The applications of stress

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<u>Chapter 5</u>: Numerical Integration

APP5. The stress developed in a rectangular bar when it is twisted can be computed if one knows the values of a torsion function U that satisfies a certain partial-differential equation. Chapter 8 describes a numerical method that can determine values of U. To compute the stress, it is necessary to integrate $\int\int U\,dx\,dy$ over the rectangular region for which the data given here apply. Determine the stress. (You may be able to simplify the integration because of the symmetry in the data.)

x \ y	0.0	0.2	0.4	0.6	0.8	1.0	1.2
0.0	0	0	0	0	0	0	0
0.2	0	2.043	3.048	3.354	3.048	2.043	0
0.4	0	3.123	4.794	5.319	4.794	3.123	0
0.6	0	3.657	5.686	6.335	5.686	3.657	0
0.8	0	3.818	5.960	6.647	5.960	3.818	0
1.0	0	3.657	5.686	6.336	5.686	3.657	0
1.2	0	3.123	4.794	5.319	4.794	3.123	0
1.4	0	2.043	3.048	3.354	3.048	2.043	0
1.6	0	0	0	0	0	0	0

The report will discuss the above question in the following aspects:

1-The domain: Stress.

2-The applied methods.

3-The MATLAB program.

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1. Introduction

Engineering Mechanics are responsible for analyzing the properties, states, and behavior of technical Bodies. These problems are ranked by the strain-stress analysis of solid bodies. The strain-stress analysis has progressed, with analytic methodologies predominating in the past being superseded by numerical tools such as the Finite Element Method, Finite Volume Method, and Boundary Element Method, among others. In contrast to traditional approaches, numerical methods are universal in that their application is independent of body shape, material properties, and so on.

Analytical and numerical approaches in strain-stress analysis both still in continues use for many reasons. Numerical approaches (for example, FEM) allow computations of numerical values of a material's strain-stress state at certain places and under specific loads. They do not, however, give any formulas for estimating the change in values as a function of load, size, stiffness, or other factors. This prediction is possible thinks to the analytical approaches that produce formulas. Regrettably, they can only be used on specific objects and load forms.

This knowledge, which is concealed in formulae produced through analytical procedures, is an effective tool for locating risky locations for laden bodies. For instance, consider the torsion of a bar having a cross-section that may be divided into many rectangles. The following conclusions can be drawn from the analysis of the shear stress of specific rectangles: in the case of an open profile, the maximum shear stress appears in rectangles with the greatest thickness, while in the case of a closed profile, it appears in rectangles with the smallest thickness. These findings may also be produced using numerical methods; however, this involves the computation of a large number of profile variations.

Now that we have a brief acknowledge on analytical and numerical approaches in calculating strain-stress we will produce our problem.

The problem states that we have a rectangular bar that is twisted, which yields a stress in it, and we were asked to calculate the stress developed in that rectangular bar with the help of the torsion function, which satisfies a PDE. we need to integrate $\int\int U\,dx\,dy$ over the rectangular area for which the data given apply; in order to compute the stress. considering the symmetry in the data will help simplifying the integration.

Simpson’s method is our best option to solve this problem, but we chose to solve it using both Simpson’s method on y-axis and trapezoidal rule on x-axis to get a better understanding about these two methods; and we wrote the algorithm as a MATLAB code for users to fill data and run the program to get the stress.

We will discuss the domain of the problem in section 2 while In Subsection 2.1 we explore why do we need to calculate stress. The definition of stress will be discussed in subsection 2.2, also in Subsection 2.3 we will explain what the types of stress are in physics and show the difference between Normal Stress and Shearing Stress ‘which is also called Tangential Stress’. (Franc˚u, 2012)

We’ve described the chosen numerical analysis methods (Trapezoidal and the Simpson's Rule) in section 3. We will present the Trapezoidal rule along with its algorithm, advantages, and disadvantages in subsection 3.1 while also in Subsection 3.2 we describe the Simpson's rule along with its algorithm, advantages, and disadvantages.

We illustrate furthermore with some examples for trapezoidal and the Simpson's Rule in Subsection 3.3. We show the Differences between trapezoidal rule and Simpson's rule in Subsection 3.4.

We explain how the method is applied in section 4. While In subsection 4.1 we show the cod in MATLAB.

2. The domain of the problem

2.1 Stress

Why do we need to calculate stress?

