

# Astronomy Answers

## Position of the Sun



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The position of the Sun in the sky as seen from a planet (such as the Earth) is determined by four things:

1. the time.
2. the motion of the planet in its orbit around the Sun, which does not happen at constant speed, because of the eccentricity of the orbit of the planet.
3. the angle between the axis of rotation of the planet and the plane of the orbit of the planet, which is not equal to 90 degrees. This causes the seasons.
4. the location of the observer on the planet, which determines how high the Sun can get in the sky.

Below, I provide formulas that take all of these things into account. I neglected small effects here and there, to keep the formulas simple. Remember that you can add multiples of 360 degrees to any angle without changing its direction.

I updated the tables on this page in October 2016 so that they are based on the relevant planet characteristics published by the IAU in 2011, rather than those of 2000. The most important change is that Pluto is no longer regarded as a major planet but now as a dwarf planet, which means that now (since 2006) the other pole of Pluto is regarded as its north pole than before. See Chapter 21.

When in this page an ecliptic or pole or coordinates such as ecliptic longitude or declination are mentioned, then these are appropriate for the planet where the observer is. Such coordinates are based on the orbit and equator of that planet and are not identical to the coordinates of the same names that we use on Earth. For example, the ecliptic of Mars, the ecliptic that is appropriate for Mars, is not the same as the ecliptic of Earth, and an earth-based atlas of the stars is of no use if you have a right ascension and declination calculated for the Sun as seen from Mars.

For examples, we'll calculate the position of the Sun for 1 April 2004 at 12:00 UTC, as seen from 52° north latitude and 5° east longitude (Netherlands) on Earth, and as seen from 14°36' south latitude and 184°36' west longitude (Gusev crater) on Mars.

## 1. Time

For this kind of astronomical calculations, it is convenient to express the date and time using an unending day numbering scheme. Such a scheme is provided by the Julian Day Number  $J$ . The Julian Day Number Calculation Page explains how you can calculate the Julian Day Number for a date in the Gregorian calendar. For the calculations of the position of the Sun you should express time as Universal Time (UTC), and this includes the Julian Date. For example, JD 2453144.5 corresponds to 0:00 hours UTC on 19 May 2004, which is (for example) equivalent to 1:00 hours Central European Time on 19 May 2004, or 20:00 hours (8 pm) Eastern Standard Time on 18 May 2004.

**Only on Earth** do the seasons repeat themselves after about one calendar year (in the western — Gregorian — calendar), so, only for the Earth, the day number  $d$  in the calendar year can be used instead of the Julian Day Number:  $d = 1$  corresponds to 0:00 UTC on January 1st, 2 to 0:00 UTC on January 2nd, 32 to 0:00 UTC on February 1st, and so on.

## 2. The Mean Anomaly

Because we see the Sun from the planet, we see the motion of the planet around the Sun reflected in the apparent motion of the Sun along the ecliptic, relative to the stars.

If the orbit of the planet were a perfect circle, then the planet as seen from the Sun would move along its orbit at a fixed speed, and then it would be simple to calculate its position (and also the position of the Sun as seen from the planet). The position that the planet would have relative to its perihelion if the orbit of the planet were a circle is called the mean anomaly, indicated in the formulas as  $M$ .

You can calculate the mean anomaly of the planets (measured in degrees) to reasonable accuracy using the following formula, for a date given as a Julian Day Number (JD)  $J$ :

$$M = (M_0 + M_1 \times (J - J_{2000})) \bmod 360^\circ \quad (1)$$

$$J_{2000} = 2451545 \quad (2)$$

You should take  $M_0$  (in degrees) and  $M_1$  (in degrees per day) from the following table:

$M_0$	$M_1$
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<a href="#">Mercury</a>	174.7948	4.09233445
<a href="#">Venus</a>	50.4161	1.60213034
<a href="#">Earth</a>	357.5291	0.98560028
<a href="#">Mars</a>	19.3730	0.52402068
<a href="#">Jupiter</a>	20.0202	0.08308529
<a href="#">Saturn</a>	317.0207	0.03344414
<a href="#">Uranus</a>	141.0498	0.01172834
<a href="#">Neptune</a>	256.2250	0.00598103
<a href="#">Pluto</a>	14.882	0.00396

For the [Earth](#), you can also use the following formula:

$$M = (-3.59^\circ + 0.98560^\circ \times d) \bmod 360^\circ \quad (3)$$

where  $d$  is the [time](#) since 00:00 [UTC](#) at the beginning of the most recent January 1st, measured in (whole and fractional) days.

The chosen date and time correspond to [Julian Day Number](#) 2453097, so  $J = 2453097$ , and also to  $d = 92.5$ . For the [Earth](#), using equation 1 we find  $M_{\text{earth}} = 1887.1807^\circ \bmod 360^\circ = 87.1807^\circ$  and with equation 3 we find  $M_{\text{earth}} = 87.58^\circ$ . Because equation 1 is a bit more accurate than equation 3, we'll use equation 1. For [Mars](#), we find  $M_{\text{mars}} = 832.6531^\circ \bmod 360^\circ = 112.6531^\circ$ .

### 3. The Equation of Center

The orbits of the [planets](#) are not perfect circles but rather ellipses, so the speed of the [planet](#) in its orbit varies, and therefore the apparent speed of the [Sun](#) along the [ecliptic](#) also varies throughout the [planet's](#) [year](#).

The true [anomaly](#) (symbol  $v$ , nu) is the angular distance of the [planet](#) from the [perihelion](#) of the planet, as seen from the [Sun](#). For a circular orbit, the mean anomaly and the true anomaly are the same. The difference between the true anomaly and the mean anomaly is called the Equation of Center, written here as  $C$ :

$$v = M + C \quad (4)$$

To calculate the Equation of Center or the true anomaly from the mean anomaly, you must first solve the Equation of [Kepler](#). This equation cannot be solved in general, but you can find approximations to the solution that are more and more accurate. See the [Calculation Page for the Equation of Kepler](#) for more information about this. If the orbit looks more like a circle than like a parabola, then the following approximation is sufficiently accurate:

$$C \approx C_1 \sin M + C_2 \sin(2M) + C_3 \sin(3M) + C_4 \sin(4M) + C_5 \sin(5M) + C_6 \sin(6M) \quad (5)$$

You can find the coefficients  $C_1$  through  $C_6$  in the following table. They depend on the [eccentricity](#)  $e$  of the planet's orbit. If a particular coefficient is not mentioned, then it is equal to zero (to four decimal places). The orbits of [Mercury](#) and [Pluto](#) deviate the most from circularity, so they need the most coefficients. The column  $E_C$  shows the maximum error that you get if you use the approximation with the coefficients from the table.

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$E_C$
<a href="#">Mercury</a>	23.4400	2.9818	0.5255	0.1058	0.0241	0.0055	0.0026
<a href="#">Venus</a>	0.7758	0.0033					0.0000
<a href="#">Earth</a>	1.9148	0.0200	0.0003				0.0000
<a href="#">Mars</a>	10.6912	0.6228	0.0503	0.0046	0.0005		0.0001
<a href="#">Jupiter</a>	5.5549	0.1683	0.0071	0.0003			0.0001
<a href="#">Saturn</a>	6.3585	0.2204	0.0106	0.0006			0.0001
<a href="#">Uranus</a>	5.3042	0.1534	0.0062	0.0003			0.0001
<a href="#">Neptune</a>	1.0302	0.0058					0.0001
<a href="#">Pluto</a>	28.3150	4.3408	0.9214	0.2235	0.0627	0.0174	0.0096

For the [Earth](#), we find

$$\begin{aligned} C_{\text{earth}} &= 1.9148^\circ \times \sin(87.1807^\circ) \\ &\quad + 0.0200^\circ \times \sin(2 \times 87.1807^\circ) \\ &\quad + 0.0003^\circ \times \sin(3 \times 87.1807^\circ) = 1.9142^\circ \end{aligned}$$

and thence  $v_{\text{earth}} = 89.0949^\circ$ . For [Mars](#) we find

$$\begin{aligned} C_{\text{mars}} &= 10.6912^\circ \times \sin(112.6531^\circ) \\ &\quad + 0.6228^\circ \times \sin(2 \times 112.6531^\circ) \\ &\quad + 0.0503^\circ \times \sin(3 \times 112.6531^\circ) \\ &\quad + 0.0046^\circ \times \sin(4 \times 112.6531^\circ) \\ &\quad + 0.0005^\circ \times \sin(5 \times 112.6531^\circ) = 9.4092^\circ \end{aligned}$$

and  $v_{\text{mars}} = 122.0623^\circ$ .

### 4. The [Perihelion](#) and the Obliquity of the [Ecliptic](#)

To find the position of the [Sun](#) in the sky we need to know what the [ecliptic longitude](#)  $\Pi$  is of the perihelion of the [planet](#), relative to the ecliptic and [vernal equinox](#) (the [ascending equinox](#)) of the [planet](#). The ecliptic of the [planet](#) is the plane of the orbit of the planet, which makes an [angle](#) with the orbit (ecliptic) of the [Earth](#) (and that angle is called the [inclination](#) of the orbit). The vernal [equinox](#) of the planet is the point where the [Sun](#) passes from south to north through the plane of the [equator](#) of the planet. We also need to know the obliquity  $\varepsilon$  of the equator of the planet compared to the orbit of the planet. These two values are listed in the following table for each planet, measured in [degrees](#).

	$\Pi$	$\varepsilon$
<a href="#">Mercury</a>	230.3265	0.0351
<a href="#">Venus</a>	73.7576	2.6376
<a href="#">Earth</a>	102.9373	23.4393
<a href="#">Mars</a>	71.0041	25.1918
<a href="#">Jupiter</a>	237.1015	3.1189
<a href="#">Saturn</a>	99.4587	26.7285
<a href="#">Uranus</a>	5.4634	82.2298
<a href="#">Neptune</a>	182.2100	27.8477
<a href="#">Pluto</a>	184.5484	119.6075

These values are different than those given on the NASA Fact Sheet (<https://nssdc.gsfc.nasa.gov/planetary/planetfact.html>), because there they are measured relative to the [ecliptic](#) of the [Earth](#) (the plane of the orbit of the [Earth](#)), while here they are measured relative to the [ecliptic](#) of the [planet](#) (the plane of the orbit of the [planet](#)).

If you use the orbital elements of a [planet](#) that are measured relative to the ecliptic of the Earth, then the [coordinates](#) that you calculate from those elements are relative to the Earth's ecliptic, too. You can then meaningfully compare those coordinates to coordinates of [stars](#) and other objects in [star](#) atlases and catalogs made for use on Earth, to see where the planet is compared to those [stars](#) and other objects, but you cannot use them to calculate where the [Sun](#) is relative to the [horizon](#) on that planet.

On the other hand, if you use orbital elements that are measured relative to the ecliptic of the planet (as on this page), then you cannot meaningfully compare the resulting coordinates with [star](#) atlases and catalogs that were made for use on Earth, but you can use them to calculate where the [Sun](#) is relative to the [horizon](#) of that planet.

## 5. The [Ecliptical Coordinates](#)

The ecliptical [longitude](#)  $\lambda$  (lambda) is the position along the [ecliptic](#), relative to the [vernal equinox](#) (so relative to the [stars](#)). The mean longitude  $L$  is the ecliptical longitude that the [planet](#) would have if the orbit were a perfect circle. That is

$$L = M + \Pi \quad (6)$$

The [ecliptic](#) longitude of the [planet](#) as seen from the [Sun](#) is equal to

$$\lambda = v + \Pi = M + \Pi + C = L + C \quad (7)$$

If you look at the [Sun](#) from the [planet](#), then you're looking in exactly the opposite direction than if you look at the planet from the [Sun](#), so those directions are  $180^\circ$  apart. So, the ecliptic longitude of the [Sun](#), as seen from the planet, is equal to

$$L_{\text{sun}} = L + 180^\circ = M + \Pi + 180^\circ \quad (8)$$

$$\lambda_{\text{sun}} = \lambda + 180^\circ = v + \Pi + 180^\circ = M + \Pi + C + 180^\circ = L_{\text{sun}} + C \quad (9)$$

The value of  $\lambda_{\text{sun}}$  determines when the (astronomical) [seasons](#) begin: when  $\lambda_{\text{sun}} = 0^\circ$ , then [spring](#) begins in the northern hemisphere, and [autumn](#) in the southern hemisphere. Each next multiple of  $90^\circ$  brings the start of the next [season](#).

The ecliptic [latitude](#)  $\beta_{\text{sun}}$  (beta) of the Sun, the perpendicular distance of the Sun from the ecliptic, is always so small that we can ignore it here. With this, we now have the ecliptic coordinates of the Sun.

As seen from [Earth](#) we find  $\lambda_{\text{sun}} = 372.0322^\circ = 12.0322^\circ \pmod{360^\circ}$  and as seen from [Mars](#)  $\lambda_{\text{sun}} = 373.0664^\circ = 13.0664^\circ \pmod{360^\circ}$ .

## 6. The [Equatorial coordinates](#)

The equatorial [coordinate system](#) in the sky is tied to the rotation axis of the [planet](#). The equatorial coordinates are the [right ascension](#)  $\alpha$  (alpha) and the [declination](#)  $\delta$  (delta). The declination determines from which parts of the [planet](#) the object can be visible, and the right ascension determines (together with other things) when the object is visible.

