

Lecture 7: Hypothesis testing part I

Statistical Methods for Data Science

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Today

1 Terminology

- Experiment and the parameter of interest
- Null hypothesis and alternative hypothesis
- Test statistic
- Null distribution $f(s \mid H_0)$
- Significance level α , power and p -value

2 Example

3 Summary

Learning outcome

- Be able to explain the following terminology
 - Null hypothesis H_0 and alternative hypothesis H_A
 - Test statistic s
 - Null distribution $f(s | H_0)$
 - Significance level α and power
 - p -value
- Be able to design and interpret the one-sample z-test

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- If company A tested drug D on 30 ducks and the average weight loss after one month is 2.2 kg, would you buy drug D instead of regular diet control?

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- What if company A tested drug D on 30 ducks and the average weight loss after one month is 2.3 kg? Would you buy drug D instead of regular diet control in this case?

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- What if company A tested drug D on 100 ducks and the average weight loss after one month is 2.3 kg?

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- What if company A tested drug D on 100 ducks and the average weight loss after one month is 2.3 kg?
- Now company B tested drug E on 30 ducks and the average weight loss after one month is 2.5 kg, while drug D results in 2.3 kg weight loss with the same setup, would you buy drug E instead of drug D?

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How would you make your decision?

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 - The test is based on sample statistics, which are computed from data
 - Hypothesis + data \rightarrow decision on rejecting/not rejecting the hypothesis

Hypothesis testing: a list to go through

- A “boring” statement
- Experiment
- Data x , random variable X
- Parameter of interest θ
- Parameter estimate $\hat{\theta}$
- Null hypothesis H_0
- Alternative hypothesis H_A
- Test statistic s
- Null distribution $f(s \mid H_0)$
- Significance level α
- p -value

Experiment and the parameter of interest

Experiment design

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Then we can use \bar{x} to approximate μ_D and check if it is greater than diet control (2.1 kg)

Experiment design (cont.)

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Then we use \bar{x} and \bar{y} to approximate μ_D and μ_E to see if they are the same

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- We make our decision by observing data; if the evidence does not support the “boring” statement, we **reject the statement**; otherwise, we **do not reject the statement**

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- The logic here is: if the statement is true, then the evidence must support the statement \iff if the evidence does not support the statement, the statement must be false \niff if the evidence supports the statement, the statement must be true

Null hypothesis and alternative hypothesis

Hypotheses H_0 and H_A

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- Example 2: drug E and drug D work the same on average (5 sec)

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 - Example 1: drug D is more effective than regular diet on average (5 sec)

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- Example 2: drug E and drug D work the same on average (5 sec)

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- **Alternative hypothesis H_A** : a complementary alternative explanation to the “boring” statement
 - Example 1: drug D is more effective than regular diet on average (5 sec)

$$H_A : \mu_D > 2.1$$

- Example 2: drug E and drug D do not work the same on average (5 sec)

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- **Alternative hypothesis H_A** : a complementary alternative explanation to the “boring” statement
 - Example 1: drug D is more effective than regular diet on average (5 sec)

$$H_A : \mu_D > 2.1$$

- Example 2: drug E and drug D do not work the same on average (5 sec)

$$H_A : \mu_D \neq \mu_E$$

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?

Hypotheses H_0 and H_A (cont.)

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Answer: an implicit assumption here is that μ_D will not be smaller than 2.1
- Question 2: can H_0 and H_A be ANYTHING I want? Like a magic mirror!?

Hypotheses H_0 and H_A (cont.)

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Answer: an implicit assumption here is that μ_D will not be smaller than 2.1

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Answer: no

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- Follow up question: what are the choices for H_0 and H_A ?

