RelaySchnorr: Anonymous, Timed and Revocable Proxy Signatures

Ghada Almashaqbeh[1], Anca Nitulescu[2]

[1]University of Connecticut, [2]Input Output (IOG)

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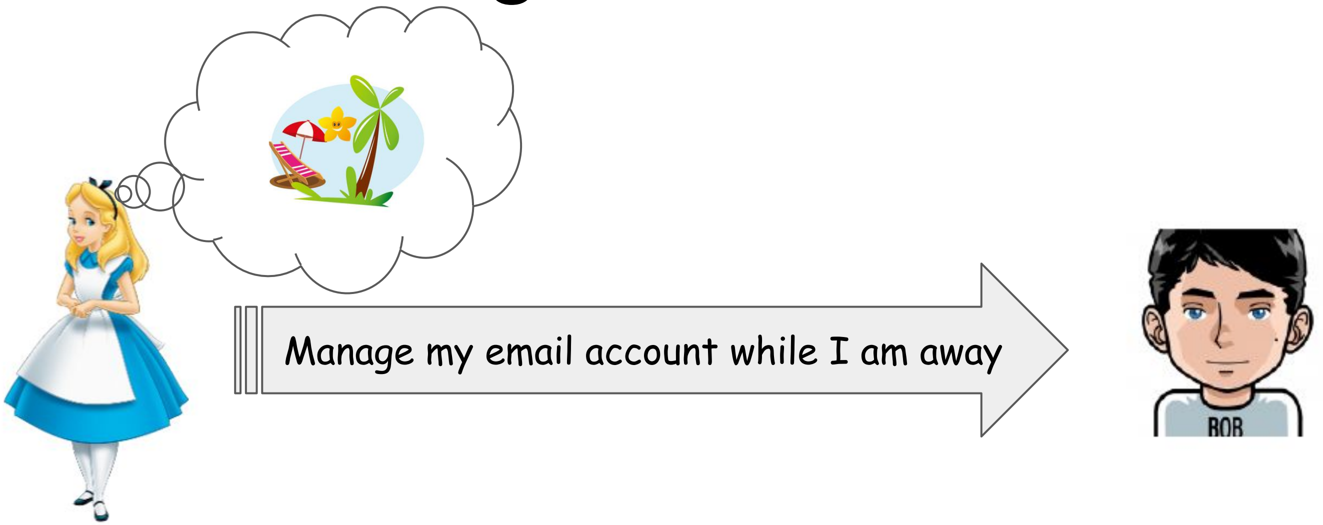
Signature Delegation (Proxy Signatures)



Signature Delegation (Proxy Signatures)

Manage my email account while I am away

Share the workload of handling emails

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Produce signed messages on Alice's behalf

Motivating Applications

Can DeFi (decentralized finance) replace traditional banking services?

