UG END SEM EXAM

Semester - Vth

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Branch : CSE - B

Subject : Graph Theory

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Q.1.

Ans → The second graph has a circuit of length 3 and the minimum length of any circuit in the first graph is 4. Hence, the given graphs are not isomorphic.

Q.2.

Ans → Given, number of nodes = 7

Total edges = 13

Sum of degrees = 2 × total edges
= 2×13
= 26

So, Sum of degrees = 26

Or, $2x + 4y + 5z = 26$ — (i)

Also, Number of nodes = 7

Or, $x + y + z = 7$ — (ii)

Comparing, we get, $x = 2$, $y = 3$, $z = 2$.

∴ The degree sequence is {2, 2, 4, 4, 4, 5, 5}

∴ This degree sequence is graphical.

Q.3.

Ans: Graphical Sequence, $\alpha = \{2,2,4,4,4,5,5\}$

(from Q.2.)

We used Havel-Hakimi algorithm to check the existence of a simple graph.

(i) Sort the sequence of non-negative integers in non-increasing order.

(ii) Delete the first element (say, E). Subtract 1 from the next E elements.

(iii) Repeat (i) and (ii) until one of the stopping conditions, mentioned below, is met:

(a) All the elements remaining are equal to zero (simple graph exists).

(b) Negative number encounter after subtraction (No simple graph exists)

(c) Not enough elements remaining for the subtraction step (No simple graph exists)

$\therefore$ Time Complexity: $O(N^2 * \log N)$

Q.8.

Ans → The given graph is not planar as the edges of $V_{10} - V_8$ and $V_7 - V_9$ intersect in another.

So, G is not planar.

Q. 40

Ans :

Suppose by contradiction that G is not connected. Then, there exists two vertices, u and v, such that there is no path between them. This implies that every other vertex can be connected to one of the two vertices but not to both. This allows us to create a bipartition of the remaining n-2 vertices.

Suppose that the number of vertices connected to u is k, which means,

$$d_G(u) = k$$

This implies that the number of vertices that could be connected to the vertex v is

$$d_G(v) <= n-k-2$$

because k vertices are connected to v, and we have to exclude u and v from the possible vertices.

This implies,

$$d_G(u) + d_G(v) <= k + (n-k-2) = n-2$$

which is incorrect because we have supposed

$$d_G(u) + d_G(v) \geq n-1$$

This proves, G is connected.

Q. 5.

Ans → Let T be a tree with n vertices.
G is a connected graph of n vertices.
Also, G is non-empty graph with $\delta(G) \geq n-1$.

Proof by induction,
Let number of edges, $m \in Z^+$.

If $m = 0, 1$, the result is obvious.

At $m = 0$, T is a single vertex.

At $m = 1$, T is a single end, and G is guaranteed to have an edge as $\delta(G) = 1$.

Assuming this to be true for all $k > 1$,
To prove this true for $k+1$.
Let T be a tree with $k+1$ edges.
Let $T' = T - \{v\}$, for some leaf v.
Let w be the neighbour of v in T.
By inductive hypothesis, T' is a subgraph of G with $\delta(G) = k+1$ (we know by the IH that T' is a subgraph of some graph with minimum degree k, so adding extra edges and necessary vertices to get minimum degree $k+1$ doesn't change this.)

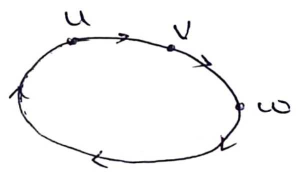
As $\deg(w) \geq k+1$ and T' has $k-1$ vertices other than v, there exists a neighbour of w not in T'. We select such a vertex from $N(w)$ and add the edge to T' to give us a tree isomorphic to T within G.
This proves T is subgraph of G. A graph is isomorphic to itself. Hence, T is isomorphic to subgraph of G. Proved

Q.6.

Ans → Tournament T is transitive iff T has no directed circuit. An asymmetric graph is a graph having directed ~~g~~ edges between each pair of vertices, if present, in only one direction. Now, since we are talking about complete asymmetric graph, there are exactly $\frac{n(n-1)}{2}$ edges with 1 in between each pair of vertices. Since the condition for transitiveness is that if $u \to v$ and $v \to w$, then $u \to w$.

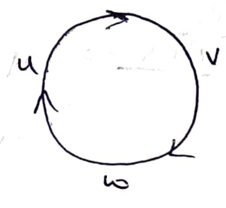
Let us assume that a directed circuit T exists.

Now since all the vertices are connected, we have n vertices in triplets each forming a circuit.

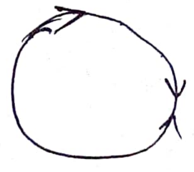


Let us assume we have a directed circuit.

Then, since we have, $u \to v$, $v \to w$ and also $w \to u$, So, $u \to w$ cannot exist since it is an asymmetric. So, graph is not transitive Tournament. This violates our initial assumption.



So, since the graph is completed, every triplet will have orientation as shown. in terms of vertices, the net direction comes to 0.



Therefore, Tournament is transitive.