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- We have seen one-sample test and two-sample test in the Q-Q plot lecture
- In practice, you can narrow down your choice of hypotheses by making a Q-Q plot

Choices for H_A

Given

$$H_0 : \theta = \beta$$

where β can be either a constant (one-sample test) or a parameter from another data distribution (two-sample test)

- **Alternative hypothesis H_A :** H_A can be **one-tailed** or **two-tailed**
 - **One-tailed:**

$$H_A : \theta > \beta$$

or

$$H_A : \theta < \beta$$

- **Two-tailed:**

$$H_A : \theta \neq \beta \iff \theta < \beta \text{ or } \theta > \beta$$

Summary: choices for H_0 and H_A

Putting everything together,

	One-sample test	Two-sample test
Two-tailed	$H_0 : \theta = c, H_A : \theta \neq c$	$H_0 : \theta_1 = \theta_2, H_A : \theta_1 \neq \theta_2$
One-tailed	$H_0 : \theta = c, H_A : \theta > c$	$H_0 : \theta_1 = \theta_2, H_A : \theta_1 > \theta_2$
	$H_0 : \theta = c, H_A : \theta < c$	$H_0 : \theta_1 = \theta_2, H_A : \theta_1 < \theta_2$

where $\theta, \theta_1, \theta_2$ are the parameters of interest and c is a constant

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Note: in this course, we only deal with null hypothesis where we are able to express the PDF/PMF $f(s | H_0)$, i.e. H_0 with an equal sign in them

Test statistic (cont.)

- Example 1. one-sample test
 - **Data:** x_1, \dots, x_N
 - **Random variable:** X_1, \dots, X_N i.i.d. **Gaussian with known σ**
 - **Parameter of interest:** μ_D
 - **Parameter estimate:** \bar{x}
 - **Null hypothesis:** $H_0 : \mu_D = 2.1$
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- Example 2. two-sample test

- **Data:** x_1, \dots, x_{N_D} and y_1, \dots, y_{N_E}
- **Random variable:** X_1, \dots, X_{N_D} i.i.d. **Gaussian with known σ_D** ; Y_1, \dots, Y_{N_E} i.i.d. **Gaussian with known σ_E** ; X_i and Y_j independent
- **Parameter of interest:** μ_D, μ_E
- **Parameter estimate:** \bar{x}, \bar{y}
- **Null hypothesis:** $H_0 : \mu_D = \mu_E \iff H_0 : \mu_D - \mu_E = 0$
- **Test statistic:** standardized $\bar{x} - \bar{y}$ assuming the null hypothesis

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\sigma_D^2/N_D + \sigma_E^2/N_E}} \text{ (explained later)}$$

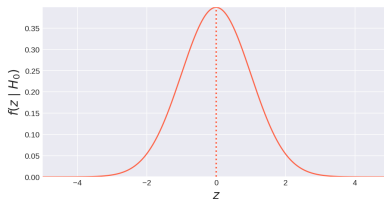
Null distribution $f(s | H_0)$

Null distribution

- **Null distribution $f(s | H_0)$:** the distribution of the test statistic given the null hypothesis
- **Example:**
 - **Data:** x_1, \dots, x_N
 - **Random variable:** X_1, \dots, X_N i.i.d. Gaussian with known σ
 - **Parameter of interest:** μ
 - **Parameter estimate:** \bar{x}
 - **Null hypothesis:** $H_0 : \mu = \mu_0$
 - **Test statistic:**

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{N}}$$

- **Null distribution:** standard Gaussian distribution



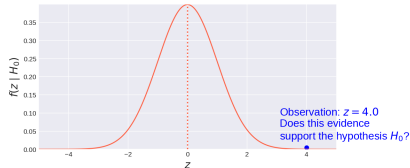
Significance level α , power and p -value

Significance level

Given a null hypothesis $H_0 : \mu = 2.1$ and the null distribution $f(s | H_0)$, we decide if we reject the hypothesis or not by observing data

- Run some experiments and collect data x_1, \dots, x_N
- Estimate the parameter of interest $\hat{\theta}$, e.g. $\hat{\mu} = \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$
- Standardize $\hat{\theta}$ assuming H_0 to compute the test statistic, e.g.

$$z = \frac{\bar{x} - 2.1}{\sigma/\sqrt{N}} = 4.0$$



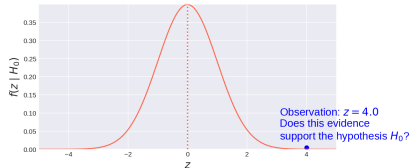
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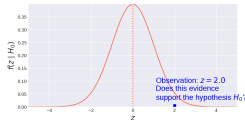
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- Does this evidence support the hypothesis H_0 ? Probably not since it's so far away from the center?

