

Sapienza University of Rome

Master in Artificial Intelligence and Robotics

Machine Learning

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10. Instance based learning

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10. Instance based learning

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Summary

- Non-parametric models
- K-NN for classification
- Locally weighted regression

References

C. Bishop. Pattern Recognition and Machine Learning. Sect. 2.5

Parametric and non-parametric models

Parametric model: Model has a fixed number of parameters

Examples:

- Linear regression
- Logistic regression
- Perceptron
- ...

Non-parametric model: Number of parameters grows with amount of data

Simple non-parametric model: **instance-based learning**

K-nearest neighbors

Classification problem: $f : X \mapsto C$ with data set $D = \{(\mathbf{x}_n, t_n)_{n=1}^N\}$

Classification with K-NN,

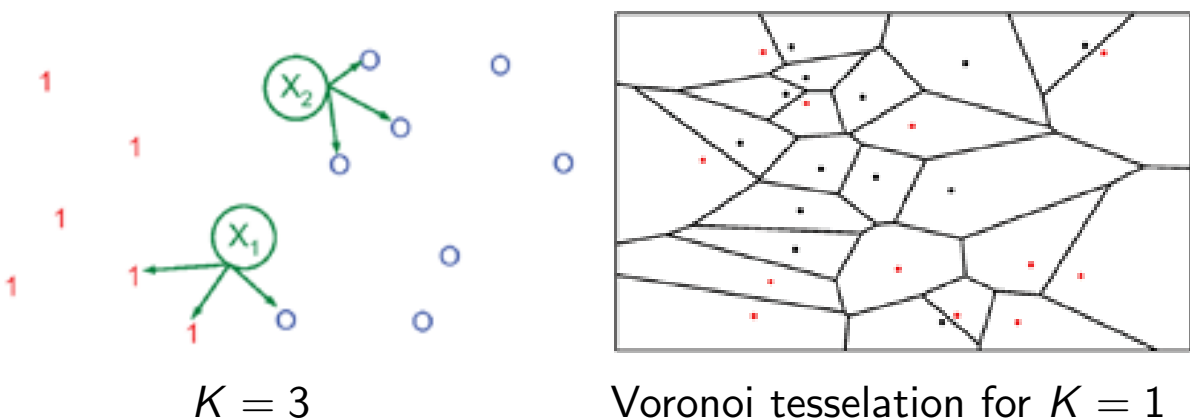
- ① Find K nearest neighbors of new instance \mathbf{x}
- ② Assign to \mathbf{x} the most common label among the majority of neighbors

Likelihood of class c for new instance \mathbf{x} :

$$p(c|\mathbf{x}, D, K) = \frac{1}{K} \sum_{\mathbf{x}_n \in N_K(\mathbf{x}, D)} \mathbb{I}(t_n = c),$$

with $N_K(\mathbf{x}_n, D)$ the K nearest points to \mathbf{x}_n and $\mathbb{I}(e) = \begin{cases} 1 & \text{if } e \text{ is true} \\ 0 & \text{if } e \text{ is false} \end{cases}$.

