

Computer graphics - Cylinder and its Normal

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1 Infinite length cylinder

Let us take a cylinder with axis vector $c + kv_a$ and radius r . A point $q = (x, y, z)$ is on the cylinder iff its distance from the axis vector $c + kv_a$ (length of the red line l on Figure 1) is equal to r . We assume that v_a is a unit vector.

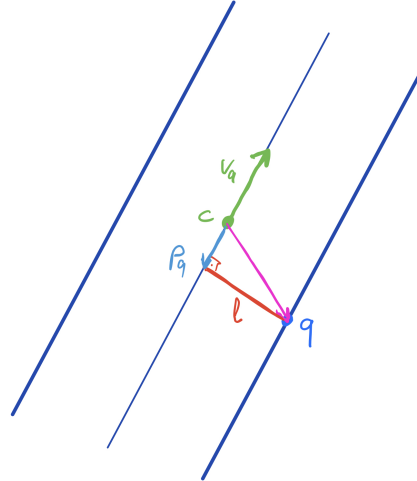


Figure 1: Cylinder

First, we need to compute p_q , v_a times the orthogonal projection of the vector \vec{cq} on v_a :

$$p_q = \langle v_a, q - c \rangle v_a$$

Then, we can compute the vector l :

$$\begin{aligned} l &= q - c - p_q \\ l &= q - c - \langle v_a, q - c \rangle v_a \end{aligned}$$

Finally, we compute the norm of l to get the distance we wanted. We can take this distance squared to simplify computation and we get the following implicit

equation for the cylinder :

$$|q - c - \langle v_a, q - c \rangle v_a|^2 - r^2 = 0$$

Now, we inject the ray parametrization of point q where v is a unit vector :

$$q = p + tv$$

$$|p - c + tv - \langle v_a, p - c + tv \rangle v_a|^2 - r^2 = 0$$

And solve this equation for t :

$$\begin{aligned} |p - c + tv - \langle v_a, p - c + tv \rangle v_a|^2 - r^2 &= 0 \\ |p - c + tv - \langle v_a, p - c \rangle v_a - t \langle v_a, v \rangle v_a|^2 - r^2 &= 0 \\ |t(v - \langle v_a, v \rangle v_a) + (p - c - \langle v_a, p - c \rangle v_a)|^2 - r^2 &= 0 \end{aligned}$$

Let us define the following vectors :

$$\begin{aligned} e &= (v - \langle v_a, v \rangle v_a) \\ f &= (p - c - \langle v_a, p - c \rangle v_a) \end{aligned}$$

Then :

$$\begin{aligned} |te + f|^2 - r^2 &= 0 \\ \langle te + f, te + f \rangle - r^2 &= 0 \\ t^2 \langle e, e \rangle + 2t \langle e, f \rangle + \langle f, f \rangle - r^2 &= 0 \end{aligned}$$

We end up with an equation of the form:

$$t^2 A + tB + C = 0$$

Where :

$$\begin{aligned} A &= \langle e, e \rangle \\ &= |v - \langle v_a, v \rangle v_a|^2 \\ B &= 2 \langle e, f \rangle \\ &= 2 \langle v - \langle v_a, v \rangle v_a, p - c - \langle v_a, p - c \rangle v_a \rangle \\ C &= \langle f, f \rangle - r^2 \\ &= |p - c - \langle v_a, p - c \rangle v_a|^2 - r^2 \end{aligned}$$

Now, we can resolve the previous equation for t using *quadraticSolve* and store the solution(s) in S .

2 Finite length cylinder

The array S may contain zero, one or two solutions. Valid solutions must be positive and if there are two valid solutions for s , we should select the smallest one, which is inside the defined cylinder (radius r and height h).

First, we remove negative values from S . Then, we keep the t_i 's such that $q = p + t_i v$ are on the finite length cylinder.

To do so, we compute the orthogonal projection $\overrightarrow{cp_q}$ of the vector \overrightarrow{cq} on v_a and check if its norm is smaller or equal to $h/2$:

$$|\langle q - c, v_a \rangle v_a|^2 \leq h/2$$

And keep the smallest t_i satisfying this inequation.

3 Normal to cylinder surface

The normal to the cylinder surface at point p_i is computed as the vector from the orthogonal projection $proj_{p_i}$ of p_i on v_a to p_i ($\overrightarrow{proj_{p_i} p_i}$).

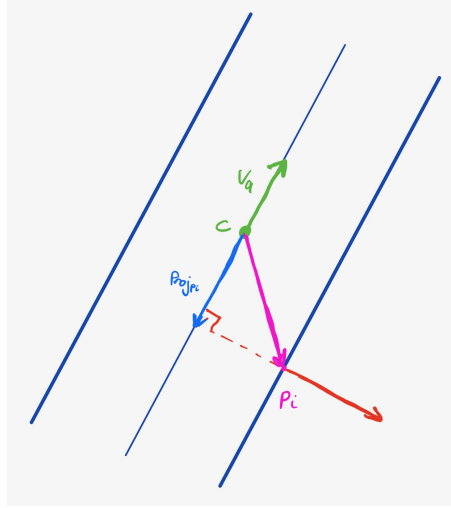


Figure 2: Normal

$$\begin{aligned} proj_{p_i} &= c + \langle p_i - c, v_a \rangle v_a \\ n &= p_i - proj_{p_i} \end{aligned}$$

As there are two possible directions for the vector v , we choose the one *pointing towards the viewer*. In other words, the dot product of v and n must be negative. Otherwise, we have to take the inverse of n by multiplying its components by -1 . It depends on whether the ray hits the cylinder on the inside or on the outside.