Lecture 2: Probability Distribution

Statistical Methods for Data Science

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Today

1. Probability distribution
   - Why probability distributions?
   - Terminology
   - Some probability distributions that you should know by heart
2. Demo
3. Summary

Learning outcome

- Be able to explain the following concepts: experiment, event, random variable, probability distribution
- Given the PDF/PMF, be able to describe the probability distribution of a continuous/discrete random variable
- Be able to compute conditional probability
- Understand Gaussian distribution and Bernoulli distribution: 1) PDF/PMF; 2) what are the parameters? 3) what happens to the shape of the distribution if we change the parameters?
- Be able to apply the learning routine to study a new probability distribution yourself

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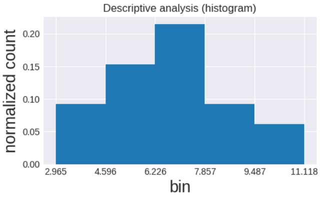
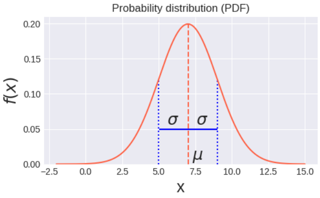
Why probability distributions?

Histogram vs probability distribution

You need to predict the average weight of your 1000 ducks without weighing all of them. You weighed 20 ducks and you plotted the histogram of the weights. Your best friend Jack suggested that you should use a **Gaussian distribution** to make a better estimation of the average. For example, you can estimate a confidence interval for the mean value, Jack said.





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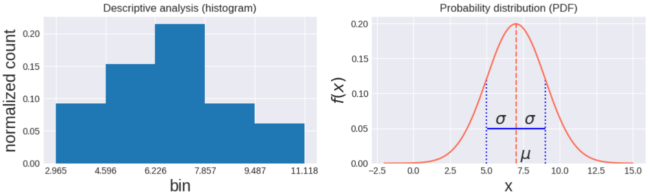

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- Question 1: Why can't I just use descriptive analysis, like the histogram? Why should I use probability distributions?

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- To address this question, let's describe the data using the histogram and a Gaussian distribution to see the difference.

Histogram vs probability distribution

Here are the weights of the 20 ducks in kg

duck id	1	2	3	4	$\cdots$	19	20
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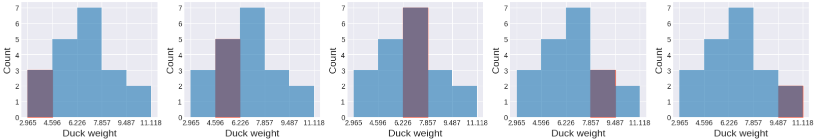

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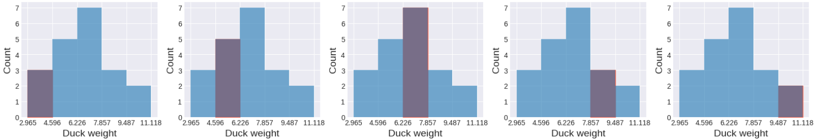

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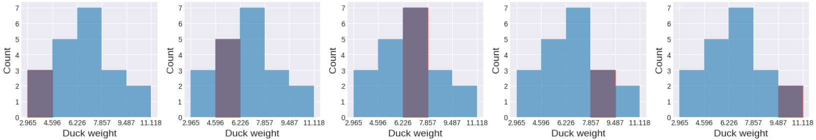

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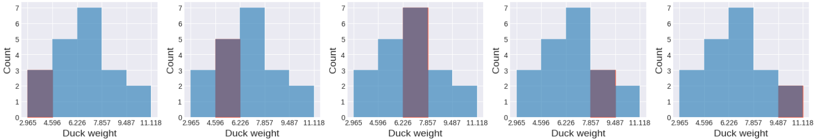

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  How about between 3.1 kg and 3.4 kg?

