Autonomous Mobile Robots

- modeling
- Control
- Planning

→ Project
  - Choose One mobile platform.
  - Extraction of EOM
  - Design a controller
  - Planner Integration.

→ Go through Syllabus for all the Course plan.

Books:

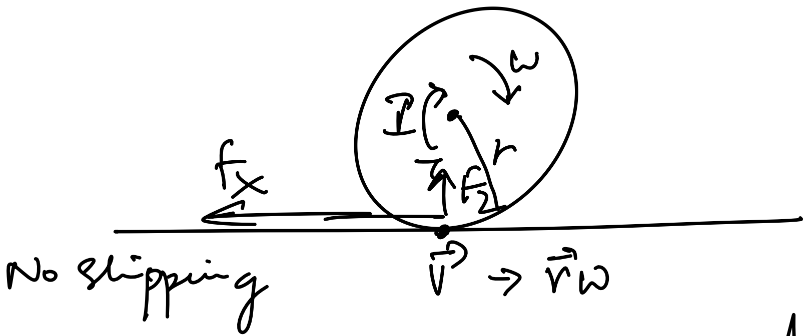
Introduction to AMR
— Illah Nourbakhsh

Control & Dynamics of Robot
— Spong

I - moment of Inertia

ω - angular velocity

r - radius



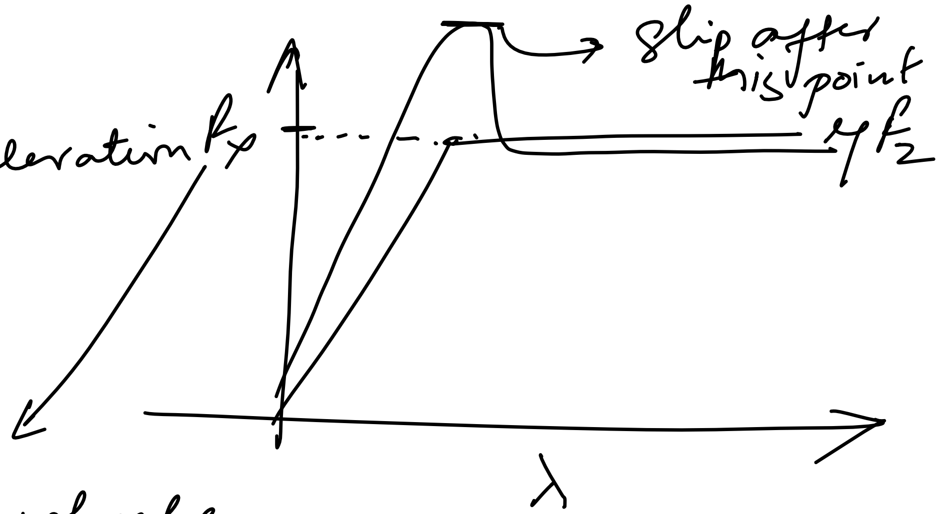
$$\lambda = \frac{r\omega - v}{\text{Max}(r\omega, v)}$$

λ = slipping factor

Limitation;

$I\vec{\alpha} = \vec{\tau}$

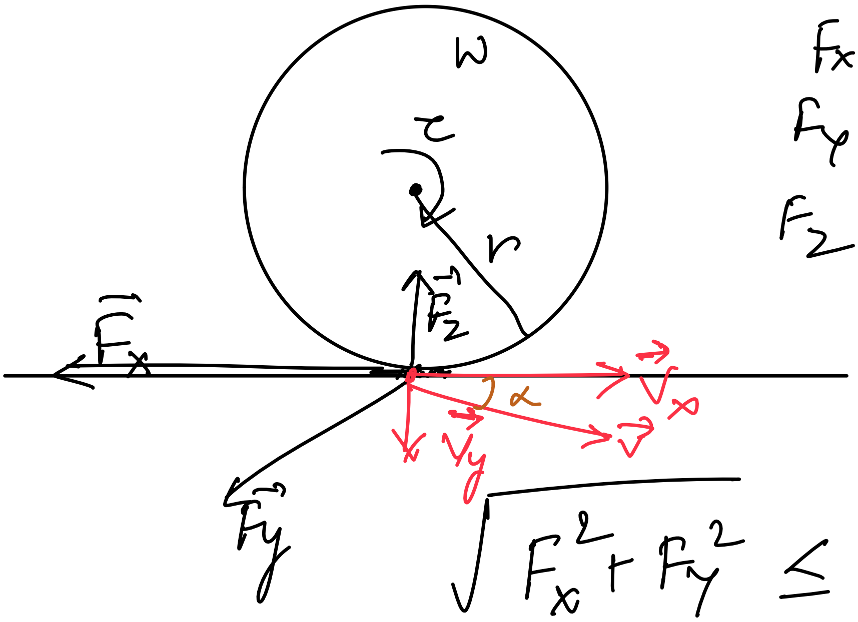
α = angular acceleration



$$F_x = \mu F_z$$

piecewise modeling.

