

# Exercises

The entailment is true. The truth table is as follows:

$\alpha$	$\beta$	$\gamma$	$\phi_1(\alpha, \beta, \gamma)$	$\phi_2(\alpha, \beta, \gamma)$	$\phi(\alpha, \beta, \gamma)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	1	1	1

# Exercises

Tell whether the following propositional formula is valid:

$$\phi(A, B) = (A \wedge B) \vee (\neg A \wedge \neg B)$$

**FALSE.** The truth table is as follows:

A	B	$\phi(A, B)$
0	0	1
0	1	0
1	0	0
1	1	1

# Exercises

(1)  $(Study \wedge Work) \Rightarrow Happy$

**correct**

(2)  $Study \wedge Work \wedge Happy$

**incorrect**

(3)  $\neg Study \vee \neg Work \vee Happy$

**correct, logically equivalent to 1. Why?**

(4)  $(Study \vee Work) \Rightarrow Happy$

**incorrect**

# Exercises

**Recall:** A knowledge base is consistent if it admits at least one model.

The knowledge base is **consistent** because there are two models:

$$\{A, B, C, D\} \text{ and } \{A, C, D\}$$

# Exercises

The new knowledge base written in **CNF** is as follows:

$$\begin{aligned} &A \vee B \\ &\neg B \vee A \\ &\neg A \vee C \\ &\neg A \vee D \end{aligned}$$

$A \vee B$  is **NOT** a Horn clause, because it has more than one positive literal.

# Exercises

Derive  $A \wedge C \wedge D$  using Resolution

Clausal form including the negated thesis:

$$\{A \vee B\}_1, \{\neg B \vee A\}_2, \{\neg A \vee C\}_3, \{\neg A \vee D\}_4, \{\neg A \vee \neg C \vee \neg D\}_5$$

Proof by **resolution**

From (1) and (2)  $\Rightarrow \{A\}_6$

From (3) and (6)  $\Rightarrow \{C\}_7$

From (4) and (6)  $\Rightarrow \{D\}_8$

From (5) and (6)  $\Rightarrow \{\neg C \vee \neg D\}_9$

From (7) and (9)  $\Rightarrow \{\neg D\}_{10}$

From (8) and (10)  $\Rightarrow \{\}$

# Exercises

$$\Gamma = \{ \{ (L \wedge V) \Rightarrow H \}, \{ L \Rightarrow V \}, \{ L \} \} \vdash_R (V \wedge H)$$

Negate the thesis:

Transform into clausal form:

$$\{ \neg L, \neg V, H \}_1, \{ \neg L, V \}_2, \{ L \}_3, \{ \neg V, \neg H \}_4$$

From (1) and (2)  $\Rightarrow \{ \neg L, H \}_5$

From (3) and (5)  $\Rightarrow \{ H \}_6$

From (4) and (6)  $\Rightarrow \{ \neg V \}_7$

From (2) and (7)  $\Rightarrow \{ \neg L \}_8$

From (3) and (8)  $\Rightarrow \{ \}$