PHSX815_Project2: Analyzing Coin Flips: Bayesian Inference and Hypothesis Testing

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1 Introduction

Coin flipping is a common practice used in statistics and probability theory. Typically, a fair coin has two possible outcomes: heads or tails, each with an equal probability of 0.5. The outcomes of coin flips can be used to test hypotheses about the fairness of the coin. The fairness of the coin could be affected by prior parameters. Therefore, this paper studies the role of the Bayesian approach to estimate the bias of the coin.

The project stimulates both fair and biased coins and performs the chi-square and log-likelihood ratio tests on them. Also, the probability of a coin flipping comes from another distribution, so it is not a constant amount.

2 Hypotheses to test the fairness of a coin

In order to determine whether there is evidence of a coin is fair or unfair, I set the following hypothesises:

1. The null hypothesis (H_o): The coin is fair with (P_0=0.5) .
2. The alternative hypothesis(H_a): The coin is biased with (P_1=0.6).

3 Code and Experimental Simulation

The code simulates the flipping of a coin a certain number of times and estimates the probability of the coin landing on heads or tails. First, the code implements a Bayesian approach to estimate the bias of a coin, given a set of observations. I use beta distribution as a prior belief distribution and set the beta parameters to have a mean around (0.6) [1]. A binomial distribution function is used to simulate flipping the coin and counting the number of heads using the prior belief distribution [2] [3]. Then, Computing the posterior distribution of the bias parameter. In order to test the null hypothesis, the code stimulates sampling a fair coin using binomial function with a probability of 0.5 .
To test the fairness, a chi-square statistic was used [4] and calculate the p-value. Also, I perform a log-likelihood ratio test for a different number of coin flips and finally calculate the power of the test. [5]. The significance level was set to be 5% .

4 Analysis

In figure. 1., the bar diagram of the outcomes from 100 coin tosses is shown for the biased coin. As we can see the heads is appearing more frequently than tails.

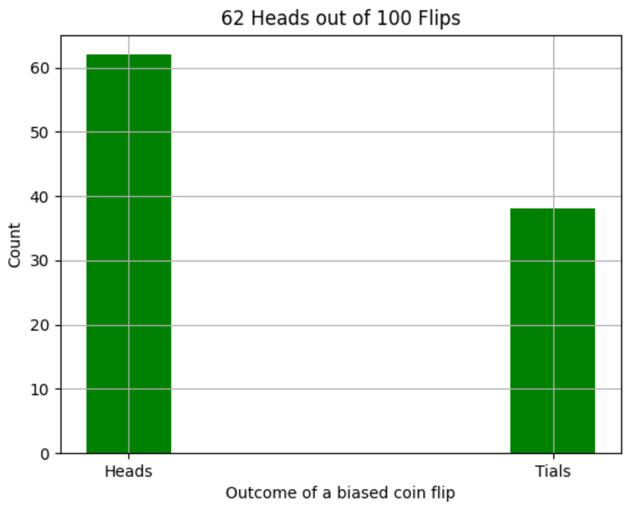

Figure 1: The outcome of 100 coin toss from the biased coin.It is evident that heads have emerged more frequently than tails.

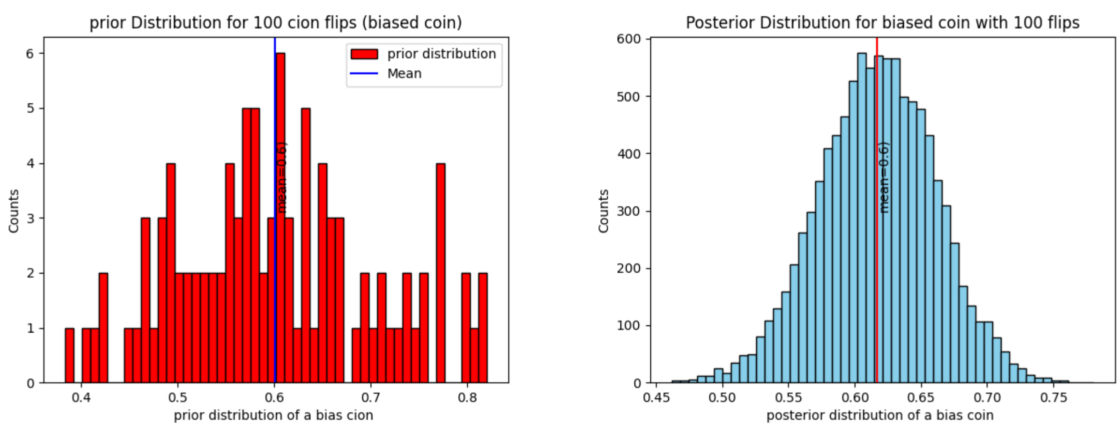

Figure 2: Visualizations of the prior and posterior distributions (beta distribution) of the biased coin with a mean of 0.6

I set beta function parameters as alpha=13.8 and beta=9.2 to get prior and posterior distributions with mean=0.6 for the biased coin as shown in figure. 2. I set beta function parameters as alpha=13.8 and beta=9.2 to get prior and posterior distributions with mean=0.6 for the biased coin as shown in figure. 2. The biased coin sampled using a binomial distribution from the prior with a number of coins flips= 10, 20, 50, 75, 100.
For the fair coin, I expect the outcomes of heads and tails to be equal with a probability of 0.5. so, a fair coin is sampled using a binomial distribution with a number of coins flips similar to the biased coin.

