CSE 3400 - Introduction to Computer & Network Security
(aka: Introduction to Cybersecurity)

Lecture 7
Hash Functions – Part II

Ghada Almashaqbeh
UConn

From Textbook Slides by Prof. Amir Herzberg
UConn

Outline

- Hash based MACs.
- Domain extension.
- Merkle digest and Merkle trees.
- Blockchains.

Hash based MAC

- Hash-based MAC is often faster than block-cipher MAC
- How? Heuristic constructions:

$$\textbf{Prepend Key: } MAC_k^{PK}(m) = h(k + m)$$

$$\textbf{Append Key: } MAC_k^{AK}(m) = h(m + k)$$

$$\textbf{Message-in-the-Middle: } MAC_k^{MitM}(m) = h(k + m + k)$$

  - Are these secure assuming CRHF ? OWF ? Both ?
    - No.
- But: all ‘secure in random oracle model’

Hash-based MAC: HMAC

- HMAC uses only the unkeyed hash function h:

$$HMAC_k(x) = h(k \oplus opad \,||\, h(k \oplus ipad \,||\, x))$$

  - *opad, ipad:* fixed sequences (of 36x, 5Cx resp.), for max hamming distance between $k \oplus opad$ and $k \oplus ipad$.
- [BCK]: secure MAC under 'reasonable assumptions' [beyond our scope]
- Widely deployed – for MAC, PRF and KDF
  - KDF – Key Derivation Function
- More results, more exposure ➔ confidence!
- Hash are useful for MACs in another way:
  - Hash then MAC.

Digest Schemes

- Generalization of collision-resistant hash
  - Input is a **sequence** of messages
  - Output is n-bit **digest**, denoted Δ
- Three types of schemes:
  - Digest-chain
  - Merkle Digest (and Merkle trees)
  - Blockchains (and Bitcoin)
- In other textbooks, this is referred to as Domain Extension.

Digest-Chain Schemes

- Generalization of collision-resistant hash
  - Input is a **sequence** of messages
  - Output is n-bit **digest**, denoted Δ

Definition *A* digest function Δ *is an efficiently computable function (in PPT) that maps blocks (finite sequences of binary strings) to n-bit binary strings, i.e.,* $\Delta : (\{0,1\}^*)^* \rightarrow \{0,1\}^*$, *where n is the* security parameter.

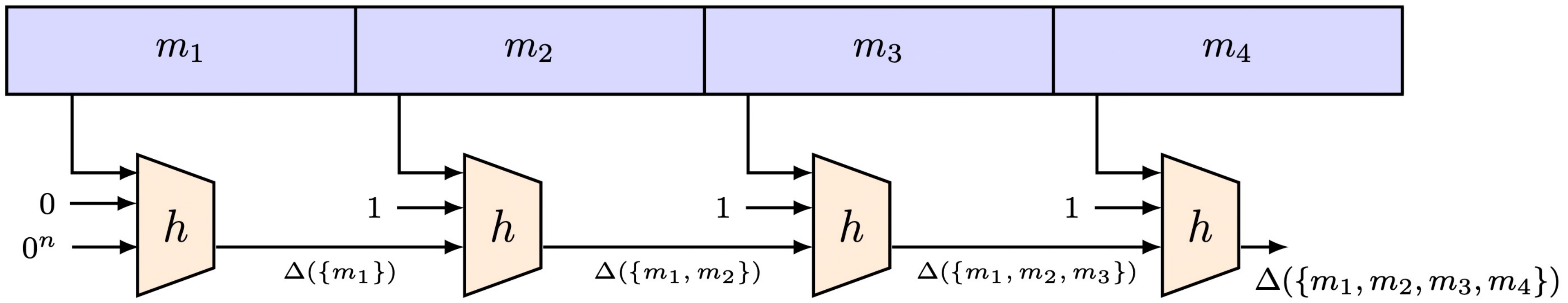
Digest function Δ is collision resistant *if the* digest collision-resistance advantage $\varepsilon^{DCR}_{\mathcal{A},\Delta}(n)$ *is negligible (in n), for every efficient adversary $\mathcal{A} \in PPT$, where:*

$$\varepsilon^{DCR}_{\mathcal{A},\Delta}(n) \equiv \Pr\left((B, B') \leftarrow \mathcal{A}(1^n)\ s.t.\ B \neq B' \wedge \Delta(B) = \Delta(B')\right)$$

