Exercises: FOL Resolution

(a)

$\forall x\, y\; horse(x) \wedge rabbit(y) \Rightarrow faster(x, y)$

$\forall x\, y\; dog(x) \wedge rabbit(y) \Rightarrow faster(x, y)$

$horse(F) \vee dog(F)$

$rabbit(B)$

$greyhound(A)$

(b) CNF:

$\neg horse(x), \neg rabbit(y), faster(x, y)$

$\neg dog(x), \neg rabbit(y), faster(x, y)$

$horse(F) \vee dog(F)$

$rabbit(B)$

$greyhound(A)$

GOAL: $\neg faster(F, B)$

(c)

$\forall x\; greyhound(x) \Rightarrow dog(x)$

Exercises: FOL Resolution

Given:

1. *The textbooks of class CA are easy*
2. *The textbooks of class CB are difficult*
3. *Mary studies (all and only) easy books*
4. *Mary passes the exam of a class if she studies at least a textbook for that class*
5. *Russel&Norvig is a textbook for class CA*
6. *Tenenbaum is a textbook for class CB*

1 Translate the sentences in FOL, in CNF and tell if it is Horn

2 Prove, using **Resolution**, that *Mary passes an exam*, by adding the appropriate knowledge (if needed)

Exercises: FOL Resolution

A straightforward translation is:

1. $\forall x \; text(CA, x) \Rightarrow easy(x)$
2. $\forall x \; text(CB, x) \Rightarrow \neg easy(x)$
3. $\forall x \; study(Mary, x) \Leftrightarrow easy(x)$
4. $\forall x \; [\exists y \; text(x, y) \wedge study(Mary, y)] \Rightarrow pass(Mary, x)$
5. $text(CA, Russel\&Norvig)$
6. $text(CB, Tenenbaum)$

Exercises: FOL Resolution

A straightforward translation in CNF is:

1. $\neg text(CA, x) \vee easy(x)$
2. $\neg text(CB, x) \vee \neg easy(x)$

3.1. $\neg study(Mary, x) \vee easy(x)$

3.2. $study(Mary, x) \vee \neg easy(x)$

4. $\neg text(x, y) \vee \neg study(Mary, y) \vee pass(Mary, x)$
5. $text(CA, Russel\&Norvig)$
6. $text(CB, Tenenbaum)$

It is Horn (at most one positive atom).

Exercises: FOL Resolution

Knowledge base for the **Resolution**:

$$\{\neg text(CA, x) \vee easy(x)\}_1,$$

$$\{\neg text(CB, x) \vee \neg easy(x)\}_2,$$

$$\{\neg study(Mary, x) \vee easy(x)\}_{3.1},$$

$$\{study(Mary, x) \vee \neg easy(x)\}_{3.2},$$

$$\{\neg text(x, y) \vee \neg study(Mary, y) \vee pass(Mary, x)\}_4,$$

$$\{text(CA, Russel\&Norvig)\}_5,$$

$$\{text(CB, Tenenbaum)\}_6,$$

$$\{\neg pass(Mary, z)\}_7$$

Exercises: FOL Resolution

From (4) and (7) with $\sigma = \{z/x\}$:

$$\{\neg text(z, y) \vee \neg study(Mary, y)\}_8$$

From (5) and (8) with $\sigma = \{z/CA;\ y/Russel\&Norvig\}$:

$$\{\neg study(Mary, Russel\&Norvig)\}_9$$

From (3.2) and (9) with $\sigma = \{x/Russel\&Norvig\}$:

$$\{\neg easy(Russel\&Norvig)\}_{10}$$

From (1) and (5) with $\sigma = \{x/Russel\&Norvig\}$:

$$\{easy(Russel\&Norvig)\}_{11}$$

From (10) and (11) $\Rightarrow \{\}$

Exercises: FOL Resolution

Transform into normal form the original formulas plus the negation of the thesis (A, B, C are Skolem constants):

