## Problem description

The necessity of an environmentally friendly clean fuel considered natural gas as an important energy source in comparison to other fossil fuels[1]. Natural gas usage increased by 96 billion cubic meters (bcm), or 3%, in 2018, the fastest increase since 2010. Hence, Global natural gas generation climbed to 131 bcm, or 4%, nearly twice the 10-year average[2]. Natural gas is transported from pre-processing plants or storage facilities to distribution systems via transmission pipeline networks that span thousands of kilometers around the world[3].

In this project, I am going to model a given gas transmission network that is shown in the following figure.

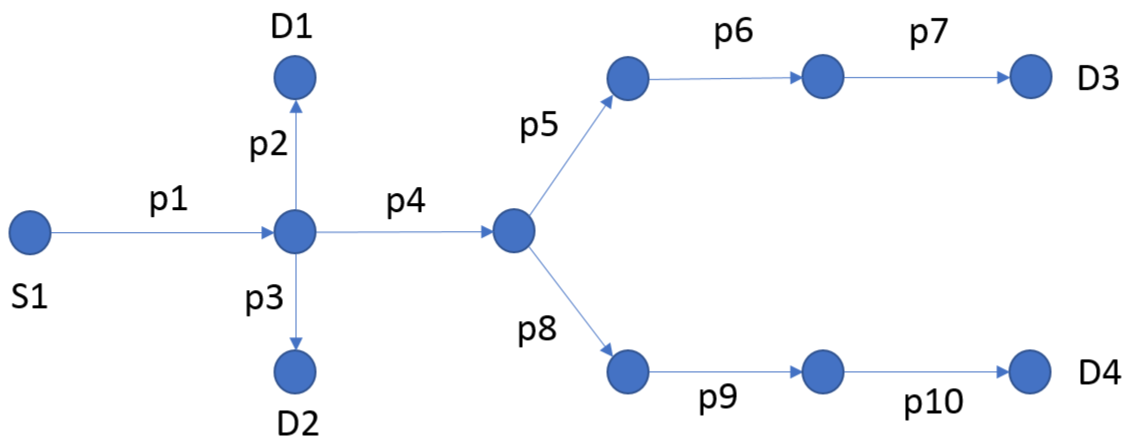


Fig 1. Given gas transmission network

The given information for this gas pipeline network is:

L for each pipeline is 10 km (could be changed in the code), pipeline diameter for all pipes is 42 inches, pressure at S1 (supply) is 12e6 Pa, and mass flow rate for all deliveries (D1, D2, D3, and D4) is 400 kg/s.

Also, I further developed the code so we can define any pipeline network that contains pipes, nodes, supplies, and deliveries.

## Methods & equations

I chose to solve a pipeline network in dynamic state mode to see the behavior of the gas flow inside the pipelines through variable time. Different types of models could be used for defining the gas flow in the pipelines and finding pressure and mass flow rate profiles. We can separate those models into non-isothermal and isothermal models. I used the isothermal model since we will have fewer parameters and it also gives us results with excellent precision. I defined an isothermal model by using continuity and momentum equations and neglecting the change in temperature as follows[4]:

$$\frac{\partial p}{\partial t} = -\frac{ZRT}{A_c}\frac{\partial \dot{m}}{\partial x} \tag{1}$$

$$\frac{\partial \dot{m}}{\partial t} = -A_c\frac{\partial p}{\partial x} - \left(\frac{f_r ZRT}{2DA_c}\right)\frac{\dot{m}|\dot{m}|}{p} - \frac{A_c pg}{ZRT}\sin\theta \tag{2}$$

Which p is pressure (Pa), t is time (s), Z is compressibility factor, R is the universal gas constant (8.314 J/mol.K), T is the temperature of the fluid (K), $A_C$ is inside the pipeline area ($m^2$), $\dot{m}$ is the mass flow rate (kg/s), x is the distance between two states (m), $f_r$ is the friction factor of the pipe, D is the diameter of the pipe (m), g is the acceleration of gravity ($m/s^2$), and $\theta$ is the angle of pipe with the ground (I assumed to be zero).