The Merkle-Damgard Digest Function

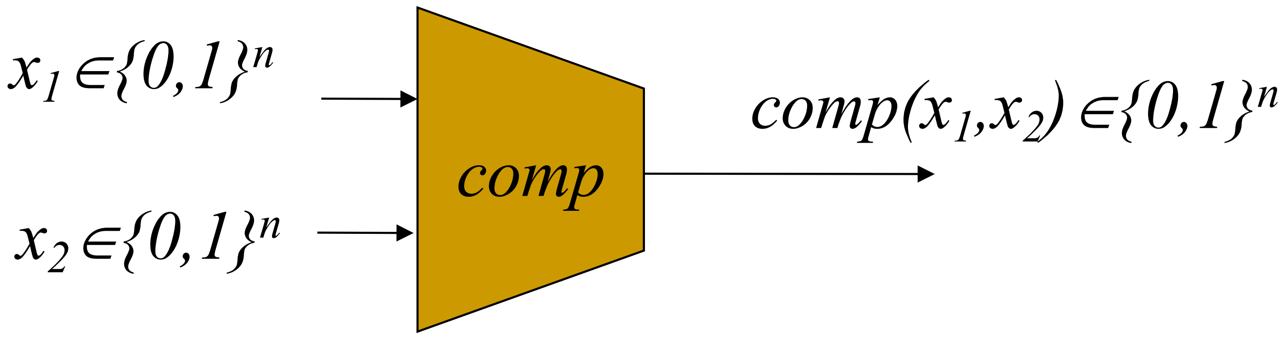
- The Merkle-Damgard construction of:
  - Collision-Resistant Digest function from CRHF
  - VIL CRHF from compression function (FIL CRHF): $|m_i| = n$
- Idea: hash iteratively, message by message:

  $\Delta(m_1, \ldots, m_l) = h(\Delta(m_1, \ldots, m_{l-1})||1||m_l)$; $\Delta(m_1) = h(0^{n+1}||m_1)$
- Lemma 4.2: if h is a CRHF, then Δ is a collision-resistant digest
- Proof… (see details in textbook)



VIL CRHF from FIL CRHF

- Recall: design and cryptanalyze simple (FIL) function, use it to construct strong (VIL) function
- Build VIL CRHF $\{0,1\}^* \rightarrow \{0,1\}^n$ from FIL CRHF (aka compression function) $comp:\{0,1\}^m \rightarrow \{0,1\}^n$
  - E.g. $m=2n$, i.e. $comp:\{0,1\}^{2n} \rightarrow \{0,1\}^n$

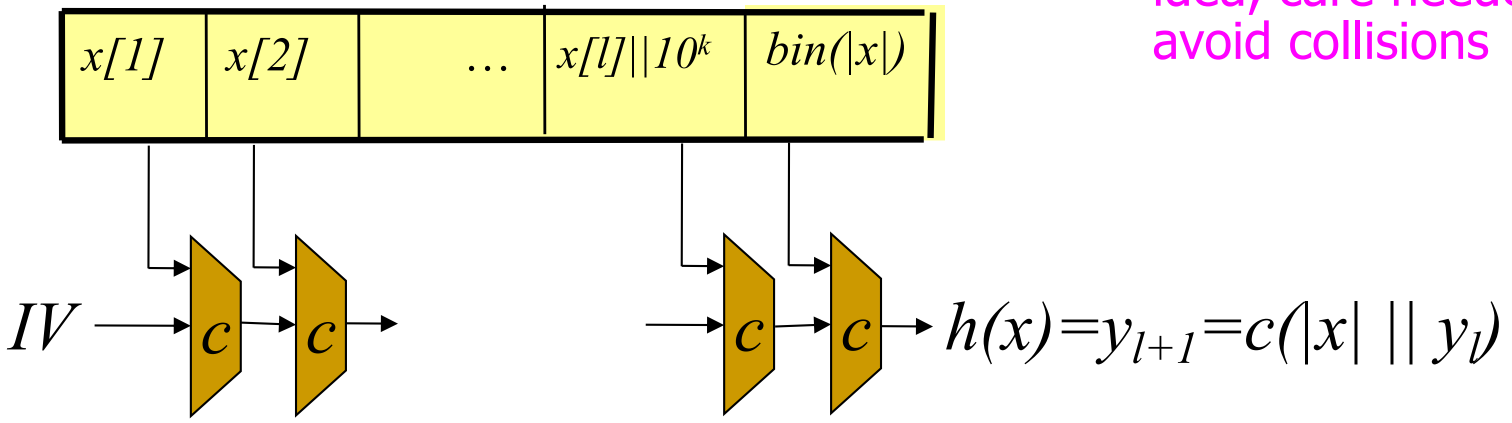


  - The Merkle-Damgard constructs a CRHF from a compression function
  - Requires `MD-strengthening' extension [see textbook]

