A Robust Optimization Approach for the Vehicle Routing Problem with Uncertain Travel Cost

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***Abstract*—The Robust Vehicle Routing problem (RVRP) with discrete scenarios is studied here to handle uncertain traveling time, where a scenario represents a possible discretization of the travel time observed on each arc at a given traffic hour. The goal is to build a set of routes considering the minimization of the worst total cost over all scenarios. A Genetic Algorithm (GA) is proposed for the RVRP considering a bounded set of discrete scenarios and the asymmetric arc costs on the transportation network. Tests on small and medium size instances are presented to evaluate the performance of the proposed GA for the RVRP. On small-size instances, a maximum of 20 customers, 3 vehicles and 30 discrete scenarios are handled. For medium-size instances, 100 customers, 20 vehicles and 20 scenarios are tested. Computational results indicate the GA produces good solutions and retrives the majority of proven optima in a moderate computational time.**

I. INTRODUCTION

The Vehicle Routing Problem (VRP) is a NP-hard problem [1] which aims at defining routes for a fleet of vehicles, such that each vehicle starts and ends its tour at the depot node, each customer is visited once, and vehicle loads comply with vehicle capacity [2]. Introduced by [3], the VRP is one of the most studied problems in combinatorial optimization. An interesting topic on solving VRP problems consists in considering parameters affected by uncertainty, making the problem more realistic in real life contexts, for example, in urban transportation.

An alternative approach to handle uncertain parameters is the robust optimization, in which one can optimize against the worst case that might arise by using a *min-max* objective function [4]. Different approaches have been proposed to deal with robust optimization [5]–[9]. In particular, for the discrete optimization problems, the authors in [9] proposed a mathematical model which aims at seeking a solution that minimize the worst case under a set of scenarios.

Some key components are involved on the application of robust optimization. These components are: (i) the way the uncertainty data is modeled (e.g. using discrete or continuous data), (ii) the selection of an appropriate optimization robust criterion, e.g *min-max* [10], [11], *min-max* regret [12], *min-max* relative regret [13], α-robustness [14], bw-robustness [13], [15], pw-robustness [16], etc, and (iii) the choice of a method to generate robust solutions. Here, we study particularly the Robust Vehicle Routing Problem (RVRP) in which uncertain data is modeled as a bounded set of discrete scenarios and the optimization relies on minimizing the total cost of arcs considering the *min-max* optimization criterion. Then, a robust solution is the one which prevents against the worst case.

Let $G = (V, A)$ be a connected and directed graph with a set $V = \{0, 1, 2...n\}$ of $n = |V|$ vertices (which can also be referred as customers), including the depot $\{0\}$, and a set $A = \{(i, j)|i, j \in V, i \neq j\}$ of arcs. The set of discrete costs associated with every arc is modeled as a set of p discrete scenarios $S = \{1, 2, ...p\}$, where a scenario $k \in S$ is defined as an assignment of a cost value $c_{ij}^k > 0$, $\forall (i, j) \in A$. It is important to mention that whenever a scenario $k \in S$ is considered, the cost value for all the arcs are set to the scenario k. In addition, a demand d_i is associated with each customer $i \in V$ and it cannot be splitted in several vehicles. The fleet of vehicles $F = \{1, 2, ..m\}$ is homogeneous, *i.e.*, each vehicle has identical capacity Q, and is located at the depot. Therefore, the RVRP aims at defining a set of routes which starts and ends at the depot, visits each customer once, and minimizes the maximum total cost considering all the scenarios.