1) $Equal(x_1, x_1)$
2) $Equal(x_2, x_3) \vee \neg Equal(x_3, x_2)$
3) $Equal(x_4, x_5) \vee \neg Equal(x_4, x_6) \vee \neg Equal(x_6, x_5)$
4) $Equal(A, B)$
5) $\neg Equal(B, C)$
6) $Equal(A, C)$

We can get the empty clause, for instance, as follows:

7) $Equal(B, A)$ resolution from (2) and (4) (substitution $\{x_3/A, x_2/B\}$)
8) $\neg Equal(A, x_5) \vee Equal(B, x_5)$ resolution from(3) and (7) (substitution $\{x_6/A, x_4/B\}$)
9) $Equal(B, C)$ resolution from (6) and (8) (substitution $\{x_5/C\}$)
10) $\{\}$ resolution from (5) and (9)

Exercises: FOL Resolution

The three sentences can be expressed as:

1) $\forall x(Rose(x) \rightarrow \exists y(Thorn(y) \wedge Has(x, y)))$
2) $\forall t(Thorn(t) \rightarrow Dangerous(t))$
3) $\forall x((\exists y(Has(x, y) \wedge Dangerous(y))) \rightarrow Dangerous(x))$

while the last sentence as:
$\forall r(Rose(r) \rightarrow Dangerous(r))$

The transformation of the formulas of KB into clauses (where $F()$ is a Skolem function) gives :
1a) $\neg Rose(x) \vee Thorn(F(x))$
1b) $\neg Rose(x) \vee Has(x, F(x))$
2) $\neg Thorn(t) \vee Dangerous(t)$
3)$\neg Has(x, y) \vee \neg Dangerous(y) \vee Dangerous(x)$
The negation of (4) (where R is a Skolem constant) gives :
4a) $Rose(R)$
4b)$\neg Dangerous(R)$
The empty clause can be derived:
5) $\neg Rose(x) \vee Dangerous(F(x))$ Res. 1a,2 $\langle t/F(x) \rangle$
6) $\neg Rose(x) \vee \neg Dangerous(F(x)) \vee Dangerous(x)$ Res. 1b,3 $\langle y/F(x) \rangle$
7) $\neg Rose(x) \vee Dangerous(x)$ Res. 5,6
8) $Dangerous(R)$ Res. 4a,7 (x R)
9) $\{\}$ Res. 4b,8

Exercises: FOL Resolution

1. $\forall x\ (HOUND(x) \rightarrow HOWL(x))$
2. $\forall x\ \forall y\ (HAVE\ (x,y) \land CAT\ (y) \rightarrow \neg \exists z\ (HAVE(x,z) \land MOUSE\ (z)))$
3. $\forall x\ (LS(x) \rightarrow \neg \exists y\ (HAVE\ (x,y) \land HOWL(y)))$
4. $\exists x\ (HAVE\ (John,x) \land (CAT(x) \lor HOUND(x)))$
5. $LS(John) \rightarrow \neg \exists z\ (HAVE(John,z) \land MOUSE(z))$

1. $\neg HOUND(x) \lor HOWL(x)$
2. $\neg HAVE(x,y) \lor \neg CAT(y) \lor \neg HAVE(x,z) \lor \neg MOUSE(z)$
3. $\neg LS(x) \lor \neg HAVE(x,y) \lor \neg HOWL(y)$
4.
    1. $HAVE(John,a)$
    2. $CAT(a) \lor HOUND(a)$
5.
    1. $LS(John)$
    2. $HAVE(John,b)$
    3. $MOUSE(b)$

Exercises: FOL Resolution

[1.,4.(b):]	6.	$CAT(a) \vee HOWL(a)$
[2,5.(c):]	7.	$\neg HAVE(x,y) \vee \neg CAT(y) \vee \neg HAVE(x,b)$
[7,5.(b):]	8.	$\neg HAVE(John,y) \vee \neg CAT(y)$
[6,8:]	9.	$\neg HAVE(John,a) \vee HOWL(a)$
[4.(a),9:]	10.	$HOWL(a)$
[3,10:]	11.	$\neg LS(x) \vee \neg HAVE(x,a)$
[4.(a),11:]	12.	$\neg LS(John)$
[5.(a),12:]	13.	$\square$