Histogram vs probability distribution

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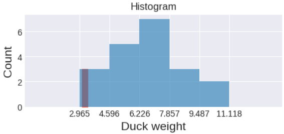

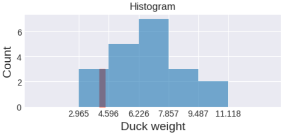

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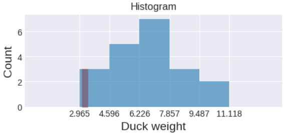

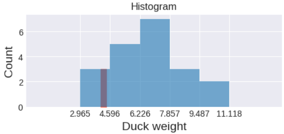

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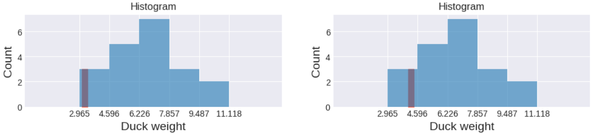

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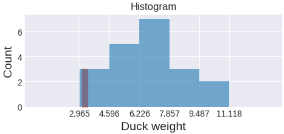

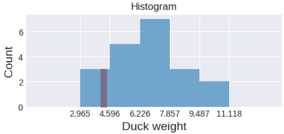

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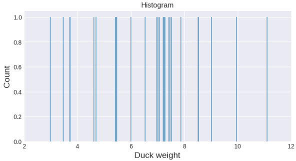

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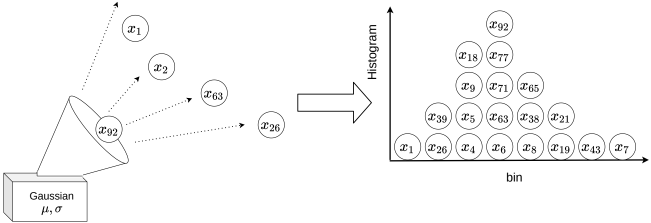
- First, we **assume** that data is **generated** from a Gaussian distribution



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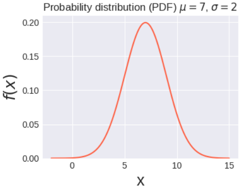

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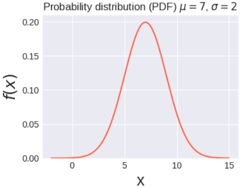

- We will replace the histogram with this function.

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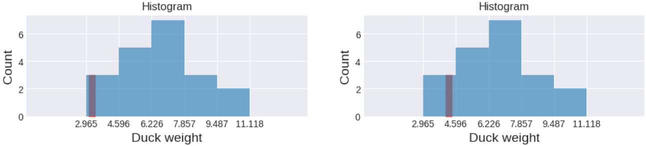

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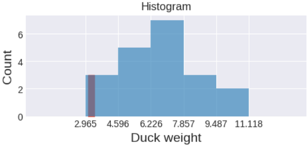

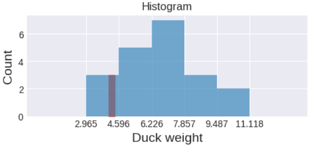

  - Gaussian distribution:
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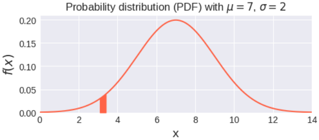

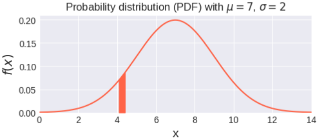

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Disclaimer: we used a continuous distribution to illustrate the comparison between a histogram and a probability distribution.

A discrete probability distribution differs from a continuous distribution.

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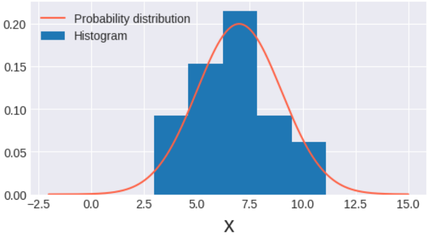

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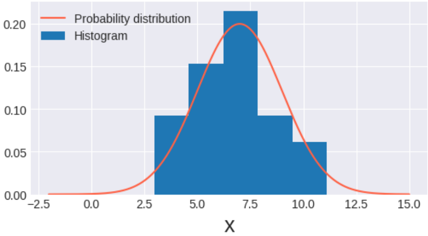
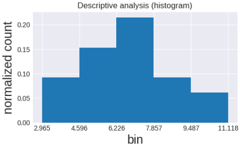
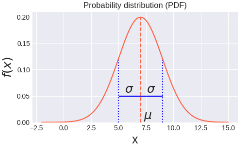


    - Long answer will be given in lecture 3.