→ max force on the wheels



F_x - Longitude force

F_y - Lateral force

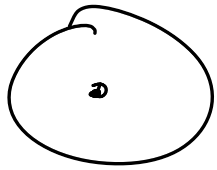
F_z - Normal force

$$\sqrt{F_x^2 + F_y^2} \leq \mu |F_z|$$

α = Side Slip Angle $\quad \alpha = \text{Atan}\left(\frac{V_y}{V_x}\right)$

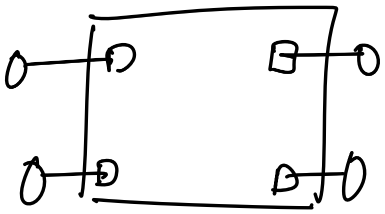
Wheels

→ Active 

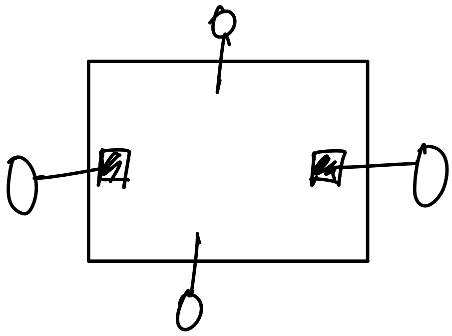
→ Passive 

Mecanum wheels

Differential wheel Drive ← Under Actuated

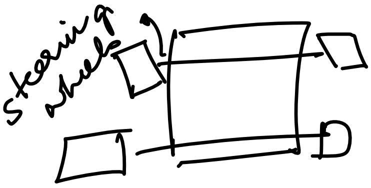


Traction



OMRON
MIR
LOCUS

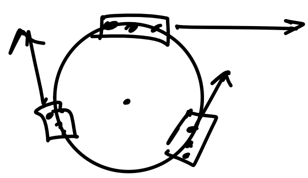
Ackerman (AV)



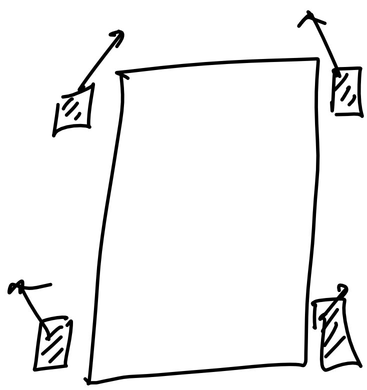
Active steering front & Back

Omni wheels Robots

- very efficient

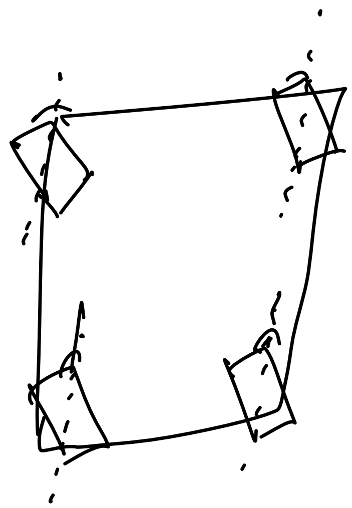


Mecanum wheels



Over Actuated

3DoF < 4 Actuators



4 Drive Motors
4 Steering

3DoF < 8 actuators
highly over actuated.

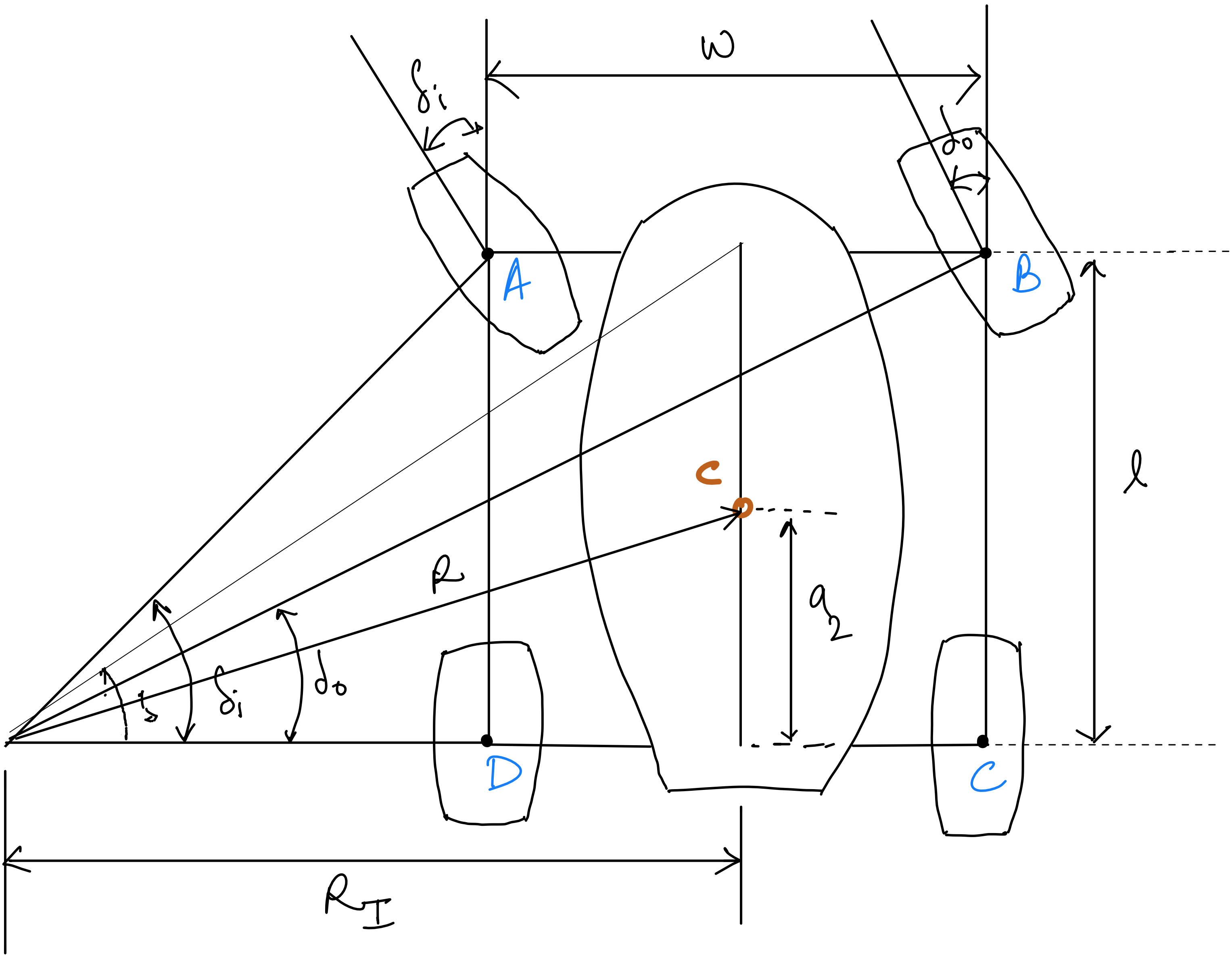
Assignment / Homework - 1

- Reading to Intro to AMR Chapter 2
- Choose AMR/AV platform. (1 – ∞ wheels)
- Justify
- Find 3 good reference papers.
  - Control
  - modeling
  - Planning

Homework / Assignment - I

Ackerman steering configuration common in automobiles. Such a vehicle typically has a turning diameter that is larger thanthe car. Furthermore, for such a vehicle to move sideways requires a parking maneuver con-sisting of repeated changes in direction forward and backward.