With these formulas you can calculate the equatorial coordinates from the [ecliptic](#) coordinates:

$$\sin \alpha \cos \delta = \sin \lambda \cos \varepsilon \cos \beta - \sin \beta \sin \varepsilon \quad (10)$$

$$\cos \alpha \cos \delta = \cos \lambda \cos \beta \quad (11)$$

$$\sin \delta = \sin \beta \cos \varepsilon + \cos \beta \sin \varepsilon \sin \lambda \quad (12)$$

For the [Sun](#) we have  $\beta_{\text{sun}} = 0$ , so then

$$\alpha_{\text{sun}} = \arctan(\sin \lambda_{\text{sun}} \cos \varepsilon, \cos \lambda_{\text{sun}}) \quad (13)$$

$$\delta_{\text{sun}} = \arcsin(\sin \lambda_{\text{sun}} \sin \varepsilon) \quad (14)$$

For future reference we define

$$\alpha_{\text{sun}} = \lambda_{\text{sun}} + S \quad (15)$$

If  $\varepsilon$  is close enough to  $0^\circ$  or  $180^\circ$ , and if we neglect small terms, then we can approximate the relationship between the right ascension  $\alpha_{\text{sun}}$  and the [ecliptic longitude](#)  $\lambda_{\text{sun}}$  of the [Sun](#) as seen from the [planet](#) by

$$\begin{aligned} \arctan(\tan(\lambda) \cos(\varepsilon)) &= \lambda - \left( \frac{1}{4} \varepsilon^2 + \frac{1}{24} \varepsilon^4 + \frac{17}{2880} \varepsilon^6 \right) \sin(2\lambda) \\ &\quad + \left( \frac{1}{32} \varepsilon^4 + \frac{1}{96} \varepsilon^6 \right) \sin(4\lambda) \\ &\quad - \frac{1}{192} \varepsilon^6 \sin(6\lambda) + O(\varepsilon^8) \end{aligned} \quad (16)$$

$$\alpha_{\text{sun}} = \lambda_{\text{sun}} + S \approx \lambda_{\text{sun}} + A_2 \sin(2\lambda_{\text{sun}}) + A_4 \sin(4\lambda_{\text{sun}}) + A_6 \sin(6\lambda_{\text{sun}}) \quad (17)$$

and the relationship between the declination  $\delta_{\text{sun}}$  and the ecliptic longitude by

$$\begin{aligned} \arcsin(\sin(\lambda) \sin(\varepsilon)) &= \left( \varepsilon - \frac{1}{6} \varepsilon^3 + \frac{1}{120} \varepsilon^5 \right) \sin(\lambda) \\ &\quad + \left( \frac{1}{6} \varepsilon^3 - \frac{1}{12} \varepsilon^5 \right) \sin^3(\lambda) \\ &\quad + \frac{3}{40} \varepsilon^5 \sin^5(\lambda) + O(\varepsilon^7) \end{aligned} \quad (18)$$

$$\delta_{\text{sun}} \approx D_1 \sin(\lambda_{\text{sun}}) + D_3 \sin^3(\lambda_{\text{sun}}) + D_5 \sin^5(\lambda_{\text{sun}}) \quad (19)$$

with  $A_2$ ,  $A_4$ ,  $A_6$ ,  $D_1$ ,  $D_3$ , and  $D_5$  (measured in [degrees](#)) from the following table. The columns  $E_A$  and  $E_D$  show the greatest errors you make for  $\alpha_{\text{sun}}$  and  $\delta_{\text{sun}}$  when you use these approximations.

	$A_2$	$A_4$	$A_6$	$E_A$	$D_1$	$D_3$	$D_5$	$E_D$
<a href="#">Mercury</a>	-0.0000			0.0000	0.0351			0.0000
<a href="#">Venus</a>	-0.0304			0.0001	2.6367	0.0009		0.0036
<a href="#">Earth</a>	-2.4657	0.0529	-0.0014	0.0003	22.7908	0.5991	0.0492	0.0003
<a href="#">Mars</a>	-2.8608	0.0713	-0.0022	0.0004	24.3880	0.7332	0.0706	0.0011
<a href="#">Jupiter</a>	-0.0425			0.0001	3.1173	0.0015		0.0034
<a href="#">Saturn</a>	-3.2338	0.0909	-0.0031	0.0009	25.7696	0.8640	0.0949	0.0010
<a href="#">Uranus</a>	-42.5874	12.8117	-2.6077	17.6902	56.9083	-0.8433	26.1648	3.34
<a href="#">Neptune</a>	-3.5214	0.1078	-0.0039	0.0163	26.7643	0.9669	0.1166	0.060
<a href="#">Pluto</a>	-19.3248	3.0286	-0.4092	0.5052	49.8309	4.9707	5.5910	0.19

The approximations for [Uranus](#) aren't very good, because [Uranus](#) lies almost on its side: You'll do better to use the full formulas for Uranus.

As seen from [Earth](#), and using equation 13, we find:

$$\alpha_{\text{sun}} = \arctan(\sin(12.0322^\circ) \times \cos(23.4393^\circ), \cos(12.0322^\circ)) = 11.0649^\circ$$

and with equation 14 we find

$$\delta_{\text{sun}} = \arcsin(\sin(12.0322^\circ) \times \sin(23.4393^\circ)) = 4.7565^\circ$$

As seen from [Mars](#), we find

$$\alpha_{\text{sun}} = \arctan(\sin(13.0664^\circ) \times \cos(25.1918^\circ), \cos(13.0664^\circ)) = 11.8605^\circ$$

and

$$\delta_{\text{sun}} = \arcsin(\sin(13.0664^\circ) \times \sin(25.1918^\circ)) = 5.5222^\circ$$

Using equations 17 and 19 we find for the [Earth](#)

$$\begin{aligned} \alpha_{\text{sun}} &= 12.0322^\circ - 2.4657^\circ \times \sin(2 \times 12.0322^\circ) \\ &\quad + 0.0529^\circ \times \sin(4 \times 12.0322^\circ) \\ &\quad - 0.0014^\circ \times \sin(6 \times 12.0322^\circ) = 11.0649^\circ \\ \delta_{\text{sun}} &= 22.7908^\circ \times \sin(12.0322^\circ) \\ &\quad + 0.5991^\circ \times \sin(12.0322^\circ)^3 \\ &\quad + 0.0492^\circ \times \sin(12.0322^\circ)^5 = 4.7565^\circ \end{aligned}$$

and for [Mars](#)

$$\begin{aligned}
\alpha_{\text{sun}} &= 13.0664^\circ - 2.8608^\circ \times \sin(2 \times 13.0664^\circ) \\
&\quad + 0.0713^\circ \times \sin(4 \times 13.0664^\circ) \\
&\quad - 0.0022^\circ \times \sin(6 \times 13.0664^\circ) = 11.8605^\circ \\
\delta_{\text{sun}} &= 24.3880^\circ \times \sin(13.0664^\circ) \\
&\quad + 0.7332^\circ \times \sin(13.0664^\circ)^3 \\
&\quad + 0.0706^\circ \times \sin(13.0664^\circ)^5 = 5.5221^\circ
\end{aligned}$$

so the approximations yield practically the same results in these cases as the full equations [13](#) and [14](#).

## 7. The Observer

Where a celestial body is in your sky depends on your [geographical coordinates](#) ([latitude](#)  $\varphi$  [phi] north, [longitude](#)  $l_w$  west), on the position of the body between the [stars](#) (its [equatorial](#) coordinates  $\alpha$  and  $\delta$ ), and on the rotation [angle](#) of the [planet](#) at your location, relative to the [stars](#). That latter angle is expressed in the [sidereal time](#)  $\theta$  (theta). The [sidereal time](#) is the [right ascension](#) that is on the celestial [meridian](#) at that moment. The sidereal time is equal to

$$\theta = \theta_0 + \theta_1 \times (J - J_{2000}) - l_w \pmod{360^\circ} \quad (20)$$

with  $\theta_0$  and  $\theta_1$  from the next table. The  $\pmod{360^\circ}$  means that on both sides of the  $=$  you can add or subtract any multiple of  $360^\circ$ . For  $\theta$  the custom is to choose the value between  $0^\circ$  and  $360^\circ$ .

	$\theta_0$	$\theta_1$
<a href="#">Mercury</a>	132.3282	6.1385025
<a href="#">Venus</a>	104.9067	-1.4813688
<a href="#">Earth</a>	280.1470	360.9856235
<a href="#">Mars</a>	313.3827	350.89198226
<a href="#">Jupiter</a>	145.9722	870.5360000
<a href="#">Saturn</a>	174.3508	810.7939024
<a href="#">Uranus</a>	29.6474	-501.1600928
<a href="#">Neptune</a>	52.4160	536.3128662
<a href="#">Pluto</a>	122.2370	56.3625225

$\theta_0$  is the [sidereal time](#) (in [degrees](#)) at [longitude](#)  $0^\circ$  at the instant defined by  $J_{2000}$ .  $\theta_1$  is the rate of change of the [sidereal time](#), in degrees per day.

For the Netherlands on [Earth](#) we find

$$\theta_{\text{aarde}} = 560529.8347^\circ - (-5^\circ) = 14.8347^\circ \pmod{360^\circ}$$

and for Gusev on [Mars](#)

$$\theta_{\text{mars}} = 544897.7392^\circ - 184.6^\circ = 33.1392^\circ \pmod{360^\circ}$$

The position of a celestial body in the sky is specified by its [altitude](#)  $h$  above the [horizon](#), and its [azimuth](#)  $A$ . The altitude is  $0^\circ$  at the [horizon](#),  $+90^\circ$  in the [zenith](#) (straight over your head), and  $-90^\circ$  in the [nadir](#) (straight down). The azimuth is the direction along the horizon, which we measure from south to west. South has azimuth  $0^\circ$ , west  $+90^\circ$ , north  $+180^\circ$ , and east  $+270^\circ$  (or  $-90^\circ$ , that's the same thing). The altitude and azimuth are the [horizontal coordinates](#). To calculate the horizontal coordinates from the [equatorial](#) coordinates, you can use the following formulas:

$$\sin A \cos h = \sin H \cos \delta \quad (21)$$

$$\cos A \cos h = \cos H \sin \varphi \cos \delta - \sin \delta \cos \varphi \quad (22)$$

$$\sin h = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H \quad (23)$$

$$H = \theta - \alpha \quad (24)$$

$$A = \arctan(\sin H, \cos H \sin \varphi - \tan \delta \cos \varphi) \quad (25)$$

The  $H$  is the [hour angle](#), which indicates how long ago (measured in sidereal time) the celestial body passed through the celestial [meridian](#).

For the Netherlands on [Earth](#) we find

$$\begin{aligned}
H &= 14.8347^\circ - 11.0649^\circ = 3.7698^\circ \\
A &= \arctan(\sin(3.7698^\circ), \cos(3.7698^\circ) \times \sin(52^\circ) \\
&\quad - \tan(4.7565^\circ) \times \cos(52^\circ)) = 5.1111^\circ \\
h &= \arcsin(\sin(52^\circ) \times \sin(4.7565^\circ) \\
&\quad + \cos(52^\circ) \times \cos(4.7565^\circ) \times \cos(3.7698^\circ)) = 42.6530^\circ
\end{aligned}$$

For Gusev on [Mars](#) we find

$$\begin{aligned}
H &= 21.2786^\circ \\
A &= \arctan(\sin(21.2786^\circ), \cos(21.2786^\circ) \times \sin(-14.6^\circ) \\
&\quad - \tan(5.5222^\circ) \times \cos(-14.6^\circ)) = 132.1463^\circ \\
h &= \arcsin(\sin(-14.6^\circ) \times \sin(5.5222^\circ) \\
&\quad + \cos(-14.6^\circ) \times \cos(5.5222^\circ) \times \cos(21.2786^\circ)) = 60.8439^\circ
\end{aligned}$$

At 12:00 [UTC](#) on 1 April 2004, the [Sun](#) as seen from the Netherlands stands about  $5^\circ$  west of south at  $43^\circ$  above the horizon, and as seen from the Gusev crater on Mars the [Sun](#) then stands about  $3^\circ$  south by northwest at  $61^\circ$  above the horizon.

## 8. [Solar Transit](#)

The [transit](#) of a celestial body is the moment at which the body passes through the celestial [meridian](#). The [transit](#) of the [Sun](#) is noon, the middle of the day, at 12 [hours solar time](#). The [hour angle](#)  $H$  of the [Sun](#) is then equal to 0. We have

$$\theta = \alpha_{\text{sun}} \pmod{360^\circ} \quad (26)$$

Using this and previous equations, you can find the [time](#) of transit by making a guess for a value of  $J$  for which equation 26 holds, calculating  $\theta$  and  $\alpha_{\text{sun}}$  for it, and then checking whether they satisfy equation 26. If they do not, then you have to adjust  $J$ . All in all, it is a big search operation for the correct value of  $J$ . Moreover, equation 26 does not help you to understand which things are most important to the time of solar transit. You do get such insight if you approximate the solution by neglecting smaller terms. The answer may then not be as accurate as the correct solution, but does indicate clearly what the solution looks like, is usually easier to calculate, and provides an excellent starting guess in the search for the real value of  $J$ .

With equations 1, 9, 5, 17, and 20, and after omitting smaller terms, we find

$$J_{\text{transit}} = J_{2000} + J_0 + l_w \frac{J_3}{360^\circ} + J_1 \sin M + J_2 \sin(2L_{\text{sun}}) \pmod{J_3} \quad (27)$$

where

$$L_{\text{sun}} = M + \Pi + 180^\circ \quad (28)$$

$$J_0 = (M_0 + \Pi + 180^\circ - \theta_0) \frac{J_3}{360^\circ} \pmod{J_3} \quad (29)$$

$$J_1 = C_1 \frac{J_3}{360^\circ} \quad (30)$$

$$J_2 = A_2 \frac{J_3}{360^\circ} \quad (31)$$

$$J_3 = \frac{360^\circ}{\theta_1 - M_1} \quad (32)$$

If you already have  $\lambda_{\text{Sun}}$ , then you can use it instead of  $L_{\text{Sun}}$  in equation 27: that is a tiny bit more accurate.  $J_0$  provides the date and time of a transit of the [Sun](#).  $J_1$  shows by how much the time of transit can vary because of the [eccentricity](#)  $e$  of the orbit.  $J_2$  indicates how much the time of transit can change because of the obliquity  $\varepsilon$  of the [ecliptic](#).  $J_3$  is the average length of the solar day (from one transit to the next). All values are measured in [Earth](#) days of 24 hours.

