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PHYSICS ASSIGNMENT- II

1. Compute the intensity of the standing electromagnetic wave given by $\vec{E}(x,t) = 2E_0 \cos kx \cos\omega t \hat{j}$, $\vec{B}(x,t) = 2B_0 \sin kx \sin\omega t \hat{k}$.

Soln: The poynting vector is,

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{1}{\mu_0}(4E_0B_0 \sin kx \sin\omega t \cos kx \cos\omega t)\hat{i}$$

$$= \frac{E_0B_0}{\mu_0} \sin 2kx \sin 2\omega t \hat{i}$$

So, The time average of S is,

$$\langle S\rangle = \frac{E_0B_0}{\mu_0} \sin 2K \langle \sin(2\omega t)\rangle = 0$$

$$\Rightarrow I = \langle S\rangle = 0$$

∴ The result is to be expected since the standing wave does not propagate.

2. Given $\vec{E} = \hat{i}E_0 \cos\omega\left(\frac{z}{c}-t\right) + \hat{j}E_0 \sin\omega\left(\frac{z}{c}-t\right)$, determine the magnetic field $\vec{B}$.

Soln:- From Maxwell's equation,

$$\vec{\nabla}\times\vec{E} = \frac{-\delta\vec{B}}{\delta t} \quad \text{—(i)}$$

Now, $\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ E_0\cos\omega(\frac{z}{c}-t) & E_0\sin\omega(\frac{z}{c}-t) & 0 \end{vmatrix}$

$$= \hat{i}\left(0 - \frac{E_o \omega}{c} \cos \omega\left(\frac{z}{c} - t\right)\right)$$

$$- \hat{j}\left(0 + \frac{E_o \omega}{c} \sin \omega\left(\frac{z}{c} - t\right)\right) + 0$$

Putting in (i), we get,

$$\frac{-\partial \vec{B}}{\partial t} = -\frac{E_o \omega}{c}\left[\cos \omega\left(\frac{z}{c} - t\right)\hat{i} + \sin \omega\left(\frac{z}{c} - t\right)\hat{j}\right]$$

Integrating on both sides, we get,

$$B = \frac{E_o}{c}\left[-\sin \omega\left(\frac{z}{c} - t\right)\hat{i} + \cos \omega\left(\frac{z}{c} - t\right)\hat{j}\right] + \underline{m}$$

Constant

3. A uniform plane wave has a wavelength of 3cm in free space and 2cm in a dielectric for which $\mu = 4.7 \times 10^{-7}\ NA^{-2}$. Determine the dielectric constant of the dielectric.

Solⁿ:- We know, $c \propto \lambda$

and, $c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$

Let, $\lambda_o = 3\ cm$, $\lambda_m = 2\ cm$

So,

$$\frac{\lambda_o}{\lambda_m} = \sqrt{\frac{\varepsilon_m \mu_m}{\varepsilon_o \mu_o}}$$

or, $$\frac{3}{2} = \sqrt{\frac{\varepsilon_m \mu_m}{\varepsilon_o \mu_o}}$$

or, $$\frac{9}{4} = \frac{\varepsilon_m \mu_m}{\varepsilon_o \mu_o}$$

Let, dielectric constant $\frac{\varepsilon_m}{\varepsilon_o} = k$ (say)

then, $K = \frac{9}{4} \times 2.7$

$\therefore K = 6.075$

40. A plane electromagnetic wave propagates from one dielectric to another at normal incidence. Find the ratio of the indices of refraction of the two electrics for which the reflection and transmission coefficients are both equal to 0.5.

Soln:- $R = T = 0.5$ (given)

Also,
$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \quad - (i)$$

and, $$T = \frac{4n_1 n_2}{(n_1+n_2)^2} \quad - (ii)$$

Using (i), we get
$$\frac{1}{2} = \left(\frac{n_1/n_2 - 1}{n_1/n_2 + 1}\right)^2$$

Let, $\frac{n_1}{n_2} = k$, then
$$\frac{k-1}{k+1} = \pm \frac{1}{\sqrt{2}}$$

for $\frac{k-1}{k+1} = \frac{1}{\sqrt{2}}$, ~~Also~~,
$$k+1 = \frac{2\sqrt{2}}{\sqrt{2}-1} \Rightarrow k = 3+2\sqrt{2}$$

Similarly for $\frac{k-1}{k+1} = \frac{-1}{\sqrt{2}}$

$$k = 3-2\sqrt{2}$$

$$\therefore k = 3 \pm 2\sqrt{2}$$

50. A uniform plane wave whose electric field is given by $E_I = 100 \cos(\omega t - 6\pi x)\,\hat{z}\ Vm^{-1}$ is incident from a region having $\varepsilon_1 = 4\varepsilon_0$, $\mu_1 = \mu_0$ normal to the plane surface of a material having $\varepsilon_2 = 9\varepsilon_0$, $\mu_2 = 4\mu_0$. Write complete expressions for the incident, reflected and transmitted electric and magnetic fields.