Some works in the literature deal with the RVRP, and most of them focused on uncertain data over the time windows and demands [17]. A survey which outlines the RVRP with uncertainty related to demands, travel times and cost coefficients can be found in [18]. One of the most studied RVRP version involves uncertain demands. For this purpose, an early work is presented by [19] for the RVRP with uncertain demands and time windows. Analytical results are reported for large scale RVRP on cluster-first route-second heuristics. The authors in [20] propose a Branch-and-Bound (B&B) algorithm considering the *min-max* optimization criterion and comparing the robust solutions against deterministic solutions. Another work in [21] introduces a Particle Swarm

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Optimization coupled with a local search. Perturbations over the demands are applied according to lower and upper bounds and the results are compared with the B&B results presented in [20]. The work [22] proposes a two-index and a three-index mathematical formulations based on the VRP, followed by a Branch-and-Cut method. Closely related problems, as the Open Vehicle Routing Problem with uncertain demands has also been investigated in the literature [23].

Strategies to solve the RVRP with uncertain data associated with time windows are found in [24]–[26]. A cutting-plane algorithm is coupled to a B&B and a column generation approach based on path inequalities and resource inequalities in [24]. As far as we know, there are few works addressing the RVRP with uncertain data associated with traveling cost (or traveling time). The work from [25] focuses on stochastic programming and the authors propose a B&B, while the method controls the maximum number of solutions set to the worst total cost. The work [26] considers uncertainty modeled as interval data and an ant colony algorithm are proposed. Moreover, perturbations are performed over travel cost towards the upper bounds of the interval data.

The following contributions are given in this study: (i) we propose a Genetic Algorithm (GA) for the RVRP; (ii) we handle uncertain data as a bounded set of discrete scenarios with asymmetric arc costs of the transportation network. These characteristics rely on more complex operations since a solution can have different evaluations according to the scenarios, while asymmetric costs raise the combinatorial choices to build solutions.

II. MATHEMATICAL FORMULATION

A Mixed Integer Linear Programming (MILP) formulation for the *min-max* RVRP is presented from (1) to (10).

$$\min \quad Z = \delta \qquad \text{subject to:} \tag{1}$$

$$\sum_{(i,j)\in A} c_{ij}^k x_{ij} \le \delta \qquad \forall k \in S \tag{2}$$

$$\sum_{i\in V} x_{ij} = 1 \qquad \forall j \in V\backslash\{0\} \tag{3}$$

$$\sum_{j\in V} x_{ij} = 1 \qquad \forall i \in V\backslash\{0\} \tag{4}$$

$$\sum_{i\in V} x_{0i} = m \tag{5}$$

$$l_j \ge l_i + d_j - Q(1 - x_{ij}) \qquad \forall (i,j) \in A,\ i,j \ne 0 \tag{6}$$

$$d_i \le l_i \le Q \qquad \forall i \in V \tag{7}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{8}$$

$$\delta \ge 0 \tag{9}$$

$$l_i \ge 0 \qquad \forall i \in V \tag{10}$$

Constraints (2) work as an upper bound considering the total cost for all the scenarios. Thus, these restrictions and the objective function (1) establish the worst total cost is minimized for the set of scenarios. The flow conservation constraints are given in (3) and (4), and determine that only one vehicle visits each customer $i \in V\backslash\{0\}$. Constraints (5) specify that m vehicles leave the depot and return to it, due to the flow conservation restrictions. Constraints (6) and (7) generalize the classical MTZ constraints for the TSP [27]. Here, they are based on vehicle loads: if a vehicle visits a client i then j, its load increases by d_j. These constraints also state that vehicle capacity is ensured. Finally, variables are defined from (8) to (10).

III. ALGORITHM DESCRIPTION FOR THE GENETIC ALGORITHM

The general components of the GA are summarized in detailed in the following sub-sections. The initial population $P(t)$ is composed of q feasible solutions. Half of the initial population is generated by an adaptation of the best insertion heuristic [28] and the remaining solutions are randomly generated. A binary tournament selection is applied to create an intermediate population P'. The fitness function is performed according to the *min-max* criterion in which the solutions are sorted according to the costs (one per scenario k) in decreasing order, giving what we call *min-max* lexicographic vector of a solution. Then, a solution is said to be better than another solution if it is lexicographically smaller. Individuals in P' referred here as offspring are obtained by applying the Order Crossover (OX) [29]. Mutation inter-routes and intra-routes are also considered according to a fixed probability. In addition, a renewal procedure is implemented to avoid premature convergence in the population. The resulting population is sorted according to the *min-max* criterion and the best solutions are selected for the next iteration preserving the population size. The algorithm stops after t iterations and the GA returns the best solution v^*.