Parameter estimation and evaluation

- Question 3: Okay fine, let's say we describe the data with a Gaussian distribution. How do I know what the parameters μ and σ are?





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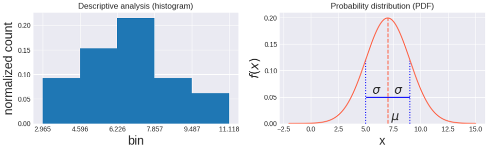

  - This is done by **parameter estimation**. In this course (lecture 3 & 4), we will talk about the **maximum likelihood estimation (MLE)** and the **maximum a posteriori estimation (MAP)**.

Terminology

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  - Heuristically, X assigns a numerical value to each outcome of the experiment:

$$X : \text{weight} \to \mathbb{R}$$

  - X follows some underlying probability distribution.
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- **Data** x: a value drawn from the **underlying distribution of** X.
  - We use a **capital letter** (e.g. X) to denote a random variable and the corresponding lower case letter (e.g. x) to denote the data generated from the underlying distribution of X.
  - Discrete random variable: categorical data or discrete numerical data
  - Continuous random variable: continuous numerical data

Probability distribution

More precisely, the probability distribution can be described by a function f_X (also denoted as f if neglecting X does not cause confusion), where

- for discrete distribution, the **probability mass function (PMF)** is used, where

$$f_X(x_i) = \boxed{P(X = x_i)}$$

  where $0 \leq f_X(x_i) \leq 1$ for all x_i.

- for continuous distribution, the **probability density function (PDF)** is used, where

$$\boxed{P(a \leq X \leq b)} = \int_a^b f_X(x)dx, \ \forall a, b \in \mathbb{R}, a \leq b$$

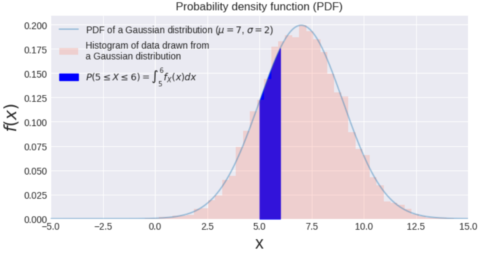
  where $f_X(x) \geq 0$ for all x.

where $\boxed{P(\text{event})}$ is the probability of the **event** occurring.

Example: continuous random variables and PDF

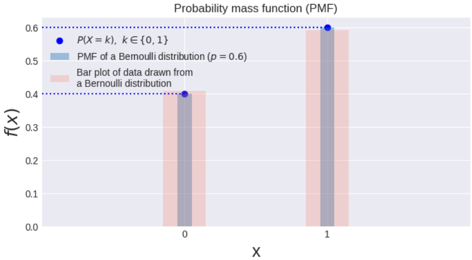
- **Experiment**: you weigh a duck and look at its weight
- **Sample space**: $0 < weight < \infty$
- **Random variable** $X : weight \to \mathbb{R}$
  - $X = x$ if the duck weighs x kg for $0 < x < \infty$
  - Assumption: X follows a Gaussian distribution with parameters μ and σ; denoted as $X \sim \mathcal{N}(\mu, \sigma^2)$
- **PDF**: $f_X(x)$

$$P(a \leq X \leq b) = \underbrace{\int_a^b f_X(x)dx}_{\text{Integral = area under the PDF curve}} \qquad \forall a, b \in \mathbb{R}, a \leq b$$