Issue a credit card for my sister
under my account

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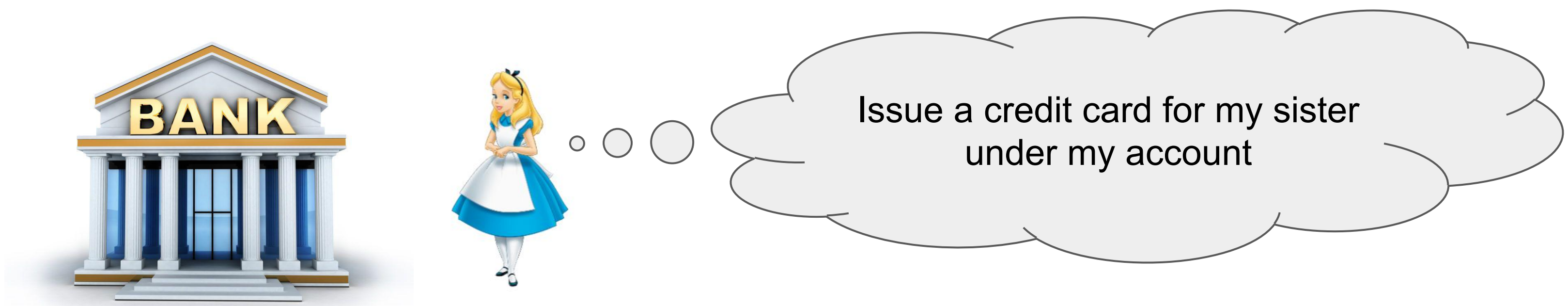
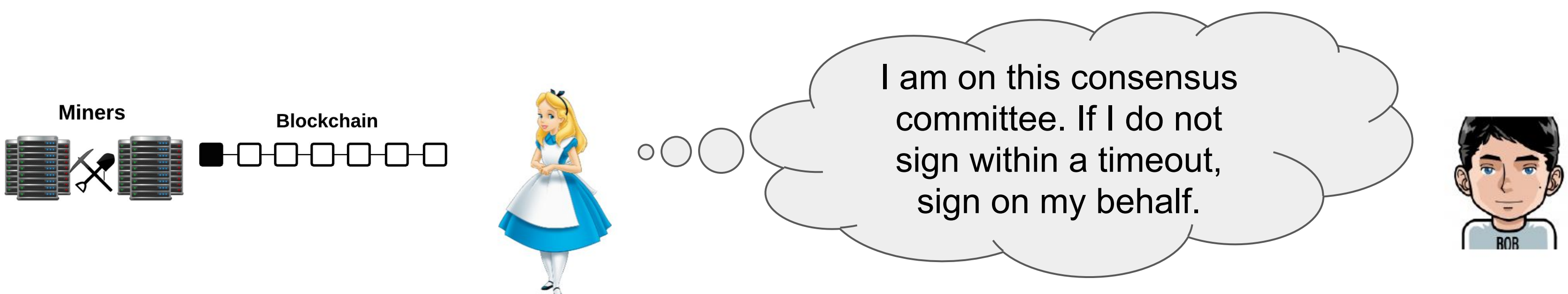


Mitigating Targeted Attacks



Desired Delegation Properties

- Anonymity of delegation.
- Timed delegation.
- Revocability.
- Policy enforcement.
- Decentralization.
- Non-interactivity.

Limitations of Prior Work

- No existing scheme achieved all these properties:
  - Many violate anonymity,
  - supported anonymity and policy enforcement without any revocation capability or timed notion,
  - or achieved revocability/timed notion at the expense of being interactive and/or involving a trusted third party.
- No formal security notion of proxy signatures encompassing all these properties.

This work

Can we do Better? … RelaySchnorr

- We define a security notion for anonymous, timed and revocable proxy signatures.
- We show a construction called **RelaySchnorr**
  - Combines Schnorr signatures, timelock encryption, and a public bulletin board.
  - Achieves all the desired properties listed before.
- We formally prove security of our scheme based on our notion.

Building Blocks - Schnorr Signatures

For a security parameter λ, let $\mathbb{G}$ be a cyclic group of a prime order q and a generator G, and $\mathsf{H} : \{0,1\}^* \times \mathbb{G}^2 \to \mathbb{Z}_q$ be a hash function. The Schnorr signature scheme is a tuple of three algorithms $\Sigma_{\mathsf{Schnorr}} = (\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ defined as follows:

- $\mathsf{Schnorr.KeyGen}(1^\lambda)$: On input the security parameter λ, choose uniform $x \in \mathbb{Z}_q$ and compute $X = G^x$. Set the secret signing key $\mathsf{sk} = x$ and the public verification key $\mathsf{vk} = X$.
- $\mathsf{Schnorr.Sign}(\mathsf{sk}, m)$: On input the secret key $\mathsf{sk} = x$ and the message m, choose uniform $k \in \mathbb{Z}_q$. Compute $K = G^k$, $X = G^x$, $c = \mathsf{H}(m, X, K)$, and $s = k + cx \mod q$. Output the signature $\sigma = (c, s)$.
- $\mathsf{Schnorr.Verify}(\mathsf{vk}, m, \sigma)$: On input the public key $\mathsf{vk} = X$, the message m, and signature $\sigma = (c, s)$ over m, compute $K = G^s \cdot X^{-c}$ and $c' = \mathsf{H}(m, X, K)$, then output 1 if $c = c'$.

