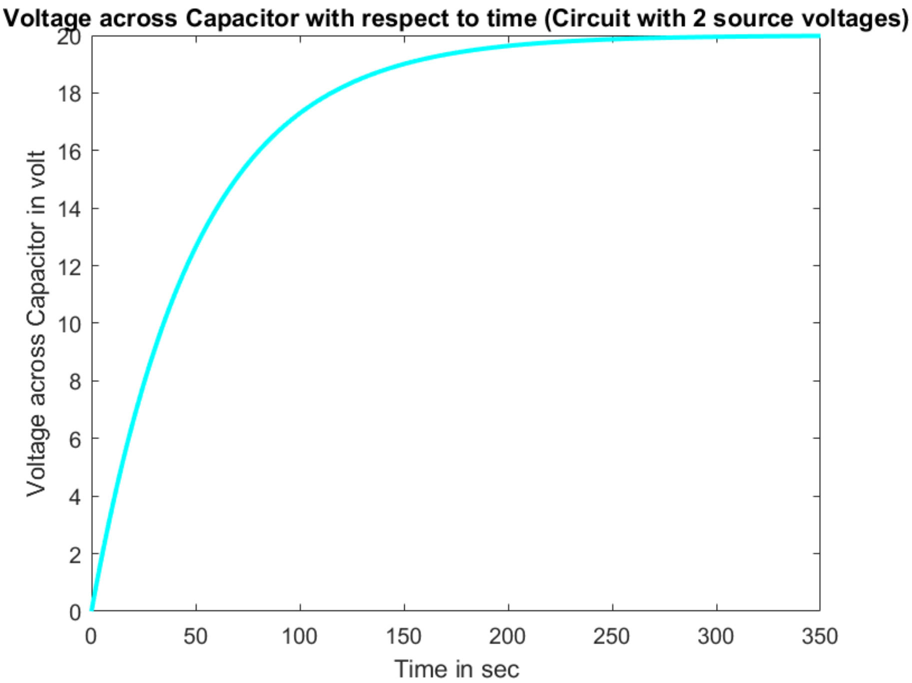
A voltage source has positive or negative polarity depending on its orientation in a circuit. In the case when a voltage across an element has negative voltage, it just means the negative terminal of the voltmeter is connected to the positive side of the circuit and the positive terminal of the voltmeter is connected to the negative side of the circuit.

It just conveys the orientation, but do not make any effect.

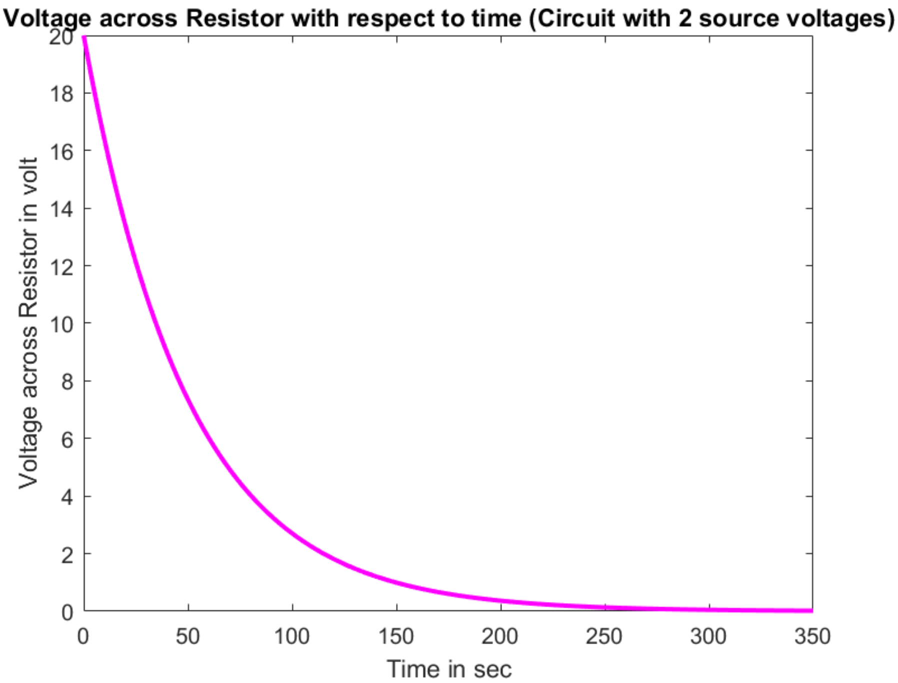
The negative value of current also shows just the opposite direction of flow of current in the circuit. Negative sign tells the direction and not affect magnitude in any way.