Merkle - Damgard Length-Padding

- Aka Merkle - Damgard Strengthening
- Let $pad(x)=1||0^k||\mathrm{bin}(|x|)$; $x'=x||pad(x)$
  - Where $bin(|x|)$ is the L–bit binary representation of $|x|$
  - And: $|x|+|pad(x)|\equiv 0 \mod L$
  - Simplify: assume $|x|\equiv 0 \mod L$, $|pad(x)|=L$
- Let $y_0=IV$ be some fixed L bits (IV=Initialization Value)
- For $i=1,..|x'|/L$ let $y_i=c(x'[i] \,||\, y_{i-1})$
- Output $MD[c]_{IV}(x)=y_{l+1}$

This is just a high level idea, care needed to avoid collisions



The Digest-Chain Extend Function

- Beyond digest and collision resistance: sequence-related integrity mechanisms
- For digest-chain, the **extend function:**
  - Input: digest and ‘next’ sequence
  - Output: digest (of entire sequence)
  - Correctness requirement:

$$Extend(\Delta_l, M_{l+1,l'}) = \Delta(M_l + M_{l+1,l'})$$

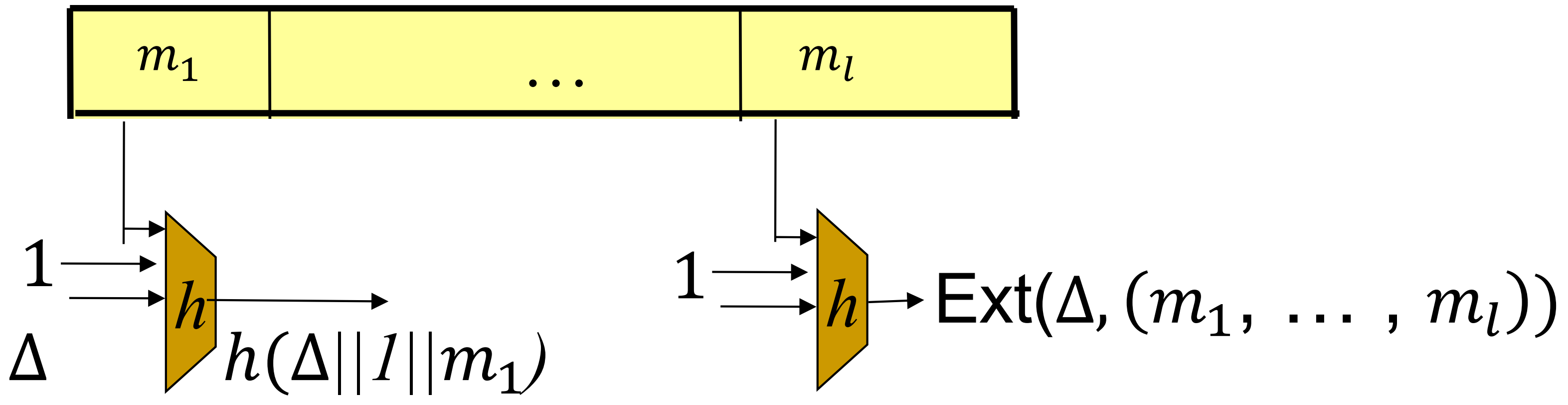
Use to (1) extend chain, (2) validate new digest (with new seq.), or (3) use digest to validate a message

The Merkle-Damgard Extend Function

- We can define Extend for Merkle-Damgard:
  - Idea: Just continue last digest!

$$\mathcal{MD}^h.Extend\,(\Delta,\{m_1,\ldots,m_l\}) \equiv \begin{cases} \text{Let } \Delta_1 \leftarrow h(\Delta + 1 + m_1) \\ \text{For } l = 1\text{: } \Delta_1 \\ \text{For } l > 1\text{:} \\ \qquad \mathcal{MD}^h.Extend\,(\Delta_1,\{m_2,\ldots,m_l\}) \end{cases}$$

- Not secure to be used to construct a MAC!



Merkle Digest Schemes

- Digest function Δ: $\{m_i \epsilon \{0,1\}^*\} \rightarrow \{0,1\}^{\boldsymbol{n}}$
    - Collision-resistance requirement
- Validation of Inclusion: PoI and $VerPoI$
    - PoI function: compute Proof of Inclusion
    - $VerPoI$ function: verify PoI
    - Both: mandatory and optimized
    - Optional, also Proof-of-Non-Inclusion (PoNI)
- Extending the Sequence: PoC and $VerPoC$
    - PoC: Proof of Consistency (from old digest to new)
    - $VerPoC$ function: verify PoC
    - Optional

Merkle digest scheme: definition

Definition (Merkle digest scheme). *A* Merkle digest scheme $\mathcal{M}$ *is a tuple of three PPT functions* $(\mathcal{M}.\Delta, \mathcal{M}.PoI, \mathcal{M}.VerPoI)$, *where:*

$\mathcal{M}.\Delta$ *is the* Merkle tree digest *function, whose input is a sequence of messages* $B = \{m_i \in \{0,1\}^*\}_i$ *and whose output is an* n*-bit* digest*:* $\mathcal{M}.\Delta : (\{0,1\}^*)^* \to \{0,1\}^n$.

