

Forward Kinematics

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Agenda

- Quick Revision

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- Implementation

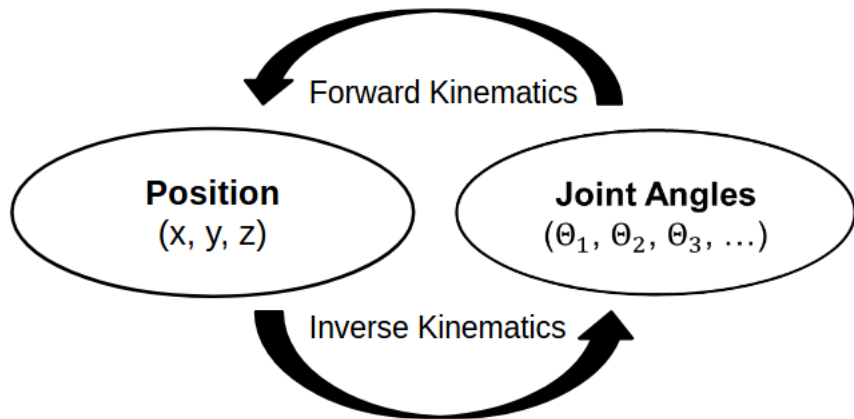
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- Quick Revision
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- Live Simulation

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- Implementation
- Live Simulation
- Deep Reinforcement Learning

What is?



Implementation

- DH Parameters.

Implementation

- DH Parameters.
- Typical Form (4 DoF).

Link	d_i	θ_i	a_i	α_i
1	d_1	θ_1	a_1	α_1
2	d_2	θ_2	a_2	α_2
3	d_3	θ_3	a_3	α_3
4	d_4	θ_4	a_3	α_4

- Transformation Matrix

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$$[X_i] = \text{Trans}_{X_i}(a_{i,i+1}) \text{Rot}_{X_i}(\alpha_{i,i+1})$$

$${}^{n-1}T_n = \text{Trans}_{z_{n-1}}(d_n) \cdot \text{Rot}_{z_{n-1}}(\theta_n) \cdot \text{Trans}_{x_n}(a_n) \cdot \text{Rot}_{x_n}(\alpha_n)$$

Implementation

$${}^{n-1}T_n = \left[\begin{array}{ccc|c} \cos \theta_n & -\sin \theta_n \cos \alpha_n & \sin \theta_n \sin \alpha_n & a_n \cos \theta_n \\ \sin \theta_n & \cos \theta_n \cos \alpha_n & -\cos \theta_n \sin \alpha_n & a_n \sin \theta_n \\ 0 & \sin \alpha_n & \cos \alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} & & & \\ & R & & T \\ & & & \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

Implemenatation

- Get Transformation Matrix.

Implemenatation

- Get Transformation Matrix.

```
1 function [T] = getTransformMatrix(theta , d , a ,  
    alpha)  
2 T = [cos(theta) -sin(theta) * cos(alpha) sin(  
    theta) * sin(alpha) a * cos(theta);  
3     sin(theta) cos(theta) * cos(alpha) -cos(  
    theta) * sin(alpha) a * sin(theta);  
4     0 , sin(alpha) , cos(alpha) , d;  
5     0 , 0 , 0 , 1];  
6 end
```


Implemenatation

- Forward Kinematics.

Implementation

- Forward Kinematics.

```
1  function [T00,T01,T12,T23,T34,T45,T56, Etip]=  
    forwardKinematics(theta1,d1,a1,alpha1,theta2,  
        d2,a2,alpha2,theta3,d3,a3,alpha3,theta4,d4,a4  
        ,alpha4,theta5,d5,a5,alpha5,theta6,d6,a6,  
        alpha6)  
2  
3  T00 = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1];  
4  T01 = getTransformMatrix(theta1,d1,a1,alpha1);  
5  T12 = getTransformMatrix(theta2,d2,a2,alpha2);  
6  T23 = getTransformMatrix(theta3,d3,a3,alpha3);  
7  T34 = getTransformMatrix(theta4,d4,a4,alpha4);  
8  T45 = getTransformMatrix(theta5,d5,a5,alpha5);  
9  T56 = getTransformMatrix(theta6,d6,a6,alpha6);  
10  
11  Etip = T00 * T01 * T12 * T23 * T34 * T45 * T56;
```

- Final Transformation Matrix.