Building Blocks - TLE

A Timelock encryption scheme $\mathcal{E}$ is a tuple of five PPT algorithms defined as follows:

$\mathsf{TLE.Setup}(1^\lambda) \to (\mathrm{pp}, s)$: Takes as input the security parameter λ, and outputs public parameters pp and a private key s.

$\mathsf{TLE.RoundBroadcast}(s, \rho) \to \pi_\rho$: Takes as input the round number ρ and a private key s, and outputs the round-related decryption information π_ρ.

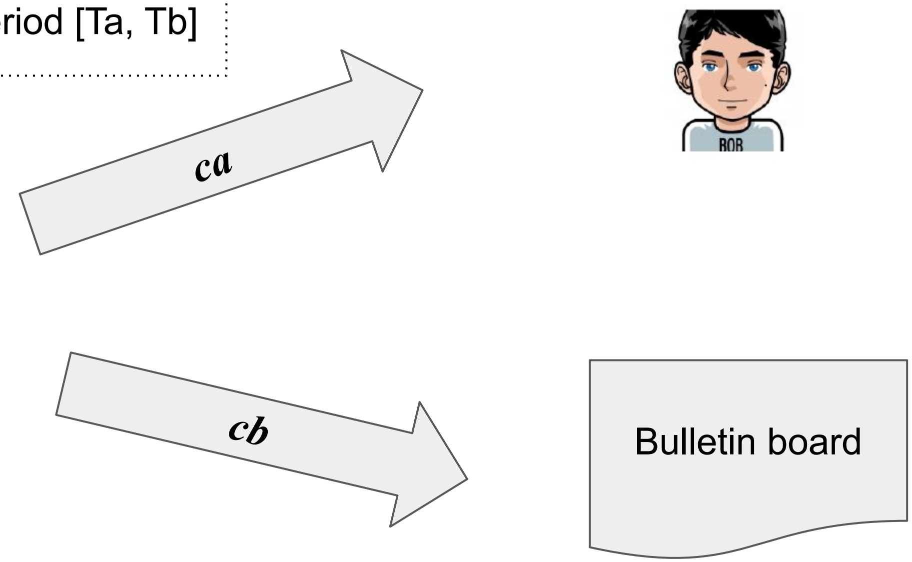
$\mathsf{TLE.Enc}(\rho, m) \to (\mathrm{ct}_\rho, \tau)$: Takes as input the round number ρ and a message m, and outputs a round-encrypted ciphertext ct_ρ, and trapdoor τ for pre-opening.

$\mathsf{TLE.Dec}(\pi_\rho, \mathrm{ct}_\rho) \to m'$: Takes as input the round-related decryption information π_ρ and a ciphertext ct_ρ, and outputs a message m'.

$\mathsf{TLE.PreOpen}(\mathrm{ct}_\rho, \tau) \to m'$: Takes as input a ciphertext ct_ρ and a trapdoor τ, and outputs a message m'.

RelaySchnorr Construction

Delegation period [Ta, Tb]



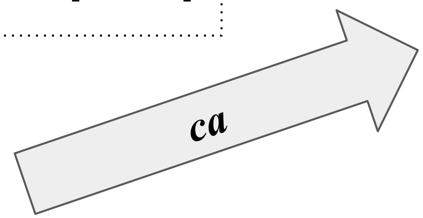
(1) Generate u random elements $k1, \ldots, ku$

(2) Generate u tokens: $t1, \ldots, tu$ (each token is a Schnorr signature over ki)

(3) Encrypt the tokens to time Ta, and the k values to time Tb

RelaySchnorr Construction

Delegation period [Ta, Tb]



