Computer graphics - cylindres

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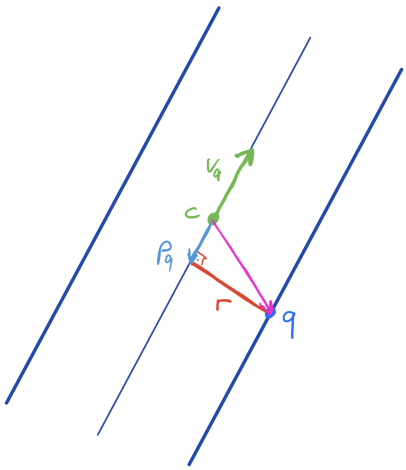
1 Infinite length cylinder

Let us take a cylinder with $c + v_a * k$ as axis and r as radius. For a point of the space $q = (x, y, z)$, it is on the cylinder iif the distance between the axis and the point (red r on the sketch) is equal to r.

Here is a sketch of the situation:



We can compute r as following: First compute the orthogonal projection of the vector cq on v_a:

$$p_q =< v_a, q - c > v_a$$

Now we can compute the vector r :

$$r = q - c - p_q$$
$$r = q - c - < v_a, q - c > v_a$$

Now we just need to compute the norm of r and we have the distance we wanted. We can take this distance squared to simplify computation and we

have the implicit equation for the cylinder:

$$|q - c - < v_a, q - c > v_a|^2 - r^2 = 0$$

for a point q.

Now we inject the ray parametrization as point q :

$$q = p + v * t$$
$$|p - c + v * t - < v_a, p - c + v * t > v_a|^2 - r^2 = 0$$

We will solve this equation for t:

$$|p - c + v * t - < v_a, p - c + v * t > v_a|^2 - r^2 = 0$$
$$|p - c + v * t - < v_a, p - c > v_a - t * < v_a, v > v_a|^2 - r^2 = 0$$
$$|t * (v + < v_a, v > v_a) + (p - c - < v_a, p - c > v_a)|^2 - r^2 = 0$$

Let us define some variables :

$$A = (v - < v_a, v > v_a)$$
$$B = (p - c - < v_a, p - c > v_a)$$

Both are vectors.

$$|t * A + B|^2 - r^2 = 0$$
$$< t * A + B, t * A + B > -r^2 = 0$$
$$t^2 * < A, A > + t * < A + B, A + B > + < B, B > -r^2 = 0$$

We have an equation of the form:

$$t^2 * C + t * E + F = 0$$

with:

$$\begin{aligned} C &= < A, A > \\ &= |v - < v_a, v > v_a|^2 \\ E &= 2 * < A, B > \\ &= 2 * < v - < v_a, v > v_a, p - c - < v_a, p - c > v_a > \\ F &= < B, B > -r^2 \\ &= |p - c - < v_a, p - c > v_a|^2 \end{aligned}$$

Now that we have these three elements, we can resolve the equation for t using *quadraticSolve* and keep only the smallest positive solution for t(to have the first intersection in front of the observer).

2 Finite length cylinder

Now that we have t for which the ray intersect the infinitely long cylinder, we can keep only intersections that this cylinder at a distance less than $h/2$ from the center.

To do so, we compute the orthogonal projection of the vector between c and the intersection point p_i and check if its norm is smaller that $h/2$.

$$| < p_i - c, v_a > v_a|^2 \leq h/2$$

$$p_i = p + v * t_i$$

where t_i is the solution of the last equation. We keep only intersections that satisfy this equation.