Nevertheless, Ackerman steering geometries have been especially popular in the hobby robotics market, where a robot can be built by starting with a remote control racecar kit and adding sensing and autonomy to the existing mechanism. In addition, the limited maneuverability of Ackerman steering has an important advantage: its directionality and steering geometry provide it with very good lateral stability in high-speed turns.



1. Slow movement of the car (vehicle) results in a kinetic condition between the inner & outer wheels that allows a slip-free turn. This condition is termed as Ackerman's Condition.

$$\boxed{\cot\delta_o - \cot\delta_i = \frac{w}{l}}$$

$\delta_o \rightarrow$ Steering angle of Outer wheels.

$\delta_i \rightarrow$ Steering angle of inner wheels.

$l \rightarrow$ wheel base of the Car.

$w \rightarrow$ width of the Car.

Q. The inner and outer wheels are determined based on the turning Centre O. The Centre of mass of the steered vehicle will turn on a circle with Radius R, which is determined as

$$\boxed{R = \sqrt{a_2^2 + l^2 \cot^2\delta}}$$

δ = Cot. average of the inner & outer steer angle.

a_2 = distance b/w Centre of the vehicle to the Centre of rear wheel axel.

$$\boxed{\cot\delta = \frac{\cot\delta_o + \cot\delta_i}{2}}$$

$$\boxed{\delta = \arctan\left(\frac{L}{R}\right)}$$

Advantages:

- minimal tire scrab & wear.
- lower steering effort
- higher efficiency.
- Used in a lot of Automobile Industry.
- Smoother, more stable in low-speed handling.
- Simple, robust, low-cost mechanism
- Better odometry & Control fidelity
  → Reduced slip leads to more accurate dead reckoning and more repeatable path tracking.

Homework 2

1) Constraints
2) Kinematic & dynamic Eqn:
3) ODD: Operational Design Domain

(4) Finding relation b/w wheel velocities to Robot velocity
Get all 4 wheels velocity into local frame & also global frame

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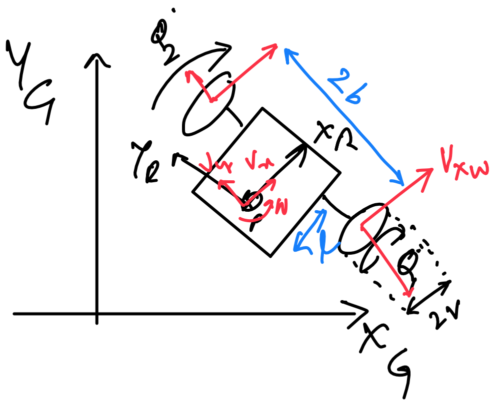


W = angular Velocity

$\dot{X} = J\dot{Q}$

$$\underbrace{q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}}_{\text{Global frame}}$$

$R(\theta) =$

$$V_x \cos\theta + V_y \sin\theta = \dot{x}$$
$$V_y \cos\theta - V_x \sin\theta = \dot{y}$$
$$\dot{\theta} = w$$

No slip effect $\longrightarrow$ $r\dot{\varphi}_1 = V_{xw_1}$

$$V_x - bw = r\dot{Q}_{w_2}$$
$$V_{xw_1} = \underbrace{V_x + bw}_{A(q)\dot{q}} = r\dot{\varphi}_{w_1}$$

w_1 - wheel 1
w_2 - wheel 2

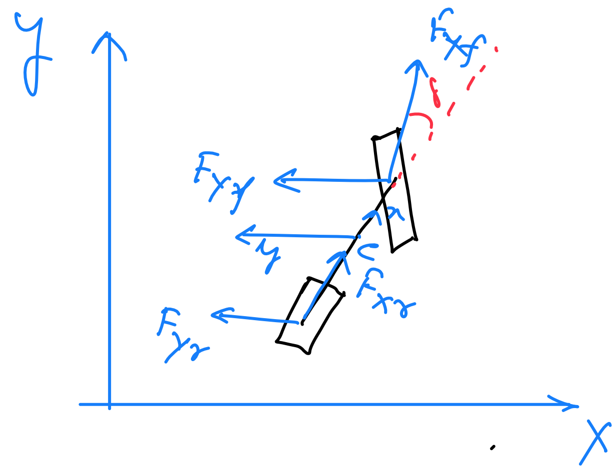
$$\begin{bmatrix} V_x \\ V_y \\ W \end{bmatrix} = \begin{bmatrix} r\dot{Q}_1 + r\dot{Q}_2 \\ 0 \\ r\dot{Q}_1 - r\dot{Q}_2 \end{bmatrix} \quad \text{— (1)}$$

No sliding effect $V_{yw_{12}} = 0$

$$V\,R(\theta) = \dot{x} \quad \text{— (2)}$$

(1) & (2)
$$\left\{ \dot{x} = \begin{bmatrix} r\dot{Q}_1 + r\dot{Q}_2 \\ 0 \\ r\dot{\varphi}_1 - r\dot{\varphi}_2 \end{bmatrix} R(\theta) \right\}$$