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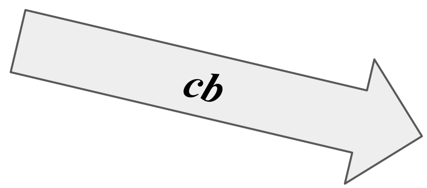
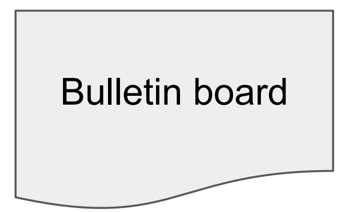
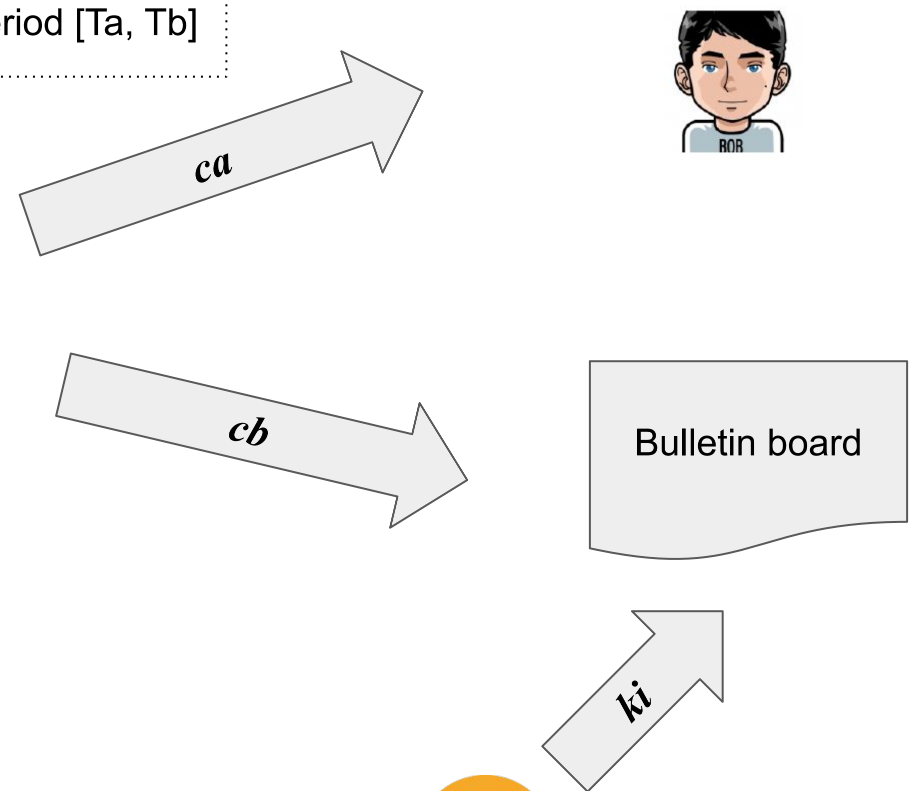




At time Ta:
(1) Decrypt the tokens.
(2) Use a token to sign message m (produce another Schnorr signature using the token).

RelaySchnorr Construction

Delegation period [Ta, Tb]



(1) Generate ***u*** random elements ***k1, …, ku***
(2) Generate ***u*** tokens: ***t1, …, tu*** (each token is a Schnorr signature over ***ki***)
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Verify a signature

RelaySchnorr Construction

One-time tokenizable Schnorr

Delegation period [Ta, Tb]

ca

(1) Generate ***u*** random elements ***k1, …, ku***
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cb

Bulletin board

ki

At time ***Ta***:
(1) Decrypt the tokens.
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Verify a signature

RelaySchnorr Construction

Delegation period [Ta, Tb]

Board validators

At time ***Tb***: Decrypt ***cb***

Publish all unused ***ki***

Bulletin board

RelaySchnorr Construction

Delegation period [Ta, Tb]

Board validators

At time ***Tb***: Decrypt ***cb***

Publish all unused ***ki***

Bulletin board

Before time ***Tb***

Publish all unused ***ki***

RelaySchnorr Construction

Delegation period [Ta, Tb]

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At time ***Tb***: Decrypt ***cb***

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Timed delegation

Automatic and on-demand revocation

Anonymity and Policy Enforcement

- **Anonymity is achieved by:**
  - Proxy signer identity is not included.
  - Delegation info is sent privately to the proxy signer.
  - The signature structure is the same for both the original or proxy signer, and verified using the same Verify algorithm.
  - Original signer mimics the behavior of having a delegation for her signatures.
- **Policy enforcement over messages:**
  - Conventional methods from the literature: public warrants and private ones (using NIZKs).

Issues in Practice

- Denial of service attacks against the signer.
- Bulletin board synchronization.
- Off-chain processing issues.
- Information lookup cost.
- Mass production of k values and delegation anonymity.

