## Written exams of Robotics 2

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All materials are in English, unless indicated (oldies are in Italian)

| Year | Date<br>(mm.dd) | Number of exercises | Topics   | Notes |
|------|-----------------|---------------------|--|-------|
| 2025 | 02.07           | 3                   | Comparison of Projected Gradient (PG), Reduced Gradient (RG) and Task Priority (TP) methods for addressing a redundant task in a 2P2R planar robot with joint limits; Reduced dynamics of a PPR robot with end effector constrained on a linear surface and a hybrid force-velocity control task (with redundancy); Dynamic modeling of a 1-dof robotic servomechanism with a DC motor and a flexible transmission, and related inverse dynamics for a rest-to-rest motion without residual vibrations |       |
| 2025 | 01.13           | 2                   | 2R polar robot: - Lagrangian dynamics and linear parametrization all known regulators and their properties, - inverse dynamics torque on a specific trajectory, - adaptive tracking control design; 3P planar robot on a 2D task: - minimum velocity norm and minimum kinetic enegy solutions; - mass distribution for a diagonal Cartesian inertia (if possible)  |       |

| 2024 | 11.07 | 3 | Dynamic model of a PPR robot in a vertical plane, linear parametrization, and upper bound on the gravity term; Inverse dynamics computation for the PPR planar robot for exact execution of a circular trajectory; Hybrid force-motion task definition: writing with a ball pen on a flat sheet  |           |
|------|-------|---|--|-----------|
| 2024 | 09.19 | 5 | Dynamic model of an RP robot with base offset and under gravity, with linear parametrization; RP robot: integral of generalized momentum on a special joint trajectory; Minimization of (twice) the kinetic energy), under constant momentum components, with application to the RP robot; RP robot in contact with a compliant wall: hybrid force-position control with linear and decoupled dynamics (using modified impedance scheme); RP robot without gravity: inverse dynamics problem and minimum motion time under torque bound  | solutions |
| 2024 | 07.08 | 4 | Dynamic model of a spatial PRR robot and its minimal linear parametrization [was part of 7 Jan 2020, without solution]; Redundancy resolution at acceleration level for a 2R planar robot in a one-dimensional task [modified from 24 Mar 2023, without solution]; Dynamic model of a two-mass system with a damped elastic joint under gravity, with design of a stabilizing feedback for regulation and of inverse dynamics torque for trajectory execution [modified from 19 Oct 2021, without solution]; Forced equilibria and local asymptotic stabilization of an underactuated 2R planar robot under gravity (Pendubot) | solutions |

|      |                            |   | <del>-</del>  |                               |
|------|----------------------------|---|---|-------------------------------|
| 2024 | 06.12                      | 4 | Kinetic energy and inertia matrix of a 3R robot given only via a DH table with linear factorization of inertial terms; Redundancy resolution for a 4R planar robot in relative pointing task (nominal or with error) with joint range optimization; Proof of global asymptotic stabiliy of a PD motor regulation law for a robot with n elastic joints without gravity; Fault detection and identification of a single actuation fault acting on the first joint of a PR robot under gravity  | solutions                     |
| 2024 | 04.24<br>(Midterm<br>Test) | 4 | 3R planar robot: i) Projected Gradient method at velocity level, with maximization of clearance from an obstacle*; ii) Reformulation with Task Priority method (two variants)*; 2R spatial robot under gravity: i) dynamic model and factorization of velocity terms*; ii) linear parametrization with a minimal set of dynamic coefficients*; iii) torque for an inverse dynamics example*; iv) equilibrium configurations; v) mechanical parameters needed to balance the robot under gravity*; vi) end-effector acceleration in response to a tip force*; Two computational uses of the recursive Newton-Euler algorithm; Maximum instantaneous reduction of the total robot energy under bounded torques: i) in a generic state with non-zero joint velocity; ii) in a state with g(q) ≠0 and zero joint velocity | solutions,<br>MATLAB<br>codes |
| 2024 | 02.16                      | 4 | Analysis of torque and acceleration limits related to the inertia matrix, with a skewed 2P planar robot example; Dynamic model of a (skewed) PPR planar robot, with linear parametrization and feedback iinearization for trajectory tracking; Sphere-in-hole task description and related hybrid force-velocity control diagram; Redundancy resolution for a (skewed) PPR robot commanded in joint velocity, with minimization of the robot kinetic energy   |                               |