Thus, on summing both the functions, as per the superposition theorem,

```
Vc_out = Vc_out1 + Vc_out2;
Vr_out = Vr_out1 + Vr_out2;
I_out = I1 + I2;

plot(t,Vc_out,"LineWidth",2,"Color",'c')
title("Voltage across Capacitor with respect to time (Circuit with 2 source voltages)")
xlabel("Time in sec")
ylabel("Voltage across Capacitor in volt")
```

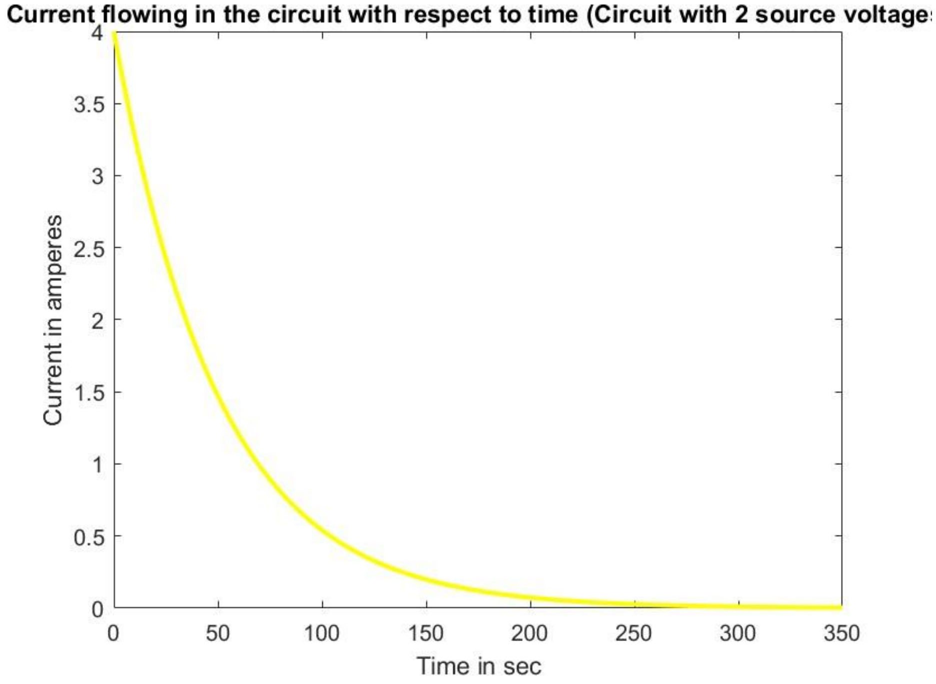



```
plot(t,Vr_out,"LineWidth",2,"Color",'m')
title("Voltage across Resistor with respect to time (Circuit with 2 source
voltages)")
xlabel("Time in sec")
ylabel("Voltage across Resistor in volt")
```




```
plot(t,I_out,"LineWidth",2,"Color",'y')
title("Current flowing in the circuit with respect to time (Circuit with 2
source voltages)")
xlabel("Time in sec")
```

```
ylabel("Current in amperes")
```



INFERENCE

We can observe that the sum of individual output voltages is equal to the output voltage when both source voltages (30 V and 10 V) are connected in the circuit.

- For Vin = 30V and 10V, we get output voltage across resistor and capacitor as Vr_out and Vc_out and current flowing through the circuit is I_out.
- For Vin = 30V, we get output voltage across resistor and capacitor as Vr_out1 and Vc_out1 and current flowing through the circuit is I_out1.
- For Vin = 10V, we get output voltage across resistor and capacitor as Vr_out2 and Vc_out2 and current flowing through the circuit is I_out2.

We observe that:

- Vr_out = Vr_out1 + Vr_out2
- Vc_out = Vc_out1 + Vc_out2
- I_out = I_out1 + I_out2

Hence, the superposition theorem is proved.

REFRENCES:

- William Flannery-Mathematical Modelling and Computational Calculus-Berkeley Science Books (2013)
- Electronics and Circuit analysis using MATLAB By JOHN O. ATTIA
- Concepts of Physics by HC Verma
- https://en.wikipedia.org/
- https://circuitglobe.com/
- https://www.electronics-tutorials.ws/
- https://www.falstad.com/circuit/directions.html
- https://byjus.com/