Security

Theorem 1. *Assuming EUF-CMA security of Schnorr signatures, the schnorr-koe assumption, a secure bulletin board, a CCA-secure TLE scheme, an EUF-CMA secure signature scheme, and a secure NIZK proof system, RelaySchnorr is an anonymous, timed and revocable proxy signature scheme (cf. Definition 2).*

- Unforgeability relies on the unforgeability of Schnorr signatures in the random oracle model, and the Schnorr knowledge of exponent assumption.
- Anonymity is achieved by having identical signature structure and behavior.
- Revocability relies on the security of timelock encryption and the bulletin board.
- Policy enforcement relies on the security of digital signatures (for public warrants) or NIZKs (for private policies), as well as security of timelock encryption and the bulletin board.

Conclusion and Future Work

- This work.
  - Delegation of signatures that achieve desired properties for Web 3.0 applications.
- Future work.
  - Explore controlled delegation for other cryptographic primitives.
    - Such as zero-knowledge proofs (i.e., private wallets).

Thank you!

Questions?

ghada@uconn.edu
https://ghadaalmashaqbeh.github.io/

Concrete Construction

Setup phase

Let λ be a security parameter, S be the original signer, P be the proxy signer, and TLE be a timelock encryption scheme. Construct an anonymous, timed and revocable proxy signature scheme $\Sigma = (\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Delegate}, \mathsf{DegSign}, \mathsf{Revoke}, \mathsf{Verify})$ as follows:

$\mathsf{Setup}(1^\lambda)$: On input the security parameter λ, set $\mathbb{G}$ to be a cyclic group of a prime order q with a generator $G \in \mathbb{G}$ and $\mathsf{H} : \{0,1\}^* \times \mathbb{G}^2 \to \mathbb{Z}_q$ to be a hash function, initialize $\mathsf{state} = \{\}$, and invoke $\mathsf{TLE.Setup}(1^\lambda)$. Output $\mathsf{pp} = (\mathsf{TLE.pp}, \mathsf{H}, \mathbb{G}, G, q, \mathsf{state})$.

$\mathsf{KeyGen}(1^\lambda)$: On input the security parameter λ, choose uniform $x \in \mathbb{Z}_q$, then compute $X = G^x$. Output the signing key $\mathsf{sk} = x$ and the verification key $\mathsf{vk} = X$.

Concrete Construction

Sign—used by original signer S

Sign(sk, m): On input the signing key sk $= x$ and some message m, do:

- Choose uniform $k, r, e \in \mathbb{Z}_q$, compute $R = G^r$, $E = G^e$
- Compute $w = \mathsf{H}(k, X, R)$, $z = (r + wx) \mod q$, and $Z = G^z$
- Compute $c = \mathsf{H}(m, Z, E)$ and $s = (e + cz) \mod q$ (if $z = 0$ or $s = 0$ start again with fresh r and e)
- Output the signature $\sigma = (w, c, s, k, Z)$

Every now and then, S either (1) populates a set klist from the stored k values and fresh ones, encrypts it as $(\mathsf{ct}_b, \tau_b) = \mathsf{TLE.Enc}(\mathsf{klist}, \rho_b)$, where ρ_b is some future round number, and posts (ρ_b, ct_b) on the board (resulting in $\mathsf{state}'[\mathsf{vk}] = \mathsf{state}[\mathsf{vk}] \parallel (\rho_b, \mathsf{ct}_b)$), or (2) posts a fresh klist on the board (resulting in $\mathsf{state}'[\mathsf{vk}] = \mathsf{state}[\mathsf{vk}] \parallel \mathsf{klist}$).