$\mathcal{M}.PoI$ *is the* Proof-of-Inclusion *function, whose input is a sequence of messages* $B = \{m_i \in \{0,1\}^*\}_i$, an integer $i \in [1, |B|]$ *(the index of one message in* B*), and whose output is a* Proof-of-Inclusion *(PoI):* $\mathcal{M}.PoI : (\{0,1\}^*)^* \times \mathbb{N} \to \{0,1\}^*$.

$\mathcal{M}.VerPoI$ *is the* Verify-Proof-of-Inclusion *predicate, whose inputs are digest* $d \in \{0,1\}^n$, *message* $m \in \{0,1\}^*$, *index* $i \in \mathbb{N}$, *proof* $p \in \{0,1\}^*$, *and whose output is a bit (*1 *for 'true' or* 0 *for 'false'):* $\mathcal{M}.VerPoI : \{0,1\}^n \times \{0,1\}^* \times \mathbb{N} \times \{0,1\}^* \to \{0,1\}$.

Merkle digest: correctness and security

A Merkle digest scheme $\mathcal{M}$ *is* correct *if for every sequence of messages* $B = \{m_i \in \{0,1\}^*\}_i$ *and every index* $i \in [1, |B|]$*, the Proof-of-Inclusion verifies correctly, i.e.:*

$$\mathcal{M}.VerPoI(\mathcal{M}.\Delta(B), m_i, i, \mathcal{M}.PoI(B, i)) = \textsc{True}$$

A Merkle digest scheme $\mathcal{M}$ *is* secure *if for every efficient (PPT) algorithm* $\mathcal{A}$*, both the* collision advantage $\varepsilon^{Coll}_{\mathcal{M},\mathcal{A}}(n)$ *and the* PoI advantage $\varepsilon^{PoI}_{\mathcal{M},\mathcal{A}}(n)$ *are* negligible *in* n*, i.e., smaller than any positive polynomial for sufficiently large* n *(as* $n \to \infty$*), where:*

$$\begin{aligned}
\varepsilon^{Coll}_{\mathcal{M},\mathcal{A}}(n) &\equiv \Pr\left[\begin{array}{c} (x, x') \leftarrow \mathcal{A}(1^n) \ s.t. \ (x \neq x') \\ \wedge(\mathcal{M}.\Delta(x) = \mathcal{M}.\Delta(x') \end{array}\right] \\
\varepsilon^{PoI}_{\mathcal{M},\mathcal{A}}(n) &\equiv \Pr\left[\begin{array}{c} (\{m_1, \ldots, m_l\}, d, m, i, p) \leftarrow \mathcal{A}(1^n) \ s.t. \ m_i \neq m \wedge \\ d = \mathcal{M}.\Delta(\{m_1, \ldots, m_l\}) \wedge \\ \mathcal{M}.VerPoI(d, m, i, p) = \textsc{True} \end{array}\right]
\end{aligned}$$

Where the probability is taken over the random coin tosses of $\mathcal{A}$.

Simply put, security means that a PPT adversary cannot find collisions and cannot forge a valid PoI

Proof of Consistency (PoC)

- A Merkle digest scheme supports PoC if it has two more functions:

$\mathcal{M}.PoC(B_C, B_N)$ *is the* Extend and Proof-of-Consistency *function* PoC*, whose input are two sequences,* B_C *and* B_N*, and whose output* $\gamma_{CN} = \mathcal{M}.PoC(B_C, B_N)$ *is a binary string which we call the* Proof-of-Consistency *from* $\Delta_C \equiv \mathcal{M}.\Delta(B_C)$ *to* $\Delta_{CN} \equiv \mathcal{M}.\Delta(B_{CN})$*.*

$\mathcal{M}.VerPoC(\Delta_C, \Delta_{CN}, l_C, l_N, p) \in \{\textbf{True}, \textbf{False}\}$ *is the* Verify-Proof-of-Consistency *predicate, whose inputs are the two digests* Δ_C, Δ_{CN}*, the numbers of entries (*l_C *and* l_N*), and a string (*PoC*)* p*.*