- Final Transformation Matrix.

$$E_{tip} = \left[\begin{array}{ccc|c} & & & \\ & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

Implemenatation

- Plotting the Robot Link.

Implementation

- Plotting the Robot Link.

```
1 function [FK_plot3D] = FK_plot3D(Th_1, Th_2,
   Th_3, Th_4, Th_5, Th_6, a_1, a_2, a_3, a_4,
   a_5, a_6, d_1, d_2, d_3, d_4, d_5, d_6, al_1,
   al_2, al_3, al_4, al_5, al_6)
2 %Plotting The workspace
3 L (1) = Link( [Th_1 d_1 a_1 al_1] );
4 L (2) = Link( [Th_2 d_2 a_2 al_2] );
5 L (3) = Link( [Th_3 d_3 a_3 al_3] );
6 L (4) = Link( [Th_4 d_4 a_4 al_4] );
7 L (5) = Link( [Th_5 d_5 a_5 al_5] );
8 L (6) = Link( [Th_6 d_6 a_6 al_6] );
9 Robot = SerialLink(L);
10 Robot.name = '6 - DoF forward Kinematics';
11 Robot.plot([Th_1 Th_2 Th_3 Th_4 Th_5 Th_6]);
```

- Computing Each Transformation Matrix.

Implementation

- Computing Each Transformation Matrix.

```
1  [T00, T01, T12, T23, T34, T45, T56, Etip]=  
    forwardKinematics(Th_1, d_1, a_1, al_1, Th_2, d_2,  
    a_2, al_2, Th_3, d_3, a_3, al_3, Th_4, d_4, a_4, al_4,  
    Th_5, d_5, a_5, al_5, Th_6, d_6, a_6, al_6);  
2  
3  T2 = T01*T12; T3 = T2*T23 ; T4 = T3*T34 ; T5 = T4  
    *T45; T6 = T5*T56;
```


- **For 6 DoF, we need at most n iteration for 24 variables.**

3D WorkSpace Plot

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- But how to implement it for generalised case?

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- But how to implement it for generalised case?
- I tried to implement for 12 variables and it took around 48,000 live lines of code.