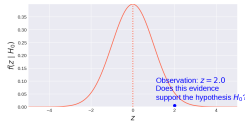
Significance level (cont.)

- What about this observation?



Significance level (cont.)

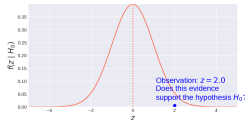
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- To be able to answer the question, you need to decide where you draw the line

Significance level (cont.)

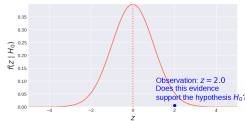
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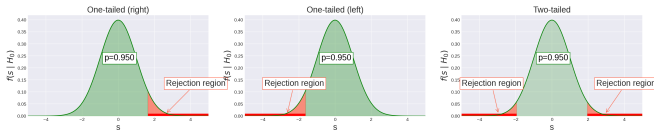
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Significance level (cont.)

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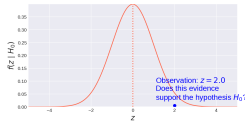
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- Significance level α** : red area under the curve



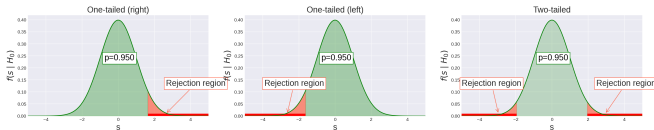
In these two images, $\alpha = 0.05$

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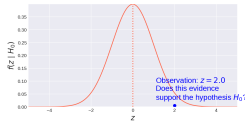


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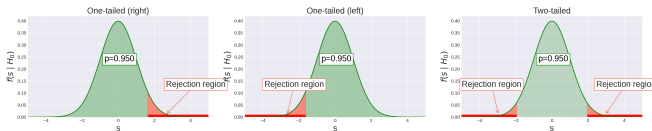
More conservative \Rightarrow less probable to reject H_0 , which indicates a smaller rejection region

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Two-tailed H_A is more conservative

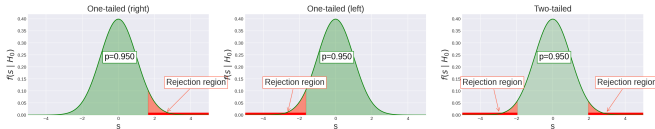
Significance level (cont.)

What is needed for choosing a meaningful α ?

- Null distribution
- H_A one-tailed or two-tailed

Interpretation of α

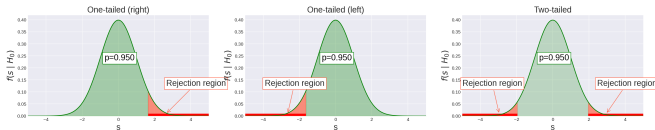
- $\alpha = P(\text{reject } H_0 \mid H_0 \text{ is true})$ - the probability of making such a mistake



- The rejection region indicates that H_0 is **unlikely**, but the probability is not zero
- It is possible that H_0 is true, but our observation happens to fall in the rejection region
- If H_0 is true and our observation falls in the rejection region, we will **mistakenly** reject H_0
- The probability of making this type of mistakes is α
- Similar to the confidence interval, $1 - \alpha$ is called the **confidence level**

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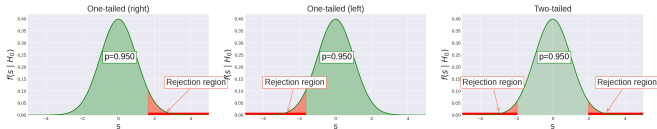
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- Similar to the confidence interval, $1 - \alpha$ is called the **confidence level** - “with 95% confidence, rejecting H_0 is the right thing to do”
- Define the significance level **before you run the experiments** so that you can't cheat!

