Computer graphics - Cylinder and its Normal

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1 Infinite length cylinder

Let us take a cylinder with axis vector $c + kv_a$ and radius r. A point $q = (x, y, z)$ is on the cylinder iif the its distance from the axis vector $c + kv_a$ (length of the red line l on the Figure 1) is equal to r. We assume that v_a is a unit vector.

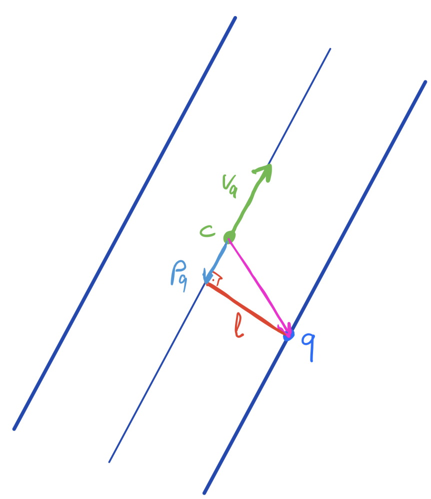

Figure 1: Cylinder

First, we need to compute p_q, the orthogonal projection times v_a of the vector $\overrightarrow{cq}$ on v_a :

$$p_q = \langle v_a, q - c \rangle v_a$$

Then, we can compute the vector l :

$$l = q - c - p_q$$
$$l = q - c - \langle v_a, q - c \rangle v_a$$

Finally, we compute the norm of l to get the distance we wanted. We can take this distance squared to simplify computation and we get the following implicit equation for the cylinder:

$$|q - c - \langle v_a, q - c\rangle v_a|^2 - r^2 = 0$$

Now, we inject the ray parametrization of point q where v is a unit vector (ray direction) :

$$\begin{gathered} q = p + tv \\ |p - c + tv - \langle v_a, p - c + tv\rangle v_a|^2 - r^2 = 0 \end{gathered}$$

And solve this equation for t :

$$\begin{gathered} |p - c + tv - \langle v_a, p - c + tv\rangle v_a|^2 - r^2 = 0 \\ |p - c + tv - \langle v_a, p - c\rangle v_a - t\langle v_a, v\rangle v_a|^2 - r^2 = 0 \\ |t(v + \langle v_a, v\rangle v_a) + (p - c - \langle v_a, p - c\rangle v_a)|^2 - r^2 = 0 \end{gathered}$$

Let us define the following vectors :

$$\begin{gathered} e = (v - \langle v_a, v\rangle v_a) \\ f = (p - c - \langle v_a, p - c\rangle v_a) \end{gathered}$$

Then :

$$\begin{gathered} |te + f|^2 - r^2 = 0 \\ \langle te + f, te + f\rangle - r^2 = 0 \\ t^2\langle e, e\rangle + t\langle e + f, e + f\rangle + \langle f, f\rangle - r^2 = 0 \end{gathered}$$

We end up with an equation of the form:

$$t^2 A + tB + C = 0$$

Where :

$$\begin{aligned} A &= \langle e, e\rangle \\ &= |v - \langle v_a, v\rangle v_a|^2 \\ B &= 2\langle e, f\rangle \\ &= 2\langle v - \langle v_a, v\rangle v_a, p - c - \langle v_a, p - c\rangle v_a\rangle \\ C &= \langle f, f\rangle - r^2 \\ &= |p - c - \langle v_a, p - c\rangle v_a|^2 \end{aligned}$$

Now, we can resolve the previous equation for t using *quadraticSolve* and store the solution(s) in S.

2 Finite length cylinder

The array S may contain zero, one or two solutions. Valid solutions must be positive and if there are two valid solutions for s, we should keep the smalllest one, which is inside the defined cylinder (radius r and height h).

First, we remove negative values from S. Then, we keep the t_i's such that $p_i = p + t_i v$ are on the finite length cylinder.

To do so, we compute the orthogonal projection of the vector between c and the intersection point p_i on v_a and check if its norm is smaller than $h/2$:

$$|\langle p_i - c, v_a \rangle v_a|^2 \leq h/2$$

And keep the smallest t_i satisfying this inequation.

3 Normal to cylinder surface

The normal to the cylinder surface at point p_i is computed as the vector from the orthogonal projection $proj_{p_i}$ of p_i on v_a to p_i.

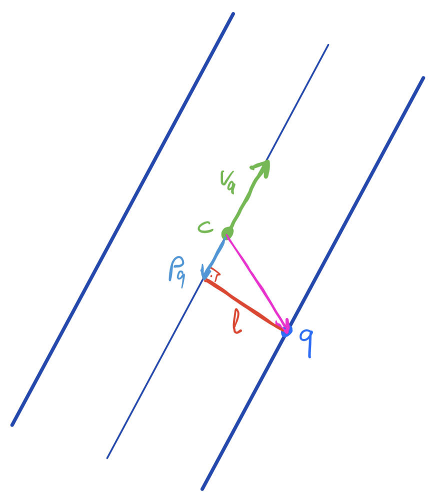

Figure 2: Normal

$$proj_{p_i} = c + \langle p_i - c, v_a \rangle$$
$$n = p_i - proj_{p_i}$$

As there are two possible directions for the vector v, we choose the one *pointing againt the ray direction*. In other words, the dot product of v and n must be negative. Otherwise, we have to take the inverse n by multiplying its components by -1.