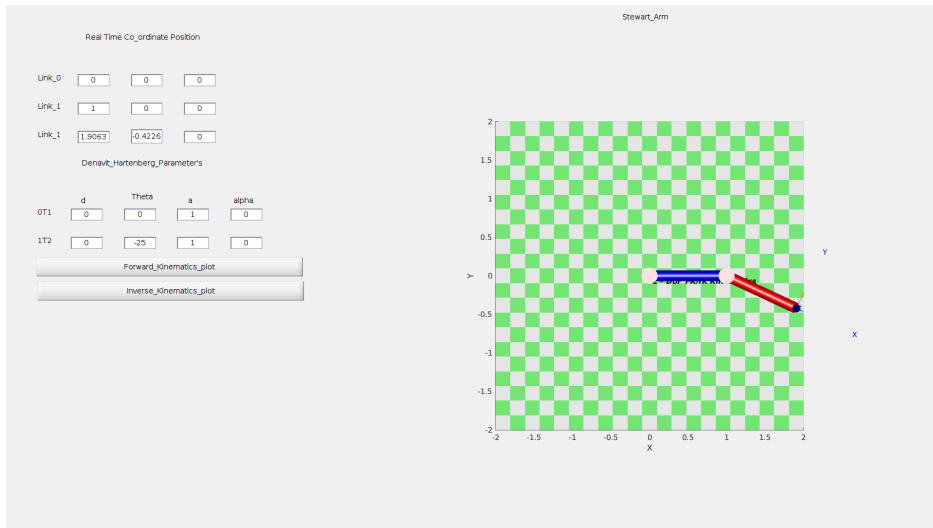
3D Workspace Plot

- For 6 DoF, we need at most n iteration for 24 variables.
- But how to implement it for generalised case?
- I tried to implement for 12 variables and it took around 48,000 live lines of code.
- Have you dared to write a generalised Workspace code for 24 variable?

- Planner Robot (2 DoF).

Link	d_i	θ_i	a_i	α_i
1	0	10	0	0
2	0	20	0	0

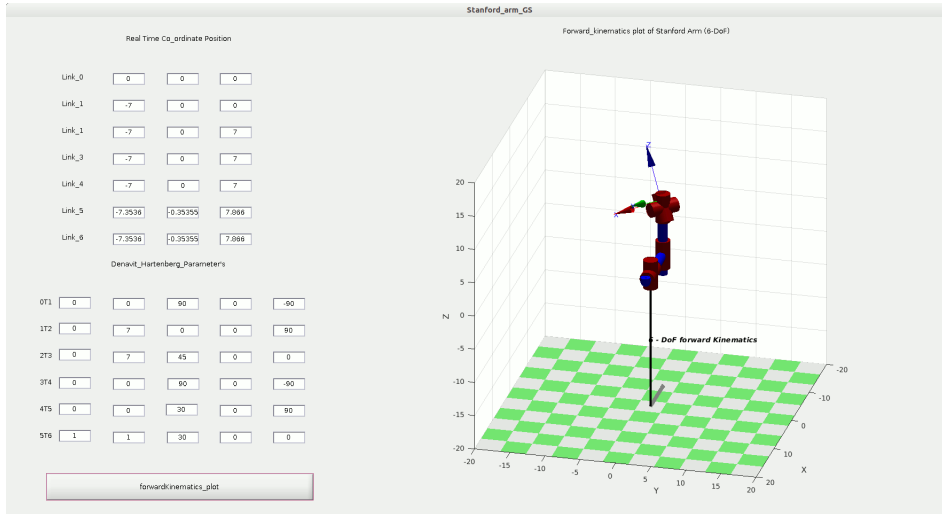
2 DoF (Open_Robo_Simulator)



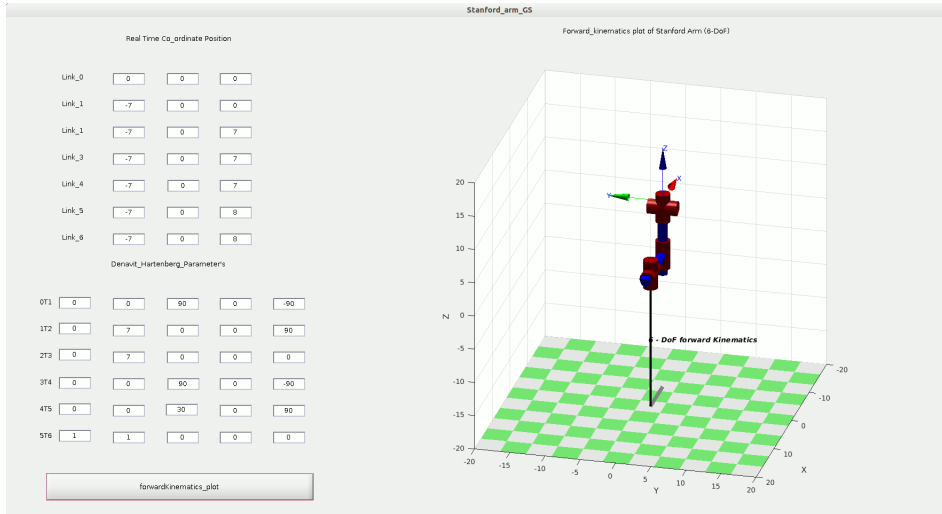
- Stanford Arm (6 DoF).