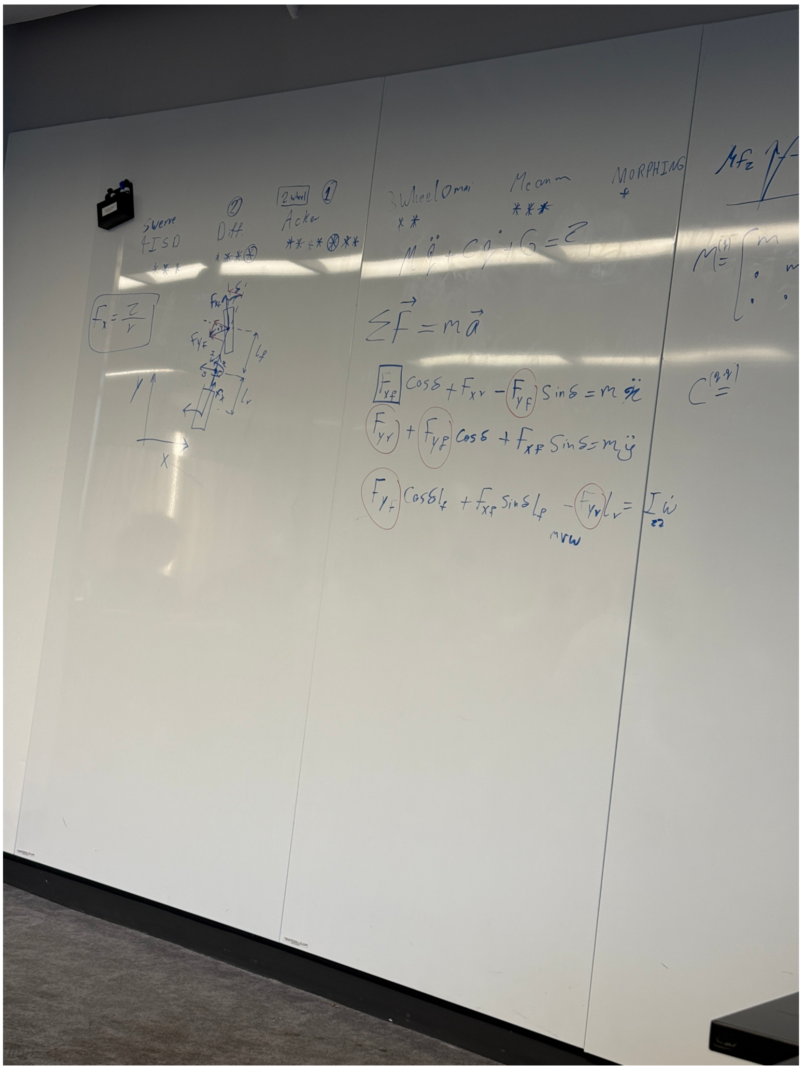
δ - turning angle



$$\sum \vec{F} = m\vec{a}$$

$$F_{xf} \cos\delta + F_{xr} - F_{yf} \sin\delta = m\ddot{x}$$

$$F_{yr} + F_{yf} \cos\delta + F_{xf} \sin\delta = m\ddot{y}$$



Homework - 2

4- wheel Ackerman Steering Model

1. <u>Constraints</u> :-

$l \rightarrow$ wheelbase of the Car.

$w \rightarrow$ width of the Car.

$L = l_r + l_f$; $(w_f = w_r = w)$

Instantaneous Centre of Rotation (ICR):

$$\tan \delta_{in} = \frac{L}{R - \frac{w}{2}} , \quad \tan \delta_{out} = \frac{L}{R + \frac{w}{2}}$$

Equivalently,

$$\boxed{\cot \delta_{out} - \cot \delta_{in} = \frac{w}{L}}$$

δ_{out} : steering angle of Outer wheel.

δ_{in} : Steering angle of inner wheel.

R : is path radius of the vehicle COG or rear axle centreline - state.

For Pure Rolling / no lateral Slip for Kinematic use, where wheel is Collinear with wheel plane.

Friction limits : $\sqrt{F_x^2 + F_y^2} \leq \mu F_z$

2) Kinematic & Dynamic Equations:

Coordinate frames & Symbols:

Interial position: (x, y, ψ) : position of vehicle COG & yaw.

Velocities: V_x & V_y, yaw rate $r = \dot{\psi}$

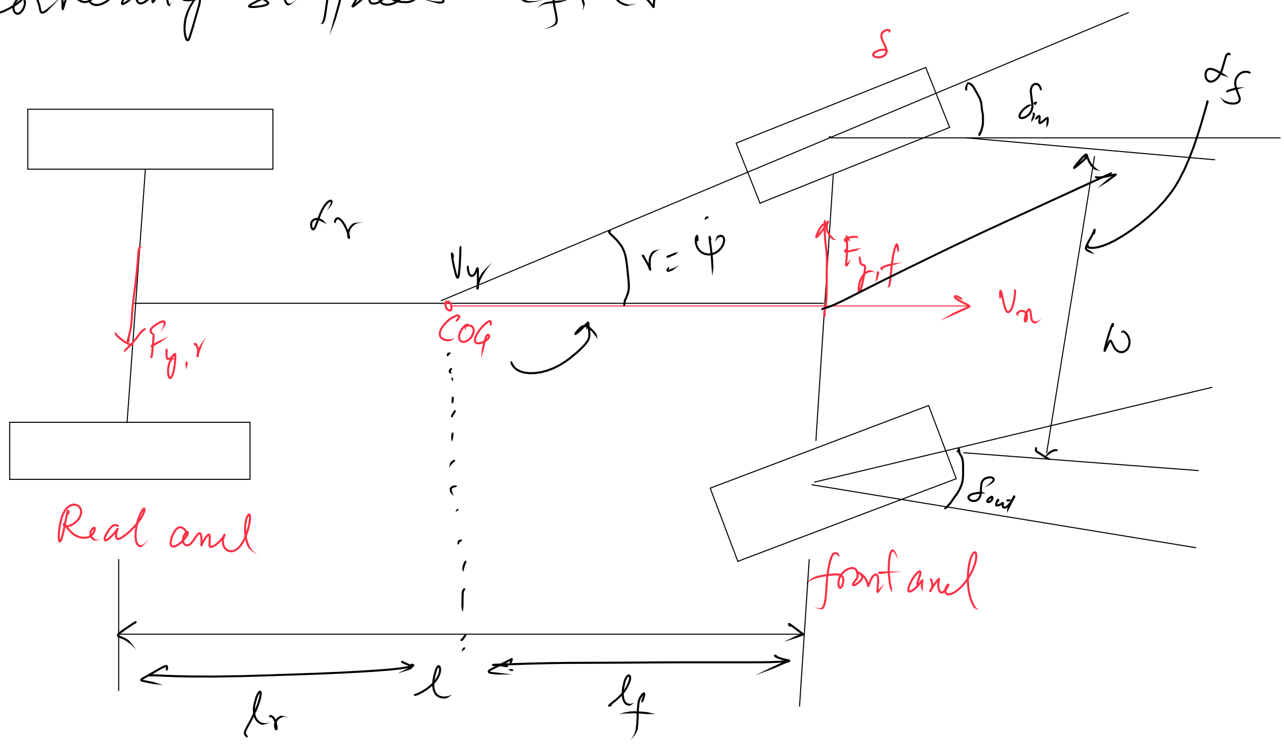
Wheelbase $L = l_f + l_r$, distance COG → front/rear axel: l_f, l_r.