	$J_0$	$J_1$	$J_2$	$J_3$
<a href="#">Mercury</a>	45.3497	11.4556		175.9386
<a href="#">Venus</a>	52.1268	-0.2516	0.0099	-116.7505
<a href="#">Earth</a>	0.0009	0.0053	-0.0068	1.0000000
<a href="#">Mars</a>	0.9047	0.0305	-0.0082	1.027491
<a href="#">Jupiter</a>	0.3345	0.0064		0.4135778
<a href="#">Saturn</a>	0.0766	0.0078	-0.0040	0.4440276
<a href="#">Uranus</a>	0.1260	-0.0106	0.0850	-0.7183165
<a href="#">Neptune</a>	0.3841	0.0019	-0.0066	0.6712575
<a href="#">Pluto</a>	4.5635	-0.5024	0.3429	6.387672

To find the date and [time](#) of a [solar transit](#) near [Julian Date](#)  $J$ , you now proceed as follows:

1. Calculate

$$n_x = \frac{J - J_{2000} - J_0}{J_3} - \frac{l_w}{360^\circ} \quad (33)$$

and then take for  $n$  the whole [number](#) nearest to  $n_x$ .

2. Then calculate

$$J_x = J + J_3 \times (n - n_x) \quad (34)$$

This  $J_x$  is a reasonable estimate for the date and time of the [transit](#) near  $J$ , except that the  $J_1$  and  $J_2$  corrections are not in it yet.

3. Calculate  $M$  and  $L_{\text{sun}}$  for  $J_x$  and then get a better estimate for the date and time of the solar [transit](#) from

$$J_{\text{transit}} \approx J_x + J_1 \sin M + J_2 \sin(2L_{\text{sun}}) \quad (35)$$

4. If you want greater precision, then calculate the [hour angle](#)  $H$  of the [Sun](#) for  $J_{\text{transit}}$  and take

$$J_{\text{transit}} - \frac{H}{360^\circ} \times J_3 \quad (36)$$

as improved value of  $J_{\text{transit}}$ . You can repeat this until  $J_{\text{transit}}$  no longer changes.

5.  $J_{\text{transit}}$  is a Julian Date, which counts days and which is equal to a whole number at 12:00 [UTC](#) (i.e., noon, UTC). So, a Julian Date like 2453096.9898 that ends in .9898 is .0102 days before the next whole number, i.e.,  $0.0102 \text{ days} = 0.0102 \times 24 = 0.245 \text{ hours} = 0.0102 \times 24 \times 60 = \text{about } 15 \text{ minutes} before 12:00 UTC, i.e., about 11:45 UTC.$

For our example, we looked near  $J = 2453097$ . Which solar transit is closest to that in the Netherlands and in Gusev crater on [Mars](#)? For the Netherlands ( $l_w = -5^\circ$ ) we find

$$n_x = \frac{2453097 - 2451545 - 0.0009}{1} - \frac{-5^\circ}{360^\circ} = 1552.0130$$

so  $n = 1552$ , so

$$J_x = 2453097 + 1 \times (1552 - 1552.0130) = 2453096.9870$$

For  $J_x$  we find  $M = 87.1679^\circ$  and then

$$L_{\text{sun}} = 87.1679^\circ + 102.9372^\circ + 180^\circ = 370.1051^\circ = 10.1051^\circ$$

With that, we find

$$J_{\text{transit}} = 2453096.9870 + 0.0053 \times \sin(87.1679^\circ) - 0.0068 \times \sin(2 \times 10.1051^\circ) = 2453096.9766$$

The repetition method provides increased accuracy. For [JD](#) 2453096.9766 we find  $H = -4.6663^\circ$ , so a better estimate is  $J_{\text{transit}} = 2453096.9766 - \frac{-4.6663^\circ}{360^\circ} \times 1 = 2453096.9895$ . For that moment we find  $H$  equal to 0 to four digits after the decimal point, which is sufficiently precise. The solar transit at  $5^\circ$  east [longitude](#) happens on 1 April 2004 around 11:45 UTC.

For the Gusev crater on [Mars](#) ( $l_w = 184.6^\circ$ ) we find

$$n_x = \frac{2453097 - 2451545 - 0.9047}{1.027491} - \frac{184.6^\circ}{360^\circ} = 1509.0822$$

so  $n = 1509$ , so

$$J_x = 2451545 + 1.027491 \times (1509 - 1509.0822) = 2453096.9155$$

For  $J_x$  we find  $M = 112.5978^\circ$  and

$$L_{\text{sun}} = 112.5978^\circ + 70.9812^\circ + 180^\circ = 363.5790^\circ = 3.5790^\circ$$

With that, we find

$$J_{\text{transit}} = 2453096.9155 + 0.0305 \times \sin(112.5978^\circ) - 0.0082 \times \sin(2 \times 3.5891^\circ) = 2453096.8945$$

Now we use the repetition method. For [JD](#) 2453096.8945 we find  $H = -15.6787^\circ$ , so a better estimate is  $J_{\text{transit}} = 2453096.8945 - \frac{-15.6787^\circ}{360^\circ} \times 1.027491 = 2453096.9393$ . That yields no further change to  $J_{\text{transit}}$  (to four digits after the decimal point) so we are done. The solar transit in Gusev crater happens on 1 April 2004 around 10:32 UTC.

## 9. Equation of Time

Above, we calculated when the [Sun](#) is highest in the sky, expressed in the Earthly time scale of the [Julian Date](#). With this, you still don't know at what time that is on your clock, and that time scale is definitely not very handy if you are on another [planet](#) where the day has a different length from the day on [Earth](#) that the Julian Date is based on.

If you are not interested in transforming to times and dates for other [planets](#), then you just want to know at what time the [Sun](#) is highest in the sky on your [planet](#) according to your clock, which is tied to your [solar time](#) and the [season](#) on your [planet](#). [Solar time](#) is the time determined by the [Sun](#). Three different kinds of solar time are relevant:

- The true solar time is a time scale in which the [Sun](#) is highest in the sky at exactly 12:00:00 [hours](#) each day, but not every day needs be equally long (as measured using something that does not care about the position of the Sun in the sky, such as an atomic clock or planetary phenomena in the sky). You can read true solar time from a well-adjusted sundial.
- The mean solar time is a time scale that runs in step with true solar time on average but in which all days are equally long. Averaged over many planet days and planet [years](#), the Sun is highest in the sky at 12:00:00 mean solar time. You can read mean solar time from a well-adjusted clock or watch. This time is often referred to as "Local Time" in these pages.
- The official clock time that you use in daily [life](#) is the mean solar time of a certain [meridian](#) that belongs to your time zone. The difference between official clock time and the mean solar time of a location depends only on the difference between the [geographical longitude](#) of that place and the longitude of the meridian that the official clock time is based on.

The answer to the question at what time the Sun is highest in the sky depends a lot on the time scale that you use. In true solar time, the answer is "always at exactly 12 o'clock", but true solar time is otherwise not very convenient at all. You can't use it if the



Sun does not shine or if it is hidden behind clouds, and a mechanical clock or watch would have to be a lot more complicated if it had to show true solar time. In mean solar time, the answer is "on average at 12 o'clock", but it may differ from 12 o'clock from day to day. In official clock time, the answer is "on average at a fixed time", but that fixed time depends on your location, though it is usually near 12 o'clock.

So how large is the difference between the mean solar time and the true solar time? This is called the Equation of Time. The Equation of Time is how much you have to add to true solar time ("sundial time") to get mean solar time ("local time").

We found earlier (equation 26) for the moment at which the Sun is highest in the sky ([transit](#)) that  $\theta = \alpha_{\text{Sun}} \pmod{360^\circ}$ . For the [right ascension](#)  $\alpha_{\text{Sun}}$  of the Sun we found  $\alpha_{\text{Sun}} = L + C + S$ . For the [sidereal time](#)  $\theta$  measured in [degrees](#) we construct a different formula:

$$\theta = (L + t + 180^\circ) \pmod{360^\circ} \quad (37)$$

Here  $t$  is the mean solar time measured in degrees ( $0^\circ$  = midnight,  $180^\circ$  = noon). The explanation for this equation is as follows: The difference between the [sidereal](#) time and the solar time reflects the motion of the planet around the Sun so it increases by  $360^\circ$  in a planet [year](#). The sidereal time is tied to the regular rotation of the planet around its axis, which does not notice the varying speed at which the planet orbits around the Sun (associated with [eccentricity](#)  $e$ ) or the position of the Sun above or below the celestial equator (associated with the obliquity  $\varepsilon$  of the [ecliptic](#)). Therefore, the sidereal time is tied to the mean longitude  $L$  of the Sun, which increases at a fixed rate by  $360^\circ$  during a planet [year](#), and which is independent of  $e$  and  $\varepsilon$ . During the [descending equinox](#) (when  $\lambda_{\text{Sun}} = L = 180^\circ$ ) the sidereal time is equal to the mean solar time, so we need the extra  $180^\circ$  in the formula. During a planet day,  $\theta$  increases by  $360^\circ$  (the planet turns once around its axis) plus or minus a small amount because the sidereal time runs a little faster or slower than the mean solar time, but that difference is already caught in  $L$ .

If we now set  $\theta$  equal to  $\alpha_{\text{Sun}}$  then we find

$$L + t + 180^\circ = \theta = \alpha_{\text{Sun}} = L + C + S \pmod{360^\circ} \quad (38)$$

or

$$t = 180^\circ + C + S \pmod{24} \quad (39)$$

The Equation of Time  $\Delta t$  is equal to the difference between  $t$  and  $180^\circ$  (because  $180^\circ$  corresponds to 12:00:00 local mean solar time):

$$\Delta t = C + S \quad (40)$$

If you divide the Equation of Time measured in degrees by 15 then you get the Equation of Time measured in hours.

$C$  depends on eccentricity  $e$  of the planet's orbit and on the mean [anomaly](#)  $M$  of the planet, i.e., on where the planet is compared to its [perihelion](#).  $S$  depends on the obliquity  $\varepsilon$  of the [ecliptic](#) of the planet and on the longitude  $\lambda_{\text{Sun}}$  of the Sun, i.e., on the season. A first-order approximation is:

$$\Delta t \approx C_1 \sin M + A_2 \sin(2\lambda_{\text{Sun}}) \quad (41)$$

where we've omitted many smaller terms. The following table shows the amplitudes, i.e., the largest contribution that the  $C$  and  $S$  terms give to the Equation of Time for each planet, measured in planet [minutes](#).

	$C$	$S$
<a href="#">Mercury</a>	94.5	0
<a href="#">Venus</a>	3.1	0.1
<a href="#">Earth</a>	7.7	9.9
<a href="#">Mars</a>	42.8	11.4
<a href="#">Jupiter</a>	22.2	0.2
<a href="#">Saturn</a>	25.4	13.0
<a href="#">Uranus</a>	21.2	178.1
<a href="#">Neptune</a>	4.1	14.1
<a href="#">Pluto</a>	114.6	69.3

For the [Earth](#), the contributions of the orbit ( $C$ ) and the [season](#) ( $S$ ) are about equally large; for [Uranus](#) and [Neptune](#) the influence of the season is much greater than that of the orbit; and for the other [planets](#) the influence of the orbit is much greater than the influence of the [seasons](#).

## 10. Sunrise and Sunset

For the [hour angle](#) that corresponds to  $h = 0$  we find

$$H = \arccos(-\tan \delta \tan \varphi) \quad (42)$$

This equation has two solutions: if  $H$  is a solution, then  $-H$  is a solution, too. The solution with  $H > 0$  is associated with sunset, and the solution with  $H < 0$  is associated with sunrise. We write  $H_+$  for the solution that has  $H \geq 0$ . By using formulas found earlier and by omitting small terms, we find

$$\begin{aligned} H_+ &\approx 90^\circ + H_1 \sin \lambda_{\text{Sun}} \tan \varphi \\ &\quad + H_3 \sin^3 \lambda_{\text{Sun}} \tan \varphi \times (3 + \tan^2 \varphi) \\ &\quad + H_5 \sin^5 \lambda_{\text{Sun}} \tan \varphi \times (15 + 10 \tan^2 \varphi + 3 \tan^4 \varphi) \end{aligned} \quad (43)$$



with

$$H_1 = \varepsilon - \frac{1}{6}\varepsilon^3 + \frac{1}{120}\varepsilon^5 \quad (44)$$

$$H_3 = \frac{1}{6}\varepsilon^3 - \frac{1}{12}\varepsilon^5 \quad (45)$$

$$H_5 = \frac{1}{40}\varepsilon^5 \quad (46)$$

where you must measure  $\varepsilon$  in radians instead of in [degrees](#). You transform from degrees to radians by multiplying the [number](#) of degrees by  $\pi/180 \approx 0.017453292$ .

The values of  $H_1$ ,  $H_3$ , and  $H_5$  are listed for all [planets](#) in the following table, measured in degrees. For [Uranus](#) and [Pluto](#) it is doubtful that the approximations yield reasonable results.

	$H_1$	$H_3$	$H_5$
<a href="#">Mercury</a>	0.035		
<a href="#">Venus</a>	2.636	0.001	
<a href="#">Earth</a>	22.137	0.599	0.016
<a href="#">Mars</a>	23.576	0.733	0.024
<a href="#">Jupiter</a>	3.116	0.002	
<a href="#">Saturn</a>	24.800	0.864	0.032
<a href="#">Uranus</a>	28.680	-0.843	8.722
<a href="#">Neptune</a>	26.668	0.967	0.039
<a href="#">Pluto</a>	38.648	4.971	1.864

[Sunrise](#) is the moment at which the top of the [solar](#) disk touches the [horizon](#) in the morning. [Sunset](#) is the same, but at night. To calculate the times of sunrise and sunset, you must take into account (besides the effects mentioned earlier) also the following things:

1. The [Sun](#) does not look like a point but like a disk, so when the center of the [Sun](#) sets ( $h = 0$ ), then half of the [Sun](#) is still above the [horizon](#). The size of the [Sun](#) must be taken into account.
2. The [atmosphere](#) of the [planet](#) (if it has one) bends [light](#) downward so that a celestial body appears slightly higher in the sky than it would have done without the atmosphere. This refraction of the light is strongest near the horizon, where the light has had to travel furthest through the air. On [Earth](#), at the horizon as seen from sea level, the refraction amounts to some  $0.57^\circ$  on average. This means that a celestial body that with the atmosphere just set would have already been  $0.57^\circ$  below the horizon without the atmosphere.