The p-values from the chi-square test for a number of a coin toss 10 to 75 tosses varies from 0.3 to 0.5 which is greater than the significance level (alpha=0.05). So, the coin is fair for the small number of flips. For 100 flips, the coin is biased because the p-value is 0.005 which is far less than the significance level. I also use the log-likelihood ratio test and the data is shown in the table below 1 and in figure.3 . The power of the test is high even for the small sample size of coin flips which means the type-II error is less likely to happen and the coin is unfair.

Table 1: Log liklihood ratio test data.

NO	N flips	1-beta
1	10	0.9
2	20	0.75
3	50	1
4	75	0.746
5	100	1

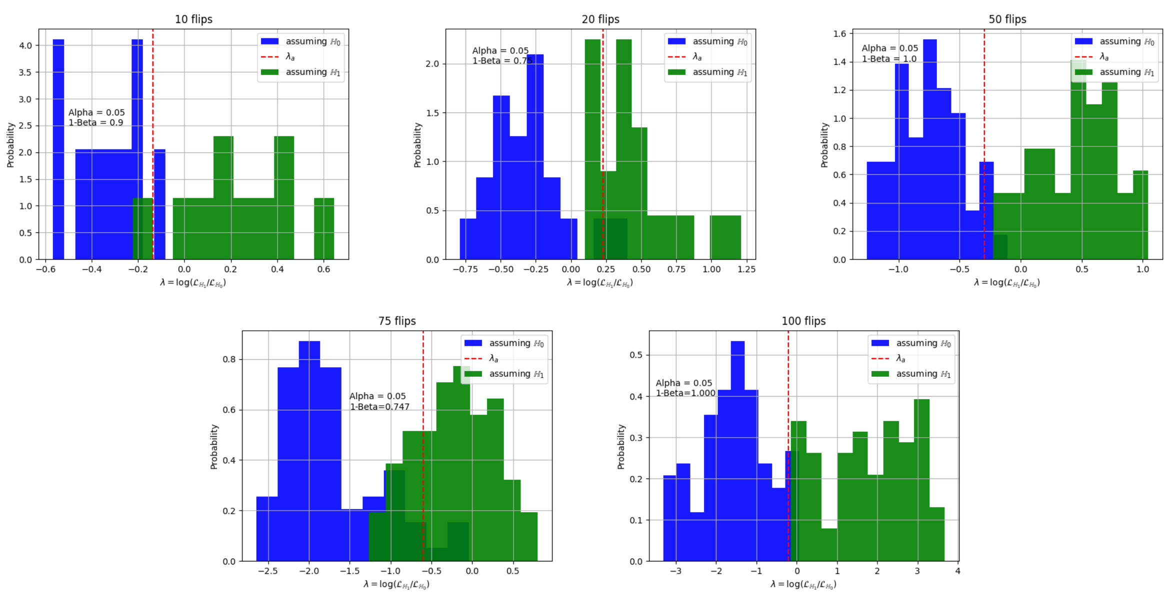

Figure 3: The figure show alpha and beta values for the log likelihood ratio test

5 Conclusion

Based on the results of this analysis, as the number of coin flips increased we tend reject the null hypothesis and accept the alternate hypothesis. using Bayesian approach decrease the number of coin flips needed to reject the null hypothesis.

Here are the references

[1] *Power of Bayesian Statistics Probability | Data Analysis (Updated 2023)* https://www.analyticsvidhya.com/blog/2016/06/bayesian-statistics-beginners-simple-english/. Accessed: 2023-03-15.

[2] *Binomial Distribution* https://www.w3schools.com/python/numpy/numpy_random_binomial.asp. Accessed: 2023-03-10.

[3] *Read a file line by line in Python* https://www.geeksforgeeks.org/read-a-file-line-by-line-in-python/. Accessed: 2023-02-06.

[4] *Chi-Squared Tests* https://rowannicholls.github.io/python/statistics/hypothesis_testing/chi_squared.html. Accessed: 2023-03-016.

[5] *PHSX815-Week2* https://github.com/crogan/PHSX815_Week2/tree/master/python. Accessed: 2023-02-01.