front steer angle: δ_L - left, δ_r - right.

Track width: W

Mass m, Interia I_z

Tire Cornering Stiffness: C_f, C_r



α_r, α_f → slip angles.

Dynamic Single track (bicycle) model

$V_x, V_y, r \rightarrow$ states

$\delta \longrightarrow$ input steer

$F_{x,f}, F_{x,r} \longrightarrow$ forces (front & rear)

→ Equation of motion (body frame):

F = force
m = mass
a = acceleration
v = velocity

$$F = ma$$

$$F = m(\dot{v})$$

$$m(\dot{v}_x - rV_y) = F_{x,f}\cos\delta - F_{y,f}\sin\delta + F_{x,r},$$

$$m(\dot{v}_y - rV_x) = F_{x,f}\sin\delta + F_{y,f}\cos\delta + F_{y,r},$$

$$I_z\dot{r} = l_f(F_{y,f}\cos\delta + F_{x,f}\sin\delta) - l_r F_{y,r}.$$

Slip angles

$$\alpha_f = \arctan\left(\frac{V_y + l_f r}{V_x}\right) - \delta, \quad \alpha_r = \arctan\left(\frac{V_y - l_r r}{V_x}\right)$$

Small angle linearization:

$$\alpha_f \approx \frac{V_y + l_f r}{V_x} - \delta, \quad \alpha_r \approx \frac{V_y - l_r r}{V_x}$$

Kinematic link to global pose:

$$\boxed{\begin{aligned} \dot{x} &= V_x\cos\psi - V_y\sin\psi \\ \dot{y} &= V_x\sin\psi + V_y\cos\psi \\ \dot{\psi} &= r \end{aligned}}$$

$$= R\psi = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$$

Tire forces:

$$F_{y,f} = -C_f \alpha_f \qquad F_{y,r} = -C_r \alpha_r$$

Per wheel Geometry:

δ_{in}, δ_{out}

$$V_{in} = \omega R_{in}, \quad V_{out} = \omega R_{out},$$

$$R_{in} = R - \frac{w}{2}, \quad R_{out} = R + \frac{w}{2}$$

Vehicle yaw rate: $\omega = \dot{\psi} = V/R$

Homework-3   16th Sept 2025

→ Give different Velocities & Simulate kinematics for 10 secs. (MatLab & Simulink)

→ Plot the position & angle of the Robot in 2D-plane

→ Input different Velocities

→ Equation of motion.

{ input Velocity
Output
atleast 2 Example }

→ Consider one uncertainity { states or parameters like M, C & G }

→ EOM in square matrix format.

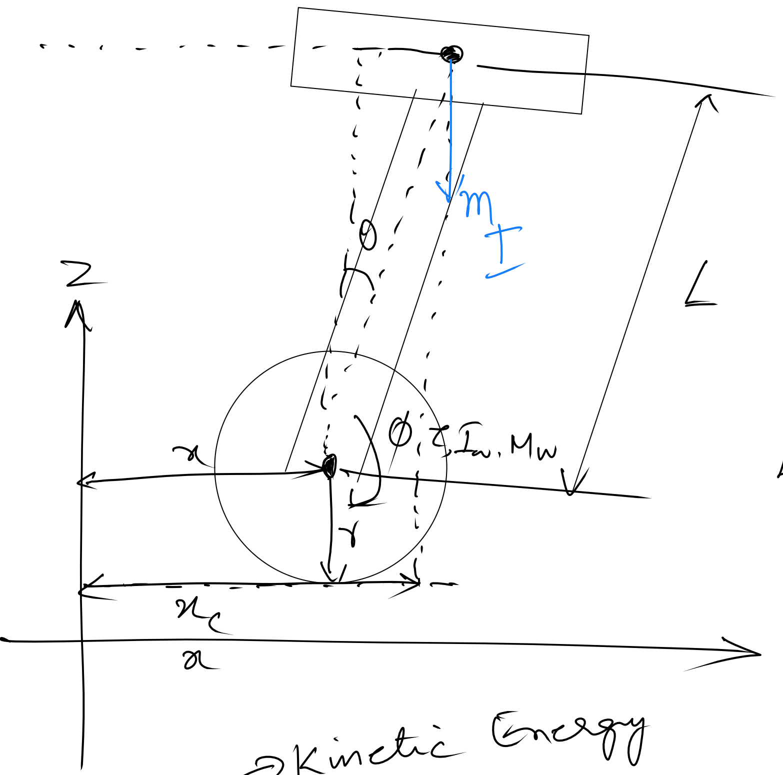
Class Notes

16th Sept 2025

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + A(q)\dot{q} = B(q)\tau$$

$$\downarrow$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B_x(q)f_x + B_y f_y$$