| 2024 | 01.24                      | 3           | State-space equations from dynamic model, using coordinates q and generalized momentum p as state variables; State-space equations with $x = (q,p)$ for the dynamics of a RP planar robot with friction and under gravity (symbolic and numeric); Questionnaire with 4 questions  |           |
|------|----------------------------|-------------|---|-----------|
| 2023 | 09.11                      | 1 (6 parts) | 4P planar robot in vertical plane: - dynamic model in joint space - Cartesian inertia - joint-space regulation with PD+ and analysis - non-zero joint torque producing no task acceleration - minimum norm joint torque for a desired task acceleration - linear parametrization and adaptive control   |           |
| 2023 | 07.10                      | 3           | Two proofs on redundant robot dynamics: dynamically consistent decomposition of joint torques, expression of the task dynamics; For a RPR planar robot: inertia matrix, gravity vector, equilibria, linear parametrization of the gravity vector, bound on the gradient of g(q); Sphere-in-Hole with natural and artificial constraints, selection matrices, handling of inconsistent measurements  |           |
| 2023 | 06.12                      | 4           | Inertia matrix for a 4R planar robot in absolute coordinates (plus some manipulation and extension); Five redundancy resolution schemes for a 4R planar robot with one or two 2-dimensional tasks, possibly with priority; Analysis of a PD + gravity compensation law for a PRR robot with balanced third link; Reduced dynamics and inversion-based motion and constraining force control of a closed kinematic chain for payload elevation | solutions |
| 2023 | 04.19<br>(Midterm<br>Test) | 6           | SNS redundancy optimization for a 3R planar robot commanded in acceleration with joint velocity and acceleration bounds; Mass and CoM distribution for achievieng a structured gravity term in a 3R planar arm;   | solutions |

|      |       |   | Minimization of kinetic energy or of joint velocity norm for a 4P planar robot in a redundant task; Inertia matrix of a RPR spatial robot (with assigned DH frames and coordinates); Symbolic computation of Coriolis and centrifugal terms, three of its factorizations for skew-symmetry (or not), and regressor for a linear parametrization of a 3-dof (RPR) robot with assigned inertia matrix; Minimum-time motion on a rest-to-rest cubic joint trajectory for a vertical PR robot under force/torque input bounds |           |
|------|-------|---|---|-----------|
| 2023 | 03.24 | 3 | Dynamic redundancy resolution for a 2R planar robot in a one-dimensional task (symbolic and numeric solutions); Adaptive trajectory tracking control of a 2R planar robot, with uniform link mass distribution (as thin rods), uncertain but equal link lengths, and unknown but equal link masses; Definition of natural and artificial constraints for a hybrid force-motion task, with choice of suitable references for the controlled variables  |           |
| 2023 | 02.13 | 3 | Three dynamic schemes of redundancy resolution for a planar 3R robot with uniform links; Minimum time swing-up for a 1-DOF pendulum along a cubic rest-to-rest trajectory under torque bounds Study of the position regulation of a system with two masses and an intermediate spring, each controlled by a PD or PD+ffw law  | solutions |
| 2023 | 01.25 | 2 | - Planar 2R robot with balanced second link:  Dynamic model of a balanced 2R robot with friction and under gravity and its linear parametrization; Minimum gains for regulation under PD+gravity compensation; Adaptive control for trajectory tracking; Minimum time rest-to-rest transfer under torque bounds keeping the second link fixed (no gravity nor friction)  - Hybrid force-velocity control task of a Cartesian robot in contact with a compliant linear surface   | solutions |

