

## Project #2: Smoothed Particle Hydrodynamics

*Professor Bindel**tbe4, dsj36*

## Introduction

Our project involved optimizing a smoothed particle hydrodynamics simulation. We started with a reference serial implementation and made two salient improvements: improving the asymptotic complexity of the simulation by employing spatial partitioning and parallelizing the codes. We used OpenMP for this project, as suggested, which allows easy management of shared memory. However, to get the code to run fast, we had to restructure a good amount of our computations to prevent the different threads from “stepping on each other’s toes,” so to speak.

## Spatial Partitioning

For each particle, the SPH simulator computes an interaction with all of the particles within some fixed radius  $h$  of the particle. This search is done for both the density and acceleration computations. The reference code used a brute force search on the particles for each particle’s computation, yielding  $O(n^2)$  complexity. As suggested, we used a form of spatial binning to improve this complexity to roughly  $O(n)$ .

Our approach is as follows. First, to improve locality, we store the particles as structures, with each containing its position, acceleration, and so forth. The reference implementation stored each attribute in a global array, so accessing the different attributes for a single particle would require looking through many different arrays. Furthermore, each particle structure has a pointer to a next particle, allowing us to create linked lists of particles easily. Using this addition, we include an array of pointers to particles (so, in essence, an array of linked lists of particles) in the state structure. These linked lists are the “bins” in our optimization.

The first order of business is to decide how many bins to have in our simulation. It is very desirable to have bins of width no less than  $2h$ , as this property guarantees that any circle of radius  $h$  within a bin can only touch at most three neighboring bins. On the other hand, we do not want a large number of particles in a single bin, as search through a bin is necessarily linear. Therefore, we compute an estimate for the number of bins where there is, on average, one particle per bin. We then take the larger of the two bounds.

Once the bins have been set up, the next step is to insert the particles into the bins. The strength of our data structure is that deciding which bin a particle should live in is  $O(1)$ , implying that inserting all the particles is  $O(n)$ . Since we know the height and width of the bins, computing which bin is simply a matter of dividing the coordinates of the particle by the dimensions of the bin and taking the floor. Once we know which bin a particle should go into, insertion into a linked list is just  $O(1)$  as well.

Now that we have the particles in their bins, we can speed up the density and acceleration computations significantly. For each particle, we receive at most four linked lists of bins from our `get_neighboring_bins` function. This function considers the circle of radius  $h$  around the point and returns all of the bins that intersect this circle. We may then search this subset of the particles in the usual fashion, adding the contributions of particles that are within  $h$  of our fixed particle. We use this optimization in the `compute_density` and `compute_accel` functions.

Although our approach described so far improves the quadratic brute force search, following the next pointers in each bin's linked list may cause our simulation to have very poor cache locality. This poor cache performance is due to there being no inherent relationship between a particle's place in the state structure's particle array and its bin. Thus, we sort the particles every 200 iterations to move particles in the same bin close to each other in memory. As suggested, we used a linear time bucket sort to rearrange the particles.

## Parallelization and Symmetry

After adding binning to our simulation, we next turned our focus to parallelizing the codes. Parallelizing the leapfrog integrator was trivial, as there are no data dependencies between the particles in that stage. Therefore, we simply added a `#pragma omp for`, which evenly splits the work for that loop over the threads.

The first optimization we made for the interaction functions was to use the symmetry inherent in the model to halve the amount of work done. The density and acceleration contributions from particle  $i$  to particle  $j$  are identically the contributions from particle  $j$  to particle  $i$ . Therefore, we may update them both in one step, halving the number of particle pairs we need to consider. To make sure we do not update a pair twice, we simply keep a flag on each particle indicating whether it has been seen already. If the second particle in a potential interaction has been seen already, the pair has already been computed.

Parallelizing this idea is not immediately straightforward, as expected. Our idea is split

the columns of bins evenly among the threads. However, each column of bins, with this symmetry handling added, needs to write to itself, the column to the left, and the column to the right. If two threads are simultaneously handling adjacent columns, they may both try to write to the same particle, causing blocking or, at worst, a race condition. Our solution to this problem is to make three passes over the bins. We do every third column in each pass, leaving two columns between any column being worked on. This pattern means that each thread can write to the column around without fear of race conditions or requiring a lock.