(No slipping)

$$\left.\begin{aligned} x &= r\phi \\ x_c &= r\phi + L\sin\theta \\ z_c &= L\cos\theta + r \end{aligned}\right\} \text{(1)}$$

$$L = K - V$$

Kinetic Energy

$$K = \frac{1}{2}m_w\dot{x}^2 + \frac{1}{2}I_w\dot{\phi}^2 + \frac{1}{2}m\dot{x}_c^2 + \frac{1}{2}m\dot{z}_c^2 + \frac{1}{2}I\dot{\theta}^2$$

Potential Energy.

$$V = Mg(r + L\cos\theta) - mg(r+L) = Lmg(\cos - 1)$$

(2)

(1) in (2)

$$L = K - V$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = f_1}$$

$$\frac{\partial L}{\partial \dot\phi} = I_T \dot\phi + mrL\cos\theta\,\dot\theta$$

I_T - Total moment of Interia

$$I_T = (I_w + M_w r^2 + mr^2)$$

$$\frac{\partial L}{\partial \phi} = 0$$

Eqn of motion I

$$\boxed{f_1 = I_T \ddot\phi + mrL\cos\theta\,\ddot\theta - mrL\sin\theta\,\dot\theta^2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = f_2$$

$$\frac{\partial L}{\partial \dot\theta} = mrL\cos\theta\,\dot\phi + I\,\dot\theta$$

$$\frac{\partial L}{\partial \theta} = -mrl\sin\theta\,\dot\phi\,\dot\theta + mgl\sin\theta$$

$$\dot x_c^2 = (r\dot\phi + \dot\theta L\cos\theta)^2$$

$$r^2\dot\phi^2 + 2rL\dot\theta\dot\phi\cos\theta + \dot\theta^2 L^2\cos\theta^2$$

$$\dot z_c^2 = (-L\dot\theta\sin\theta)^2$$

$$= L^2\dot\theta^2\sin^2\theta$$

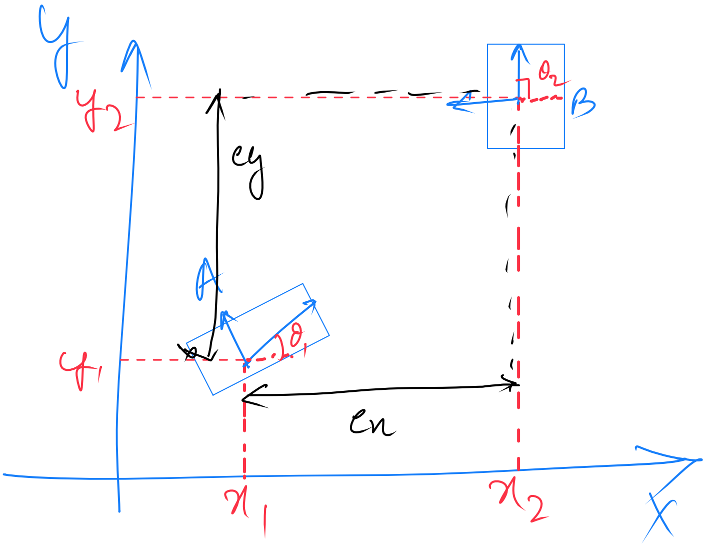
$$\boxed{I\ddot\theta + mrl\cos\theta\,\ddot\phi - mrl\sin\theta = f_2}$$

Eqn of motion II

$$q = \begin{bmatrix}\phi\\ \theta\end{bmatrix} \quad M(q) = \begin{bmatrix} I_T & mrl\cos\theta \\ mrl\cos\theta & -I\end{bmatrix}$$

$$C(q.\dot q) = \begin{bmatrix} 0 & -mrl\sin\theta\dot\theta \\ 0 & 0\end{bmatrix} \quad G(q) = \begin{bmatrix} 0 \\ -mrl\sin\theta\end{bmatrix}$$

Perception



- GPS (Lat & Long) } Low Cost AMR
- IMU, Encoders
- Point cloud matching.
- Sensor fusion
- Vision Based Models

$X_{GT} = \bar{x} + \Delta x \rightarrow$ error

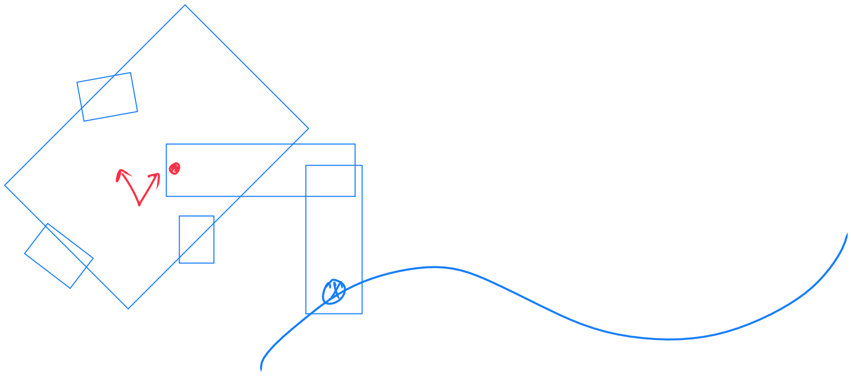
$\bar{x}$ ↳ measure.