- Correct PoC:

$$\mathcal{M}.VerPoC\,(\mathcal{M}.\Delta(B_C), \mathcal{M}.\Delta(B_C + B_N), l_C, l_N, \mathcal{M}.PoC(B_C, B_N)) = \textsc{True}$$

Secure Proof of Consistency

We say that $\mathcal{M}$ *has* secure PoC, *if for every efficient (PPT) algorithm* $\mathcal{A}$, *the* PoC-advantage $\varepsilon^{PoC}_{\mathcal{M},\mathcal{A}}(n)$ *is* negligible *in* n, *where:*

$$\varepsilon^{PoC}_{\mathcal{M},\mathcal{A}}(n) \quad\equiv\quad \Pr\left[\begin{array}{c} (B_C, B_A, l_C, l_A, p) \leftarrow \mathcal{A}(1^n) \;\; s.t. \\ \mathcal{M}.VerPoC(\mathcal{M}.\Delta(B_C), \mathcal{M}.\Delta(B_A), l_C, l_A, p) = \textsc{True} \wedge \\ \wedge \; B_C \textit{ is not a prefix of } B_A \end{array}\right]$$

Where the probability is taken over the random coin tosses of $\mathcal{A}$.

To be consistent with previous slides, replace B_A with B_{CN}

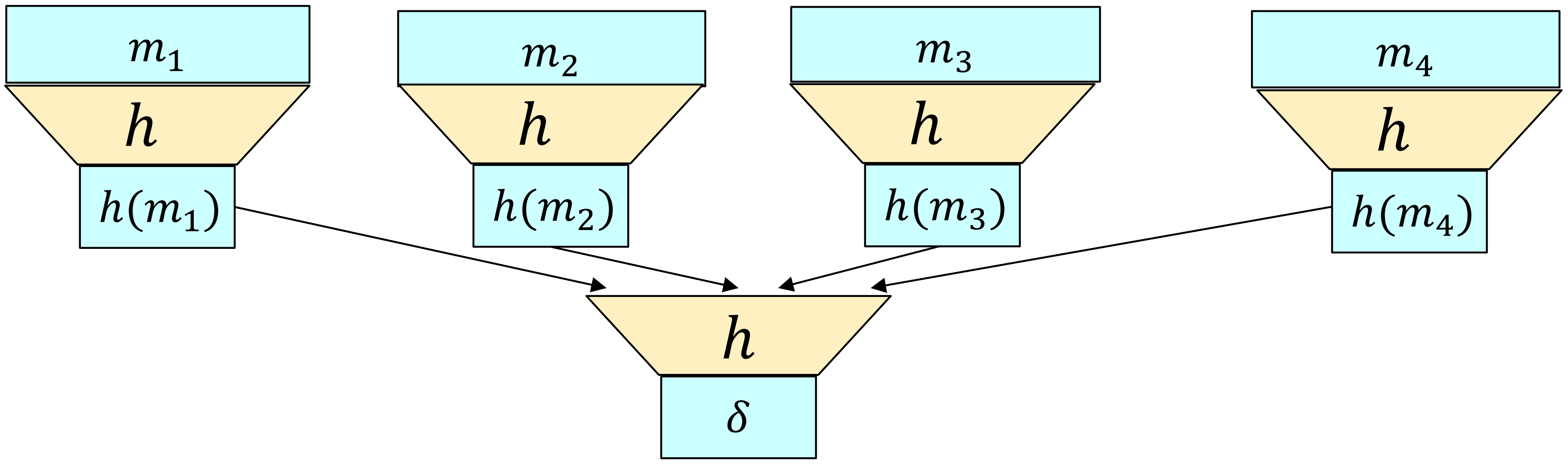
Simply put, the above says that a PPT adversary cannot forge a valid PoC

Two-layered Merkle tree

- Short digest validates integrity of large object
  - Often, object consists of multiple ‘files’
- Merkle tree : integrity for many ‘messages’
  - Hash each ‘message’ in block, then hash-of-hashes