| 2022 | 10.21 | 6 | All exercises are related to a PR robot under gravity: Dynamic model; Minimal linear parametrization; Regulation with PD + constant gravity compensation; Computation of the associated initial acceleration; Inverse dynamics command along a desired joint trajectory; Minimum time motion under torque bounds along a special prescribed joint path   | solutions                     |
|------|-------|---|--|-------------------------------|
| 2022 | 09.09 | 3 | Constrained minimization of joint range function for a planar 3R robot*; Task control for a planar 2R robot at the torque level, with specified transients for the errors along tangential/normal directions to the linear path (with numerical evaluation)*; Dynamic modeling and design of PID and iterative learning control laws for a two-mass system under gravity (plus dynamic modeling with a flexible cable) | solutions;<br>MATLAB<br>codes |
| 2022 | 07.08 | 3 | Change of coordinates in the dynamic model a generic 3R robot so as to match the work of the actuating torques*; Reduced dynamics and control problem for a generic robot, with the first joint variable being constrained $(q_1=k)$ ; Minimum-time rest-to-rest motion of a Cartesian PP robot under gravity and with bounded input forces*   | solutions;<br>MATLAB<br>codes |
| 2022 | 06.10 | 4 | Adaptive control of a PR robot horizontal plane with regressor Y, given M(q)*; Algorithmic singularities for a planar 4R micro-macro robot performing two tasks, and its task priority solution*; Natural impedance control for elastic joint robots under preliminary joint elastic torque feedback; Minimum-time rest-to-rest motion of an actuated pendulum with bounded torque and bang-bang acceleration profile  | solutions;<br>MATLAB<br>codes |

| 2022 | 04.13<br>(Midterm<br>Test) | 6 | Calibration of link lengths with data for a planar 2R robot; Discrete-time redundancy acceleration control minimizing norm of next velocity; Inertia matrix of a 3R spatial robot (polar robot mounted on a rotating base); Projected gradient at velocity level for a planar 3R robot minimizing H_range (with joint limits and task scaling); Minimum and weighted norm of torques for a PR robot on a 1-dimensional task; Gravity term for a vertical PR robot, with bound on the norm of its gradient                                  | solutions |
|------|----------------------------|---|--|-----------|
| 2022 | 02.03                      | 3 | Cartesian regulation of a RPR robot under gravity, with minimal factorization of the gravity term and analysis of singular situation for the control law; Minimization of the acceleration norm with a null-space velocity damping in a 4R planar robot, with a 2-dimensional and then 3-dimensional augmented task; Reduced dynamic model of a constrained planar 2R robot under gravity with only one torque (mimicking an elevator along the y-axis), with inverse dynamics for a static condition and a rest-to-rest motion trajectory | solutions |
| 2022 | 01.11                      | 3 | Minimization of kinetic energy and additional obstacle avoidance for a planar RPR robot; Joint torque control to smoothly stop a robot (at equilibrium) in time T and uniform scaling in presence of an additional acceleration bounds; Reduced dynamic model and control of a planar PR robot with its endeffector constrained on a line segment  | solutions |

| 2021 | 10.19                                | 2 | Robust redundancy resolution for the two-arm DLR Justin robot, for independent or coordinated tasks of the two end effectors, at the velocity or acceleration level; Lagrangian dynamic model of a visco-elastic robot joint, with the design of a stabilizing feedback for regulation and inverse dynamics for trajectory execution [updated version from 23 March 2007]  |                               |
|------|--------------------------------------|---|--|-------------------------------|
| 2021 | 09.10                                | 3 | Deceleration control for a planar 3R redundant robot under input bounds*;<br>Cartesian inertia matrix for the same planar 3R redundant robot*;<br>Impedance control design for a 2-dof Cartesian robot without F/T sensing   | solutions;<br>MATLAB<br>codes |
| 2021 | 07.12                                | 3 | Acceleration control problems for a planar 3R robot under input bounds; Modeling and analysis of an incipient blocking actuator fault in a robot; Dynamic modeling and adaptive control of a planar RPR robot, with partially known dynamic parameters   | solutions                     |
| 2021 | 06.11                                | 5 | Weighted pseudoinverse using the pinv function of MATLAB*; Dynamic model of an elastically suspended link and two related dynamic problems; Guessing a robotic structure from inertial and gravity terms (a 3P portal robot); Transformation of generalized forces in two systems of coordinates (planar 3R); Trajectory tracking using feedback linearization along a circle, with transient behaviors specified in a time-varying task frame | solutions;<br>MATLAB code     |
| 2021 | 04.14<br>(Remote<br>Midterm<br>Test) | 3 | Dynamics of a planar 2R robot under gravity whose links have a non-symmetric CoM, with its linear parameterization; Comparative methods of redundancy resolution at the velocity level in a planar 3R robot performing 2 tasks, possibly with priority: pseudoinversion of the Extended Jacobian, damped least squares on the Extended Jacobian, task priority method;   | solutions                     |