$\boxed{\dot{x} = f(x \cdot u)} + \boxed{\Delta f(x \ldots)} \rightarrow$ uncertainities

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix}$$

Equations of Motion

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B\tau$$



$$q_v = \begin{bmatrix} \alpha \\ y \\ \psi \end{bmatrix} \qquad q_a = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}$$

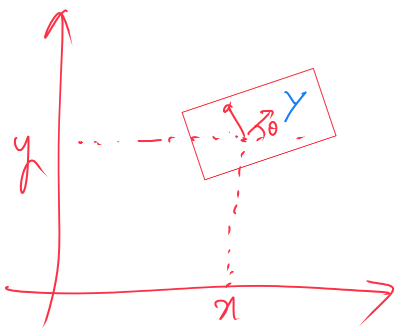
$$M_a \ddot{q}_a + C_a \dot{q}_a + G_a = B_a \tau_a$$

$$\underset{3\times3}{M_v}\,\underset{3\times1}{\ddot{q}_v} + C_v \dot{q}_v + G_v = \underset{3\times3}{B_v}\,\underset{3\times1}{\tau_v}$$

Transpose of Each other.

$$M = \begin{bmatrix} M_a & M_{ra} \\ M_{av} & M_r \end{bmatrix} \qquad C = \begin{bmatrix} C_a & C_{ra} \\ C_{ar} & C_r \end{bmatrix}$$

$$G = \begin{bmatrix} G_a \\ G_r \end{bmatrix} \qquad B = \begin{bmatrix} B_a & 0 \\ 0 & B_r \end{bmatrix} \qquad q = \begin{bmatrix} q_a \\ q_v \end{bmatrix}$$



$$Y = \begin{bmatrix} x + d\cos\theta & y + d\sin\theta \end{bmatrix}$$

$$\underset{2\times1}{Y} = T_{2\times3}\,\dot{X}_{3\times1}$$

T - Transformation Matrix

$$\underset{2\times2}{M}\ddot{Y} + C\dot{Y} = B$$

$$\mathring{q} = T\mathring{\phi}$$

Meccanum :

$$\underset{4\times3}{M}\,\underset{3\times1}{\ddot{q}} + \underset{4\times3}{C}\dot{q} + \underset{4\times1}{G}q = \underset{4\times4}{B}\,\underset{4\times1}{\tau}$$

To make $\bar{B}_{3\times4}$, $\bar{M}_{3\times3}$, $\bar{C}$

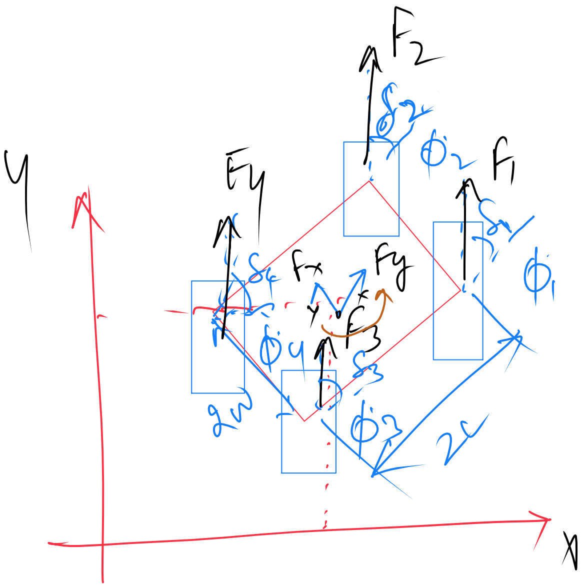
$$\underset{3\times4}{\bar{B}} = \underset{3\times4}{T}\,\underset{4\times4}{B}$$

$$\underset{4\times3}{TM}\ddot{q} + \underset{4\times3}{TC}\dot{q} + \underset{4\times1}{TGq} = \boxed{\underset{3\times4}{T}\cdot\underset{4\times4}{B}}\,\underset{4\times1}{\tau}$$

$$\bar{M} = TM$$

$$\bar{C} = TC$$

$$\bar{G} = TG$$



Centre of Gravity
→ no Centrifugal force.

① No sliding on Each wheel

② No Slipping on Each wheel

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B\tau$$

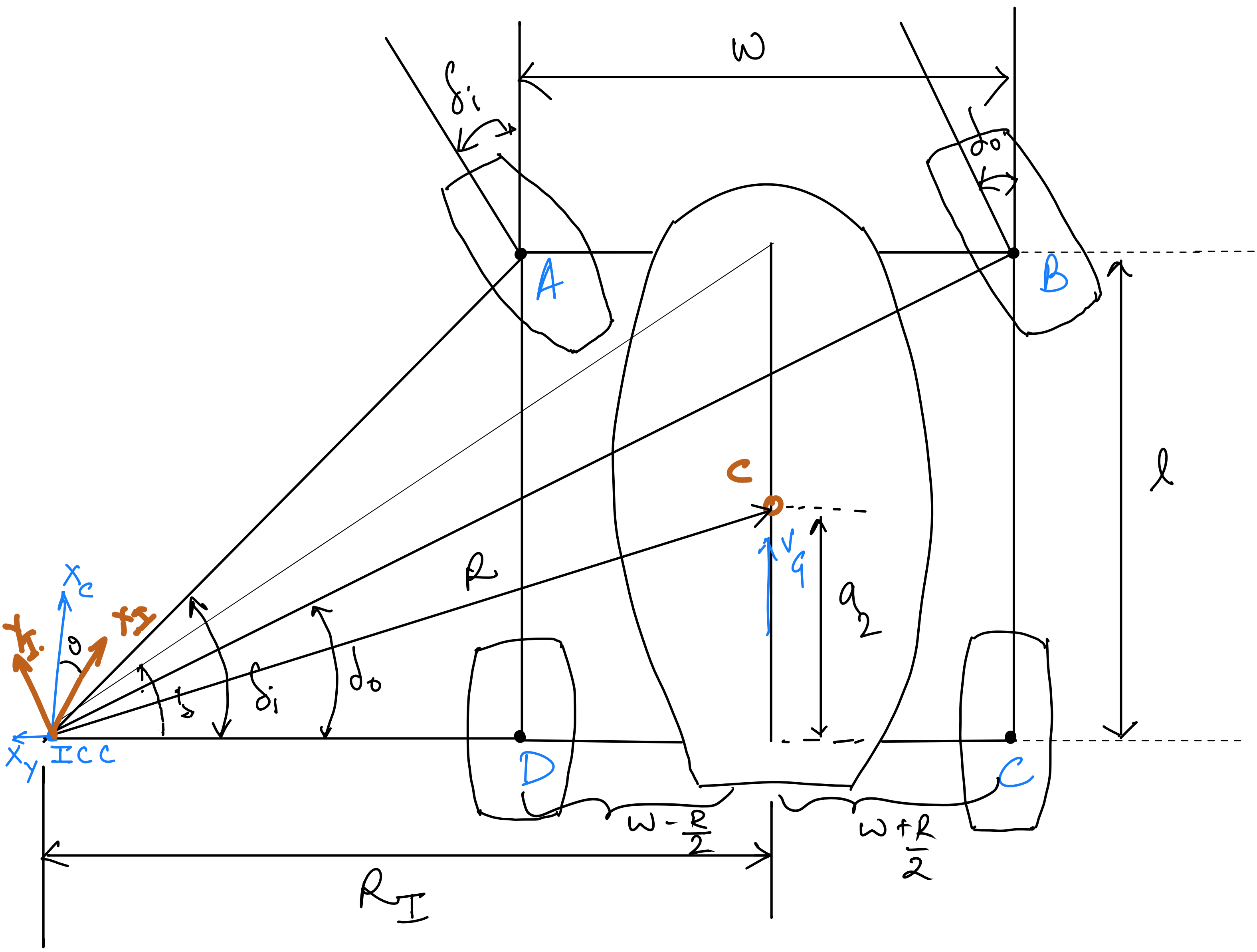
$$\sum F_x = m\ddot{x} = F_1 \cos\delta_1 + F_2 \cos\delta_2 + F_3 \cos\delta_3 + F_4 \cos\delta_4$$