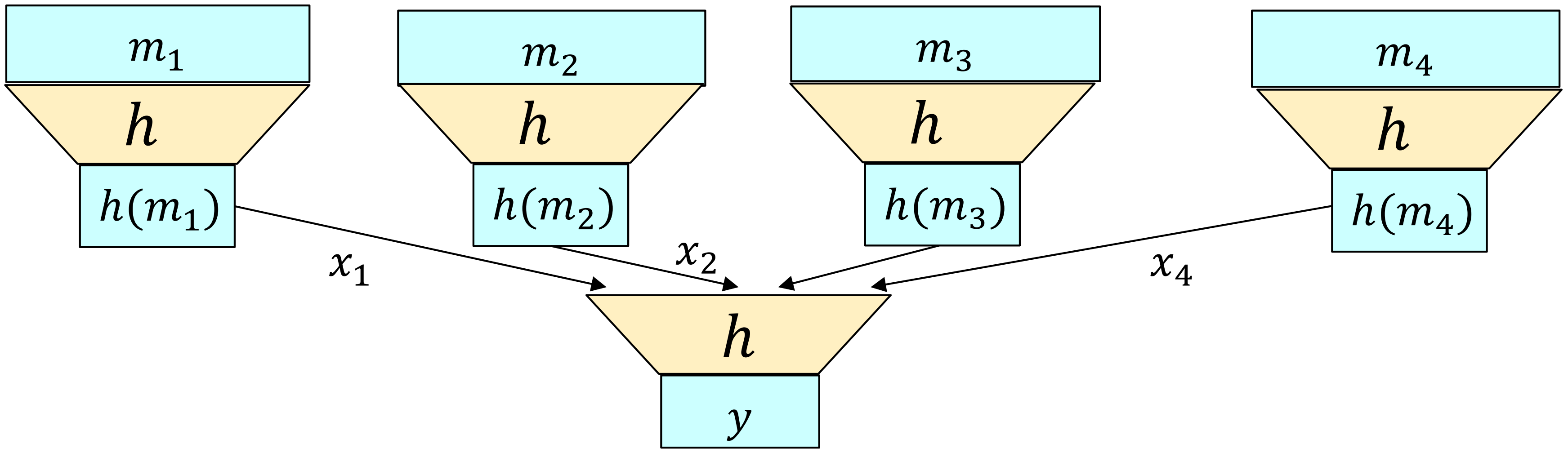
$$\delta = h(h(m_1)||h(m_2)||h(m_3)||h(m_4))$$

  - Validate each ‘message’ independently
    - Advantages: **efficiency** (computation, communication) and **privacy**



Two-layered Merkle tree

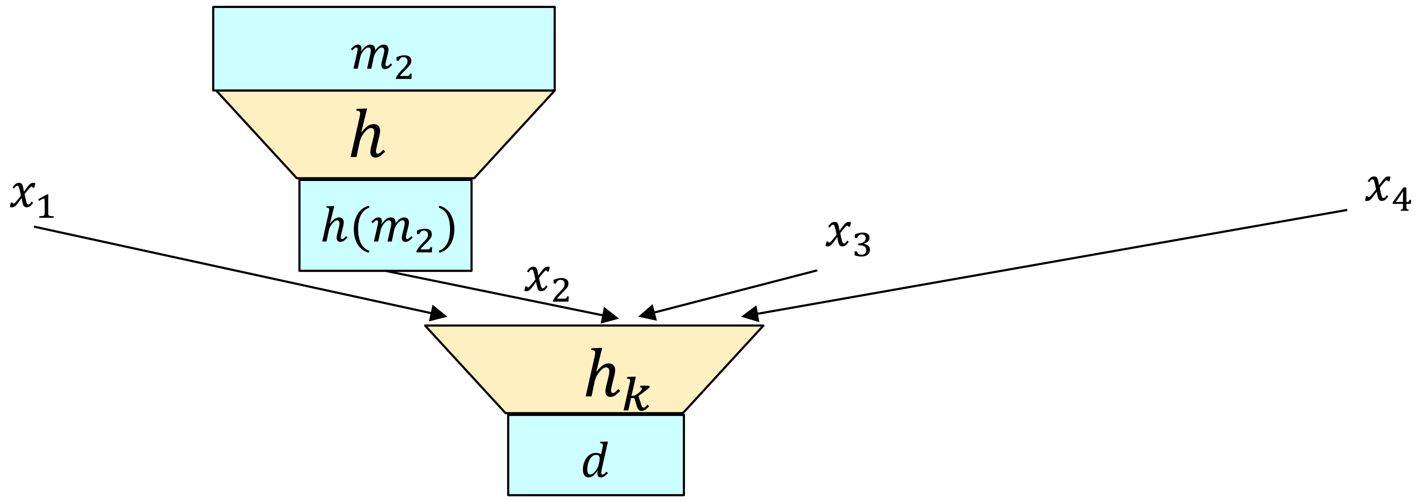
$$\begin{aligned}
2l\mathcal{MT}.\Delta(m_1,\ldots,m_l) &\equiv h\left[h(m_1) \mathbin{+\!\!+} \ldots \mathbin{+\!\!+} h(m_l)\right] \\
2l\mathcal{MT}.PoI((m_1,\ldots,m_l),j) &\equiv \{h(m_i)\}_{i=1}^{l} \\
2l\mathcal{MT}.VerPoI(d,m,i,\{x_i\}_{i=1}^{l}) &\equiv \left[\begin{array}{c} \textsc{True}\ \textit{if}\ x_i = h(m),\ \textit{and} \\ d = h(x_1 \mathbin{+\!\!+} \ldots \mathbin{+\!\!+} x_l) \end{array}\right]
\end{aligned}$$



Allows each user to receive, validate only required items. How?