Example: discrete random variables and PMF

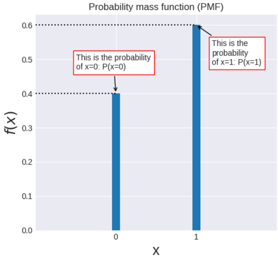
- **Experiment**: you measure the color of the duck.
- **Sample space**: the color can be only red or blue
- **Random variable** $X : color \rightarrow \mathbb{R}$
  - $X = \begin{cases} 0, & \text{if a duck is red} \\ 1, & \text{if a duck is blue} \end{cases}$
  - Assumption: X follows a Bernoulli with parameter p; denoted as $X \sim Bernoulli(p)$
- **PMF**: $f_X(x_i) = P(X = x_i)$

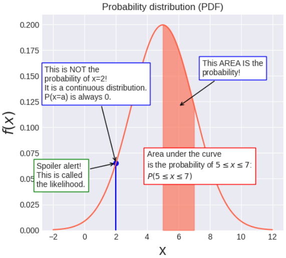


Probability distribution

Differences between PMF and PDF

- Discrete distribution $f_X(x_i) = P(X = x_i)$:
  - y-axis represents the probability itself
- Continuous distribution:
  - $P(a \leq X \leq b) = \int_a^b f_X(x)dx$: **y-axis $f(x)$** DOES NOT represent the probability itself.
  - For continuous distributions, **the probability at any given value is always 0**, i.e. $P(X = a) = P(a \leq X \leq a) = \int_a^a f_X(x)dx \equiv 0$. Example: what is the probability of a duck weighing exactly 4.32028374... kg?





Conditional probability

Given events A and B,

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

Conditional probability example

Example:

- **Experiment**: You ask your ducks to stand in a row again and look at their colors and sizes.
- **Sample space**: The color can be either red or blue; the size can be either slim or chonker.
- **Data**:

duck id	1	2	3	4	5	6
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size	chonker	slim	slim	chonker	chonker	slim

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Conditional probability example (cont.)

An alternative way to estimate $P(A \mid B)$:

- Count the chonkers: 3
- Count blue ducks within the chonker gang: 2
- $P(A \mid B) = \frac{2}{3}$

Conditional probability example (cont.)

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Side note: from this calculation, can you make a bold statement of the probability distribution of all the ducks in the world? No. It is only an estimation based on the data you got.

Conditional probability example (cont.)

As an exercise, let's define the random variables.

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Write the conditional probability in terms of the random variables X and Y (10 secs), i.e.

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Independent events

Two events A and B are independent if and only if

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B)$$

$\iff \quad P(A \mid B) = P(A),\ P(B \mid A) = P(B)$ (conditional probability)

$\iff \quad log\,(P(A \text{ and } B) = \log\,(P(A \cap B)) = \log(P(A)) + \log(P(B))$

Bayes' rule

Given events A and B,

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Just a heads-up!

Summary: terminologies

- Experiment
- Sample space
- Event
- Random variable:
    - Discrete random variable
    - Continuous random variable
- Data
- Probability distribution:
    - Discrete distribution: P(event) is described by the probability mass function (PMF)
    - Continuous distribution: P(event) is described by the probability density function (PDF)

  What are their differences?
- Conditional probability of events
- Independent events
- Bayes' rule

Some probability distributions that you should know by heart

Probability distributions

Probability distribution	Continuous/discrete	Apply to data type
Bernoulli distribution	Discrete	Categorical (nominal)
Categorical distribution	Discrete	Categorical (nominal)
Discrete uniform	Discrete	Numerical (discrete)
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For each distribution, you need to know:

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- its applications
- how to estimate the parameters (next lecture)

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- What is the PMF?
  Merge these two equations:

$$P(X = k) = f_X(k) \equiv f_X(k \mid p) = pk + (1 - p)(1 - k), \;\; k \in \{0, 1\}, p \in [0, 1]$$

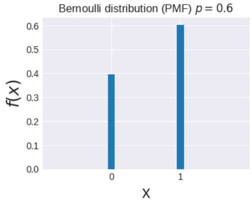
Note: here we use a $|$ to indicate that the parameter p is given.

Bernoulli distribution

- Discrete distribution
- Applies to nominal data with 2 categories
- PMF:
  - Equation

$$f_X(k \mid p) = pk + (1-p)(1-k), k \in \{0,1\}, p \in [0,1]$$

  - Shape



- Parameters: p

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  $P(\text{a duck is gray}) = P(X = 4) = p_4$
- What is the PMF?