|      |                   |   | Minimum time motion under bounded inputs for a PR robot tracing a circular path  |           |
|------|-------------------|---|--|-----------|
| 2021 | 02.04<br>(Remote) | 4 | Inertia and gravity terms of a RRPR robot, with their linear parameterization; Minimum-time motion on a linear path for two cooperating Cartesian robots under motor torque bounds; Dynamic self-motion task for a 3R robot with null-space projection or joint space decomposition; 1-dof force regulation with zero error (using PI control) for a rigid robot and a rigid environment   | solutions |
| 2021 | 01.12<br>(Remote) | 5 | Kinematic control in a self-motion task for a redundant planar 3R robot so as to minimize the potential energy due to gravity; Torque control for the same planar 3R robot and self-motion task; Dynamic model of a polar RR robot; Two dynamic problems for the same polar RR robot: analysis of a steady-state condition and minimal linear parametrization; Collision detection, isolation and identification for a PPR robot in the horizontal plane | solutions |
| 2020 | 10.23<br>(Remote) | 4 | Regressor for adaptive control of a PR robot moving on a horizontal plane; Minimum weighted torque solutions for 1-dimensional task of the same PR robot; Sufficient conditions for asymptotic stability of a PD plus gravity compensation law on the same PR robot in the vertical plane; Definition of a force-motion hybrid task for a sphere rolling on a plane while following a desired trajectory of the contact point                            |           |

| 2020 | 09.11<br>(Remote)                    | 5    | Control formulation for the simultaneous trajectory execution and obstacle avoidance of a planar 3R robot arm; Interaction matrix for the geometric barycenter of three point-features in IBVS, with null space computation; Cartesian inertia matrix of a RP robot, with tip acceleration in response to a force; Reduced dynamics for a planar 3R robot under Cartesian constraints on last link; Comparison of torques between feedback linearization and global Lyapunov-based trajectory tracking control laws | solutions                                       |
|------|--------------------------------------|------|---|---|
| 2020 | 07.15<br>(Remote)                    | 5    | Inertia matrix of a 4-dof (PRRR) planar robot using absolute coordinates; Task augmented Jacobian for two tasks (on the same 4-dof robot) with minimum norm solutions and analysis of algorithmic singularity; Natural and artificial constraints for a hybrid task of moving a cylinder on a plane; Uniform time scaling to recover torque feasibility for a 2R robot under gravity; Three questions on regulation controllers with specified asymptotic or exponential transients for the errors                  | solutions                                       |
| 2020 | 06.05<br>(Remote)                    | 5    | SNS algorithm under velocity bounds for a 3R robot; Analysis of physical conditions for a 2-dof dynamic model; Bound on the gradient of the gravity term for an RP robot under gravity with limited range of the prismatic joint; Dynamic modeling and adaptive control for a 2P robot in the vertical plane with joint friction and payload; Natural and artificial constraints for the hybrid task of closing a door  | solutions                                       |
| 2020 | 04.15<br>(Remote<br>Midterm<br>Test) | Q 10 | Questionnaire with 10 questions, with replies to be elaborated*   | solutions;<br>MATLAB<br>codes (for Q2<br>to Q9) |