To verify inclusion of m_2 ...

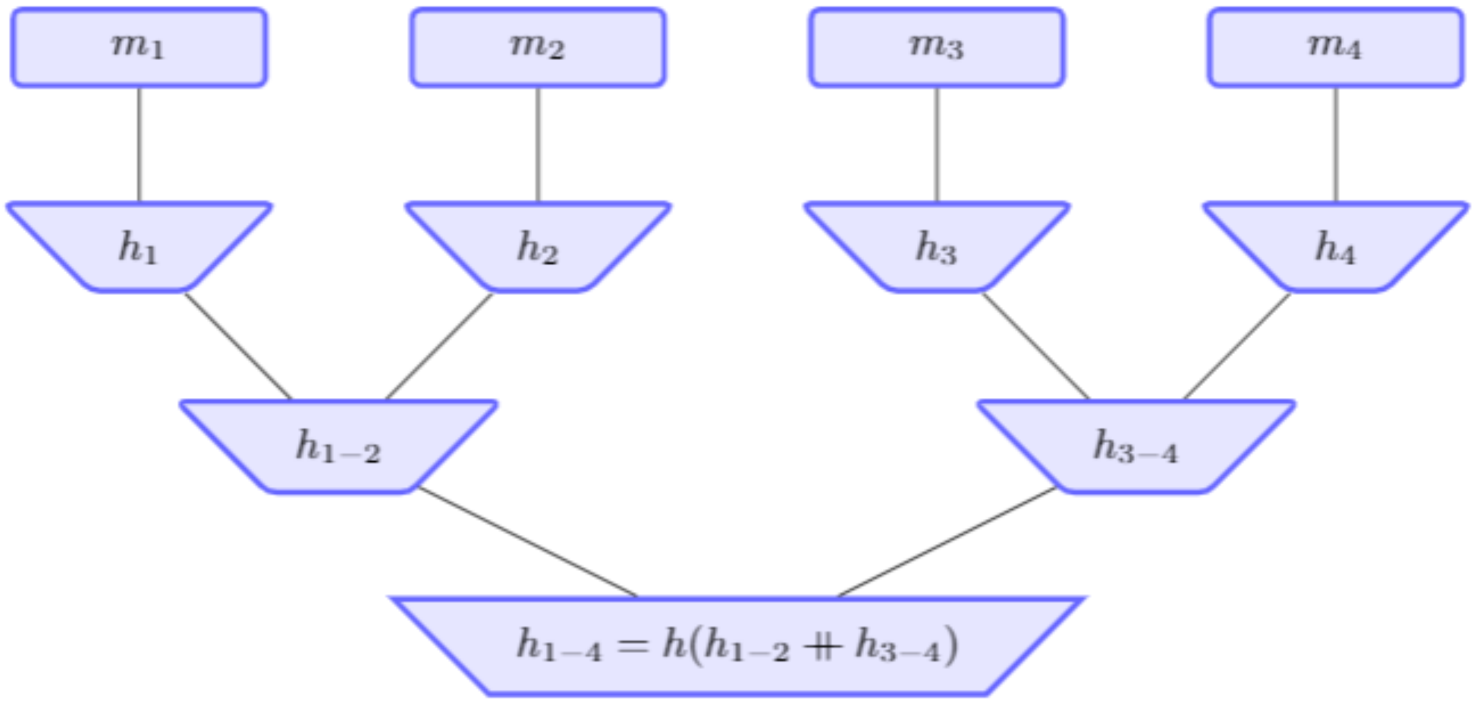
$$\begin{aligned}
2l\mathcal{MT}.\Delta(m_1,\ldots,m_l) &\equiv h\left[h(m_1) \mathbin{+\!\!+} \ldots \mathbin{+\!\!+} h(m_l)\right] \\
2l\mathcal{MT}.PoI((m_1,\ldots,m_l),j) &\equiv \{h(m_i)\}_{i=1}^{l} \\
2l\mathcal{MT}.VerPoI(d,m,i,\{x_i\}_{i=1}^{l}) &\equiv \left[\begin{array}{c} \textsc{True}\ \textit{if}\ x_i = h(m),\ \textit{and} \\ d = h(x_1 \mathbin{+\!\!+} \ldots \mathbin{+\!\!+} x_l) \end{array}\right]
\end{aligned}$$



Receive and validate only m_2. Other hashes still required, though.