In order to have a perfect perception of stress we have to know the importance of stress and whether we deal with it in our real life, the following two examples will illustrate this:

The first example, picture a traffic sign mounted on a single column on a windy day, the wind causes the sign to twist, and this twist causes shear stress which a type of stress we will discuss later in this section.

The second example, when we open a thin jar, we actually use our hands to apply a force equal in magnitude and opposite in direction over the jar. The question here how much force need to apply until open it without broken. In the mechanics of material, they consider all bodies are deformable and those applied external force generate internal stress. Calculating stress will prevent this problem from appearing. [see figure 1] (A.I.M Voorbij)

2.2 What is stress?

The measure of the external force affecting the cross-sectional area of an object is called Stress. It is a physical quantity can expression through the units of force per area: $\boldsymbol{N}/\ \boldsymbol{m}^{\mathbf{2}}$. The commonly SI units are referred as pascals.

This unit is used in many fields, not limited to physics. In Architectural Engineering, the stress must be calculated to find out how much force is applied to the particles of a particular material. In other word, each material has an ultimate stress- a measure of how much stress the material can withstand before failing.

There are two types of stress that any construction can experience:

- Normal Stress.
- Shear Stress.

(P.Holmes)

Figure 1

2.3 Types of Stress

In physics, there are various types of stress, however they are divided into two main types: Normal Stress and Tangential or Shearing Stress. The next paragraphs go over some of the many forms of stress.

1. Normal Stress:

When the direction of the deforming force is perpendicular to the cross-sectional area of the body, stress is said to be normal stress. The stress will be normal as the wire length, or the body volume varies.

Normal stress is further divided into two subcategories based on the force dimension.

- Longitudinal stress
- Bulk Stress or Volumetric stress

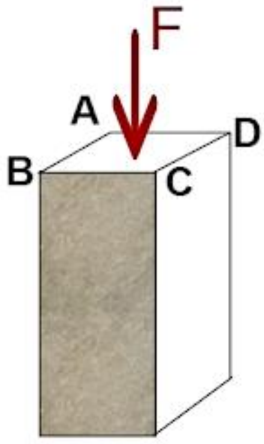

Figure 2

- Longitudinal Stress:

When a body is under longitudinal stress, the deforming force affects the length of the body, just as the name implies.

The length of the body changes as a result of longitudinal tension. there will be a little difference in diameter of the object.

The Longitudinal Stress stretches or compresses the object and its whole length. therefore, based on the direction of deforming force, it may be further categorized into two types:

- Tensile stress
- Compressive stress

Tensile Stress

Tensile stress is defined as a stress that occurs when a deforming force or applied force causes an increase in the object's length. When a rod or wire is stretched. such as, an equal and opposing forces are applied at both ends (outwards).

Compressive Stress

When a deforming force or applied force causes an object's length to decrease, the resulting tension is known as compressive stress. When a rod or wire is compressed/squeezed by pulling it (inwards) with equal and opposite pressures at both ends, for example.

- Bulk Stress or Volume Stress

When a deforming force or applied force acts on an object in all dimensions, causing a change in its volume, this is referred to as volumetric stress or bulk stress. Volume stress occurs when the volume of a body changes as a result of a deforming force.

2. Shearing Stress or Tangential Stress:

when the force acts parallel to the item's surface area. The stress created in the object is called shear stress or tangential stress. It results in changing the shape of the body.

As shown in the diagram, the force F acts parallel to the surface area ABCD, causing shear stress to form in the item. (Stress, 2021)

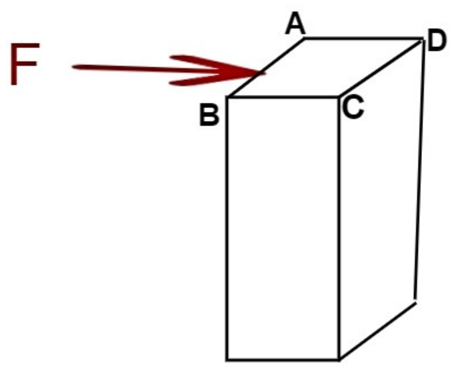

Figure3

3. The methods

we have several numerical methods for evaluating Definite integrals. In this report, we are going to talk about the Trapezoidal and the Simpson's Rule.

3.1 trapezoidal rule:

The trapezoidal Rule, a Newton-cotes formula, is an approach to evaluate the region under the curve. It is named the "Trapezoidal" rule because it divides the area into n Trapezoids instead of rectangles.