To compensate for these two effects, you can set  $h$  equal to the  $h_0$  from the following table (measured in [degrees](#)), instead of  $0^\circ$ . The refraction has been added in to the value for the [Earth](#), but not for the other [planets](#), because they have either no atmosphere of any consequence, or else they have one that is so dense that you cannot see the Sun at all from the surface. The table also gives the average diameter  $d_{\text{Sun}}$  of the solar disk (in degrees).

	$h_0$	$d_{\text{Sun}}$	$\sin(h_0)$
<a href="#">Mercury</a>	-0.69	1.38	-0.0120
<a href="#">Venus</a>	-0.37	0.74	-0.0064
<a href="#">Earth</a>	-0.83	0.53	-0.0146
<a href="#">Mars</a>	-0.17	0.35	-0.0031
<a href="#">Jupiter</a>	-0.05	0.10	-0.0009
<a href="#">Saturn</a>	-0.03	0.06	-0.0005
<a href="#">Uranus</a>	-0.01	0.03	-0.0002
<a href="#">Neptune</a>	-0.01	0.02	-0.0002
<a href="#">Pluto</a>	-0.01	0.01	-0.0001

With equation [23](#) you can then find the [hour angle](#) at which the [Sun](#) with that  $\alpha_{\text{Sun}}, \delta_{\text{Sun}}$  rises or sets:

$$H_t = \arccos\left(\frac{\sin h_0 - \sin \varphi \sin \delta}{\cos \varphi \cos \delta}\right) \quad (47)$$

Here again,  $H = H_t > 0$  holds for [sunset](#) and  $H = -H_t < 0$  for [sunrise](#). The estimated [JD](#) of sunrise and sunset are then

$$J_{\text{rise}} = J_{\text{transit}} - \frac{H_t}{360^\circ} J_3 \quad (48)$$

$$J_{\text{set}} = J_{\text{transit}} + \frac{H_t}{360^\circ} J_3 \quad (49)$$

You can improve these estimates by repetition similar to how we did it for the [transit](#). For  $J_{\text{rise}}$  or  $J_{\text{set}}$  calculate the [hour angle](#)  $H$  and [declination](#)  $\delta_{\text{Sun}}$  of the [Sun](#), and then use equation [47](#) to calculate the hour angle  $\pm H_t$  that the [Sun](#) at those [equatorial coordinates](#) should have to be rising or setting. Then replace the estimates by

$$J_{\text{rise}} - \frac{H + H_t}{360^\circ} J_3 \quad (50)$$

$$J_{\text{set}} - \frac{H - H_t}{360^\circ} J_3 \quad (51)$$

until they don't change anymore, to the desired precision.

We found earlier that the [transit](#) of the [Sun](#) as seen from the [Earth](#) occurs at  $J_{\text{transit}} = 2453096.9895$ . The declination of the Sun is then  $\delta_{\text{Sun}} = 4.7545^\circ$ . According to the above table, we have for the [Earth](#)  $\sin h_0 = -0.0146$ . With  $\phi = 52^\circ$  we then find from equation 47 that

$$H_t = \arccos\left(\frac{-0.0146 - \sin(52^\circ)\sin(4.7545^\circ)}{\cos(52^\circ)\cos(4.7545^\circ)}\right) = 97.4841^\circ$$

The first estimate for the preceding sunrise is then

$$J_{\text{rise}} = 2453096.9895 - \frac{97.4841}{360} \times 1 = 2453096.7187$$

For that instant of [time](#) we calculate the position of the Sun and find  $H = -97.5043^\circ$  and  $\delta_{\text{sun}} = 4.6501^\circ$ , and hence  $H_t = 97.3483^\circ$ . Then the [JD](#) correction is  $-\frac{-97.5043+97.3483}{360} \times 1 = 0.0004$ , so a better estimate is  $J_{\text{rise}} = 2453096.7187 + 0.0004 = 2453096.7191$ .

If we repeat this improvement procedure once more then the correction is equal to 0 (to 4 digits after the decimal point), so we're done.

Sunrise occurs around 05:15 [UTC](#).

The first estimate for the following sunset is

$$J_{\text{set}} = 2453096.9895 + \frac{97.4841}{360} \times 1 = 2453097.2603$$

For that instant of time we calculate the position of the Sun and find  $H = 97.5042^\circ$  and  $\delta_{\text{sun}} = 4.8587^\circ$ , and hence  $H_t = 97.6199^\circ$ . Then the JD correction is  $-\frac{97.5042-97.6199}{360} \times 1 = 0.0003$ , so a better estimate is  $J_{\text{set}} = 2453097.2603 + 0.0003 = 2453097.2606$ .

If we repeat this improvement procedure once more then the correction is equal to 0 (to 4 digits after the decimal point), so we're done.

Sunset occurs on 1 April 2004 around 18:15 UTC.

We found earlier that the [transit](#) of the Sun as seen from Gusev crater on [Mars](#) occurs at  $J_{\text{transit}} = 2453096.9389$ . The declination of the Sun is then  $\delta_{\text{sun}} = 5.5000^\circ$ . According to the above table, we have for [Mars](#)  $\sin h_0 = -0.0031$ . With  $\phi = -14.6^\circ$  we then find from equation 47 that

$$H_t = \arccos\left(\frac{-0.0031 - \sin(-14.6^\circ)\sin(5.5000^\circ)}{\cos(-14.6^\circ)\cos(5.5000^\circ)}\right) = 88.7472^\circ$$

The first estimate for the preceding sunrise is then

$$J_{\text{rise}} = 2453096.9389 - \frac{88.7472}{360} \times 1.027491 = 2453096.6856$$

For that instant of time we calculate the position of the Sun and find  $H = -88.7690^\circ$  and  $\delta_{\text{sun}} = 5.4494^\circ$ , and hence  $H_t = 88.7605^\circ$ . Then the JD correction is  $-\frac{-88.7690+88.7605}{360} \times 1.027491 = 0.0000$ , so that first estimate was already accurate enough.

Sunrise in Gusev crater happens on 1 April 2004 around 04:27 UTC.

The first estimate for the following sunset is

$$J_{\text{set}} = 2453096.9389 + \frac{88.7472}{360} \times 1.027491 = 2453097.1922$$

For that instant of time we calculate the position of the Sun and find  $H = 88.7690^\circ$  and  $\delta_{\text{sun}} = 5.5507^\circ$ , and hence  $H_t = 88.7339^\circ$ . Then the JD correction is  $-\frac{88.7690-88.7339}{360} \times 1.027491 = -0.0001$ , so a better estimate is  $J_{\text{set}} = 2453097.1922 - 0.0001 = 2453097.1921$ . Another repeat does not improve this result anymore.

Sunset in Gusev crater happens on 1 April 2004 around 16:37 UTC.

If you watch sunrise or sunset from great [altitude](#), then there are two more corrections to be made:

3. The [horizon](#) appears to be lower, by an amount in [degrees](#) that, on Earth, is approximately equal to the square root of the altitude in kilometers. For an altitude of 10 km (6 mi), the [horizon](#) is about  $3.2^\circ$  lower.
4. There is more refraction of sunlight, because the sunlight almost touches the ground but then goes back up through the [atmosphere](#) to get to the observer. The total refraction is at most twice as great as it is at sea level, and I expect that the refraction from jet cruising altitude (10 km) above the Earth is almost at that maximum, so it is about  $0.5^\circ$  more than at sea level.

The correction  $\Delta H$  to equation 42 that is necessary because of the large Sun and the atmosphere is approximately equal to

$$\Delta H \approx - \frac{h_0}{\sqrt{\cos^2(\varphi) - \sin^2(\delta_{\text{sun}})}} \quad (52)$$

For the Netherlands and Belgium (with  $\varphi$  about  $50^\circ$ ) this is about 5 to 6 [minutes](#).

## 11. The Duration of [Sunrise](#) and [Sunset](#)

To calculate the duration of sunrise or sunset you must calculate the moment that the top of the [solar](#) disk touches the [horizon](#), as described above (with  $h_0$ ), and also the moment that the bottom of the solar disk touches the [horizon](#). To calculate that last one, you should use  $h_0 + d_0$  instead of  $h_0$ .

## 12. [Earth](#)

For places on [Earth](#), we can combine various approximations into the following (with results measured in [hours UTC](#)):

$$\begin{aligned} t_{\text{transit}} &\approx 12\text{h}00\text{m} + \frac{l_w}{15^\circ} + 24 \times (J_0 + J_1 \sin M + J_2 \sin 2L_{\text{Sun}}) \\ &= 12\text{h}01\text{m} + \frac{l_w}{15^\circ} + 7.6\text{m} \sin M - 9.9\text{m} \sin 2L_{\text{Sun}} \end{aligned} \quad (53)$$

$$\begin{aligned} t_{\text{rise}} &\approx t_{\text{transit}} - \frac{H}{15^\circ} \\ &\approx 6\text{h}00\text{m} + \frac{l_w}{15^\circ} + 24 \times (J_0 + J_1 \sin M + J_2 \sin 2L_{\text{Sun}}) \\ &\quad - \frac{H_1 \tan \varphi \sin L_{\text{Sun}} + H_3 \tan \varphi (3 + \tan^2 \varphi) \sin^3 L_{\text{Sun}}}{15^\circ} \\ &= 6\text{h}01\text{m} + \frac{l_w}{15^\circ} + 7.6\text{m} \sin M - 9.9\text{m} \sin 2L_{\text{Sun}} \\ &\quad - \left( 1\text{h}31\text{m} \tan \varphi \sin L_{\text{Sun}} + 2.2\text{m} \tan \varphi (3 + \tan^2 \varphi) \sin^3 L_{\text{Sun}} + \frac{\Delta H}{15^\circ} \right) \end{aligned} \quad (54)$$

$$\begin{aligned} t_{\text{set}} &\approx t_{\text{transit}} + \frac{H}{15^\circ} \\ &\approx 18\text{h}00\text{m} + \frac{l_w}{15^\circ} + 24 \times (J_0 + J_1 \sin M + J_2 \sin 2L_{\text{Sun}}) \\ &\quad + \frac{H_1 \tan \varphi \sin L_{\text{Sun}} + H_3 \tan \varphi (3 + \tan^2 \varphi) \sin^3 L_{\text{Sun}}}{15^\circ} \\ &= 18\text{h}01\text{m} + \frac{l_w}{15^\circ} + 7.6\text{m} \sin M - 9.9\text{m} \sin 2L_{\text{Sun}} \\ &\quad + \left( 1\text{h}31\text{m} \tan \varphi \sin L_{\text{Sun}} + 2.2\text{m} \tan \varphi (3 + \tan^2 \varphi) \sin^3 L_{\text{Sun}} + \frac{\Delta H}{15^\circ} \right) \end{aligned} \quad (55)$$

## 13. The Netherlands

For a spot at  $\varphi = 50^\circ$  and  $l_w = -5^\circ$  (in the middle of the Netherlands) we find, approximately, measured in Central European [Time](#) (CET, equal to [UTC](#) plus one [hour](#)):

$$t_{\text{rise}} \approx 6\text{h}34\text{m} - 1\text{h}48.6\text{m} \sin L - 13.2\text{m} \sin^3(L) + 7.6\text{m} \sin M - 9.9\text{m} \sin(2L) \quad (56)$$

$$t_{\text{transit}} \approx 12\text{h}40\text{m} + 7.6\text{m} \sin M - 9.9\text{m} \sin(2L) \quad (57)$$

$$t_{\text{set}} \approx 18\text{h}46\text{m} + 1\text{h}48.6\text{m} \sin L + 13.2\text{m} \sin^3(L) + 7.6\text{m} \sin M - 9.9\text{m} \sin(2L) \quad (58)$$

These formulas, though approximations, are reasonably accurate. A comparison, for all days of the [year](#) 2000, between the results of the full formulas and the results of the last approximate formulas shows differences of at most 3.5 [minutes](#) for the times of [sunrise](#) and [sunset](#), and less than 1 [minute](#) for the time of [transit](#).

## 14. Local dependence on location

From the preceding formulas we can deduce how the times of [sunrise](#), [transit](#), and [sunset](#) depend on the location (near any chosen spot). The times of sunrise, [transit](#), and sunset (measured in a fixed [time](#) zone) get earlier by 4 [minutes](#) for each [degree](#) that you go to the east. This corresponds to  $\frac{2.16}{\cos \varphi}$  [seconds](#) per kilometer. The time of [transit](#) (i.e., high noon) does not depend on the [geographical latitude](#). In addition, sunrise gets earlier and sunset later by a [number](#) of minutes per degree that you go north that is approximately equal to  $\frac{1.59 \sin(L)}{\cos^2(\varphi)}$  and that is equivalent to about  $\frac{0.86 \sin(L)}{\cos^2(\varphi)}$  seconds per kilometer. The shift of the times of sunrise and sunset with latitude is greatest at the beginning of [summer](#) and [winter](#). At the beginning of [spring](#) and [autumn](#) it is 0, so then the times of sunrise and sunset do not depend on the latitude (just like the time of transit).

