Exercises: DPLL

$\Delta_1 = \{\{\neg A, B, C\}, \{\neg B, \neg C\}, \{\neg A, \neg C, \neg D\}, \{C, \neg D\}, \{A, D\}, \{A, \neg C, \neg D\}\}$

1. Splitting rule:

   1a. $A \mapsto F$
   $\{\{\neg B, \neg C\}, \{C, \neg D\}, \{D\}, \{\neg C, \neg D\}\}$

   2a. Unit propagation: $D \mapsto T$
   $\{\{\neg B, \neg C\}, \{C\}, \{\neg C\}\}$

   3a. Unit propagation: $C \mapsto T$
   $\{\{\neg B\}, \square\}$

   1b. $A \mapsto T$
   $\{\{B, C\}, \{\neg B, \neg C\}, \{\neg C, \neg D\}, \{C, \neg D\}\}$

   2b. Splitting rule:

      1ba. $B \mapsto F$
      $\{\{C\}, \{\neg C, \neg D\}, \{C, \neg D\}\}$

      2ba. Unit propagation: $C \mapsto T$
      $\{\{\neg D\}\}$

      3ba. Unit propagation: $D \mapsto F$
      $\{\}$

Satisfying assignment: $A, \neg B, C, \neg D$

Exercises: DPLL

1. Splitting rule:

   1a. $A \mapsto F$
   $\{\{B, C, D\}, \{\neg B, \neg C\}, \{\neg D\}, \{C, \neg D\}, \{B, \neg C\}, \{\neg B, C\}\}$

   2a. Unit propagation: $D \mapsto F$
   $\{\{B, C\}, \{\neg B, \neg C\}, \{B, \neg C\}, \{\neg B, C\}\}$

   3a. Splitting rule:

      1aa. $B \mapsto T$
      $\{\{\neg C\}, \{C\}\}$

      2aa. Unit propagation: $C \mapsto T$
      $\{\square\}$

      $\rightarrow$ Learned clause: $\neg B$

         i. add $\neg B$ to Δ

         ii. Go back to last splitting rule ($B \mapsto T$)

         iii. Continue: $\{\{B, C\}, \{\neg B, \neg C\}, \{B, \neg C\}, \{\neg B, C\}, \{\neg B\}\}$

Exercises: DPLL

1ab. Unit propagation: $B \mapsto F$
$\{\{C\}, \{\neg C\}\}$

2ab. Unit propagation: $C \mapsto T$
$\{\square\}$

$\rightarrow$ Learned clause: A

i. add A to Δ

ii. Go back to last splitting rule ($A \mapsto F$)

iii. Continue: $\{\{A, B, C, D\}, \{\neg A, \neg B\}, \{\neg B, \neg C\}, \{\neg A, \neg D\}, \{A, \neg D\}, \{C, \neg D\}, \{B, \neg C\}, \{\neg B, C\}, \{\neg A, C, D\}, \{A\}, \{\neg B\}\}$

1b. Unit propagation: $A \mapsto T$
$\{\{\neg B\}, \{\neg B, \neg C\}, \{\neg D\}, \{C, \neg D\}, \{B, \neg C\}, \{\neg B, C\}, \{C, D\}\}$

2b. Unit propagation: $B \mapsto F$
$\{\{\neg D\}, \{C, \neg D\}, \{\neg C\}, \{C, D\}\}$

2b. Unit propagation: $C \mapsto F$
$\{\{\neg D\}, \{D\}\}$

3b. Unit propagation: $D \mapsto T$
$\{\square\}$

There is no satisfying assignment.

Exercises: FOL

Solution:

1 F
3 B
4 D
5 C

No correspondence for the formulas 2 and 6, and for the sentences A and E.

Exercises: FOL Skolemization

- Is $P(c)$ the Skolemized version of $\exists x\ P(x)$?
  correct
- Is $P(c, x)$ the Skolemized version of $\forall x\, \exists y\ P(y, x)$?
  incorrect, see next Skolemization
- Is $P(f(x), x)$ the Skolemized version of $\forall x\, \exists y\ P(y, x)$?
  correct
- Is $P(c_1, c_2)$ the Skolemized version of $\exists x\, \exists y\ P(x, y)$?
  correct
- Is $P(c, y, f(y))$ the Skolemized version of $\exists x\, \forall y\, \exists z\ P(x, y, z)$?
  correct
- Is $P(x, y, f(y))$ the Skolemized version of $\forall x\, \forall y\, \exists z\ P(x, y, z)$?
  incorrect, z depends on (x, y) hence $f(y) \Rightarrow f(x, y)$

Exercises: FOL Normal Forms

$$\neg Animal(y) \vee \neg Kills(x, y) \vee \neg Loves(z, x)$$

1a) $\neg Rose(x) \vee Thorn(F(x))$
1b) $\neg Rose(x) \vee Has(x, F(x))$