The Merkle Tree Construction

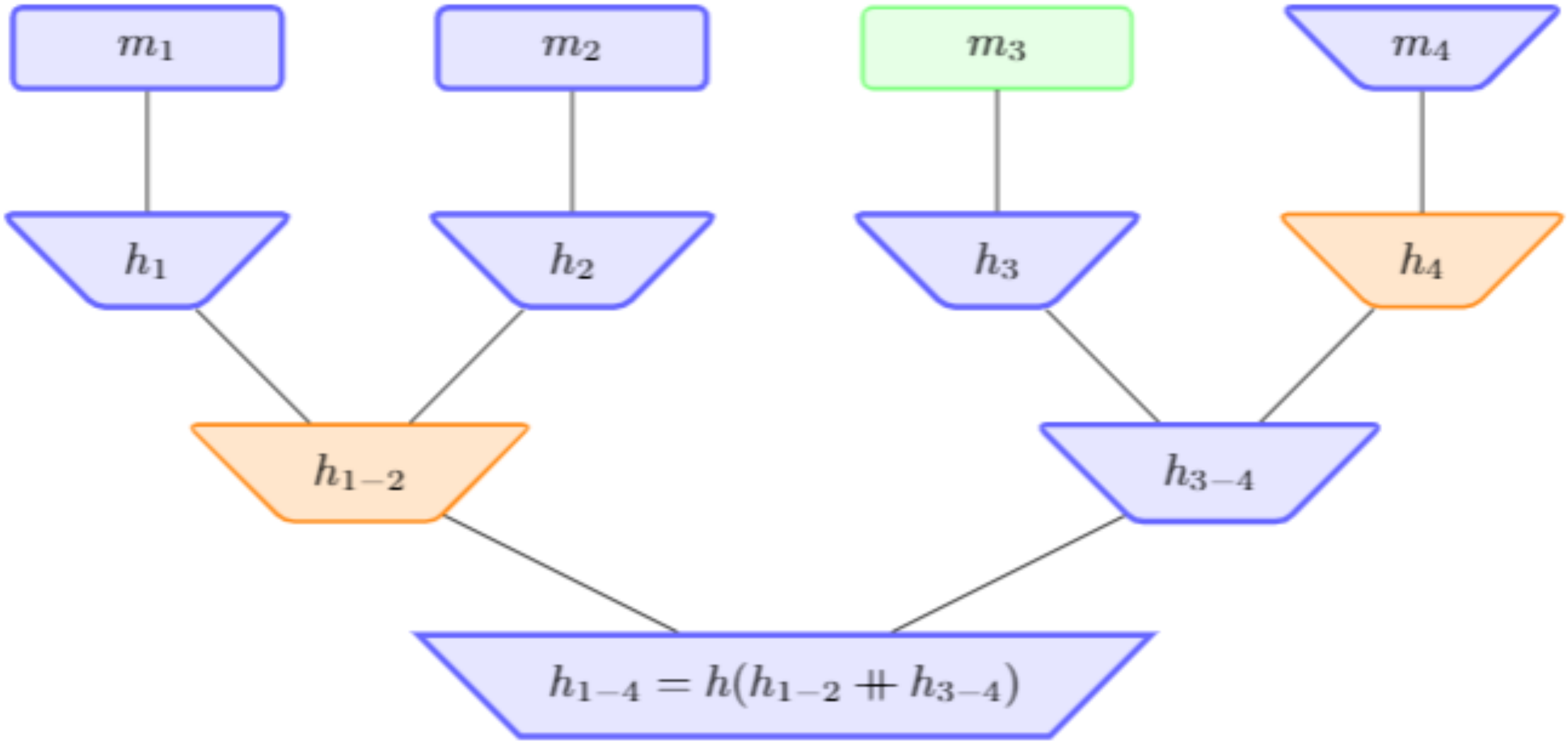
- Reduce length of ‘proofs’ – send less hashes of ‘other msgs’



$$\mathcal{MT}.\Delta(M) \equiv \begin{cases} \text{If } L = 0: & h(m_1) \\ \text{Else} & h\left(\mathcal{MT}.\Delta\left(m_1, \ldots, m_{2^{L-1}}\right) + \right. \\ & \quad \left. + \mathcal{MT}.\Delta\left(m_{2^{L-1}+1}, \ldots, m_{2^L}\right)\right) \end{cases}$$

Merkle Tree: Proof of Inclusion (PoI)

- To prove inclusion of m_3 , send also 'proofs': h_{1-2}, h_4



Blockchains

- Next slides set.

Covered Material From the Textbook

- Chapter 3
  - Sections 3.8, 3.9, and 3.10

Thank You!