Algorithm 1 General description of the GA

```
Require: G = (V, A), m, Q, S, d_i ∀i ∈ V
  t ← 1
  Generate population P(t)
  Evaluation of individuals considering the min-max criterion
  in P(t)
  while stopping criterion is not met do
    Initialize a temporary population P'
    for i ← 1 to q do
      Perform Binary Tournament
      Perform crossover
      Perform mutation
      Place the offspring into P'
    end for
    Update P(t) according to fitness
    t ← t + 1
    if renewal criterion is met then
      Perform partial renewal P(t)
      Perform fitness P(t)
    end if
  end while
  Output the solution v*
```

1) Chromosomes and fitness: For the proposed GA, a chromosome is encoded as a set of routes [30], in which the customers appear in the order they are visited in a route. Delimiters are used to identify a route. Figure 1 illustrates a chromosome for a feasible route with $n = 7$ clients with $m = 3$ vehicles, and the delimiters are identified by $\{0\}$.

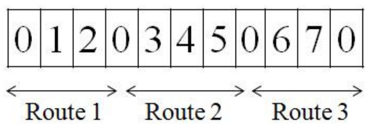

Fig. 1. Example of a chromosome representation

The fitness function is composed of two evaluation criteria for the RVRP. Initially, each chromosome is evaluated over all scenarios and the maximum total cost $max\{C^k : \forall k \in S\}$ is kept. Considering these computed values for each individual, a second evaluation is performed to classify all individuals in the population, following a minimization.

2) Initial Population: The generation of the initial population is built using an adaptation of the best insertion heuristic [28] and with a random generation of solutions. The principle of the best insertion heuristic is to create a set of routes by selecting unserviced customers and inserting them in one of the partial routes already created. The heuristic builds routes in parallel as follows: a customer i is selected and inserted in the first route, the second customer is selected in the second route, etc. until m initial customers (seed customers) has been included per route. Then, unserviced clients are sorted in decreasing order of demands. Considering this order of unserviced customers, the best insertion heuristic expands the current routes by inserting one unserviced client at a time, considering the smallest impact on the cost and the vehicles capacity. For the RVRP, a random scenario k is necessary to select in order to evaluate the best insertion. Since demands cannot be splitted, the insertion heuristic can fail to use only m vehicles. Thus, extra vehicles can be added to the solution in order to ensure each customer is visited by a vehicle. This violation on the number of vehicles is not repaired, and it is left to be managed by the GA selection.

On the random generation, routes are built in a sequential way, on contrary to the best insertion heuristic. Thus, unserviced customers are assigned to a vehicle until the vehicle capacity is not violated. Whenever any customers can enter in the first route, only then a second route is created. The procedure stops when all customers are attended by one vehicle.

3) Genetic operators: Such as genetic operators, crossover and mutation have been developed for the RVRP. For the crossover, the selection of solutions are done following the binary tournament selection [31]. This strategy is applied since it works as a diversification mechanism. Initially, two solutions are randomly selected and the best solution in terms of the *min-max* cost is kept to be the first parent for the crossover. This operation is repeated to obtain the second one.

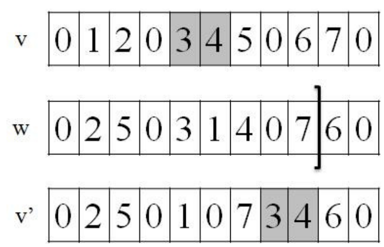

Fig. 2. Example of the ordered crossover

For the crossover, a modification of the ordered crossover (OX) [29] is applied. Let v and w be two individuals selected by the binary tournament. Then, the main idea of the OX is to select a sequence of customers from a parent and insert it in the other one. Figure 2 shows an example of the OX. A sequence of customers from a route in v (gray color) is considered. Then, an offspring v' is obtained by the best insertion of the sequence selected from v in w considering the *min-max* criterion. Repeated customers are dropped from the offspring v' to preserve that each customer i is visited once.