Link	d_i	θ_i	a_i	α_i
1	0	-90	0	-90
2	7	90	0	90
3	7	0	0	0
4	0	90	0	-90
5	0	45	0	90
6	1	45	0	0

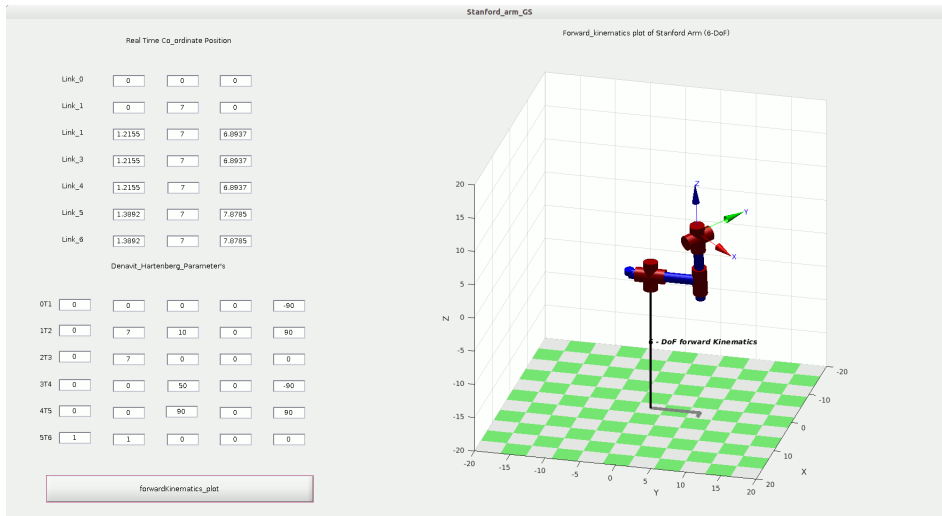
6 DoF (Open_Robo_Simulator)



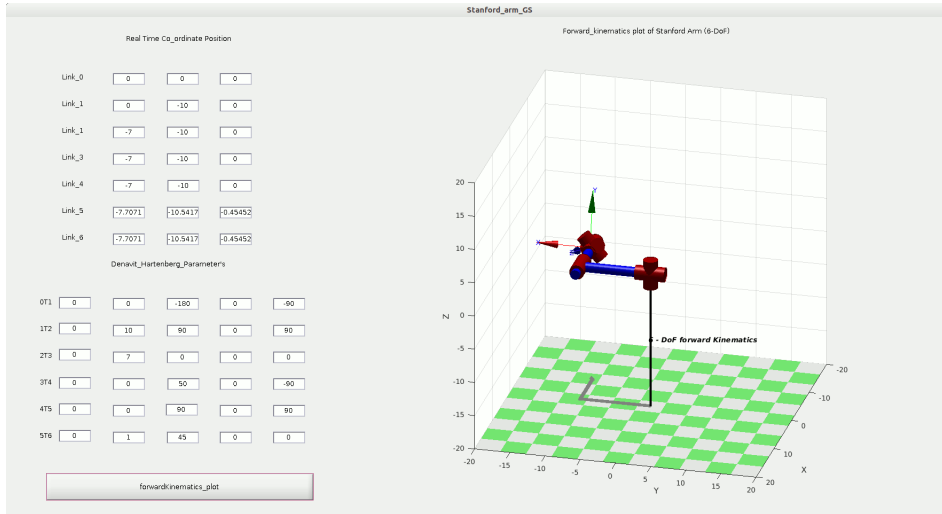
6 DoF (Open_Robo_Simulator)