This method will solve the definite integral $\int_a^b f(x)dx$ by using the linear approximation of the function f(x). In figure 4, the curve f(x) is approximated to the linear function P(x). (Weisstein E. W., 2021)

The Trapezoidal Rule Formula: $\int_a^b f(x) = h\left(\frac{f(a)+f(b)}{2}\right)$, where $h = (b-a)/n$

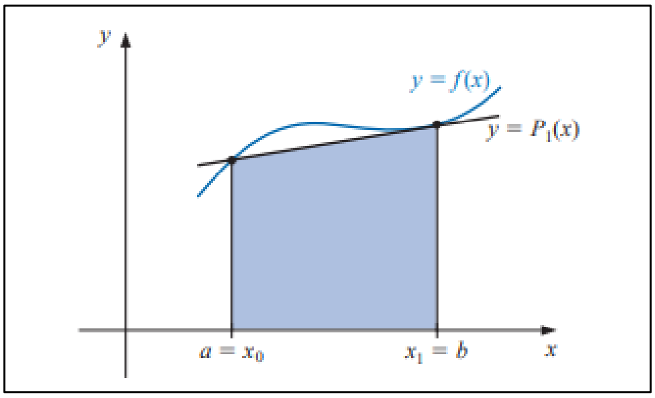

Figure 4

3.1.1 Algorithm

1. Given function *f(x)*
2. Read data points:

   (x0, y0), (x1, y1), ⋯ ⋯ ⋯ (xn, yn)
3. Suppose the lower limit of integration is *a* and the upper limit of integration is *b*, let *a* = *x0*, and *b* = *xn*
4. Divide the interval [a, b] into *n* subinterval

   Calculate step size *h* = *(b − a) / n*
5. Set integration variable *I* =*[f(a) + f(b)]/2*
6. Suppose i = 1 // number of intervals
7. While *i* < *n* Calculate *xi* = *a*+*h*∗*i*
8. Add r to integration variable *I* = *I* + *r*
9. Increment i
10. Calculate integration variable after multiplying it with step size I = I ∗ h (Gerald Wheatly, 2004)

3.1.2 Advantages

- The trapezoidal rule's simplicity makes it appropriate for many numerical integration applications.

- for piecewise linear curves like a ROC curve, the trapezoidal rule is exact and also fast! [see figure 5] (OpenEye Scientific Software, 2019)

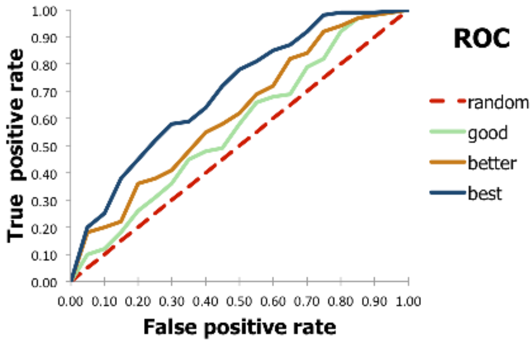

Figure 5

- To tell the truth the trapezoid rule is so old and also not accurate, but scientists use it till now cause its efficient with these situations:

1. Although the trapezoid rule is inefficient in general, it can be incredibly effective when used to periodic functions.

2. For analytic functions that go to zero quickly, such as double exponential functions, the trapezoid method can be astonishingly efficient. (Wicklin, 2011)

3. Till now you can use the trapezoid rule to divertive a publication, A doctor re-invented the trapezoid rule in a paper in 1994. Not only did the editors miss this old algorithm, but the paper has been cited numerous times since it was published.

3.1.3 Disadvantages

- In the trapezoid, the rate of convergence is low, and we need more number of subspaces to get more accurate approximate results.

