Computer graphics - cylindres

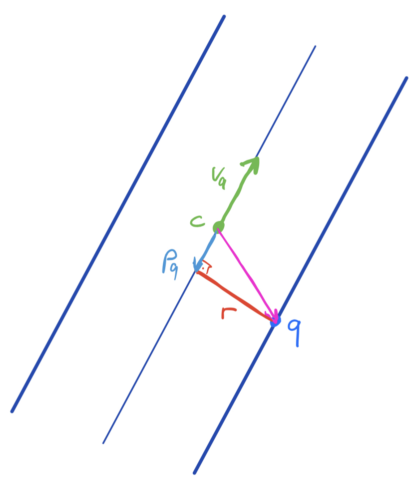
samuel.chassot

February 2019

1 Infinite length cylinder

Let us take a cylinder with $c + v_a * k$ as axis and r as radius. For a point of the space $q = (x, y, z)$, it is on the cylinder iif the distance between the axis and the point (red r on the sketch) is equal to r.

Here is a sketch of the situation:



We can compute r as following: First compute the orthogonal projection of the vector cq on v_a:

$$p_q = \langle v_a, q - c \rangle v_a$$

Now we can compute the vector r :

$$r = q - c - p_q$$
$$r = q - c - \langle v_a, q - c \rangle v_a$$

Now we just need to compute the norm of r and we have the distance we wanted. We can take this distance squared to simplify computation and we have the implicit equation for the cylinder:

$$|q - c - \langle v_a, q - c \rangle v_a|^2 - r^2 = 0$$

for a point q.

Now we inject the ray parametrization as point q :

$$q = p + v * t$$
$$|p - c + v * t - \langle v_a, p - c + v * t \rangle v_a|^2 - r^2 = 0$$

We will solve this equation for t:

$$|p - c + v * t - \langle v_a, p - c + v * t \rangle v_a|^2 - r^2 = 0$$
$$|p - c + v * t - \langle v_a, p - c \rangle v_a - t * \langle v_a, v \rangle v_a|^2 - r^2 = 0$$
$$|t * (v + \langle v_a, v \rangle v_a) + (p - c - \langle v_a, p - c \rangle v_a)|^2 - r^2 = 0$$

Let us define some variables :

$$A = (v - \langle v_a, v \rangle v_a)$$
$$B = (p - c - \langle v_a, p - c \rangle v_a)$$

Both are vectors.

$$|t * A + B|^2 - r^2 = 0$$
$$\langle t * A + B, t * A + B \rangle - r^2 = 0$$
$$t^2 * \langle A, A \rangle + t * \langle A + B, A + B \rangle + \langle B, B \rangle - r^2 = 0$$

We have an equation of the form:

$$t^2 * C + t * E + F = 0$$

with:

$$\begin{aligned} C &= \langle A, A \rangle \\ &= |v - \langle v_a, v \rangle v_a|^2 \\ E &= 2 * \langle A, B \rangle \\ &= 2 * \langle v - \langle v_a, v \rangle v_a, p - c - \langle v_a, p - c \rangle v_a \rangle \\ F &= \langle B, B \rangle - r^2 \\ &= |p - c - \langle v_a, p - c \rangle v_a|^2 \end{aligned}$$

Now that we have these three elements, we can resolve the equation for t using *quadraticSolve* and keep only the smallest positive solution for t(to have the first intersection in front of the observer).

2 Finite length cylinder

Now that we have t for which the ray intersect the infinitely long cylinder, we can keep only intersections that this cylinder at a distance less than $h/2$ from the center.

To do so, we compute the orthogonal projection of the vector between c and the intersection point p_i on v_a and check if its norm is smaller than $h/2$.

$$|\langle p_i - c, v_a \rangle v_a|^2 \leq h/2$$
$$p_i = p + v * t_i$$

where t_i is the solution of the last equation. We keep only intersections that satisfy this equation.

3 Normal to cylinder surface

The normal to the cylinder surface at point p_i is computed as the vector from the orthogonal projection $proj_{p_i}$ of p_i on v_a to p_i.

$$proj_{p_i} = c + \langle p_i - c, v_a \rangle$$
$$n = p_i - proj_{p_i}$$

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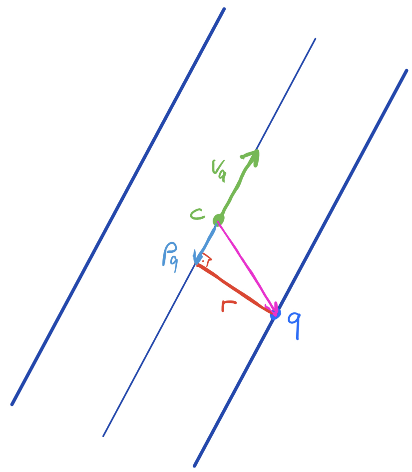

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As there are two possible directions for the vector v, we choose the one *pointing againt the ray direction.* In other words, if the dot product of v and n must be negative. Otherwise, we have to take the inverse n by multiplying its components by -1.