K-nearest neighbors examples

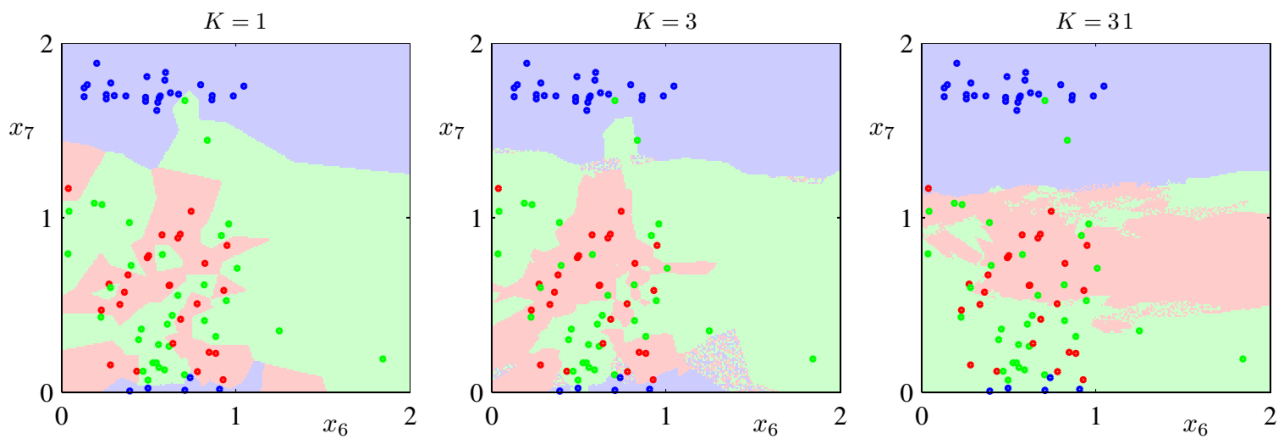


Requires storage of all the data set!

Depends on a distance function!

K-nearest neighbors

Increasing K brings to smoother regions (reducing overfitting)



Kernelized nearest neighbors

Distance function in computing $N_K(\mathbf{x}, D)$

$$\|\mathbf{x} - \mathbf{x}_n\|^2 = \mathbf{x}^T \mathbf{x} + \mathbf{x}_n^T \mathbf{x}_n - 2\mathbf{x}^T \mathbf{x}_n.$$

can be kernelized by using a kernel $k(\mathbf{x}, \mathbf{x}_n)$

Locally weighted regression

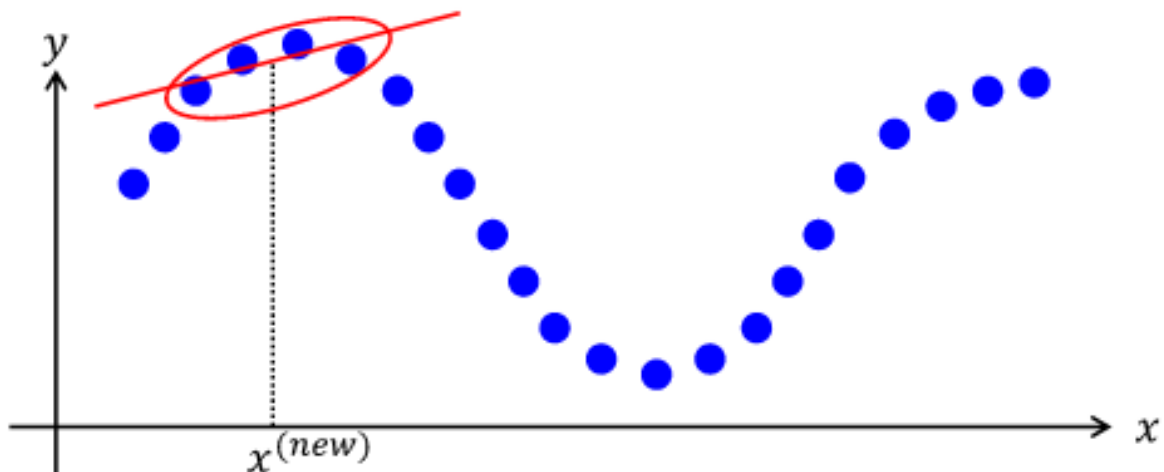
Regression problem $f : X \mapsto \mathbb{R}$ with data set $D = \{(x_n, t_n)_{n=1}^N\}$

Fit a local regression model around the query sample \mathbf{x}_q

- ① Compute $N_K(\mathbf{x}_q, D)$: K-nearest neighbors of \mathbf{x}_q
- ② Fit a regression model $y(\mathbf{x}; \mathbf{w})$ on $N_K(\mathbf{x}_q, D)$
- ③ Return $y(\mathbf{x}_q; \mathbf{w})$

Locally weighted regression

Example with linear kernel



Summary

- ① Non-parametric models based on storing data (lazy approaches)
- ② No explicit model
- ③ Sensitive to parameters and distance function
- ④ Require storage of all data