- in the following graph, the decreasing gradient (from x = 1 to x = 3), the trapezium rule gives an underestimate, while for an increasing gradient (from x = 3 to x = 5), it will give an overestimate. This results always appear with this method. (Wikiversity contributors, 2016)

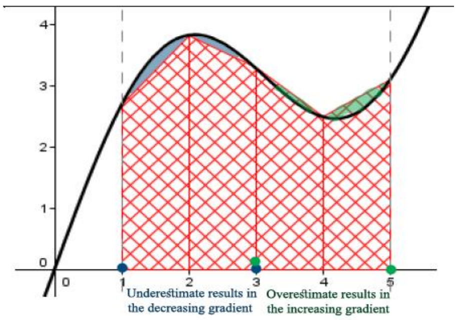

Figure 6

3.2 Simpson's method:

The Simpson's Rule is a Newton-cotes formula to find the definite integral $\int_a^b f(x)\,dx$ by using quadratic polynomial functions.
In the Simpson's Rule, we will estimate the area under the curve by dividing the area into parabolas (instead of straight lines as in the Trapezoidal Rule). In figure 7, the curve f(x) is approximated to the quadratic function P(x). (Weisstein E. W., 2021)

The Simpson's Rule Formula: $\int_{x_0}^{x_2} f(x) = \frac{h}{3}(f(x_0) + 4f(x_1) + f(x_2),$

where $h = (b - a)/n$

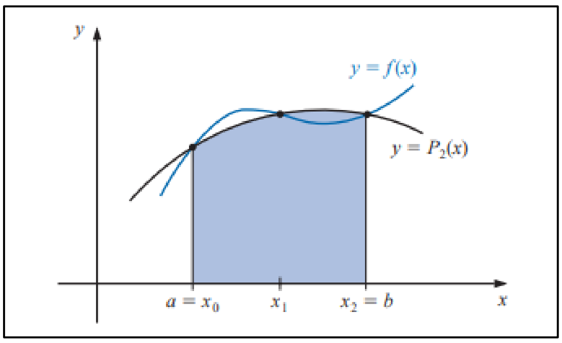

Figure 7

3.2.1 Algorithm

1. Given function $f(x)$
2. Read data points:

   $(x0, y0), (x1, y1), \cdots \cdots \cdots (xn, yn)$

3. Suppose the lower limit of integration is a and the upper limit of integration is b

   let $a = x0$, and $b = xn$

4. Divide the interval [a, b] into n subinterval

   Calculate step size $h = (b - a) / n$

5. Set integration variable $I = [f(a) + f(b)]/2$
6. Suppose $i = 1$ // number of intervals
7. If I mod 2 = 0 then go to
8. Calculate: $k = a + i * h$
9. If i mod 2 = 0 then

   $I = I + 2 * f(k)$

10. Otherwise

    $I = I + 4 * f(k)$

11. Increment i by 1 and go to step 7
12. Calculate: $I = I * h/3$

(Gerald Wheatly, 2004)

3.2.2 Advantages

- We can apply both rules in the figure 8 below. There are seven x values, and thus six intervals. Now let's talk about each interval .Every interval is taken as it is in trapezoidal, but we divide it into two pieces in Simpson's and then use the formula so, as a result, Simpson's is more accurate

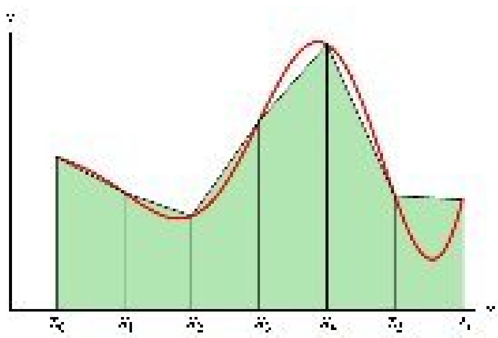

Figure 8

- When the underlying function is smooth, the trapezoidal rule is less accurate than Simpson's Rule because Simpson's Rule applies quadratic approximations instead of linear approximation, so it has a smaller error.

- Simpson's Rule has the advantage that, whereas rectangles and trapezoids are preferable for linear functions, it works well on curves.

3.2.3 Disadvantages

- Simpson's method can be applied to only even number of finite intervals between the limits. So, the value of 'n' in Simpson's rule is always an even number. n=2i; where (i=1,2,3,4....). This gives a better approximation than trapezoidal method. However, since 3 points are required to define a parabola, an even number of strips is required for the formula to work.

- it is easy to integrate quadratic equations by integrate the quadratic equation over the intervals. It ends with a very accurate approximation, but it is also needing a lot of math! (web stanford, n.d.)