The mutation operator can be applied to an offspring, according to a specified probability. Two different mutations have been developed. The first mutation is performed intra-routes and randomly swaps two customers from the same route. The second mutation applies a swap inter-routes, and it is performed only if the vehicle capacity of both vehicles are ensured. Both mutations generate feasibility on the solution.

4) Renewal Procedure and stopping criterion: The renewal procedure is a restarting method for the population, and its goal is to scape from local optima. This procedure is applied whenever the best solution found so far does not change after a pre-specified number of iterations. Considering the initial population, a percentage of the population is replaced by new individuals generated with the best insertion method. Then, $P(t)$ is re-sorted after each renewal procedure. The GA stops after t iterations.

IV. COMPUTATIONAL EXPERIMENTS

Computational experiments were performed on a Dell Precision 6600M, Intel Core i7-2720QM, 2.2 GHz with 16GB of RAM. The proposed GA was implemented in $C{+}{+}$ on Visual Studio Express 2013 and the mathematical model was solved using GLPK (GNU Linear Programming Kit) version 4.47 (www.gnu.org/software/glpk) under default parameters. OPL-Studio, based on CPLEX solver was also considered, but this software could not found better results than GLPK for the *min-max* model and the instances used in this work. Experiments for the mathematical formulation were carried out with a runtime limit of four hours (14,400s).

Test over small and medium size instances were used to calibrate and evaluate the performance of the proposed GA for the RVRP. For the set of small-size, 18 instances with a size of 10, 15, and 20 customers; 2 or 3 vehicles; and 10, 20 and 30 scenarios are considered. The demand per client and the travel cost of each arc were randomly generated in [1, 50]. The vehicle capacity is selected to ensure a slack of $0.2Q$ to $0.8Q$ between the total demand d and fleet capacity mQ. The instance names is related to the numerical values of the number of clients n, the number of m vehicles and the number of p scenarios.

The set of medium-size set contains 24 instances with a size of 50 and 100 customers; 5 or 10 vehicles; and 10, 20 scenarios. Each node i has its coordinates randomly selected in $[0, 1000]$. The arc costs are travel costs (times) proportional to the Euclidean distance e_{ij}, using $[e_{ij}, (1+\theta/100)\times e_{ij}]$, where $\theta \in \{10, 50, 100\}$ is the maximum deviation (%) to the travel cost. The vehicle capacity is $Q = 1000$, while the demands are random integers to ensure approximately $0.9\times$ mQ. Each node i has its coordinates randomly selected in $[0, 1000]$. The file name format is the same as in the small-size instances, except that the value of θ is added at the end.

A. Calibration

For the set of medium-size instances, calibration with different configuration of parameters were used to see which produces the best results from the GA. To select the parameters, we used a fine-tuning procedure, beginning from a promising configuration using 90% of partial renewal when no improvement is found after 25 iterations, and a 10% of probability of mutation. Then, this configuration is used on a subset of 4 representative instances varying the number of iterations, and the population size. The number of iterations were stated to 100, 150. The population size was defined according to the number of customers per each instance [32]. In addition, 5 runs per instance were taken into account using different seeds and the results are summarized in Figure 3.

Figure 3 contains the Best Cost solutions (BS) and the Average (Avg) over the 4 configurations tested per instance, considering a population size of $q = |V|$ with 100 and 150 iterations, respectively. Figure 4 corresponds to a population size of $q = 2|V|$ testing with 100 and 150 iterations, respectively. The results show that the configuration with promising results is the one with $q = 2|V|$ and 150 iterations, which can achieve better cost solutions and good average costs for the *min-max* criterion. Thus, these parameters were kept for the complete set of medium-size instances, using $q = 2|V|$, 150 iterations, 90% of partial renewal, 25 iterations for the partial renewal and a 10% of probability of mutation.