Significance level and power

- Contingency table:

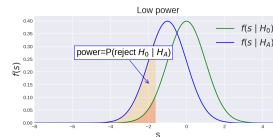
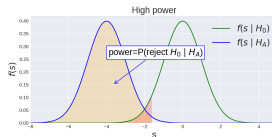
	$y = H_A$	$y = H_0$
$\hat{y} = \text{reject } H_0$	TP	FP (Type I error)
$\hat{y} = \text{do not reject } H_0$	FN (Type II error)	TN

- Significance level α : incorrectly rejecting H_0

$$\alpha = P(\text{type I error})$$

- Power: correctly rejecting H_0

$$\text{power} = P(\text{reject } H_0 | H_A) = 1 - P(\text{type II error})$$



- What is needed for computing the power?

Significance level and power

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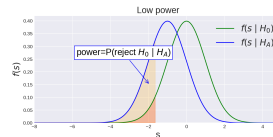
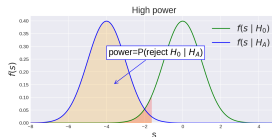
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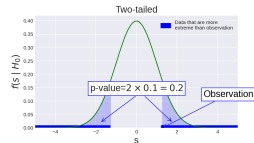
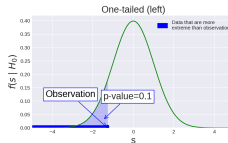
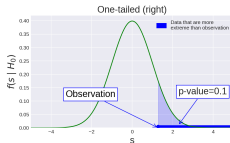


- What is needed for computing the power? $f(s | H_0)$, $f(s | H_A)$

p -value

- p -value:
 - One-tailed:
 - Right tail: $p = P(S \geq s | H_0)$
 - Left tail: $p = P(S \leq s | H_0)$
 - Two-tailed: $p = 2 \min(P(S \leq s | H_0), P(S \geq s | H_0))$
- Note: if $f(s | H_0)$ is symmetric,

$$p = 2P(S \leq s | H_0)$$

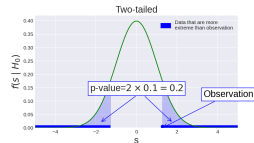
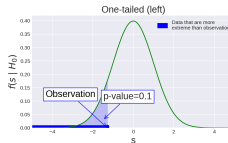
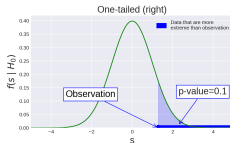


- What is needed for computing the p -value? (10 sec)

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- What is needed for computing the p -value? (10 sec)
 - Null distribution
 - Alternative hypothesis H_A to know one-tailed or two-tailed
 - Observation - test statistic computed from data

Summary: steps for hypothesis testing

- Step 1 Make a “boring” statement
- Step 2 Design an **experiment**
- Step 3 Describe the **data** generated from the experiment and the corresponding random variables
- Step 4 Describe the parameter of interest and their estimates
- Step 5 Translate the “boring” statement into a statistical hypothesis and call it the **null hypothesis H_0**
- Step 6 Find the expression for the **test statistic s**
- Step 7 Find the expression for the **null distribution**
- Step 8 Define **an alternative hypothesis H_A** : one-tailed or two-tailed
- Step 9 Choose a **significance level α** (the tail), which defines the rejection region
- Step 10 Collect **data**
- Step 11 Compute the test statistic from data
- Step 12 Compute the p -value
- Step 13 If $p\text{-value} < \alpha$, i.e. the test statistic falls in the rejection region of the null distribution, then we reject the hypothesis H_0 .

Today

- 1 Terminology
- 2 Example
- 3 Summary

Example

Recall example: if you control the diet of your ducks, they lose 2.1 kg after one month on average. Company A has developed a drug D to help ducks lose weight. They claim that on average the drug works better than diet control. Here is the set up for the experiment.