3.3 Examples on the Trapezoidal & Simpson's rule

we are going to solve some definite integral using the trapezoidal and Simpson's method:

a. $\int_0^2 x^2\, dx$ b. $\int_0^2 \sqrt{1+x^2}\ dx$ c. $\int_1^2 \frac{1}{1+x}\ dx$

(Faires, 2011)

solution for (a):

Using trapezoidal method: $\int_a^b f(x) = h\ (\frac{f(a)+f(b)}{2})$, where $h = b - a$

$h = 2 - 0 = 2$

$f(0) = 0^2 = 0$

$f(2) = \ 2^2 = 4$

$$\int_0^2 x^2\, dx \ = \ (2)\left(\frac{0+4}{2}\right) = 4.000$$

Using Simpson's method: $\int_{x_0}^{x_2} f(x) = \frac{h}{3}(f(x_0) + 4f(x_1) + f(x_2),\ h = (x_2 - x_0)/2$

Here we have $x_0 = 0, x_1 = 1,\ x_2 = 2$

$$h = \frac{(2-0)}{2} = 1$$

$f(0) = 0^2 = 0$

$f(1) = 1^2 = 1$

$f(2) = \ 2^2 = 4$

$$\int_0^2 x^2\, dx = \frac{1}{3}(0 + 4(1) + 4) \approx 2.667$$

solution for (b):

Using trapezoidal method:

$h = 2 - 0 = 2$

$f(0) = \sqrt{1+0^2} = 1$

$f(2) = \ \sqrt{1+2^2} \approx 2.2360$

$$\int_0^2 \sqrt{1+x^2}\, dx \ = \ (2)\left(\frac{1+2.2360}{2}\right) \ \approx 3.236$$

Using Simpson's method:

Here we have $x_0 = 0, x_1 = 1, \; x_2 = 2$

$$h = \frac{(2-0)}{2} = 1$$

$$f(0) = \sqrt{1+0^2} = 0$$

$$f(1) = \sqrt{1+1^2} \approx 1.4142$$

$$f(2) = \sqrt{1+2^2} \approx 2.2360$$

$$\int_0^2 \sqrt{1+x^2}\, dx = \frac{1}{3}(0 + 4(1.4142) + 2.2360) \approx 2.964$$

<u>solution for (c):</u>

Using trapezoidal method:

$$h = 2 - 1 = 1$$

$$f(1) = \frac{1}{1+1} = 0.5$$

$$f(2) = \frac{1}{1+2} \approx 0.3333$$

$$\int_1^2 \frac{1}{1+x} dx = (1)\left(\frac{0.5 + 0.3333}{2}\right) \approx 0.4166$$

Using Simpson's method:

Here we have $x_0 = 1, x_1 = 1.5, \; x_2 = 2$

$$h = \frac{(2-1)}{2} = 0.5$$

$$f(1) = \frac{1}{1+1} = 0.5$$

$$f(1.5) = \frac{1}{1+1.5} \approx 0.4$$

$$f(2) = \frac{1}{1+2} \approx 0.3333$$

$$\int_1^2 \frac{1}{1+x} dx = \frac{1}{3}(0.5 + 4(0.4) + 0.3333) \approx 0.4055$$

	(a)	*(b)*	*(c)*
Exact valve	2.667	2.958	0.4055
Trapezoid rule	4.000	3.263	0.1446
Simpson's rule	2.667	2.964	0.4055

3.4 Differences between trapezoidal rule and Simpson's rule:

- trapezoidal rule referred to as Trapezium rule while Simpson's Rule referred to as Parabolic Rule.

- The boundary in the trapezoidal rule between ordinates is considered straight so that why the result in trapezoidal not affected. While in Simpson's rule are parabolic and that why the result in Simpson's is greater or lesser as the curve

- There are no restrictions in trapezoidal so it can be used with any number of ordinates While in Simpson's rule the number of divisions should be even.

- An approximate result given by the trapezoidal rule while in the result in Simpson's rule is more accurate than the trapezoidal rule but its slow so that mean that the trapezoidal rule fast but less accurate. (Difference between Trapezoidal and Simpson's Rule, n.d.)

  Let us take a good example to show you how the accuracy differences in the two methods so, the equations are:

$$(a)\ x^2 \qquad (b)\ x^{\ 2} \qquad (c)\ (x\ +\ 1) - 1 \qquad (d)\ \sqrt{1 + x^2} \qquad (e)\ sinx \qquad (f)\ e^x$$

After we applied both methods on these functions let's compare the results :

$F(x)$	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	*(f)*
Exact value	2.667	6.400	1.099	2.958	1.416	6.389
Trapezoid rule	4.000	16.000	1.333	3.326	0.909	8.389
Simpson's rule	2.667	6.667	1.111	2.964	1.425	6.421

Hence: we notice an important note that, the Simpson’s rule is more accurate even when we applied any polynomial of degree.