## 15. Comparison with Planetarium Programs

The planetarium programs [Redshift 3](#), [Redshift 5](#), [Stellarium 0.15.0](#), the NASA web page [JPL HORIZONS](#), and the current procedure (AA) yield the following values for the [azimuth](#) and [altitude](#) of the [Sun](#) in the sky at [JD](#) 2451545.0 (1-1-2000, 12:00 [UTC](#)) as seen from the location with [latitude](#) and [longitude](#) equal to zero on each of the [planets](#):

[Redshift 5](#)

[Redshift 3](#)

[AA](#)

[Stellarium 0.15.0](#)

[JPL HORIZONS](#)

	A	h	A	h	A	h	A	h	A	h
<a href="#">Mercury</a>	89 59 36	-4 28 58	90 00	-4 21 27	01 32	-4 29 32	89 59 42.8	-4 22 03.8	90.0256	-4.4830
<a href="#">Venus</a>	263 39 25	-70 00 13	263 39	-70 00	83 39 27	-69 59 26	92 14 33.2	-14 59 18.5	263.6545	-70.0006
<a href="#">Earth</a>	178 03 33	+66 57 05	177 37	+66 57 35	18 25	+66 56 23	178 02 03.4	+66 57 37.5	178.0722	+66.9528
<a href="#">Mars</a>	233 17 03	+44 46 23	233 21	+44 41	53 03 40	+45 04 15	233 20 09.0	+44 43 21.0	233.2109	+44.8716
<a href="#">Jupiter</a>	275 36 38	-56 47 58	273 19	+22 27	93 19 50	+23 06 54	283 53 42.5	+77 07 12.8	273.3132	+22.3831
<a href="#">Saturn</a>	115 08 60	+33 21 18	115 08	+33 18 29	48 03	+32 59 53	115 08 41.7	+33 21 41.8	115.1490	+33.3541
<a href="#">Uranus</a>	223 07 12	+45 26 37	223 09	+45 25	44 13 02	+45 46 42	213 51 11.4	-51 47 54.9	223.1205	+45.4433
<a href="#">Neptune</a>	217 28 20	-54 09 28	218 03	-54 32	37 49 37	-54 44 36	217 55 18.7	-53 54 11.7	217.4714	-54.1581
<a href="#">Pluto</a>	125 32 00	-43 59 37	125 32	-44 00	125 35 22	-42 06 27	119 07 54.6	+30 48 30.8	305.5559	-42.1332

And here are similar results for [JD 2453097.0](#) (1-4-2004, 12:00 [UTC](#)):

	<a href="#">Redshift 5</a>		<a href="#">Redshift 3</a>		<a href="#">AA</a>		<a href="#">Stellarium 0.15.0</a>		<a href="#">JPL HORIZONS</a>	
	A	h	A	h	A	h	A	h	A	h
<a href="#">Mercury</a>	89 52 29	-87 19 22	89 01	-87 06 26	9 40	-87 19 34	89 56 12.4	-87 12 34.2	89.3290	-87.3182
<a href="#">Venus</a>	266 46 35	+35 02 25	266 47	+35 03	86 46 41	+35 02 37	264 45 51.5	-59 43 08.3	266.7781	+35.0387
<a href="#">Earth</a>	11 08 57	+85 07 29	11 10	+85 07 19	48 06	+85 05 14	11 13 57.3	+85 07 14.8	11.1353	+85.1259
<a href="#">Mars</a>	77 36 54	-63 14 20	77 46	-62 54 25	30 37	-63 34 42	77 38 47.5	-63 07 28.1	77.5625	-63.3588
<a href="#">Jupiter</a>	92 10 30	+46 10 15	91 36	+20 10 27	1 35 20	+19 04 53	96 19 20.0	+76 11 48.7	91.5977	+19.6703
<a href="#">Saturn</a>	231 05 23	+47 32 41	231 24	+47 11	50 42 48	+47 56 56	231 16 56.8	+47 20 24.3	231.0880	+47.5457
<a href="#">Uranus</a>	143 35 09	-72 11 32	144 19	-72 22 32	1 08 50	-72 43 10	109 18 45.9	+41 28 47.3	143.5871	-72.1924
<a href="#">Neptune</a>	173 45 46	-61 31 02	172 54	-61 57 35	2 58 27	-61 58 26	174 30 03.9	-61 33 40.9	173.7614	-61.5171
<a href="#">Pluto</a>	135 36 03	-41 02 35	135 37	-41 04 13	5 41 23	-39 00 02	127 19 52.2	+27 17 46.0	315.6817	-39.0309

If you take into account that I measure [azimuth](#) from the south but the other sources measure azimuth from the north (corresponding to a difference of 180°), then the results fit to within about a [degree](#), except that the position from [Jupiter](#) as calculated by [Redshift 5](#) is very different from that calculated by the other sources, and that the azimuth of the [Sun](#) from [Pluto](#) calculated by [JPL HORIZONS](#) differs by 180° from that of the other sources. I therefore suspect that [Redshift 5](#) is in error for [Jupiter](#). My azimuth for [Pluto](#) differs about 180° from that of [JPL](#) because I count azimuth from the south and [JPL](#) counts it from the north. The other sources are older than, or presumably haven't been updated for, the change in which of [Pluto's poles](#) is regarded as the north [pole](#).

## 16. Derivation of $\Pi$ and $\varepsilon$

Table [1](#) mentions  $\Pi$  en  $\varepsilon$ . How do you calculate those?

The orientation of the orbit of a [planet](#) in space is expressed by three independent [angles](#) relative to a chosen fundamental plane and a chosen primary direction in that fundamental plane. These angles are part of the orbital elements. We use the [inclination](#)  $i$ , the [longitude](#)  $\Omega$  of the ascending [node](#), and the argument  $\omega$  of the [perihelion](#) as the three angles. The inclination is the [angle](#) between the orbit and the fundamental plane. The ascending node is one of the two intersections between the orbit and the fundamental plane. The longitude of the ascending node is the angle between the primary direction and the ascending node, as seen from the relevant focus of the orbit. Likewise, the argument of the perihelion is the angle between the ascending node and the perihelion of the orbit.

The calculation of the position of the [Sun](#) in the sky of a [planet](#) is easiest if the fundamental plane is chosen to be equal to the plane of the orbit of the [planet](#), and the primary direction is chosen to be the intersection of the fundamental plane and the extension of the planet's [equator](#) into the sky. In the remainder of this text, we assume that that choice has been made. In the case of the [Earth](#), the fundamental plane is the plane of the [ecliptic](#), and the primary direction is the [vernal equinox](#), and the [coordinates](#) are with respect to "the equator and [equinox](#) of the date".

Then the inclination of the orbit is zero, and the [nodes](#) aren't defined, and the [ecliptic latitude](#) of the [Sun](#) is zero. In that case the only angle needed to specify the orientation of the orbit is the angle between the primary direction and the [perifocus](#) of the orbit, measured in the plane of the orbit. That is the planet-based ecliptic longitude of the perifocus. We use symbol  $\Pi$  for that angle.

Once the planet-based ecliptic coordinates of the [Sun](#) are known, then we convert them into planet-based [equatorial](#) coordinates. For this we need to know the angle  $\varepsilon$  between the fundamental plane (chosen to be the plane of the orbit of the planet) and the equator of the planet. For the [Earth](#), that angle is called the obliquity of the ecliptic.

So, to calculate the [Sun's](#) equatorial coordinates relative to the planet's equator, we need to know  $\Pi$  and  $\varepsilon$  of that planet.

How do we calculate  $\varepsilon$ ?  $\varepsilon$  is the angle between the plane of the planet's orbit and the plane of the planet's equator. This is also the angle between the primary [pole](#) of the planet (perpendicular to the planet's equator) and the primary pole of the planet's orbit (perpendicular to the planet's orbit). The planet and its orbit each have two [poles](#). One of those two has been chosen as the primary pole. Usually it is the north pole. See Chapter [21](#).

We calculate the direction of the planet's primary pole from its Earth-based equatorial coordinates, which are available on the internet. And we calculate the direction of the planet orbit's primary pole from its Earth-based orbital elements.

How do we calculate  $\Pi$ ?  $\Pi$  is the angle between the planet's primary direction and the perifocus of the planet's orbit. We can calculate the direction of the perifocus of the planet in Earth-based coordinates from the widely available Earth-based orbital elements of the planet, so we need to calculate the Earth-based coordinates of the planet's primary direction. That primary direction is one of the intersections of the plane of the planet's orbit (the fundamental plane) and the plane of the extension of the planet's equator, so it is perpendicular to the primary pole of the planet and to the primary pole of the orbit of the planet.

Here are the mathematical details. We assume that the following quantities are given:

$\alpha_p$  [right ascension](#) of north [pole](#)  
 $\delta_p$  [declination](#) of north [pole](#)  
 $\Omega$  [longitude](#) of ascending [node](#)  
 $i$  [inclination](#) of the orbit  
 $\omega$  argument of [perifocus](#)  
 $W_0$  rotation [angle](#) at [epoch](#)  
 $W_1$  rotation speed at [epoch](#)  
 $\varepsilon_0$  obliquity of the [ecliptic](#) of [Earth](#)

All of these orbital elements except for  $W_0$ ,  $W_1$  are assumed specified with respect to the [ecliptic](#) (= orbit) or [equator](#) of the [Earth](#), not those of the [planet](#).

We use the following notation:

- $C_{mn}$  is an [orthogonal matrix](#) of 3 by 3 elements that represents the conversion of cartesian [coordinates](#) from [coordinate system](#)  $n$  to [coordinate system](#)  $m$ .
- $\vec{v}_n$  for any  $\vec{v}$  is a column [vector](#) with 3 elements that represents the cartesian coordinates of a direction in space, with the coordinates specified in coordinate system  $n$ . The three elements of the vector are indicated as  $v_{1n}$ ,  $v_{2n}$ ,  $v_{3n}$ . The length of all vectors named below is equal to 1.
- $\vec{x}_{mn}$ ,  $\vec{y}_{mn}$ ,  $\vec{z}_{mn}$  are column vectors that represent three mutually perpendicular coordinate axes of coordinate system  $m$ , with the coordinates specified in coordinate system  $n$ .
- $\varepsilon_{mn}$  is the [angle](#) over which to rotate the equator of coordinate system  $n$  to get coordinate system  $m$ . The axis around which to rotate depends on the context.

The following directions are of interest. They are defined in terms of lines and planes in space, which should be imagined extended out to infinity. Parallel lines in space then correspond to two opposite points on the celestial sphere, of which one is called the primary one. Parallel planes in space then correspond to a [great circle](#) on the celestial sphere. All lines perpendicular to those planes correspond to two opposite points on the celestial sphere, which are called the [poles](#) of the plane, of which one is called the primary [pole](#) of the planes. Intersecting planes correspond to two great circles on the celestial sphere, which intersect in two points, of which one is called the primary intersection,

- The primary pole of the object. We define the primary pole to be the one from above which the object seems to rotate around its axis in the counterclockwise direction. For most objects that is what the [IAU](#) calls the north pole, but for [Venus](#) and [Uranus](#) it is what the IAU calls the south pole. See Sec. [21](#).
- The primary pole of the orbit of the object. The two poles of the orbit are the opposite directions perpendicular to the plane of the orbit of the object. The below formulas and values of  $i$  and  $\Omega$  define which of the two poles is the primary one.
- The primary direction of the orbit of the [planet](#). This is one of the two intersections of the plane of the equator of the [planet](#) with the plane of the orbit of the planet. The below formulas define which of the two intersections is the primary one. For the Earth, the [vernal equinox](#) is the primary direction.
- The primary location of the planet. This is the location with planetographic [longitude](#) and [latitude](#) equal to zero, which is the intersection of the [prime meridian](#) of the planet (longitude zero) and the equator of the planet (latitude zero).
- The primary intersection of the equator of the planet with the equator of the Earth. The below formulas define which of the two intersections is the primary one. For the Earth, the primary intersection is not defined.

The coordinate systems that are associated with the Earth have a lowercase letter for symbol, and the coordinate systems that are associated with the other planet have the corresponding uppercase letter for symbol. The coordinate systems are:

- $q$  is the Earth-based [equatorial](#) coordinate system. It uses the north pole of the Earth as its z axis, and the Earth-based primary direction (the vernal [equinox](#)) as its x axis.
- $Q$  is likewise the planet-based equatorial coordinate system, with the primary pole of the planet as its z axis, and the planet-based primary direction as its x axis.
- $c$  is the Earth-based ecliptic coordinate system. It uses the primary pole of the Earth orbit as its z axis, and the Earth-based primary direction (the vernal [equinox](#)) as its x axis.
- Likewise,  $C$  is the planet-based ecliptic coordinate system, with the primary pole of the planet orbit as its z axis, and the planet-based primary direction as its x axis.
- $E$  is the planet-based hybrid equatorial coordinate system, which uses the primary pole of the planet as its z axis, and uses the primary intersection of the equator of the Earth and the equator of the planet as its x axis.  $E$  is needed because the IAU uses it to define the rotation angle  $W$  of the planet.
- $T$  is the planet-based topographic coordinate system, which uses the primary pole of the planet as its z axis, and uses the primary location on the planet as its x axis. Coordinate system  $T$  rotates around its z axis at a rate of once per [sidereal](#) planet day, relative to coordinate systems  $Q$  and  $E$ .

We derive formulas for converting between all of those coordinate systems and the Earth-based ecliptic coordinate system, which means we can convert between all of them. All of the described coordinate systems are right-handed.

The relevant quantities to begin with for all major [planets](#) and [Pluto](#) are listed in the following table. All [angles](#) are given in [degrees](#).  $W_1$  is given in degrees per Earth day (of 86400 [seconds](#)).  $\alpha_p$ ,  $\delta_p$ ,  $W_0$ ,  $W_1$  come from *Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements: 2009* and *Erratum to: Reports of the IAU Working Group on Cartographic Coordinates and Rotational Elements: 2006 & 2009*, found at [/astrogeology.usgs.gov/groups/iau-wgccre](https://astrogeology.usgs.gov/groups/iau-wgccre).  $\Omega$ ,  $i$ ,  $\omega$  come from

[Meeus] and are based on the [VSOP87 model](#) of Betagnon en Francou, except for [Pluto](#). All values are given for the J2000.0 [epoch](#) (JD 2451545).