| -    |       |   |  | 1                             |
|------|-------|---|--|-------------------------------|
| 2020 | 02.12 | 3 | Dynamic model of a planar RPR robot under gravity, linear parametrization of the gravity vector, bound on its gradient; Two redundancy resolution problems for the same robot above, assuming joint velocity as input; A 1-dof force control problem with combinations of P, I and feedforward laws  |                               |
| 2020 | 01.07 | 2 | Dynamic model of a spatial PRR robot, linear parametrization, and adaptive control design; Questionnaire with 4 questions  |                               |
| 2019 | 09.11 | 3 | Control law to assign a desired dynamic behavior to the robot kinetic energy in the closed-loop system, with application to a 3R robot moving on the horizontal plane; Minimum-time motion under joint torque/force bounds for a RP planar arm moving its end-effector along a circular path; Impact of two masses, with conservation of kinetic energy and total momentum | solutions                     |
| 2019 | 07.11 | 3 | Task priority control of a 3R planar robot for two tasks, with analysis of their compatibility*; Residual vector computation for collision detection and isolation in a RP planar robot; Minimum-time motion under torque bound for an actuated pendulum under gravity on a special class of trajectories with bang-coast-bang acceleration profile*                       | solutions;<br>MATLAB<br>codes |
| 2019 | 06.17 | 6 | Gravity term in the dynamic model of the 6-dof Kawasaki S030 robot (with simplified assumptions); Check for a 3x3 matrix to be an inertia matrix; Comparison of Jacobian pseudoinverse and Jacobian transpose velocity commands in redundant robots; Assignment of a linear and decoupled Cartesian error dynamics to a 2R robot in a regulation task;                     | solutions                     |

|      |                            |   | Natural and artificial constraints in a hybrid force-velocity control task formulation   |           |
|------|----------------------------|---|--|-----------|
| 2019 | 04.29<br>(Midterm<br>Test) | 5 | Check for 2x2 matrices to be inertia matrices; Dynamic model of a PRP planar robot and its linear parametrization; Gravity term in the dynamic model of a nR planar robot, with equilibrium conditions and balancing conditions; Factorizations of the Coriolis/centrifugal terms of a 2-dof robot and a 3-dof robot; Execution of a one-dimensional task with a 2R robot, with minimization of its kinetic energy; Acceleration resolution for a redundant 3R planar robot, under hard joint acceleration constraints | solutions |
| 2018 | 07.11                      | 4 | Inertia matrix and its linear parametrization for a 2R planar robot with elastic joints; Saturated solution for a 4R planar robot executing a Cartesian acceleration under joint acceleration bounds (variant of the SNS algorithm); Interaction matrix for the average position of n point features; Reduced dynamics of a robot under the geometric constraint that a subset of coordinates is constant $(q_b = q_{bd})$   | solutions |
| 2018 | 06.11                      | 3 | Explicit expression of the dynamic terms in the residual vector for collision detection and isolation in a RP robot moving in the vertical plane, and analysis of the possible issues in detection/isolation; Adaptive control law for a 1-dof actuated pendulum, when also the current-to-torque drive gain is unknown; Feedback/feedforward control schemes for force regulation of a mass in contact through a load cell with a stiff environment: equilibria and stability analyses                                | solutions |

| 2018 | 04.26<br>(Midterm<br>Test) | 4 | Dynamic model of a 2P2R planar robot under gravity and its linear parametrization*; Factorizations of the Coriolis/centrifugal terms in a planar 2R robot plus an inverse dynamics problem for the same robot*; General derivation of the Hamiltonian model equations of a robot manipulator, using generalized coordinates q and generalized momentum p as state variables; Minimum norm velocity solution for a (redundant) PPR planar robot with the associated unit inconsistency problem, and use of a weighted pseudoinversion to resolve the issue* | solutions;<br>MATLAB<br>codes |
|------|----------------------------|---|--|-------------------------------|
| 2018 | 03.27                      | 2 | Dynamic model, linear parametrization, approximate linearization and partial feedback linearization control of a 2-dof automated crane (with passive swinging link/payload);  PPR planar robot with end-effector in contact with a linear, frictionless surface: nominal commands for a hybrid force/velocity dynamic task addressing also robot redundancy  | solutions                     |
| 2018 | 02.05                      | 3 | Formulation of the task Jacobian and of the related kinematic control problem when the task function is the norm of the task error, and its application to a visual servoing problem with two point-features; Dynamic model of a PRP (cylindrical-like) robot, with a horizontal first prismatic axis, and adaptive trajectory tracking control design; Iterative learning control of an actuated pendulum (single-link under gravity), with convergence analysis of the PD feedback plus (updated) feedforward law  | solutions                     |
| 2018 | 01.11                      | 3 | Redundancy resolution schemes for a planar RP robot in a one-<br>dimensional task;<br>Lagrangian dynamics, analysis, and nonlinear control of a Boulton-Watt<br>centrifugal regulator;<br>Impedance and force control design alternatives for a single mass subject<br>to an external force  | solutions                     |

