PHSX815_Project3:
Estimating the Probability of getting Heads in a Coin Flip Experiment

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1 Introduction

Parameter estimation is a fundamental concept in statistical inference and one method for estimating the parameter is the maximum likelihood estimation (MLE) approach. The likelihood function, which assesses how well the observed data fit a particular probability distribution with unknown parameters, is a function that the MLE approach aims to optimize. In the context of coin flips, the likelihood function would be the probability of observing a particular sequence of heads and tails given the unknown probability of success. Coin flips follow Bernoulli distribution and the likelihood function for Bernoulli distribution (an individual toss):

$$L_x(p) = p^x(1-p)^{1-x} \tag{1}$$

where p here represents the probability of heads. In general, for n coin flips with h heads:

$$L(p) = p^h(1-p)^{n-h} \tag{2}$$

To find the estimate p parameter, take the log of the likelihood function and find the derivative of the resulting function with respect to p. Then, Setting the derivative equal to 0. This results in [1] :

$$p = \frac{h}{n} \tag{3}$$

Since the coin flip experiment is a common method of understanding probability and statistical inference, the objective of this project is to simulate the coin toss experiment and perform statistical inference to estimate the probability of getting heads.

2 Code and Experimental Simulation

Let's look at some simulated data: First, I define a class named CoinFlips that contains methods to simulate the coin toss experiment by using Numpy' binomial function, estimate the probability of getting heads, and calculate confidence intervals. The class takes an alpha value, which is the probability of getting heads, and a seed value. Also, The methods in the class include:
The 'likelihood' method and the 'estimate alpha' method that returns the estimated alpha value that maximizes the likelihood. Then, the confidence intervals was calculated for different number of coin flips experiment. The alpha set to be 0.65(I assume the coin is unfair)

3 Analysis

The simulation of likelihood functions for different number of flips was visualized in Figure. 1. It is evident that likelihood functions have peaks near the true value of alpha. Also, for larger sample size, the peaks becomes narrower and more regular shape. The figure shows the true and estimated probability of getting heads (alpha) that represented by red and black dashed lines, respectively. The distance between the two lines gets closer as the number of flips increases, which means more accurate alpha with large size samples.

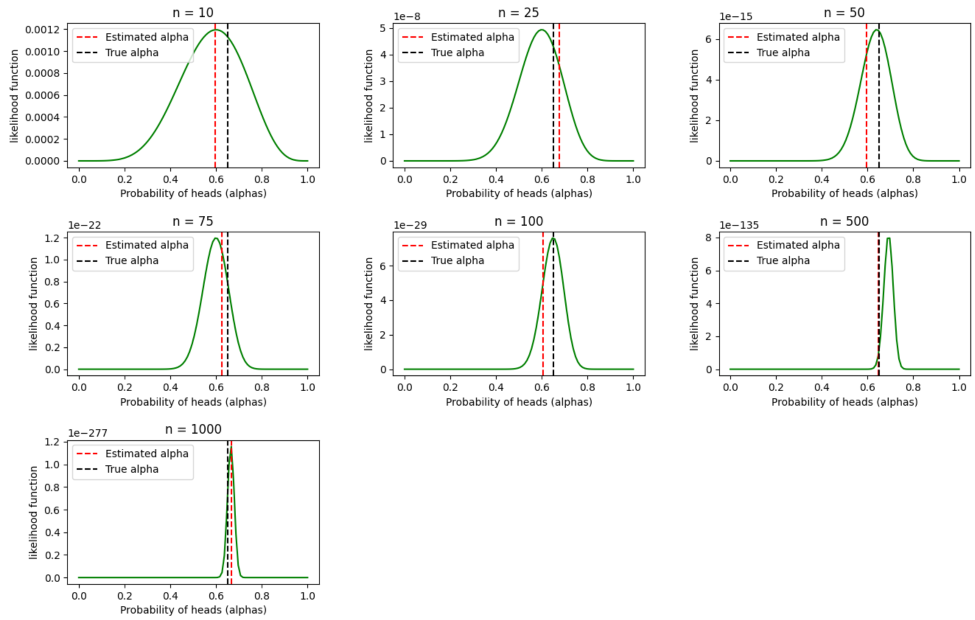

Figure 1: The likelihood functions for a coin flip experiment with different number of trials.

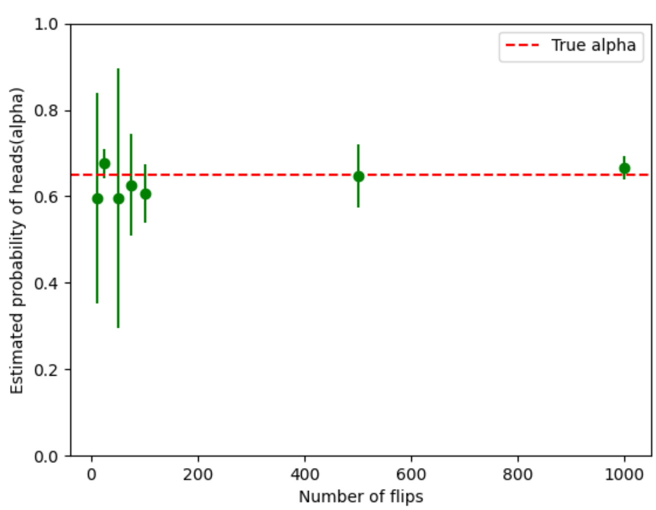

Figure 2: The confidence intervals of the estimated probability of heads for number of different flips

The simulation results show that as the number of flips increases, the estimated alpha value becomes more accurate and the confidence interval becomes narrower. As you can see from Fig.2 .The estimated alpha values represented by error bars are close to the true alpha value of 0.65 (the dashed red line), and the confidence intervals become tighter as the number of flips increases. The number of tosses increased gradually from 10 to 1000 toss.

4 Conclusion

The simulation and statistical inference performed in this project demonstrate the usefulness of probability and statistical inference in understanding the coin flip experiment. The project results show that as the sample size increases (number of flips), the estimates become more accurate, and the confidence intervals become narrower.

Here are the references

[1] W. Fleshman, *Fundamentals of Machine Learning (Part 2)* `https://towardsdatascience.com/maximum-likelihood-estimation-984af2dcfcac`, 2019. Accessed: 04.06.2023.