

# Computer graphics - cylindres

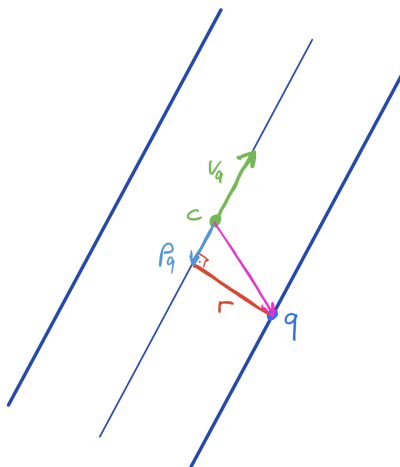
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February 2019

## 1 Infinite length cylinder

Let us take a cylinder with  $c + v_a * k$  as axis and  $r$  as radius. For a point of the space  $q = (x, y, z)$ , it is on the cylinder iff the distance between the axis and the point (red  $r$  on the sketch) is equal to  $r$ .

Here is a sketch of the situation:



We can compute  $r$  as following: First compute the orthogonal projection of the vector  $cq$  on  $v_a$ :

$$p_q = \langle v_a, q - c \rangle v_a$$

Now we can compute the vector  $r$  :

$$\begin{aligned} r &= q - c - p_q \\ r &= q - c - \langle v_a, q - c \rangle v_a \end{aligned}$$

Now we just need to compute the norm of  $r$  and we have the distance we wanted. We can take this distance squared to simplify computation and we

have the implicit equation for the cylinder:

$$|q - c - \langle v_a, q - c \rangle v_a|^2 - r^2 = 0$$

for a point  $q$ .

Now we inject the ray parametrization as point  $q$  :

$$\begin{aligned} q &= p + v * t \\ |p - c + v * t - \langle v_a, p - c + v * t \rangle v_a|^2 - r^2 &= 0 \end{aligned}$$

We will solve this equation for  $t$ :

$$\begin{aligned} |p - c + v * t - \langle v_a, p - c + v * t \rangle v_a|^2 - r^2 &= 0 \\ |p - c + v * t - \langle v_a, p - c \rangle v_a - t * \langle v_a, v \rangle v_a|^2 - r^2 &= 0 \\ |t * (v + \langle v_a, v \rangle v_a) + (p - c - \langle v_a, p - c \rangle v_a)|^2 - r^2 &= 0 \end{aligned}$$

Let us define some variables :

$$\begin{aligned} A &= (v - \langle v_a, v \rangle v_a) \\ B &= (p - c - \langle v_a, p - c \rangle v_a) \end{aligned}$$

Both are vectors.

$$\begin{aligned} |t * A + B|^2 - r^2 &= 0 \\ \langle t * A + B, t * A + B \rangle - r^2 &= 0 \\ t^2 * \langle A, A \rangle + t * \langle A + B, A + B \rangle + \langle B, B \rangle - r^2 &= 0 \end{aligned}$$

We have an equation of the form:

$$t^2 * C + t * E + F = 0$$

with:

$$\begin{aligned} C &= \langle A, A \rangle \\ &= |v - \langle v_a, v \rangle v_a|^2 \\ E &= 2 * \langle A, B \rangle \\ &= 2 * \langle v - \langle v_a, v \rangle v_a, p - c - \langle v_a, p - c \rangle v_a \rangle \\ F &= \langle B, B \rangle - r^2 \\ &= |p - c - \langle v_a, p - c \rangle v_a|^2 \end{aligned}$$

Now that we have these three elements, we can resolve the equation for  $t$  using *quadraticSolve* and keep only the smallest positive solution for  $t$  (to have the first intersection in front of the observer).

## 2 Finite length cylinder

Now that we have  $t$  for which the ray intersect the infinitely long cylinder, we can keep only intersections that this cylinder at a distance less than  $h/2$  from the center.

To do so, we compute the orthogonal projection of the vector between  $c$  and the intersection point  $p_i$  on  $v_a$  and check if its norm is smaller than  $h/2$ .

$$|\langle p_i - c, v_a \rangle v_a|^2 \leq h^2/4$$
$$p_i = p + v * t_i$$

where  $t_i$  is the solution of the last equation. We keep only intersections that satisfy this equation.

## 3 Normal to cylinder surface

The normal to the cylinder surface at point  $p_i$  is computed as the vector from the orthogonal projection  $proj_{p_i}$  of  $p_i$  on  $v_a$  to  $p_i$ .

$$proj_{p_i} = c + \langle p_i - c, v_a \rangle v_a$$
$$n = p_i - proj_{p_i}$$

As there are two possible directions for the vector  $v$ , we choose the one *pointing against the ray direction*. In other words, the dot product of  $v$  and  $n$  must be negative. Otherwise, we have to take the inverse  $n$  by multiplying its components by  $-1$ .