B. Results for the RVRP

Table I summarizes the results for the MILP and the GA for the set of small-size instances. The MILP results correspond to the headings of LR (Linear Relaxation), LB (best Lower Bound), UB (best Upper Bound), *Gap* (percentage deviation of the optimum or the best upper bound to LB), and T (computational time in seconds). The results for the GA are indicated with the headings of the *Gap'* which is the percentage gap between the upper bound produced by the GA and the lower bound achieved by GLPK. Optimal values are identified in bold for the GA and GLPK. Results in Table I

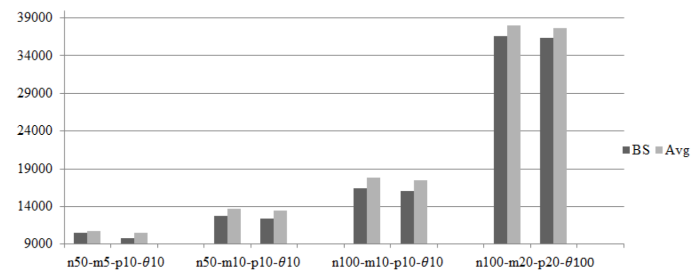

Fig. 3. Calibration of medium-size instances with $q = |V|$

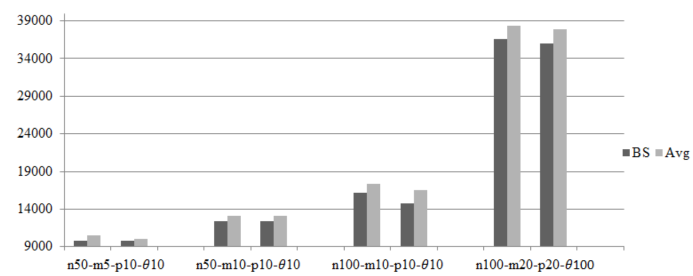

Fig. 4. Calibration of medium-size instances with $q = 2|V|$

demonstrate that the RVRP is a very hard problem to solve. In fact, the results show that on instances with up to 15 customers and 30 scenarios, the mathematical model cannot solve them to optimality using a maximum computational time of 4 hours, in which, the proposed GA could found good solutions within 8 seconds in a small average *Gap'* of 3.14%. Moreover, our GA retrieves 7 optima out of 10 optima values that were met using the MILP.

Table II summarizes the results for the GA over the set of the medium-size instances. The results correspond to the headings of BS, Avg and Avg(T) (Average time) using 5 runs per instance with different seeds. Results shows a good performance of the GA, which takes 118.4 seconds in average to solve instances with $n = 50$, and 573.21 seconds to solve instances with $n = 100$. It is also noticed that whenever n and p increase and the loads of the vehicle are tighted, the complexity of the problem and the computational time to solve the RVRP raise. Considering the impact of the scenarios on the robustness of solutions, it seems that when we handle more scenarios that does not improve the robustness according to the *min-max* criterion.

V. Conclusion

This article considers the RVRP with uncertain data related to the travel costs/travel times which covers a number of important applications in urban transportation. A GA is proposed to solve the problem considering the *min-max* criterion. Such methods deal with a bounded set of discrete scenarios and asymmetric arc costs.