I developed a network grid for each pipe and used discretization of the spatial differential operators and reduction of the (PDAE) index in the networked system, to have a system of ordinary differential equations (ODE)[4]:

$$\frac{\partial \dot{m}}{\partial t} = -A_C\frac{p_{in} - p_{out}}{L} - \left(\frac{f_r ZRT}{2DA_c}\right)\frac{\dot{m}|\dot{m}|}{p} - \frac{A_c pg}{ZRT}\sin\theta \tag{3}$$

$$\frac{\partial p}{\partial t} = -\frac{ZRT}{A_C}\frac{\dot{m}_{in} - \dot{m}_{out}}{L} \tag{4}$$

I utilized forward, backward and central differentiation based on the position of each segment node. As we can see in the segments in Fig. 2, at the first segment of the pipeline I used the forward difference approximation, at the last segment I utilized the backward difference approximation, and on other segments, I applied the central difference approximation of spatial differential operators.

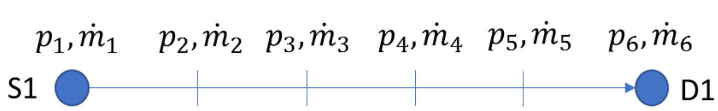


Fig 2. Showing a single pipeline with its states variables and defined segments

I had four different types of equations based on the type of the initial and the final node of each pipeline and I defined each part of the equations separately. A node in the pipeline could be a supply node (with specified pressure), a junction node (the internal nodes of pipelines with no specified values of pressure or mass flow rate), or a deliver node (with specified mass flow rate). Hence, the combination of the first and last node for each pipe could be a) from a supply node to a junction node, b) from a junction node to another junction node, c) from a junction node to a delivery node, and d) from a supply node to a delivery node.

Also, for calculating the compressibility factor I used the American Gas Association (AGA) compressibility factor[4]:

$$z(p,T) = 1 + 0.257\frac{p}{p_c} - 0.533\frac{pT}{p_c T_c} \tag{5}$$

Moreover, for solving the pressure of each junction I used the following equation[5]:

$$\frac{\partial p_j}{\partial t} = \frac{\sum_{i=1}^{n_i} q_i - \sum_{i=1}^{n_o} q_i + q_{supply} - q_{demand}}{\tau} \tag{6}$$

Where $p_j$ is the node pressure and $\tau$ is the equation's time constant, which is determined by the hold-up volume and average vessel parameters. If the nodal volume is small enough to allow for very rapid pressure establishment, this assumption yields reasonable results at all times.

I used ode23s in MATLAB to solve a set of ODEs for each pressure sate and mass flow rate state. ode23s can be more efficient than ode15s at problems with crude error tolerances. It can solve some stiff problems for which ode15s is not effective. ode23s computes the Jacobian in each step, so it is beneficial to provide the Jacobian via odeset to maximize efficiency and accuracy.

## Conclusion

As we can see through the results, the model could by solving ODEs show the behavior of a given gas transmission network in a dynamic state starting from arbitrary initial states. Hence, we could have pressure and mass flow rate profiles of each node and inner node of all pipelines and the pressure profile of every junction.

# References

1. Mokhatab, S.; Poe, W. A., *Handbook of natural gas transmission and processing*. Gulf professional publishing: 2012.
2. Dudley, B., BP statistical review of world energy. *BP Statistical Review, London, UK, accessed Aug* **2018,** *6* (2018), 00116.
3. Ríos-Mercado, R. Z.; Borraz-Sánchez, C., Optimization problems in natural gas transportation systems: A state-of-the-art review. *Applied Energy* **2015,** *147*, 536-555.
4. Cejudo Grano de Oro, J. E., Coupling of isothermal models in gas networks. **2019**.
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