| 2017 | 10.27 | 1<br>(4 parts) | Definition of four different optimization schemes for redundant robots by local minimization of suitable norms (with application of the first two schemes to a planar RP robot): generalized momentum difference; velocity difference with respect to the (anti)gradient of a function; torque; weighted combination of torque and task acceleration error  |                               |
|------|-------|----------------|---|-------------------------------|
| 2017 | 09.21 | 2              | Dynamics of RP robot in a tilted plane, with computation of the maximum and minimum norm of the tip acceleration as function of the configuration, when starting at rest and under bounds on the command inputs; Definition of control laws and transition conditions among states for a robot operating in the presence of a human, with a Cartesian trajectory task in normal conditions and detection/reaction to mild or severe collisions  | solutions                     |
| 2017 | 07.11 | 3              | Dynamic terms of an RPP (cylindrical) robot used in a tracking control law; Derivation of a reduced dynamics model and control of a Cartesian 2P robot in a vertical plane, with its end-effector motion being constrained to a linear surface; Steady-state analysis of a planar 3R robot under a Cartesian position regulation law, while in contact with an obstacle: equilibrium control torques, torques at the joints due to contact, momentum-based residual, and estimation of the Cartesian contact force                                    | solutions                     |
| 2017 | 06.06 | 3              | Inverse differential solution with minimum norm of the joint jerk for a 3R planar robot using absolute coordinates (nominal case and feedback control to correct initial mismatch with Cartesian trajectory)*; Choice of PD gains in a regulation control law with gravity cancelation for the same 3R planar arm of Ex#1 and under joint torque bounds; Dynamic model of a PRP planar robot, with factorization of the quadratic velocity terms, equilibrium configurations, and linear bound on the Hessian of the potential energy due to gravity* | solutions;<br>MATLAB<br>codes |

| 2017 | 05.29<br>(Final<br>Test)   | 4 | Presentation of known regulation control laws in the presence of gravity, with two more specific questions; Interaction matrix for a point feature in an IBVS problem when using polar coordinates in the image plane, and its characteristics*; Gravity term and its parametrization in an adaptive trajectory tracking controller for a PRP planar robot when the other inertial parameters are already known; Definition of task frame and of natural and artificial constraints for a hybrid force/motion control of a robotized surface polishing task | solutions;<br>MATLAB code     |
|------|----------------------------|---|---|-------------------------------|
| 2017 | 03.29<br>(Midterm<br>Test) | 5 | Inertia matrix of a 3R spatial robot and its linear parametrization*; Proof of a weighted pseudoinverse in case of rank deficiency of the Jacobian; Saturated solution for a 4R planar robot executing a Cartesian velocity under joint velocity bounds (SNS algorithm)*; Reduced Gradient and Task Augmented solutions for a 3R planar robot*; Calibration equations for a 2R planar robot with uncertain parameters*  | solutions;<br>MATLAB<br>codes |
| 2017 | 01.11                      | 3 | Interpretation of the inertia matrix and all possible regulation control laws for a 2R robot, with their design conditions and convergence/stability properties; Use of recursive Newton-Euler algorithm for computing kinematic quantities (similar to Ex#2, 22.09.2014); Redundancy resolution for planar 3R robot at the velocity level in the presence of an obstacle (same as Ex#1, 10.06.2014)  |                               |
| 2016 | 10.28                      | 2 | Inertia matrix and gravity vector (with equilibrium configurations) of a RP robot in a vertical plane, with second prismatic joint axis skewed; Dynamic modeling, determination of unforced equilibrium states and of steady-state conditions under constant input force, and regulation control for a mechanical system of three masses interconnected by springs with viscous friction  | solutions                     |