- Finally, I will talk about the similarity of trapezoidal rule and Simpson’s rule that are Both of them are latter consisting of a collection of equations of varying complexity, are used to analyze complex curves and make modeling easier.

4. How the method is applied

- $\iint U(x,y)dxdy$ (4.1)
- $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$ (4.2)

What does the first equation express (4.1)? when back to chapter 8 and which is talking about the partial differential equations, we find that the equation in (4.2) represents differential equation, we can't use the analytical method to solve this type of equation, so resorting to the numerical methods is the best to get the particular value of $f(x,y)$ that satisfies the equation. When looking to the equation in (4.2) we see that U represents the derivative in terms of x and y. Therefore, we can define the function as $U(x,y)$. In the equation given to us (Click here to see the question), we do not need to calculate the values of the derivative using in detail, but we can calculate the stress direct through the periods mentioned in the table using double integration.

Let us use the trapezoidal rule in x-direction and $\frac{1}{3}$ Simpson's rule; to understand the application way of both rules, or we can apply Simpson's rule in both directions:

y=0.2 then we have $\int_{0.2}^{1.4} U(x,y)\,dx = \int_{0.2}^{1.4} U(x,0.2)\,dx$

$= \frac{h}{2}(f1 + 2f2 + 2f3 + 2f4 + 2f5 + 2f6 + f7)$

$= \frac{0.2}{2}$ [2.043 + 2(3.123)+2(3.657)+2(3.818)+2(3.657)+2(3.123)+2.043]

=3.8842

y=0.4 then we have $\int_{0.2}^{1.4} U(x,y)\,dx = \int_{0.2}^{1.4} U(x,0.4)\,dx$

$= \frac{h}{2}(f1 + 2f2 + 2f3 + 2f4 + 2f5 + 2f6 + f7)$

$= \frac{0.2}{2}[3.048 + 2(4.794) + 2(5.686) + 2(5.960) + 2(5.686) + 2(4.794) + 3.048]$

= 5.9936 which is the stress

y=0.6 then we have $\int_{0.2}^{1.4} U(x,y)\,dx = \int_{0.2}^{1.4} U(x,0.6)\,dx$

$= \frac{0.2}{2}[3.354 + 2(5.319) + 2(6.335) + 2(6.647) + 2(6.336) + 2(5.319) + 3.354]$

= 6.662

y=0.8 then we have $\int_{0.2}^{1.4} U(x,y)\,dx \quad = \int_{0.2}^{1.4} U(x,0.8)\,dx$

$$\frac{0.2}{2}[3.048 + 2(4.794) + 2(5.686) + 2(5.960) + 2(5.686) + 2(4.794) + 3.048]$$

=5.9936

y=1.0 then we have $\int_{0.2}^{1.4} U(x,y)\,dx \quad = \int_{0.2}^{1.4} U(x,1)\,dx$

$\frac{0.2}{2}[2.043 + 2(3.123)$+2(3.657)+2(3.818)+2(3.657)+2(3.123)+2.043]

= 3.8842

We now apply Simpson's rule: in y-direction

$\int U(x,y)\,dx$ =$\frac{0.2}{3}[3.8842 + 4(5.9936) + 2(6.662) + 4(5.9936) + 3.8842]$