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- Step 3 Describe the **data** and **random variables** with assumptions about their distributions (5 secs):

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$$z_0 = \frac{2.2 - 2.1}{0.6/\sqrt{30}} = 0.91$$

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- For $H_A : \mu_D > 2.1$ (one-tailed): $p = P(Z \geq z_0 \mid H_0) = 0.1807 > \alpha$

Example (cont.)

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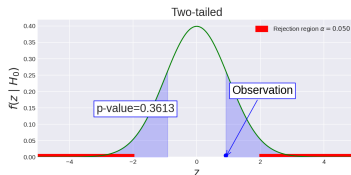
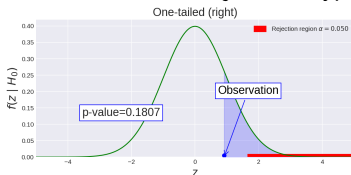
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Example (cont.)

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Do not reject H_0 for both one-tailed and two-tailed H_A

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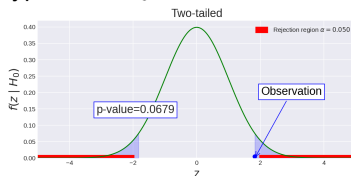
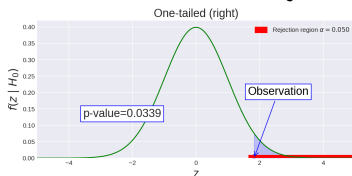
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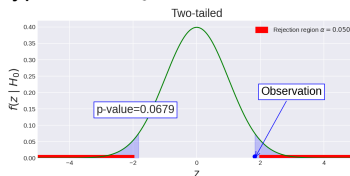
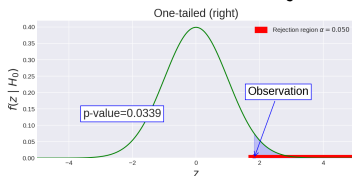
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Reject H_0 for one-tailed H_A ; do not reject H_0 for two-tailed H_A
for the same confidence level $1 - \alpha = 5\%$

Example (cont.)

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Reject H_0 for one-tailed H_A ; do not reject H_0 for two-tailed H_A for the same confidence level $1 - \alpha = 5\%$

Note: the two-tailed test is more conservative - if the data passes a two-tailed test, it is more conclusive than one-tailed test for the same confidence level

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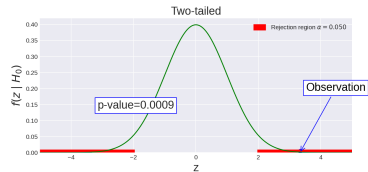
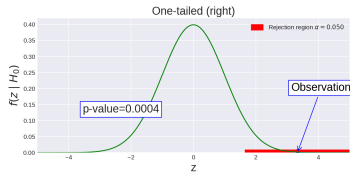
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Step 12 Compute the p -value (20 secs):

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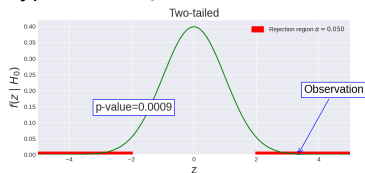
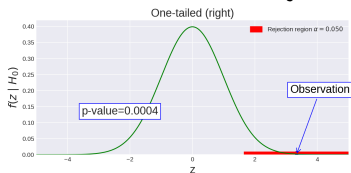
Reject H_0 for both one-tailed and two-tailed H_A

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- With more data, it becomes more certain that we should reject H_0 in favor of H_A given the observation $\bar{x} = 2.3$

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This test is called **one-sample z-test**

Today

- 1 Terminology
- 2 Example
- 3 Summary

Summary

So far:

- Data types and data containers
- Descriptive data analysis: descriptive statistics, visualization
- Probability distributions, events, random variables, PMF, PDF, parameters
- CDF, Q-Q plot, how to compare two distributions (data vs theoretical, data vs data)
- Modeling
- Parameter estimation: maximum likelihood estimation (MLE) and maximum a posteriori estimation (MAP)
- Classification, multinomial naive Bayes classifier, Gaussian naive Bayes classifier
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Before next lecture:

- Steps for hypothesis testing