Experimental results show that the RVRP is a hard problem and its complexity increases according to the number of

TABLE I
RESULTS SMALL-SIZE INSTANCES

Instance Name	d	Q	GLPK					GA		
			LR	LB	UB	*Gap*	T	BS	*Gap*'	T
n10-m2-p10	264	150	195.5	217	**217**	0.00	8.2	**217**	0.00	2.7
n10-m2-p20	264	175	250.6	284	**284**	0.00	32.2	**284**	0.00	4.1
n10-m2-p30	264	200	272.1	301	**301**	0.00	35.8	**301**	0.00	4.1
n15-m2-p10	346	200	320.2	346	**346**	0.00	313.2	**346**	0.00	2.7
n15-m2-p20	346	230	339.3	373	**373**	0.00	6,560	374	0.27	6.2
n15-m2-p30	346	260	368.7	398	404	1.51	14,400.0	409	2.76	6.2
n20-m2-p10	441	250	402.3	419	423	0.95	14,400.0	447	6.68	7.5
n20-m2-p20	441	300	443.3	460	470	2.17	14,400.0	493	5.87	4.1
n20-m2-p30	441	350	463.7	481	501	4.16	14,400.0	520	8.11	4.8
n10-m3-p10	264	95	227.0	255	**255**	0.00	10.4	**255**	0.00	6.8
n10-m3-p20	264	105	279.5	316	**316**	0.00	24.7	**316**	0.00	4.1
n10-m3-p30	264	115	302.0	337	**337**	0.00	38.9	**337**	0.00	4.1
n15-m3-p10	346	120	349.1	381	**381**	0.00	2,200.0	405	6.30	8.3
n15-m3-p20	346	140	365.2	399	**399**	0.00	6,871.0	411	3.01	6.2
n15-m3-p30	346	160	394.4	426	433	1.64	14,400.0	436	2.35	6.2
n20-m3-p10	441	160	427.3	443	448	1.13	14,400.0	484	9.26	6.2
n20-m3-p20	441	175	466.3	481	497	3.33	14,400.0	520	8.11	6.2
n20-m3-p30	441	190	489.6	508	528	3.94	14,400.0	551	8.46	6.8
Average						1.05	9,350.4		3.47	4.1

TABLE II
RESULTS MEDIUM-SIZE INSTANCES

Instance Name	Load	GA		
		BS	Avg	Avg(T)
n50-m5-p10-θ10	4.48	9,797	10,049	106.5
n50-m5-p10-θ50	4.43	12,002	12,994	121.1
n50-m5-p10-θ100	4.47	13,891	14,960	122.3
n50-m5-p20-θ10	4.49	9,302	9,740	140.8
n50-m5-p20-θ50	4.44	11,040	12,093	133.7
n50-m5-p20-θ100	4.54	13,358	13,943	125.4
n50-m10-p10-θ10	9.05	12,394	13,073	93.0
n50-m10-p10-θ50	8.96	15,469	16,253	111.9
n50-m10-p10-θ100	8.97	18,413	19,653	102.8
n50-m10-p20-θ10	8.97	13,060	13,293	114.6
n50-m10-p20-θ50	8.94	15,893	16,790	124.1
n50-m10-p20-θ100	8.99	16,802	18,272	124.8
n100-m10-p10-θ10	8.87	14,813	16,532	572.8
n100-m10-p10-θ50	8.94	19,812	20,939	598.9
n100-m10-p10-θ100	8.92	21,386	23,840	649.4
n100-m10-p20-θ10	8.89	16,980	17,738	705.0
n100-m10-p20-θ50	9.01	19,878	21,204	580.7
n100-m10-p10-θ100	8.93	24,647	25,323	548.1
n100-m20-p10-θ10	17.97	26,596	27,418	479.0
n100-m20-p10-θ50	17.88	30,954	31,847	532.6
n100-m20-p20-θ100	17.96	35,056	36,556	552.6
n100-m20-p20-θ10	18.10	23,553	24,592	641.9
n100-m20-p20-θ50	18.11	28,594	29,869	574.3
n100-m20-p20-θ100	17.88	36,000	37,877	443.3

scenarios, the numbers of vehicles and the numbers of clients. The proposed GA, could found good solutions and retrieve 7 optima out of 10 instances within 8 seconds.

Concerning future works, other heuristics and metaheuristics are considered for the RVRP. An extension over the impact of the scenarios on the robustness can be explored. Multiple uncertain parameters will be handled. Moreover, the adaptation for large scale RVRP will be also considered.

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