Categorical distribution

In Jack's town, ducks have FOUR colors: blue, red, green and gray. What is the probability distribution of duck colors in Jack's town?

- Given p_1 the probability of a duck being blue, p_2 the probability of a duck being red, p_3 the probability of a duck being green and p_4 the probability of a duck being gray. Note that $p_1 + p_2 + p_3 + p_4 = 1$.
- Let X be a discrete random variable $X = \begin{cases} 1 & \text{a duck is blue} \\ 2 & \text{a duck is red} \\ 3 & \text{a duck is green} \\ 4 & \text{a duck is gray} \end{cases}$
- Now we can express the probability distribution as follows:

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- What is the PMF?

$$P(X = k) = f_X(k) \equiv f_X(k \mid p_1, p_2, p_3, p_4) = p_k, \ \sum_{i=1}^{4} p_i = 1, p_i \geq 0, \ k \in \{1, \cdots, 4\}$$

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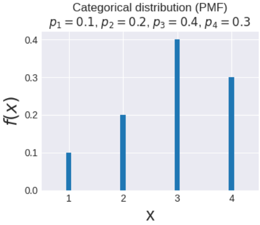
Note: categorical distribution is also called multinoulli distribution. It is a generalization of the Bernoulli distribution.

Categorical distribution

- Discrete distribution
- Applies to nominal data with $n > 0$ categories
- PMF:
  - Equation

$$f_X(k \mid p_1, p_2, \cdots, p_n) = p_k, \ \sum_{i=1}^{n} p_i = 1, p_i \geq 0, \ k \in \{1, \cdots, n\}$$

  - Shape



- Parameters: p_k, $k \in \{1, \cdots, n\}$ for given n.

Probability distributions

Probability distribution	Continuous/discrete	Apply to data type
Bernoulli distribution	Discrete	Categorical (nominal)
Categorical distribution	Discrete	Categorical (nominal)
Discrete uniform	Discrete	Numerical (discrete)
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Discrete uniform distribution

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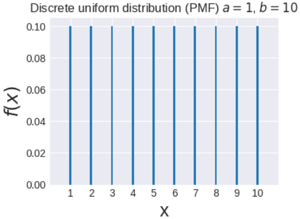
Discrete uniform distribution

- Discrete distribution
- Applies to discrete numerical data
- PMF:
  - Equation

$$f_X(k \mid a, b) = \frac{1}{b - a + 1}, \quad a \le b, \quad a, b \text{ integers}$$

  - Shape



- Parameters: integers a, b

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- What is the PDF of a Gaussian distribution with $\mu = 7$ and $\sigma = 2$?

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- What is the PDF of a Gaussian distribution with $\mu = 7$ and $\sigma = 2$?

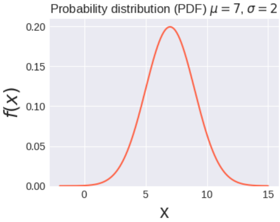
$$f_X(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{\sqrt{8\pi}} e^{-\frac{1}{2}\left(\frac{x-7}{2}\right)^2}$$

Gaussian (normal) distribution

- Continuous distribution
- Applies to continuous numerical data
- PDF:
  - Equation

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \ \mu \in \mathbb{R}, \ \sigma \in \mathbb{R}^+$$

  - Shape



- Parameters: $\mu, \ \sigma$

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Hooray!

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Check out what data types they apply to!

We are going to talk about more applications in the future

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Check out what data types they apply to!

We are going to talk about more applications in the future (even though they won't be as important as ducks)

Today

1. Probability distribution
2. Demo
3. Summary

Demo

Code demo

- Image processing
- Natural language processing
- Table with numerical data
- Table with categorical data

Today

1. Probability distribution
2. Demo
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- Probability distributions, sample space, events, random variables, PMF, PDF, parameters

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Before next lecture:

- Quantile
- Probability distributions from today and their parameters
- PMF and PDF

Stay safe!