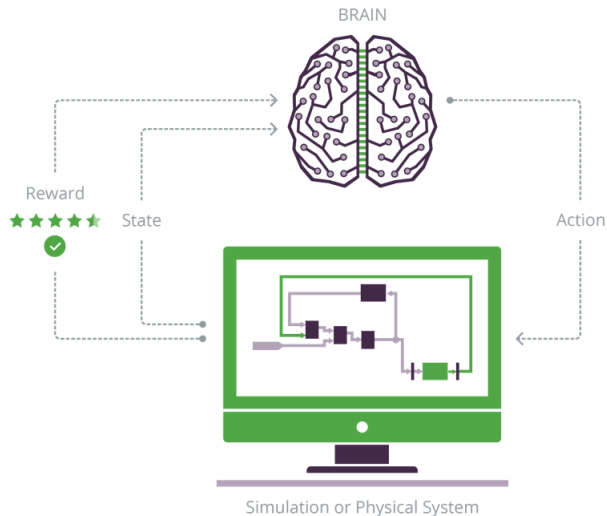
6 DoF (Open_Robo_Simulator)



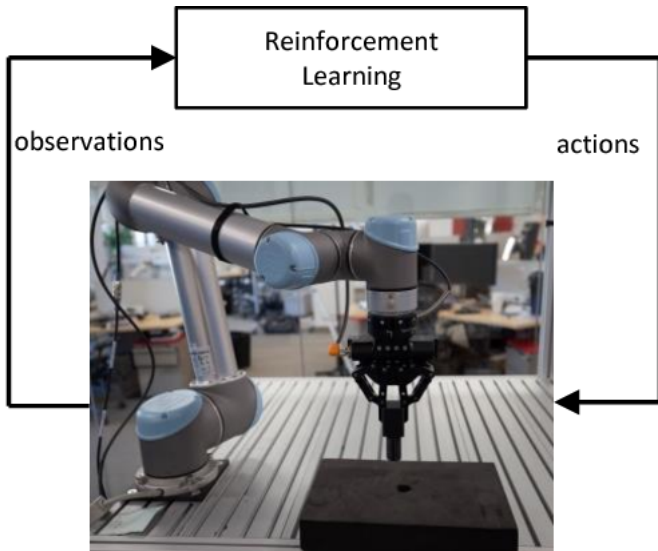
6 DoF (Open_Robo_Simulator)



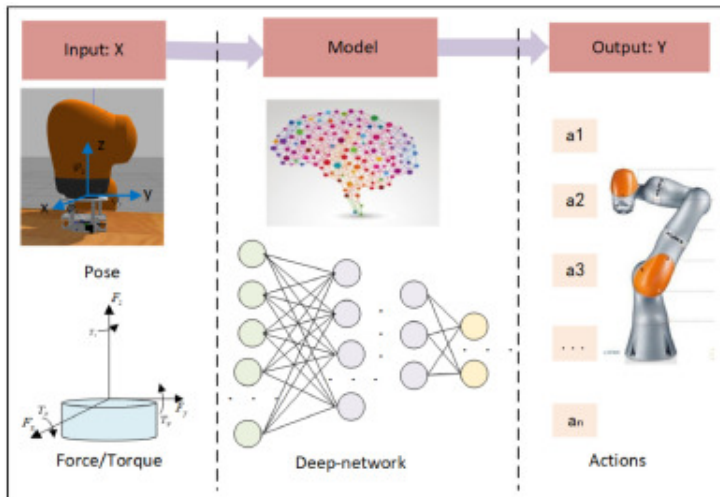
Deep Reinforcement Learning



Deep Reinforcement Learning



Deep Learning



- 1) **ROBOTICS: FUNDAMENTAL CONCEPTS AND ANALYSIS**
- 2) **Introduction to Robotics: Analysis, Control, Applications, 3rd Edition(Saeed B. Niku)**
- 3) **Denavit–Hartenberg parameters(Wikipedia)**