

# Computer graphics - Cylinder and its Normal

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## 1 Infinite length cylinder

Let us take a cylinder with axis vector  $c + kv_a$  and radius  $r$ . A point  $q = (x, y, z)$  is on the cylinder iff its distance from the axis vector  $c + kv_a$  (length of the red line  $l$  on Figure 1) is equal to  $r$ . We assume that  $v_a$  is a unit vector.

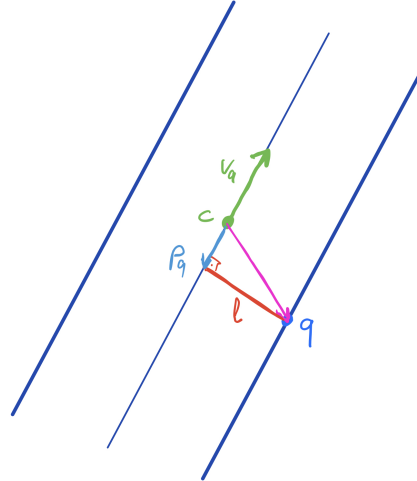


Figure 1: Cylinder

First, we need to compute  $p_q$ ,  $v_a$  times the orthogonal projection of the vector  $\vec{cq}$  on  $v_a$  :

$$p_q = \langle v_a, q - c \rangle v_a$$

Then, we can compute the vector  $l$  :

$$\begin{aligned} l &= q - c - p_q \\ l &= q - c - \langle v_a, q - c \rangle v_a \end{aligned}$$

Finally, we compute the norm of  $l$  to get the distance we wanted. We can take this distance squared to simplify computation and we get the following implicit

equation for the cylinder :

$$|q - c - \langle v_a, q - c \rangle v_a|^2 - r^2 = 0$$

Now, we inject the ray parametrization of point  $q$  where  $v$  is a unit vector :

$$q = p + tv$$

$$|p - c + tv - \langle v_a, p - c + tv \rangle v_a|^2 - r^2 = 0$$

And solve this equation for  $t$  :

$$\begin{aligned} &|p - c + tv - \langle v_a, p - c + tv \rangle v_a|^2 - r^2 = 0 \\ &|p - c + tv - \langle v_a, p - c \rangle v_a - t \langle v_a, v \rangle v_a|^2 - r^2 = 0 \\ &|t(v - \langle v_a, v \rangle v_a) + (p - c - \langle v_a, p - c \rangle v_a)|^2 - r^2 = 0 \end{aligned}$$

Let us define the following vectors :

$$\begin{aligned} e &= (v - \langle v_a, v \rangle v_a) \\ f &= (p - c - \langle v_a, p - c \rangle v_a) \end{aligned}$$

Then :

$$\begin{aligned} &|te + f|^2 - r^2 = 0 \\ &\langle te + f, te + f \rangle - r^2 = 0 \\ &t^2 \langle e, e \rangle + 2t \langle e, f \rangle + \langle f, f \rangle - r^2 = 0 \end{aligned}$$

We end up with an equation of the form:

$$t^2 A + tB + C = 0$$

Where :

$$\begin{aligned} A &= \langle e, e \rangle \\ &= |v - \langle v_a, v \rangle v_a|^2 \\ B &= 2 \langle e, f \rangle \\ &= 2 \langle v - \langle v_a, v \rangle v_a, p - c - \langle v_a, p - c \rangle v_a \rangle \\ C &= \langle f, f \rangle - r^2 \\ &= |p - c - \langle v_a, p - c \rangle v_a|^2 - r^2 \end{aligned}$$

Now, we can resolve the previous equation for  $t$  using *quadraticSolve* and store the solution(s) in  $S$ .

## 2 Finite length cylinder

The array  $S$  may contain zero, one or two solutions. Valid solutions must be positive and if there are two valid solutions for  $s$ , we should select the smallest one, which is inside the defined cylinder (radius  $r$  and height  $h$ ).

First, we remove negative values from  $S$ . Then, we keep the  $t_i$ 's such that  $q = p + t_i v$  are on the finite length cylinder.

To do so, we compute the orthogonal projection  $\overrightarrow{cp_q}$  of the vector  $\overrightarrow{cq}$  on  $v_a$  and check if its norm is smaller or equal to  $h/2$  :

$$|\langle q - c, v_a \rangle v_a|^2 \leq h/2$$

And keep the smallest  $t_i$  satisfying this inequation. If there is no, keep no solution which means the ray does not intersect the cylinder.

### 3 Normal to cylinder surface

The normal to the cylinder surface at point  $p_i$  is computed as the vector from the orthogonal projection  $proj_{p_i}$  of  $p_i$  on  $v_a$  to  $p_i$  ( $\overrightarrow{proj_{p_i} p_i}$ ).

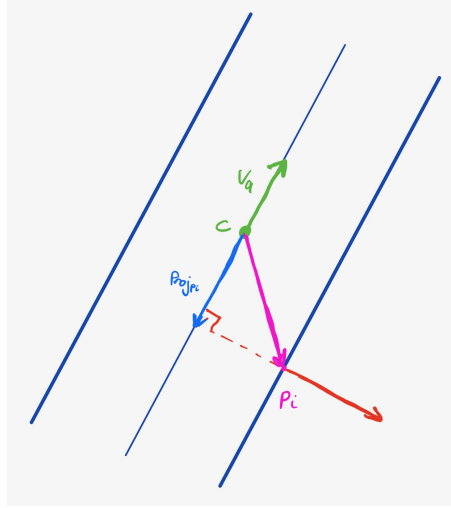


Figure 2: Normal

$$\begin{aligned} proj_{p_i} &= c + \langle p_i - c, v_a \rangle v_a \\ n &= p_i - proj_{p_i} \end{aligned}$$

As there are two possible directions for the vector  $v$ , we choose the one *pointing towards the viewer*. In other words, the dot product of  $v$  and  $n$  must be negative. Otherwise, we have to take the inverse of  $n$  by multiplying its components by  $-1$ . It depends on whether the ray hits the cylinder on the inside or on the outside.