National Institute of Technology, Silchar

Mid-Semester (UG) Examinations, March 2021

Subject Code: MA221, Subject: Introduction to Stochastic Process

Semester: Fourth, Department: Mathematics.

Duration: One Hour, Total Marks: 20

Figure in the right hand margin indicates full marks for the question.

All questions are compulsory.

1. The joint PDF of (X, Y) is given by **10**

$$f_{XY}(x,y) = \begin{cases} kx^2(4-y) & x < y < 2x, 0 < x < 2 \\ 0 & otherwise \end{cases}$$

where k is a constant.

(a) Find the value of k.
(b) Find the marginal density functions.
(c) Find the conditional probability density functions.
(d) Find the conditional means and conditional variances.
(e) Find the covariance of X and Y and the correlation coefficient.
(f) Are X and Y independent? Justify your answer.

2. Let X and Y be independent random variables, each having uniformly distributed over $(0,1)$. Let $Z = X + 2Y$ and $W = X - Y$. **5**

(a) Find the joint pdf of Z and W.
(b) Find the joint moment generating function of X and Y.

3. Consider the Markov chain with state $\{0, 1, 2\}$ and transition probability matrix **3**

$$P = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix}$$

Is state 0 periodic? Justify your answer.

4. Consider a two-state Markov chain with the transition probability matrix **2**

$P = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Find the n-step transition probability matrix P^n.

*****End*****