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Serial: 06	Page No. 15	Date : 16/11/2021	Remarks:

Q.1. Find the Hamming distance between two pairs of words.

(i) The Hamming distance $d(000, 011)$?

(ii) The Hamming distance $d(10101, 11110)$?

Solution: (i) $000 \oplus 011 = 011$

As there are two 1s in 011,

$\therefore d(000, 011) = 2$.

(ii) $10101 \oplus 11110 = 01011$

As there are three 1s in 01011

$\therefore d(10101, 11110) = 3$

Q.2. Find the minimum Hamming distance of the coding scheme in the following table.

Datawords	Codewords
00	000
01	011
10	101
11	110

Solution: $000 \oplus 011 = 011$ (two 1s)

$000 \oplus 101 = 101$ (two 1s)

$000 \oplus 110 = 110$ (two 1s)

$011 \oplus 101 = 110$ (two 1s)

$011 \oplus 110 = 101$ (two 1s)

$101 \oplus 110 = 011$ (two 1s)

$\therefore d_{min} = 2$.

Q.3. Find the minimum Hamming distance of the coding scheme in the following table.

Dataword	Codeword
00	00000
01	01011
10	10101
11	11110

Solution: 00000 ⊕ 01011 = 01011 (three 1s)
00000 ⊕ 10101 = 10101 (three 1s)
00000 ⊕ 11110 = 11110 (four 1s)
01011 ⊕ 10101 = 11110 (four 1s)
01011 ⊕ 11110 = 10101 (three 1s)
10101 ⊕ 11110 = 01011 (three 1s)

As three 1s is the minimum hamming distance,
$\therefore$ $d_{min} = 3$

Q.4. A code scheme has a Hamming distance $d_{min} = 4$. What is the error detection and correction capability of this scheme?

Solution: $d_{min} = 4$

$s = d_{min} - 1 = 4 - 1 = 3$

$t = (d_{min} - 1)/2 = 3/2 = 1$ (floor value)

$\therefore$ The code scheme can detect upto 3 errors.
$\therefore$ The code can correct up to 1 error.

Q.5. We need a dataword of at least 7 bits. Calculate values of k and n that satisfy this requirement.

Solution: We need to make $k = n - m \geq 7$
Using trial and error method,

1. Let, $m=1$, so, $n=2^1-1=1$; $K=1-1=0$ (Rejected)
2. Let, $m=2$, so, $n=2^2-1=3$; $K=3-2=1$ (Rejected)
3. Let, $m=3$, so, $n=2^3-1=7$; $K=7-3=4$ (Rejected)
4. Let, $m=4$, so, $n=2^4-1=15$; $K=15-4=11$ (Accepted)

$K=11$ satisfies the condition.

$\therefore K=11$; $n=15$

$\therefore$ Dataword of at least 7 bits can be obtained with a Hamming Code $C(15,11)$.

Q.6. Which of the following $g(x)$ values guarantees that a single-bit error is caught? For each case, what is the error that cannot be caught? (a) $x+1$ (b) x^3 (c) 1

Solution: (a) $g(x) = x+1$. No x^i can be divisible by $x+1$.

$\therefore$ Any single-bit error can be caught.

(b) $g(x) = x^3$. x^i can be divisible by x^3 only if the value of $i \geq 3$.

$\therefore$ All single-bit errors in positions 1 to 3 are caught.

(c) $g(x) = 1$. All x^i can be divisible by 1.

$\therefore$ None of the single-bit errors can be caught.

Q.7. Find the status of the following generators related to two isolated, single-bit errors. (a) $x+1$ (b) x^4+1 (c) x^7+x^6+1 (d) $x^{15}+x^{14}+1$.

Solution: (a) $x+1$ is a very poor choice for a generator. Any two errors next to each other cannot be detected.

(b) x^4+1 cannot detect two errors at four positions apart.

(c) x^7+x^6+1 is a good choice for two isolated, single bit errors

(d) $x^{15}+x^{14}+1$ cannot divide x^t+1 if t is less than 32,768. A codeword with two isolated errors upto 32,768 bits apart can be detected by $x^{15}+x^{14}+1$

Serial: 06 | Page No. 18 | Date: 16/11/2021 | Remarks:

Q.8. Find the suitability of the following generators in relation to burst errors of different lengths.

(a) x^6+1 (b) $x^{18}+x^7+x+1$ (c) $x^{32}+x^{23}+x^7+1$

Solution: (a) x^6+1 can detect all burst errors with a length less than or equal to 6 bits. 3 out of 100 burst errors with length 7 will slip by. 16 out of 1000 burst errors of length 8 and more will slip by.

(b) $x^{18}+x^7+x+1$ can detect all burst errors with a length less than or equal to 18 bits. 8 out of 1 million burst errors with length 19 will slip by. 4 out of 1 million burst errors of length 20 or more will slip by.

(c) $x^{32}+x^{23}+x^7+1$ can detect upto 32 bits; i.e, all burst errors with a length less than or equal to 32. 5 out of 10 billion burst errors with length 33 will slip by. 3 out of 10 billion burst errors of length 34 or more will slip by.

Q.9. What are the criteria for a good polynomial generator?

Solution: A good polynomial generator needs to have the following characteristics:

(i) It should have at least two terms.

(ii) The coefficient of the term x^0 should be 1.

(iii) It should not divide x^t+1, for t between 2 and $n-1$.

(iv) It should have the factor $x+1$.

Serial: 06 | Page No.: 19 | Date: 16/11/2021 | Remarks:

Q. 10. How can we represent the number 21 in one's complement arithmetic using only four bits?

Solution:

$(21)_{10} = (10101)_2$

It has five bits. But we need four bits.
So, wrapping the leftmost bit and adding it to the four rightmost bits, we get,

$(0101 + 1)_2 = (0110)_2 = (6)_{10}$

Q. 11. How can we represent the number −6 in one's complement arithmetic using only four bits?

Solution:

$(6)_{10} = (0110)_2$

Negative numbers are found by inverting all bits.
So, $(-6)_{10} = (1001)_2 = (9)_{10}$
i.e., complement of 6 is 9.
Also, $(-6)_{10} = (1001)_2$ which is four bits.
Hence, −6 in one's complement arithmetic is 1001