= 4.6027

4.1 Code in MATLAB

```
disp('_______________________________________________________________')
disp('             Hello in The Stress Calculator Program             ')
disp('_______________________________________________________________')
% In this program will calculate the stress in 2D
%To get the stress result we must integrate The given value using some methods
%The methods adopted here are:
%In y-axis: Trapazoidal rule
%In X-axis:Simpson's rule


a=input('\n> Enter the lower limit of x-axis: ');
b=input('> Enter the upper limit of x-axis: ');
stripsX=input('> How many bar that you want to use in x-axis? ');
rows=input('> How many rows do you want? ');


c=input('> Enter the lower limit of y-axis: ');
d=input('> Enter the upper limit of y-axis: ');
stripsY=input('> How many bar that you want to use in y-axis? ');
columns=input('> How many columns do you want? ');
%-----------------------------------
intervalX=linspace(a,b,rows);
intervalY=linspace(c,d,columns);
%-----------------------------------




hX=(b-a)/stripsX;
hY=(d-c)/stripsY;

Matrix=zeros(rows,columns);
numOfVal=rows*columns;
fprintf('Please Enter %d value(You can ignore zero symmetric data on the axis): \n',numOfVal)
```

```
for i=1:rows
    for j=1:columns
        value=input('> Enter the data: ');
        Matrix(i,j)=value;
        disp(Matrix)
    end
end



for j=1:columns
    sum=0;
    for i=2:rows-1

        sum=Matrix(i,j)+sum;

    end

      y=hX/2*(Matrix(1,j)+2*sum+Matrix(rows,j));
      Matrix2(j)=y;

end
%Temporary matrix to store the values of x-axis after applied trapezoid
%rule on 1D

fprintf('\nThe results of the trapazoidal rule in x-direction are:\n')
disp(Matrix2)
```

```
len=length(Matrix2);
sum2=0;
for i=2:2:len-1
 sum2=sum2+Matrix2(i);
end


sum3=0;
for i=3:2:len-2
 sum3=sum3+Matrix2(i);
end




evenNumber=mod(stripsY,2);
if(evenNumber==0)
fprintf('Now,We applied Simpson`s rule in Y-direction\n')
I=hY/3*(Matrix2(1) + 4*sum2+ 2*sum3 + Matrix2(end));
fprintf('The stress is: %d\n\n',I)

else
    warning('The number of strips is not even,we can not apply the Simpson`s rule.')
end




fprintf('-------------------------------------------\n')
disp('    Thank You for using this program      ')
disp('-------------------------------------------')
```

5. Conclusion

In conclusion, let’s mention what we are doing in sequence order to give you the whole idea in this report in very briefly way, the first section in this report discuss the domain
Of problem which is the stress with two examples given. Moreover, we mentioned several types of stress. The next section was the methods that uses to solve this problem, the methods are trapezoidal rule and Simpson’s rule with more explaining and examples, advantages and disadvantages and in this part, we concluded the most important thing that the Simpson rule is more accurate than trapezoidal, also we go to their algorithms and know how the methods are applied. In the next section, we solve stress problem by the mentioned methods previously. After that, MATLAB code and implementation to these methods come lastly. As expected, it was reasonable with our early assumptions.

6. References

(n.d.). Retrieved from web stanford: https://web.stanford.edu/group/sisl/k12/optimization/MO-unit4-pdfs/4.2simpsonintegrals.pdf

(2019). Retrieved from OpenEye Scientific Software: https://docs.eyesopen.com/toolkits/cookbook/python/plotting/roc.html

A.I.M Voorbij, L. S. (n.d.). The twisting force of aged consumers when opening a jar. In *Applied Ergonomics* (pp. 105-109).

Difference between Trapezoidal and Simpson's Rule. (n.d.). Retrieved from steadyrun: https://www.steadyrun.com/difference-trapezoidal-simpsons-rule

Faires, R. L. (2011). *Numerical Analysis NINTH EDITION.* Boston.

Franc˚u, J. (2012). TORSION OF A NON-CIRCULAR BAR. In *Engineering MECHANICS* (pp. 40-50).

Gerald Wheatly. (2004). *Applied Numerical Analysis.*

john. (n.d.). *Three surprises with the trapezoid rule*. Retrieved from johndcook: https://www.johndcook.com/blog/2010/12/02/three-surprises-with-the-trapezoid-rule/

P.Holmes, D. (n.d.). *mechanics of materials stress.* Retrieved from Back2Bu: https://www.bu.edu/moss/mechanics-of-materials-stress/

Stress. (2021). Retrieved from byjus: https://byjus.com/physics/stress/

Weisstein, E. W. (2021). *SimpsonsRule*. Retrieved from mathworld: https://mathworld.wolfram.com/SimpsonsRule.html

Weisstein, E. W. (2021). *TrapezoidalRule*. Retrieved from mathworld: https://mathworld.wolfram.com/TrapezoidalRule.html

Wicklin, R. (2011). *The trapezoidal rule of integration*. Retrieved from blogs sas: https://blogs.sas.com/content/iml/2011/06/01/the-trapezoidal-rule-of-integration.html

Wikiversity contributors. (2016). *University of Florida/Egm6341/s10.team3.aks/HW4*. Retrieved from Wikiversity: https://en.wikiversity.org/w/index.php?title=University_of_Florida/Egm6341/s10.team3.aks/HW4&oldid=1517769