$$\sum F_y = m\ddot{y} = F_1 \sin\delta_1 + F_2 \sin\delta_2 + F_3 \sin\delta_3 + F_4 \sin\delta_4$$

$$\sum \tau = I\ddot{\theta} = F_1(\cos\delta_1 W + \sin\delta_1 L) + F_2(-\cos\delta_2 W + \sin\delta_2 L) +$$

$$F_3(\cos\delta_3 W - \sin\delta_3 L) + F_4(-\cos\delta_4 W - \sin\delta_4 L)$$

Ackerman's 4 - wheel Differential Forward Kinematics



$$\dot{x} = V_G \cos\theta$$

$$\dot{y} = V_G \sin\theta$$

$$\dot{\theta} = \frac{V_G}{R} = \frac{V_G}{\frac{L}{\tan(\delta)}} = \frac{V_G \tan(\delta)}{L}$$

$$\dot{x} = V_G \cos\theta = V_G \cos\left(\frac{V_G}{R} t\right) \Rightarrow x = R \sin\left(\frac{V_G}{R} t + \theta_0\right) + x_0$$

$$\dot{y} = V_G \sin\theta = V_G \sin\left(\frac{V_G}{R} t\right) \Rightarrow y = -R \cos\left(\frac{V_G}{R} t + \theta_0\right) + R + y_0$$

$$\tan\delta = \frac{L}{R} = \text{Const}$$

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$\dot{\theta} = \frac{V_G}{R} = \text{Const}$$

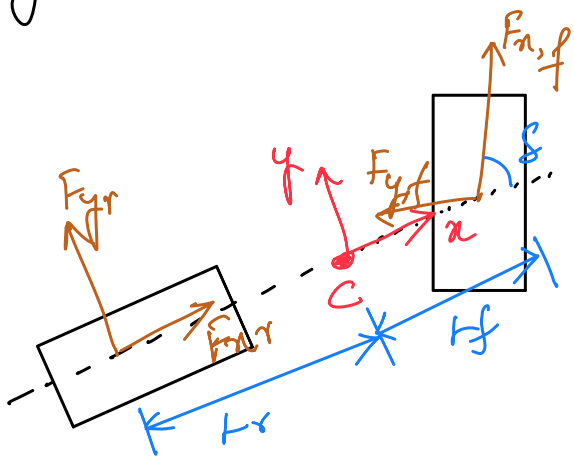
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{V_G}{R}} = \frac{2R\pi}{V_G}$$

If Speed is constant & steering angle is constant
then the car is moving circle motion.

Equation of motion

→ Ackerman's steering Bicycle model

F_{xf}, F_{yf} : Longitudinal & lateral forces
$F_{x,r}, F_{y,r}$: Longitudinal & lateral forces



WkT

$$\sum F = m \cdot a$$

$$\sum M = I \cdot \dot{w}$$

Equation of Motion :

Longitudinal (x-axis)

$$m V_y w + F_{xf} \cos\delta - F_{yf} \sin\delta + F_{xr} = m\ddot{x}$$

Lateral (y-axis)

$$-m V_x w + F_{yr} + F_{yf} \cos\delta + F_{xf} \sin\delta = m\ddot{y}$$

Yaw (Rotation)

$$F_{yf} \cos\delta \cdot l_f + F_{xf} \sin\delta \cdot l_f - F_{yr} l_r = I\dot{w}$$

Matrix Representation

$$M(q) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$C\dot{q} = \begin{bmatrix} m V_y \omega \\ -m V_x \omega \\ G \end{bmatrix}$$

$$M\ddot{q} + C\dot{q} + G = B_y F_y + B_x F_x$$

$Mq \rightarrow$ mass / Interia matrix

$Cq \rightarrow$ Centrifugal & Coriolis matrix

$G \rightarrow$ Gravity (often 0 on flat Surface)

$B_y F_y \rightarrow$ effect of lateral tire force.

$B_x F_x \rightarrow$ Effect of longitudinal force.

$$B_y F_y = \begin{bmatrix} -\sin\delta & 0 \\ \cos\delta & 1 \\ l_f \cos\delta & -l_r \end{bmatrix} \cdot \begin{bmatrix} F_{yf} \\ F_{yr} \end{bmatrix}$$