	$\alpha_p$	$\delta_p$	$\Omega$	$i$	$\omega$	$W_0$	$W_1$
<a href="#">Mercury</a>	281.0097	61.4143	48.330893	7.004986	29.125226	329.5469	6.1385025
<a href="#">Venus</a>	272.76	67.16	76.679920	3.394662	54.883787	160.20	-1.4813688
<a href="#">Earth</a>	0	90	174.873174	0	288.064174	190.147	360.9856235
<a href="#">Mars</a>	317.68143	52.88650	49.558093	1.849726	286.502141	176.630	350.89198226
<a href="#">Jupiter</a>	268.056595	64.495303	100.464441	1.303270	273.866868	284.95	870.5360000
<a href="#">Saturn</a>	40.589	83.537	113.665524	2.488878	339.391263	38.90	810.7939024
<a href="#">Uranus</a>	257.311	-15.175	74.005947	0.773196	98.999212	203.81	-501.1600928
<a href="#">Neptune</a>	299.36	43.46	131.784057	1.769952	276.339634	253.18	536.3128492
<a href="#">Pluto</a>	132.993	-6.163	110.307	17.140	113.768	302.695	56.3625225

For the rotation [angle](#)  $W$  of the [planet](#) at the [time](#) given by [Julian Day Number](#)  $J$  we have

$$W = W_0 + W_1 \times (J - J_{2000}) \quad (59)$$

As an example, we'll calculate  $\Pi, \varepsilon, \theta_0$  for [Mars](#). I show all [coordinates](#) rounded to 7 digits after the decimal marker, and all [angles](#) to 4 digits after the decimal marker. For the obliquity of the [ecliptic](#)  $\varepsilon_0$  I assume the value  $23.4392911^\circ$ .

- Figure 1 shows the [Earth](#)-based [equatorial coordinate system](#)  $q$  in [fuchsia](#), the [Earth](#)-based [ecliptic coordinate system](#)  $c$  in [red](#), the [planet](#)-based ecliptic coordinate system  $C$  in [yellow](#), and the [planet](#)-based equatorial coordinate system  $Q$  in [blue](#).

The black circle represents the celestial sphere of infinite size, to which the [Sun](#) and [stars](#) and [planets](#) appear to be attached.

All displayed ellipses are actually projections of circles on the celestial sphere, seen at an angle. Each circle represents the intersection of the celestial sphere and a flat plane of interest. The part of each circle on the near side of the celestial sphere is displayed as a solid curve, and the part on the far side of the celestial sphere is displayed as a dotted curve.

All displayed straight lines from the center of the celestial sphere are projections of lines from the center of the sphere to a point on the surface of the sphere. Each of those lines represents the infinite extension of an axis or [pole](#) of interest.

Actually, the circles and straight lines from the center represent entire families of parallel planes and lines in space. Because the celestial sphere is infinitely large, parallel lines in space seem to meet in a single point on the celestial sphere, just like parallel railroad tracks seem to meet at the [horizon](#). Likewise, parallel planes in space seem to meet in a single [great circle](#) on the celestial sphere. In the current discussion we're not interested in differences in the origins (the zero points in space) of the coordinate systems, but only in their orientations in space. For the purposes of this discussion, we move the coordinate systems so that their origins coincide, without rotating them.

The [fuchsia](#) curve represents the plane of the [equator](#) of the Earth.  $\vec{z}_q$  represents the primary pole of the Earth, which is the north pole.  $\vec{x}_q$  is the primary direction of the orbit of the Earth, one of the two intersections of the equator of the Earth and the orbit of the Earth. In the case of the Earth, it is called the [vernal equinox](#).  $\vec{y}_q$  is perpendicular to both  $\vec{x}_q$  and  $\vec{z}_q$  and completes the Earth-based equatorial coordinate system.

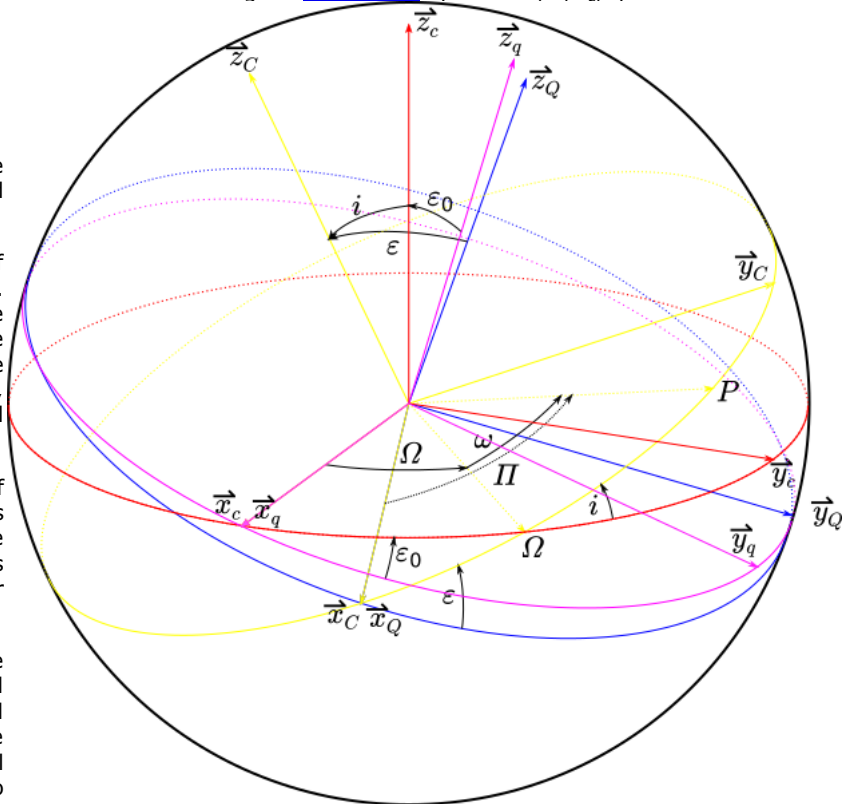
The [red](#) curve represents the plane of the orbit of the Earth, i.e., the ecliptic.  $\vec{z}_c$  represents the primary pole of the orbit of the Earth, i.e., the north pole of the ecliptic.  $\vec{y}_c$  completes the Earth-based ecliptic coordinate system.  $\varepsilon_0$  is the angle over which to rotate the equatorial coordinate system  $q$  around its x axis to get the ecliptic coordinate system  $c$ . In the case of the Earth, it is called the obliquity of the ecliptic.

The [yellow](#) curve represents the plane of the orbit of the planet.  $\vec{z}_C$  represents the primary pole of the orbit of the planet.  $\vec{x}_C$  is the primary direction of the orbit of the planet, one of the two intersections of the equator of the planet and the orbit of the planet.  $\vec{y}_C$  completes the planet-based ecliptic coordinate system.

The [blue](#) curve represents the plane of the equator of the planet.  $\vec{z}_Q$  represents the primary pole of the planet.  $\vec{x}_Q = \vec{x}_C$  is the primary direction, and  $\vec{y}_Q$  completes the planet-based equatorial coordinate system.

$\varepsilon_0 = \varepsilon_{cq}$  is the angle over which to rotate the Earth-based equatorial coordinate system  $q$  around its x axis to get the Earth-based ecliptic coordinate system  $c$ . It is called the obliquity of the ecliptic.

Fig. 1: [Coordinate](#) systems  $C, c, Q, q$





$\varepsilon = \varepsilon_{CQ}$  is the angle over which to rotate the planet-based equatorial coordinate system  $Q$  around its x axis to get the planet-based ecliptic coordinate system  $C$ . One might call it the planet-based obliquity of the ecliptic.

Point  $\Omega$  is the ascending [node](#) of the orbit of the planet (relative to the Earth's orbit). Angle  $\Omega$  is the angle from the Earth-based primary direction (the vernal [equinox](#)) to the ascending node.

$i = \varepsilon_{C_c}$  is the [inclination](#) of the orbit of the planet (relative to the Earth's orbit). It is the angle over which to rotate the plane of the Earth's orbit around the ascending node to get the plane of the planet's orbit.

Point  $P$  is the [perifocus](#) of the orbit of the planet: the point at which it is closest to the [Sun](#).

$\omega$  is the argument of the perifocus of the planet. It is the angle over which to rotate the ascending node around the z axis of the orbit of the planet to get  $P$ .

$\Pi$  is the planet-based ecliptic [longitude](#) of the perifocus of the planet. It is the angle over which to rotate the planet-based primary direction around the z axis of the orbit of the planet to get  $P$ .

2.  $\vec{z}_{Qq}$  are the coordinates of the primary pole of the planet, expressed in Earth-based equatorial coordinates.

$$\vec{z}_{Qq} = \begin{pmatrix} \cos \alpha_p \cos \delta_p \\ \sin \alpha_p \cos \delta_p \\ \sin \delta_p \end{pmatrix} \quad (60)$$

For [Mars](#) we find

$$\vec{z}_{Qq} = \begin{pmatrix} +0.4461587 \\ -0.4062376 \\ +0.7974418 \end{pmatrix}$$

3. Converting from Earth-based equatorial coordinates to Earth-based ecliptic coordinates goes via a rotation around the x axis by angle  $-\varepsilon_0$ :

$$C_{cq} = \text{Rotx}(-\varepsilon_0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon_0 & \sin \varepsilon_0 \\ 0 & -\sin \varepsilon_0 & \cos \varepsilon_0 \end{pmatrix} \quad (61)$$

and converting in the opposite direction, from Earth-based ecliptic coordinates to Earth-based equatorial coordinates goes via the opposite rotation:

$$C_{qc} = \text{Rotx}(\varepsilon_0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon_0 & -\sin \varepsilon_0 \\ 0 & \sin \varepsilon_0 & \cos \varepsilon_0 \end{pmatrix} \quad (62)$$

$\vec{z}_{Qc}$  are the coordinates of the primary pole of the planet, expressed in Earth-based ecliptic coordinates.

$$\vec{z}_{Qc} = C_{cq} \vec{z}_{Qq} = \begin{pmatrix} \cos \alpha_p \cos \delta_p \\ \sin \varepsilon_0 \sin \delta_p + \cos \varepsilon_0 \sin \alpha_p \cos \delta_p \\ \cos \varepsilon_0 \sin \delta_p - \sin \varepsilon_0 \sin \alpha_p \cos \delta_p \end{pmatrix} \quad (63)$$

For Mars we find

$$\vec{z}_{Qc} = \begin{pmatrix} +0.4461587 \\ -0.0555116 \\ +0.8932306 \end{pmatrix}$$

which corresponds to Earth-based ecliptic longitude  $352.9076^\circ$  and [latitude](#)  $+63.2821^\circ$ .

4.  $\vec{z}_{Cc}$  are the coordinates of the primary pole of the planet orbit, expressed in Earth-based ecliptic coordinates.

$$\vec{z}_{Cc} = \text{Rotz}(\Omega) \text{Rotx}(i) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sin \Omega \sin i \\ -\cos \Omega \sin i \\ \cos i \end{pmatrix} \quad (64)$$

where  $\text{Rotz}$  represents rotation around the z axis, similar to  $\text{Rotx}$  described before:

$$\text{Rotz}(\Omega) = \begin{pmatrix} \cos \Omega & -\sin \Omega & 0 \\ \sin \Omega & \cos \Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (65)$$



For Mars we find

$$\vec{z}_{Cc} = \begin{pmatrix} +0.0245658 \\ -0.0209381 \\ +0.9994789 \end{pmatrix}$$

which corresponds to Earth-based ecliptic longitude  $319.5581^\circ$  and latitude  $+88.1503^\circ$ .

5.  $\varepsilon$  is the obliquity of the planet's orbit, which is the angle between the primary pole of the planet and the primary pole of the orbit of the planet. It has a value between 0 and  $180^\circ$ . Its cosine is equal to the inner product of the two [poles](#):

$$\begin{aligned} \cos \varepsilon &= \vec{z}_{Cc} \cdot \vec{z}_{Qc} \\ &= \sin i (-\cos \Omega (\sin \varepsilon_0 \sin \delta_p + \cos \varepsilon_0 \sin \alpha_p \cos \delta_p) + \sin \Omega \cos \alpha_p \cos \delta_p) \\ &\quad + \cos i (\cos \varepsilon_0 \sin \delta_p - \sin \varepsilon_0 \sin \alpha_p \cos \delta_p) \end{aligned} \quad (66)$$

but going via the cosine leads to loss of precision for small values of  $\varepsilon$  (see [the page about Distances in the Sky](#)). For this, a better alternative is to go via

$$t^2 = \tan^2 \left( \frac{1}{2} \varepsilon \right) = \frac{\|\vec{z}_{Cc} - \vec{z}_{Qc}\|^2}{\|\vec{z}_{Cc} + \vec{z}_{Qc}\|^2} \quad (67)$$

If after that you are interested in  $\sin \varepsilon$  or  $\cos \varepsilon$ , then they can be calculated from

$$\sin \varepsilon = \frac{2t}{1 + t^2} \quad (68)$$

$$\cos \varepsilon = \frac{1 - t^2}{1 + t^2} \quad (69)$$

For Mars we find

$$\begin{aligned} \cos \varepsilon &= \begin{pmatrix} +0.0245658 \\ -0.0209381 \\ +0.9994789 \end{pmatrix} \cdot \begin{pmatrix} +0.4461587 \\ -0.0555116 \\ +0.8932306 \end{pmatrix} \\ &= 0.0245658 \times 0.4461587 + (-0.0209381) \times (-0.0555116) + 0.9994789 \times 0.8932306 = 0.9048877 \\ \varepsilon &= \arccos(0.9048877) = 25.1918^\circ \\ t^2 &= \frac{\left\| \begin{pmatrix} +0.0245658 \\ -0.0209381 \\ +0.9994789 \end{pmatrix} - \begin{pmatrix} +0.4461587 \\ -0.0555116 \\ +0.8932306 \end{pmatrix} \right\|^2}{\left\| \begin{pmatrix} +0.0245658 \\ -0.0209381 \\ +0.9994789 \end{pmatrix} + \begin{pmatrix} +0.4461587 \\ -0.0555116 \\ +0.8932306 \end{pmatrix} \right\|^2} = \frac{\left\| \begin{pmatrix} -0.4215929 \\ +0.0345735 \\ +0.1062484 \end{pmatrix} \right\|^2}{\left\| \begin{pmatrix} +0.4707245 \\ -0.0764497 \\ +1.8927095 \end{pmatrix} \right\|^2} \\ &= \frac{(-0.4215929)^2 + 0.0345735^2 + 0.1062484^2}{0.4707245^2 + (-0.0764497)^2 + 1.8927095^2} = \frac{0.1902247}{3.8097754} = 0.0499307 \\ \varepsilon &= 2 \arctan(\sqrt{0.0499307}) = 25.1918^\circ \\ \cos \varepsilon &= \frac{1 - t^2}{1 + t^2} = \frac{1 - 0.0499307}{1 + 0.0499307} = 0.9048877 \\ \sin \varepsilon &= \frac{2t}{1 + t^2} = \frac{2 \times \sqrt{0.0499307}}{1 + 0.0499307} = 0.4256504 \end{aligned}$$

6.  $\vec{x}_{Cc}$  are the coordinates of the primary direction of the planet, expressed in Earth-based ecliptic coordinates. The primary direction is perpendicular to the primary pole of the planet and also to the primary pole of the orbit of the planet. We calculate this using the [vector cross product](#) of the two poles. The cross product is perpendicular to both factors. The order of the two factors determines which of the two perpendicular directions is chosen.

$$\vec{x}_{Cc} = \frac{\vec{z}_{Qc} \times \vec{z}_{Cc}}{\|\vec{z}_{Qc} \times \vec{z}_{Cc}\|} = \frac{\vec{z}_{Qc} \times \vec{z}_{Cc}}{|\sin \varepsilon|} \quad (70)$$

The division by the length of the cross product or  $|\sin \varepsilon|$  is necessary because the length of this vector cross product isn't necessarily equal to 1.