| 2016 | 09.12                      | 2              | Inertia matrix and gravity vector (with equilibrium configurations) of a RPR robot in a vertical plane; Dynamic modeling and determination of the equilibrium states for a mechanical system of two masses interconnected by a nonlinear spring  | solutions                                      |
|------|----------------------------|----------------|--|--|
| 2016 | 07.11                      | 3              | Dynamic model of a planar 2R robot in absolute coordinates and input torque transformation; Task frame and natural/virtual constraints definition for a square peg-inhole insertion; Compliance and force control laws in one contact direction, with robustness analysis  | solutions                                      |
| 2016 | 06.06                      | 2              | Dynamic modeling of a polar 2R robot and its adaptive trajectory tracking control law; All possible regulation control laws for this robot, with their design conditions and stability properties  | only solution<br>to Ex#1                       |
| 2016 | 06.01<br>(Final<br>Test)   | 1<br>(4 parts) | Planar 2R robot subject to a single holonomic constraint: reduced dynamic model and its features; equilibrium torques (and associated constraint force); simulation set-up for the reduced model; hybrid force/motion control task   | solution                                       |
| 2016 | 04.13<br>(Midterm<br>Test) | 3              | Inertia matrix and Coriolis/centrifugal terms of a planar PRR robot;<br>Gravity vector, its linear parametrization, equilibria and mechanical<br>balance of a planar 4R robot;<br>Singularities of a planar 4R robot for position/orientation tasks, and a joint<br>velocity solution minimizing the distance from the joint range midpoints | solutions                                      |
| 2015 | 04.15                      | 2              | Equilibrium configurations, approximate linearization, and regulation control of RP robot; Motion of a planar 3R robot using redundancy to avoid Cartesian obstacles   | modified<br>from<br>2006.07.13<br>(in Italian) |
| 2014 | 10.27                      | 2              | Expression of the residual for collision detection and isolation in a planar PRR;  | solutions                                      |

|      |       |                           | Maximum contact force in norm applied at the tip of the planar PRR robot that can be balanced in the presence of hard bounds on the actuator torques  |           |
|------|-------|---------------------------|---|-----------|
| 2014 | 09.22 | 2                         | Dynamic model of a DC motor with elastic transmission, planning of a rest-to-rest trajectory, and its inverse dynamics solution; Use of recursive Newton-Euler algorithm for computing end-effector differential kinematic quantities |           |
| 2014 | 07.15 | 1                         | Analysis of gravity terms and unforced equilibrium configurations for a KUKA LWR with last three joints frozen  |           |
| 2014 | 06.10 | 2                         | Redundancy resolution for planar 3R robot at the velocity level in the presence of an obstacle; Dynamic modeling and control of a two-mass system under gravity and with an elastic transmission                                      |           |
| 2014 | 04.02 | 2                         | Definition of kinematic controllers for the two-arm Justin robot;<br>Acceleration analysis for a planar 3R robot subject to an end-effector force   |           |
| 2013 | 09.19 | 1                         | Inertia matrix and acceleration analysis for a planar RPPR robot  |           |
| 2013 | 07.15 | 1                         | Analysis and estimation of a contact force in three different cases for a planar 3R robot   | solution  |
| 2013 | 06.10 | 1                         | Kinetic energy using the recursive algorithm with moving frames, inertia matrix and its linear parametrization for a 4-dof robot for which only the DH table is given   |           |
| 2013 | 02.06 | 1                         | Reduced dynamic model for a planar RP arm, with its end-effector constrained on a line, and associated hybrid force/motion regulation law   | solution  |
| 2013 | 01.09 | 2                         | Jacobians for two tasks, algorithmic singularities, and task priority solution for a planar 3R arm; Constrained dynamic model for a planar RP arm, with its end-effector constrained on a line  | solutions |
| 2012 | 07.05 | 1<br>(2 parts +<br>bonus) | Dynamics of a 3-dof portal robot for aeronautical industry; Controller for a regulation task; (bonus) Generalized coordinates for the closed kinematic loop   | solution  |