Concrete Construction

Delegate—invoked by original signer S

Delegate(sk, vk, degspec): On input the keypair ($\mathsf{sk} = x, \mathsf{vk} = X$) and delegation specifications $\mathsf{degspec} = (u, [\rho_a, \rho_b])$, where $u \in \mathbb{N}$ and $[\rho_a, \rho_b]$ is the delegation period, do the following:

- Set $\mathsf{klist} = \{\}$
- Do the following for $i \in \{1, \ldots, u\}$:
  - Choose uniform $k_i, r_i \in \mathbb{Z}_q$
  - Compute $R_i = G^{r_i}$ and $w_i = \mathsf{H}(k_i, X, R_i)$
  - Compute $z_i = (r_i + w_i x) \mod q$ (if $z_i = 0$ start again with fresh r_i)
  - Set $t_i = (z_i, w_i, k_i)$ and $\mathsf{klist} = \mathsf{klist} \cup \{k_i\}$
- Compute two ciphertexts: $(\mathsf{ct}_a, \tau_a) = \mathsf{TLE.Enc}(t_1 \| \cdots \| t_u, \rho_a)$ and $(\mathsf{ct}_b, \tau_b) = \mathsf{TLE.Enc}(\mathsf{klist}, \rho_b)$ (where τ_b is the revocation key rk).
- Set $\mathsf{degInfo} = (\rho_a, \rho_b, \mathsf{ct}_a)$
- Output $(\mathsf{degInfo}, \mathsf{ct}_b \| \tau_b)$

S stores ciphertext ct_b and trapdoor τ_b to be used for revocation if needed (τ_a is dropped as it is not needed), posts (ρ_b, ct_b) on the board (resulting in $\mathsf{state}'[\mathsf{vk}] = \mathsf{state}[\mathsf{vk}] \| (\rho_b, \mathsf{ct}_b)$), and sends degInfo to P.

Concrete Construction

Delegate Sign—used by proxy signer P

DegSign(m, degInfo): On input a message m and delegation information degInfo, P does the following (let ρ_{now} = state.round be the current round number):

- If $\rho_{now} < \rho_a$ or $\rho_{now} > \rho_b$, then do nothing
- If $\rho_a \leq \rho_{now} \leq \rho_b$, then:
  - If degInfo $= (\rho_a, \rho_b, \mathsf{ct}_a)$, then retrieve π_{ρ_a} from the board (π_{ρ_a} = state.roundInfo(ρ_a)) and set degInfo $= (\rho_a, \rho_b, \mathsf{TLE.Dec}(\pi_{\rho_a}, \mathsf{ct}_a))$
  - Pick an unused signing token $t = (z, w, k)$ from degInfo
  - Compute $Z = G^z$
  - Choose uniform $e \in \mathbb{Z}_q$ and compute $E = G^e$
  - Compute $c = \mathsf{H}(m, Z, E)$, and $s = e + cz \mod q$ (if $s = 0$ start again with a fresh e)
  - Output the signature $\sigma = (w, c, s, k, Z)$

Concrete Construction

Automatic/On demand revoke—invoked by validators or original signer S
Verify—Invoked by a verifier for any signature

Revoke(degInfo, rk, state[vk]): On input degInfo $= (\rho_b, \mathsf{ct}_b)$, revocation key rk, and revocation state state[vk], do (let $\rho_{now} = \mathsf{state.round}$ be the current round number):

- If $\rho_{now} \geq \rho_b$, then retrieve π_{ρ_b} from the board ($\pi_{\rho_b} = \mathsf{state.roundInfo}(\rho_b)$) and compute klist $= \mathsf{TLE.Dec}(\pi_{\rho_b}, \mathsf{ct}_b)$
- If $\rho_{now} < \rho_b$, then use $\mathsf{rk} = \tau_b$ to compute klist $= \mathsf{TLE.PreOpen}(\mathsf{ct}_b, \tau_b)$
- Add all k values such that $k \in \mathsf{klist} \wedge k \notin \mathsf{state[vk]}$ to the board state state[vk] associated with vk resulting in an updated state state[vk]$'$.

Verify(vk, $m, \sigma = (w, c, s, k, Z)$, revState = state[vk]): On input the verification key $\mathsf{vk} = X$, the message m, signature $\sigma = (w, c, s, k, Z)$ over m, and the revocation state state[vk], if $k \in \mathsf{state[vk]}$, then output 0. Else, add k to state[vk] (resulting in $\mathsf{state}'[\mathsf{vk}] = \mathsf{state}[\mathsf{vk}] \,\|\, k$) and do the following:

- Compute $R = Z \cdot X^{-w}$ and $E = G^s \cdot Z^{-c}$
- Output 1 if and only if $w = \mathsf{H}(k, X, R) \wedge c = \mathsf{H}(m, Z, E)$.