For Mars we find

$$\vec{x}_{Cc} = \frac{\begin{pmatrix} +0.4461587 \\ -0.0555116 \\ +0.8932306 \end{pmatrix} \times \begin{pmatrix} +0.0245658 \\ -0.0209381 \\ +0.9994789 \end{pmatrix}}{0.4256504} = \frac{\begin{pmatrix} -0.0367801 \\ -0.4239833 \\ -0.0079780 \end{pmatrix}}{0.4256504} = \begin{pmatrix} -0.0864092 \\ -0.9960834 \\ -0.0187432 \end{pmatrix}$$

which corresponds to Earth-based ecliptic longitude  $265.0421^\circ$  and latitude  $-1.0740^\circ$ .

7. Now that we have  $\vec{x}_{Cc}$  and  $\vec{z}_{Cc}$ , we can complete the coordinate basis  $C_{Cc}$  for converting from Earth-based ecliptic coordinates to planet-based ecliptic coordinates. We combine the three vectors as rows of the matrix.

$$\vec{y}_{Cc} = \vec{z}_{Cc} \times \vec{x}_{Cc} \quad (71)$$

$$C_{Cc} = \begin{pmatrix} \vec{x}'_{Cc} \\ \vec{y}'_{Cc} \\ \vec{z}'_{Cc} \end{pmatrix} \quad (72)$$

where the  $'$  symbol represents the transposition operator which turns rows of a matrix into columns and columns into rows.

The coordinate basis  $C_{cC}$  for converting from planet-based ecliptic coordinates to Earth-based ecliptic coordinates is the inverse of  $C_{Cc}$ , which inverse is equal to

$$C_{cC} = C_{Cc}^{\text{inv}} = C_{Cc}' \quad (73)$$

For Mars we find

$$\vec{y}_{Cc} = \begin{pmatrix} +0.0245658 \\ -0.0209381 \\ +0.9994789 \end{pmatrix} \times \begin{pmatrix} -0.0864092 \\ -0.9960834 \\ -0.0187432 \end{pmatrix} = \begin{pmatrix} +0.9959568 \\ -0.0859037 \\ -0.0262788 \end{pmatrix}$$

which corresponds to Earth-based ecliptic longitude  $355.0703^\circ$  and latitude  $-1.5058^\circ$ . Then

$$C_{Cc} = \begin{pmatrix} -0.0864092 & -0.9960834 & -0.0187432 \\ +0.9959568 & -0.0859037 & -0.0262788 \\ +0.0245658 & -0.0209381 & +0.9994789 \end{pmatrix}$$

$$C_{cC} = \begin{pmatrix} -0.0864092 & +0.9959568 & +0.0245658 \\ -0.9960834 & -0.0859037 & -0.0209381 \\ -0.0187432 & -0.0262788 & +0.9994789 \end{pmatrix}$$

8.  $\vec{P}_c$  are the coordinates of the perifocus of the orbit of the planet, expressed in Earth-based ecliptic coordinates.

$$\vec{P}_c = \text{Rotz}(\Omega) \text{Rotx}(i) \text{Rotz}(\omega) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \\ \cos \Omega \sin \omega \cos i + \sin \Omega \cos \omega \\ \sin \omega \sin i \end{pmatrix} \quad (74)$$

For Mars we find

$$\vec{P}_c = \begin{pmatrix} +0.6486767 & -0.7610641 & 0 \\ +0.7610641 & +0.6486767 & 0 \\ 0 & 0 & +1 \end{pmatrix} \begin{pmatrix} +1 & 0 & 0 \\ 0 & +0.9994789 & -0.0322782 \\ 0 & +0.0322782 & +0.9994789 \end{pmatrix} \begin{pmatrix} +0.2840512 & +0.9588091 & 0 \\ -0.9588091 & +0.2840512 & 0 \\ 0 & 0 & +1 \end{pmatrix} \begin{pmatrix} +1 \\ 0 \\ 0 \end{pmatrix} =$$

which corresponds to Earth-based ecliptic longitude  $336.0684^\circ$  and latitude  $-1.7735^\circ$ .

9.  $\vec{P}_C$  are the coordinates of the perifocus of the orbit of the planet, expressed in planet-based ecliptic coordinates. We find it by converting  $\vec{P}_c$  from Earth-based ecliptic coordinates to planet-based ecliptic coordinates.

$$\vec{P}_C = C_{Cc} \vec{P}_c \quad (75)$$

The perifocus is part of the orbit of the planet, so it lies in the planet-based plane of the ecliptic, so its planet-based ecliptic  $z$  coordinate and latitude are equal to 0.

For Mars we find

$$\vec{P}_C = \begin{pmatrix} -0.0864092 & -0.9960834 & -0.0187432 \\ +0.9959568 & -0.0859037 & -0.0262788 \\ +0.0245658 & -0.0209381 & +0.9994789 \end{pmatrix} \begin{pmatrix} +0.9135923 \\ -0.4054519 \\ -0.0309486 \end{pmatrix} = \begin{pmatrix} +0.3255013 \\ +0.9455416 \\ 0 \end{pmatrix}$$

which corresponds to Earth-based ecliptic longitude  $71.0041^\circ$  and latitude  $0^\circ$ .

10.  $\Pi$  is the planet-based ecliptic longitude of the perifocus.

$$\Pi = \arctan(P_{2C}, P_{1C}) \quad (76)$$

For Mars we find

$$\Pi = \arctan(0.9455416, 0.3255013) = 71.0041^\circ$$

11. We can now finish the planet-based equatorial coordinate basis. We already had  $\vec{z}_{Qc}$  and the primary direction is shared between the planet-based equatorial coordinate system and the planet-based ecliptic coordinate system, so  $\vec{x}_{Qc} = \vec{x}_{Cc}$  and we already had  $\vec{x}_{Cc}$ . We complete the coordinate basis  $C_{Qc}$ .

$$\vec{y}_{Qc} = \vec{z}_{Qc} \times \vec{x}_{Qc} \quad (77)$$

$$C_{Qc} = \begin{pmatrix} \vec{x}'_{Qc} \\ \vec{y}'_{Qc} \\ \vec{z}'_{Qc} \end{pmatrix} \quad (78)$$

$$C_{cQ} = C'_{Qc} \quad (79)$$

For Mars we find

$$\vec{y}_{Qc} = \begin{pmatrix} +0.4461587 \\ -0.0555116 \\ +0.8932306 \end{pmatrix} \times \begin{pmatrix} -0.0864092 \\ -0.9960834 \\ -0.0187432 \end{pmatrix} = \begin{pmatrix} +0.8907726 \\ -0.0688209 \\ -0.4492080 \end{pmatrix}$$

$$C_{Qc} = \begin{pmatrix} -0.0864092 & -0.9960834 & -0.01874317 \\ +0.8907726 & -0.0688209 & -0.4492080 \\ +0.4461587 & -0.0555116 & +0.8932306 \end{pmatrix}$$

$$C_{cQ} = \begin{pmatrix} -0.0864092 & +0.8907726 & +0.4461587 \\ -0.9960834 & -0.0688209 & -0.0555116 \\ -0.0187432 & -0.4492080 & +0.8932306 \end{pmatrix}$$

## 17. Derivation of $\theta_0$

Figure 2 shows the [Earth-based equatorial coordinate system](#)  $q$  in fuchsia, the [planet-based equatorial coordinate system](#)  $Q$  in blue, the [planet-based ecliptic coordinate system](#)  $C$  in yellow (all three as in Figure 1), the [planet-based hybrid equatorial coordinate system](#)  $E$  in olive, and the [planet-based topographic coordinate system](#)  $T$  in maroon.

Coordinate systems  $q$ ,  $Q$ , and  $C$  were described in detail earlier.

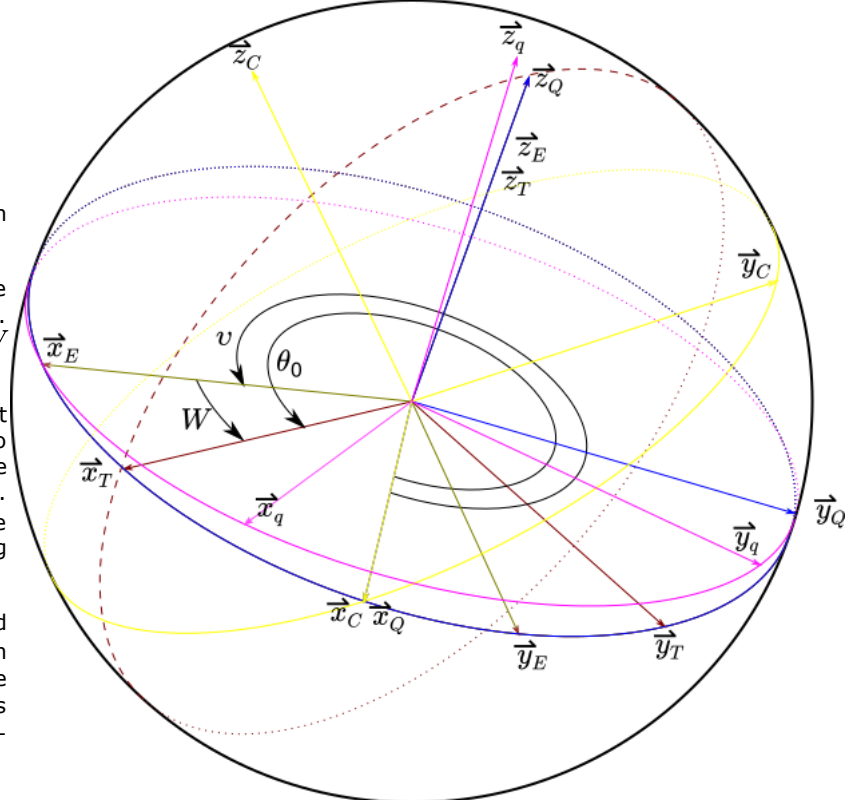
$\vec{x}_E$  represents the primary intersection of the [equator](#) of the [Earth](#) and the equator of the planet. This is the basis for the planet rotation [angle](#)  $W$  used by the [IAU](#).

$\vec{x}_T$  represents the primary location on the planet (with topographic [latitude](#) and [longitude](#) equal to zero). It rotates around the rotation axis of the planet ( $\vec{z}_Q = \vec{z}_E = \vec{z}_T$ ) once per [sidereal](#) planet day. The planet's [prime meridian](#) is indicated by the dashed (front) and dotted (rear) ellipse passing through  $\vec{x}_T$  and  $\vec{z}_T$ .

The [sidereal time](#) angle  $\theta_0$  says which planet-based [right ascension](#) passes through the [zenith](#) of an observer at the planet's prime location  $\vec{x}_T$  at the [epoch](#) ( $J_{2000}$ ). This is equal to how far the planet's prime location  $\vec{x}_T$  has rotated beyond the planet-based primary direction  $\vec{x}_C = \vec{x}_Q$ .

The angle  $v = \theta_0 - W_0$  says how far the planet's primary intersection  $\vec{x}_E$  is beyond the planet's primary direction  $\vec{x}_C = \vec{x}_Q$ .

Fig. 2: [Coordinate](#) systems  $C$ ,  $E$ ,  $Q$ ,  $q$ ,  $T$



How do you calculate  $\theta_0$ ?

1.  $\vec{x}_{Eq}$  are the [coordinates](#) of the primary intersection of the planet's equator with the Earth's equator, expressed in Earth-based equatorial coordinates, as defined by the IAU.

$$\vec{x}_{Eq} = \begin{pmatrix} \cos(\alpha_p + 90^\circ) \cos 0 \\ \sin(\alpha_p + 90^\circ) \cos 0 \\ \sin 0 \end{pmatrix} = \begin{pmatrix} -\sin \alpha_p \\ \cos \alpha_p \\ 0 \end{pmatrix} \quad (80)$$

For [Mars](#) we find

$$\vec{x}_{Eq} = \begin{pmatrix} +0.6732522 \\ +0.7394129 \\ 0 \end{pmatrix}$$

which corresponds to Earth-based [ecliptic](#) longitude  $47.6814^\circ$  and latitude  $0^\circ$ .

2.  $\vec{x}_{Ec}$  are the coordinates of the primary intersection expressed in Earth-based ecliptic coordinates.

$$\vec{x}_{Ec} = \text{Rotx}(-\varepsilon_0) \vec{x}_{Eq} = \begin{pmatrix} -\sin \alpha_p \\ \cos \varepsilon_0 \cos \alpha_p \\ -\sin \varepsilon_0 \cos \alpha_p \end{pmatrix} \quad (81)$$

For [Mars](#) we find

$$\vec{x}_{Ec} = \begin{pmatrix} +0.6732522 \\ +0.6783981 \\ -0.2941216 \end{pmatrix}$$

which corresponds to Earth-based ecliptic longitude  $45.21813$  and latitude  $-17.1049^\circ$ .