| 2012 | 06.11 | 2 | Null space of visual interaction matrix for a point feature (MATLAB code included); Regulation task for a planar 3R robot subject to a contact force   | solutions                       |
|------|-------|---|--|---------------------------------|
| 2011 | 09.12 | 1 |  | solution                        |
| 2011 | 09.12 | 1 | Gravity balance and PD + constant gravity compensation of a 2R robot in the vertical plane with an additional payload  | Solution                        |
| 2011 | 07.04 | 1 | Dynamic modeling and feedback/feedforward force regulation of a two-<br>mass, two-spring system  |                                 |
| 2011 | 06.17 | 1 | Collision reaction for a 2R planar robot and use of the residual   | solution                        |
| 2010 | 09.15 | 1 | Inertia matrix and its minimal linear parameterization for a planar RPR robot  | solution                        |
| 2010 | 07.07 | 1 | Dynamic model of a two-mass/spring/damper system and analysis of a contact force loop  | solution                        |
| 2010 | 06.15 | 1 | Dynamic model of RP under gravity and minimum torque solution for a one-dimensional acceleration task  | solution                        |
| 2009 | 09.10 | 2 | Dynamic control of a linear Cartesian trajectory with decoupled error along the tangential and normal directions to the path; Uniform scaling of unfeasible trajectory with respect to torque constraints (without gravity) (in Italian) | solutions (in Italian)          |
| 2009 | 07.10 | 2 | Feasible acceleration region for a 2R planar robot with torque constraints; Definition of natural and artificial constraints for a hybrid task and its realizability with a SCARA robot (in Italian)                                     | solutions (in Italian)          |
| 2009 | 06.10 | 2 | Inertia matrix of a planar PRP robot;<br>Solution with minimum weighted norm of the torque and dynamic control<br>in the task space for redundant robots (in Italian)  | solutions (in Italian)          |
| 2008 | 09.11 | 1 | Adaptive control of planar 2R robot with payload and minimal parameterization (in Italian)   | sketch of solution (in Italian) |
| 2008 | 03.20 | 1 | Dynamic model of planar PRR robot under gravity and linear parameterization (in Italian)   |                                 |

| 2007 | 09.13 | 2                | General structure of the minimal parameterization of the gravity term for planar nR robots; Hybrid task of surface finishing on a sphere with a 3P or 3R robot (in Italian)  |   |
|------|-------|------------------|--|---|
| 2007 | 04.19 | 2                | Dynamic model of PRP robot under gravity; Robot regulation control with assignment of linear error dynamics (in Italian)   | solutions (in Italian)                    |
| 2007 | 03.23 | 2                | Redundancy resolution for the two-arm DLR Justin robot; Lagrangian dynamic model of a visco-elastic robot joint (in Italian)   |   |
| 2006 | 09.11 | 1                | Planar 3R robot: inertia matrix, optimization of manipulability, minimum norm joint velocity computation (in Italian)  |   |
| 2006 | 07.13 | 2                | Equilibrium configurations and approximate linearization of RP robot;<br>Motion of a planar 3R robot among obstacles with Cartesian artificial<br>potentials (in Italian)  |   |
| 2006 | 06.30 | 2                | End-effector forces and torque at second joint that produce zero acceleration at the joints of a planar 3R robot under gravity;  Voronoi diagram (in Italian)  | only solution<br>to ex #2 (in<br>Italian) |
| 2005 | 09.22 | 1                | Kinematic control of a mobile manipulator (unicycle + single link arm) (in Italian)  |   |
| 2005 | 04.14 | 1<br>(two parts) | Dynamic model of a polar 2R robot; regulation with PD plus constant gravity compensation law (in Italian)  |   |
| 2005 | 04.05 | 2                | Dynamic model of a planar RP robot, equilibrium configurations under gravity, minimal parameterization and adaptive control; Stabilization of a linear mass in the presence of limited motion range, using artificial potentials (Lyapunov proof) (in Italian) | solutions (in Italian)                    |
| 2004 | 04.06 | 1                | Motion planning with approximate cell decomposition (in Italian)   | solution (in Italian)                     |
| 2004 | 03.25 | 1                | Dynamic model of an offset RP robot under gravity, adaptive control, partial feedback linearization of the first joint dynamics when the second joint is passive (or vice versa) (in Italian)  | solution (in Italian)                     |

Note: For these\* problems, MATLAB codes for computing solutions and/or for graphics are either embedded in the solution text or available to the students of the course upon request (contact <a href="mailto:deluca@diag.uniroma1.it">deluca@diag.uniroma1.it</a>).