3. We now have the x axis  $\vec{x}_{Ec}$  of the planet-based hybrid equatorial coordinate system, in terms of Earth-based ecliptic coordinates. The z axis is the planet's primary [pole](#)

$$\vec{z}_{Ec} = \vec{z}_{Qc} \quad (82)$$

Now that we have two of the three axes, we can complete the coordinate basis  $C_{Ec}$  for converting from Earth-based ecliptic coordinates to planet-based hybrid equatorial coordinates.

$$\vec{y}_{Ec} = \vec{z}_{Ec} \times \vec{x}_{Ec} \quad (83)$$

$$C_{Ec} = \begin{pmatrix} \vec{x}'_{Ec} \\ \vec{y}'_{Ec} \\ \vec{z}'_{Ec} \end{pmatrix} \quad (84)$$

$$C_{cE} = C_{Ec}' \quad (85)$$

Normalization of the cross product isn't needed here because the two factors are perpendicular and of length 1.

For Mars we find

$$\vec{y}_{Ec} = \begin{pmatrix} +0.4461587 \\ -0.0555116 \\ +0.8932306 \end{pmatrix} \times \begin{pmatrix} +0.6732522 \\ +0.6783981 \\ -0.2941216 \end{pmatrix} = \begin{pmatrix} -0.5896388 \\ +0.7325944 \\ +0.3400465 \end{pmatrix}$$

$$C_{Ec} = \begin{pmatrix} +0.6732522 & +0.6783981 & -0.2941216 \\ -0.5896388 & +0.7325944 & +0.3400465 \\ +0.4461587 & -0.0555116 & +0.8932306 \end{pmatrix}$$

$$C_{cE} = \begin{pmatrix} +0.6732522 & -0.5896388 & +0.4461587 \\ +0.6783981 & +0.7325944 & -0.0555116 \\ -0.2941216 & +0.3400465 & +0.8932306 \end{pmatrix}$$

4.  $\vec{x}_{EQ}$  are the coordinates of the primary intersection of the [equators](#) of Earth and the planet, expressed in planet-based equatorial coordinates.

$$\vec{x}_{EQ} = C_{Qc} \vec{x}_{Ec} \quad (86)$$

The z coordinate of that vector is equal to 0 by definition, because the primary intersection is on the celestial equator of the planet.

For Mars we find

$$\vec{x}_{EQ} = \begin{pmatrix} -0.0864092 & -0.9960834 & -0.0187432 \\ +0.8907726 & -0.0688209 & -0.4492080 \\ +0.4461587 & -0.0555116 & +0.8932306 \end{pmatrix} \begin{pmatrix} +0.6732522 \\ +0.6783981 \\ -0.2941216 \end{pmatrix} = \begin{pmatrix} -0.7284035 \\ +0.6851484 \\ 0 \end{pmatrix}$$

which corresponds to Earth-based ecliptic longitude  $136.7527^\circ$  and latitude  $0^\circ$ .

5.  $v$  is the planet-based right ascension of the primary intersection  $\vec{x}_E$ . This is the angle between the planet's primary direction and the primary intersection.

$$v = \arctan(x_{2EQ}, x_{1EQ}) \quad (87)$$

For Mars we find

$$v = \arctan(0.6851484, -0.7284035) = 136.7527^\circ$$

6. Calculate  $\theta_0$  from  $W_0$  and  $v$ :

$$\theta_0 = W_0 + v \quad (88)$$

For Mars we find

$$\theta_0 = 176.63^\circ + 136.7527^\circ = 313.3827^\circ$$

## 18. Planet-based Topographic Coordinates

We derive the transformation of coordinates between planet-based topographic coordinate system  $T$  and Earth-based ecliptic coordinate system  $c$ . Coordinate system  $T$  is the only one of the coordinate systems discussed on this page that rotates. The orientation of coordinate system  $T$  depends on the rotation angle  $W$ .

$$W = W_0 + W_1 \times (J - J_{2000}) \quad (89)$$

The conversion from  $T$  to  $c$  goes as follows.

1.  $\vec{z}_{Tc}$  are the coordinates of the primary pole of the planet, expressed in Earth-based ecliptic coordinates.

$$\vec{z}_{Tc} = \vec{z}_{Ec} \quad (90)$$

2.  $\vec{x}_{Tc}$  are the coordinates of the primary location on the planet, expressed in Earth-based ecliptic coordinates. We find  $\vec{x}_{Tc}$  from  $\vec{x}_{Ec}$  by converting that to the planet-based equatorial coordinate system  $Q$ , then rotating over angle  $W$  around the  $z$  axis of  $Q$  (the rotation axis of the planet), and then converting back to the Earth-based ecliptic coordinate system.

$$\vec{x}_{Tc} = C_{cQ} \text{Rotz}(W) C_{Qc} \vec{x}_{Ec} = C_{cQ} \begin{pmatrix} \cos W & -\sin W & 0 \\ \sin W & \cos W & 0 \\ 0 & 0 & 1 \end{pmatrix} C_{Qc} \vec{x}_{Ec} \quad (91)$$

3. We find  $\vec{y}_{Tc}$  from  $\vec{x}_{Tc}$  and  $\vec{z}_{Tc}$ , which are at right angles.

$$\vec{y}_{Tc} = \vec{z}_{Tc} \times \vec{x}_{Tc} \quad (92)$$

4. Now that we have all three coordinate axes we can construct the coordinate basis  $C_{Tc}$  for converting Earth-based ecliptic coordinates to planet-based topographic coordinates, and  $C_{cT}$  for the opposite conversion.

$$C_{Tc} = \begin{pmatrix} \vec{x}'_{Tc} \\ \vec{y}'_{Tc} \\ \vec{z}'_{Tc} \end{pmatrix} \quad (93)$$

$$C_{cT} = C'_{Tc} \quad (94)$$

$C_{Tc}$  and  $C_{cT}$  vary with time, because they depend on  $W$  which varies with time.

For Mars for  $J = J_{2000}$  we find

$$\begin{aligned}
\vec{z}_{Tc} &= \vec{z}_{Ec} = \vec{z}_{Qc} = \begin{pmatrix} +0.4461587 \\ -0.0555116 \\ +0.8932306 \end{pmatrix} \\
\vec{x}_{Tc} &= \begin{pmatrix} -0.0864092 & +0.8907726 & +0.4461587 \\ -0.9960834 & -0.0688209 & -0.0555116 \\ -0.0187432 & -0.4492080 & +0.8932306 \end{pmatrix} \begin{pmatrix} -0.9982707 & -0.0587837 & 0 \\ +0.0587837 & -0.9982707 & 0 \\ 0 & 0 & +1 \end{pmatrix} \begin{pmatrix} -0.0864092 & -0.9960834 & -0.0187432 \\ +0.8907726 & -0.0688209 & -0.4492080 \\ +0.4461587 & -0.0555116 & +0.8932306 \end{pmatrix} \\
\vec{y}_{Tc} &= \begin{pmatrix} +0.4461587 \\ -0.0555116 \\ +0.8932306 \end{pmatrix} \times \begin{pmatrix} -0.7067491 \\ -0.6341604 \\ +0.3136021 \end{pmatrix} = \begin{pmatrix} +0.5490429 \\ -0.7712063 \\ -0.3221690 \end{pmatrix} \\
C_{Tc} &= \begin{pmatrix} -0.7067491 & -0.6341604 & +0.3136021 \\ +0.5490429 & -0.7712063 & -0.3221690 \\ +0.4461587 & -0.0555116 & +0.8932306 \end{pmatrix} \\
C_{cT} &= \begin{pmatrix} -0.7067491 & +0.5490429 & +0.4461587 \\ -0.6341604 & -0.7712063 & -0.0555116 \\ +0.3136021 & -0.3221690 & +0.8932306 \end{pmatrix}
\end{aligned}$$

## 19. Planet-Bound Horizontal Coordinates

From the [planet-bound topographic coordinate system](#)  $T$  we can deduce the [planet-bound horizontal coordinate system](#)  $H$  for location  $A$  at [latitude](#)  $\Phi$  and [longitude](#)  $\Lambda$  on the planet, assuming the planet to be spherical. If the primary [pole](#) is the north pole, then  $\Phi$  indicates north latitude and  $\Lambda$  indicates east longitude.

The  $z$  axis of  $H$  points to the [zenith](#) of an observer at that location. The  $y$  axis of  $H$  at that location points along the surface toward the primary pole. The  $x$  axis is perpendicular to the  $y$  and  $z$  axes and makes the coordinate system right-handed. If the north pole is the primary pole, then the  $x$  axis points to the east.

Coordinate system  $T$  is equal to coordinate system  $H$  of the primary location on the planet, except that the  $x$  axis of  $T$  is the  $z$  axis of  $H$ , the  $z$  axis of  $T$  is the  $y$  axis of  $H$ , and the  $y$  axis of  $T$  is the  $x$  axis of  $H$ .

For location  $A$  on the planet, we find  $H$  from  $T$  as follows:

- Rotate by [angle](#)  $-\Lambda$  around the  $z$  axis so that the  $x$  axis points from the center of the planet to the [meridian](#) of the location.
- Rotate by angle  $\Phi$  around the  $y$  axis such that the  $x$  axis points from the center of the planet to the location.
- Substitute the axes as follows:  $(x, y, z) \rightarrow (z, x, y)$ .

Combined this yields:

$$C_{HT} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \text{Rot}_y(\Phi) \text{Rot}_z(-\Lambda) = \begin{pmatrix} -\sin \Lambda & \cos \Lambda & 0 \\ -\cos \Lambda \sin \Phi & -\sin \Lambda \sin \Phi & \cos \Phi \\ \cos \Lambda \cos \Phi & \sin \Lambda \cos \Phi & \sin \Phi \end{pmatrix} \quad (95)$$

$$C_{TH} = C'_{HT} = \begin{pmatrix} -\sin \Lambda & -\cos \Lambda \sin \Phi & \cos \Lambda \cos \Phi \\ \cos \Lambda & -\sin \Lambda \sin \Phi & \sin \Lambda \cos \Phi \\ 0 & \cos \Phi & \sin \Phi \end{pmatrix} \quad (96)$$

Let's check. The direction from the center of the planet to location  $A$  at latitude  $\Phi$  and longitude  $\Lambda$  is, expressed in topographic coordinates:

$$\vec{r}_{AT} = \begin{pmatrix} \cos \Lambda \cos \Phi \\ \sin \Lambda \cos \Phi \\ \sin \Phi \end{pmatrix} \quad (97)$$

and that is indeed equal to the  $z$  axis (3rd column) of  $C_{TH}$ , so equal to the direction of the [zenith](#) of point  $A$ , expressed in topographic coordinates.

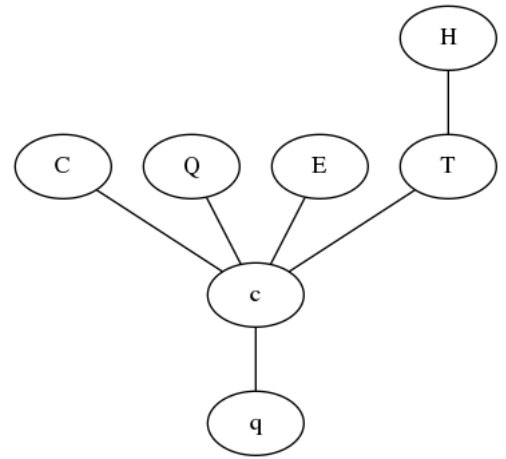
## 20. Coordinate Transformations Between All Systems

We now have a network of coordinate transformations with which we can convert [coordinates](#) from each of the coordinate systems  $q, c, Q, C, E, T$  to all other systems from that list. Figure 3 shows the coordinate systems. A line between two coordinate systems in that figure means we have described a direct transformation between the two systems, earlier on this page. By combining two direct transformations we can connect all other coordinate systems with each other. For example:

- $C_{Cq} = C_{Cc} C_{cq}$
- $C_{EC} = C_{Ec} C_{cC}$

Fig. 3: Coordinate Transformations

$$\bullet C_{Hq} = C_{HT}C_{Tc}C_{cq}$$



## 21. North [Pole](#), South Pole, Primary Pole

The [poles](#) of a celestial object are the two intersections of the rotation axis of the object and the surface of the object. We call one of them the north pole and the other one the south pole, but which is which? Not everyone agrees about that. There are basically two ways to choose which pole is the north pole and which is the south pole:

1. The north pole is the pole that points to the north side of a suitable fundamental plane.
2. The north pole is the pole seen from above which the object appears to rotate counterclockwise around its axis.

The [IAU](#) uses the first of these definitions for the [planets](#) of the [Solar](#) System. The fundamental plane is then the Invariable Plane of the Solar System — of which the poles are the average rotation axis for all orbital motion and rotation in the Solar System. The north side of that plane is the side to which the north pole of the [Earth](#) points. According to the IAU definition, the north poles of all [planets](#) of the Solar System point to the north side of the Invariable Plane of the Solar System.

For all other celestial objects, including dwarf [planets](#) and [asteroids](#) and [comets](#) in our Solar System, and for all celestial objects outside of the Solar System (including major planets), the IAU uses the [second](#) definition.

The great advantage of the second definition is that you need to look only at the celestial object itself, and do not need to search for some fundamental plane with which to compare the pole. The orientation of the fundamental plane might not even be known very accurately.

For most planets in the Solar System both definitions yield the same result — but not for [Venus](#) and [Uranus](#). If you look down from above the IAU-defined north pole of [Venus](#) or [Uranus](#), then those planets rotate clockwise around their axis.

Both definitions don't yield the same results for [Pluto](#), either. Until 2006, [Pluto](#) was regarded as a small one of the major planets, to which the first definition applies, so the pole of [Pluto](#) that points to the north of the Invariable Plane of the Solar System was regarded as its north pole. Since 2006, Pluto is regarded as a [dwarf planet](#), to which the second definition applies, so now the pole from above which Pluto appears to rotate in the counterclockwise direction is regarded as its north pole — which is the opposite pole from what was regarded as its north pole until 2006.

Because it isn't always clear which pole is the north pole, I use the term "primary pole" instead in the above text, so that you cannot be led astray if your idea of which pole is the north pole were different from mine.



languages: [\[en\]](#) [\[nl\]](#)

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