Solⁿ:-

$$E_I = 100 \cos(\omega t - 6\pi x)\,\hat{z}\ Vm^{-1}$$

$$\Rightarrow E_I = 100 \cos\{-(6\pi x - \omega t)\}\,\hat{z}$$

$$\Rightarrow E_I = 100 \cos(6\pi x - \omega t)\,\hat{z} \qquad \{\because \cos(-\theta) = \cos\theta\}$$

$$\therefore \vec{v} = v(\hat{x})$$

$$\therefore B_I = \frac{1}{v_1}(\hat{x}) \times E(\hat{z}) = \frac{-1}{v_1} 100 \cos(6\pi x - \omega t)\,\hat{y}$$

for Medium I,

$$\varepsilon_1 = 4\varepsilon_0 \;;\; \mu_1 = \mu_0$$

for Medium II, $\varepsilon_2 = 9\varepsilon_0$; $\mu_2 = 4\mu_0$

Now,

$$100 = E_{0\,im} \qquad \therefore E_{0\,reflected} = \left(\frac{1-\beta}{1+\beta}\right) E_{0\,im}$$

where,

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \left(\frac{\varepsilon_2 \mu_1}{\varepsilon_1 \mu_2}\right)^{\frac{1}{2}} = \left(\frac{9\varepsilon_0 \mu_0}{4\varepsilon_0 4\mu_0}\right)^{1/2}$$

$$\therefore \beta = \frac{3}{4}$$

$$\therefore E_{0\,reflection} = \left(\frac{1 - \frac{3}{4}}{1 + \frac{3}{4}}\right) \times 100 = \frac{100}{7}$$

$$\therefore E_{0\,transmitted} = \left(\frac{2}{1+\beta}\right) \times 100 = \frac{2}{7/4} \times 100 = \frac{8}{9} \times 100$$

$\therefore$ Equation for reflected wave;

$$E_I \text{ reflected} = \frac{100}{7} \cos(-6\pi x - \omega t)(-\hat{z})$$

$$B_I \text{ reflected} = \frac{1}{V_1}\left(\frac{100}{7}\right)\cos(-6\pi x - \omega t)(-\hat{y})$$

$\therefore$ Equation for transmitted wave:

$$E_I \text{ transmitted} = \left(\frac{800}{7}\right)\cos(6\pi x - \omega t)(\hat{z})$$

$$B_I \text{ transmitted} = \frac{1}{V_2}\left(\frac{800}{7}\right)\cos(6\pi x - \omega t)(-\hat{y})$$

Now,

$$V_1 = \frac{1}{\sqrt{\varepsilon_1 \mu_1}} = \frac{1}{\sqrt{4\varepsilon_0 \mu_0}} = \frac{c}{\alpha}$$

$$V_2 = \frac{1}{\sqrt{\varepsilon_2 \mu_2}} = \frac{1}{3\times 2\sqrt{\varepsilon_0 \mu_0}} = \frac{c}{\varepsilon}$$

6. Find the momentum of (a) a 10.0-MeV gamma ray (b) a 25-KeV X-ray (c) a 1.0-μm infrared photon. (d) a 150-MHz radio-wave photon. Express the momentum in kg m/s and eV/c.

Sol^n: We know,

$$E^2 = (pc)^2 + (mc^2)^2$$

Here, $m = 0$, So,

$$E = pc \Rightarrow p = \frac{E}{c}$$

(a)

$$p = \frac{10 \times 10^6}{3 \times 10^8} \times 1.6 \times 10^{-19}$$

$$\Rightarrow p = 0.53 \times 10^{-20} \text{ kg.m/s}$$

Again, $p = \frac{E}{c} = 10 \times 10^6 = 10^7 \frac{eV}{c}$

(b)

$$p = \frac{E}{c} = \frac{25 \times 10^3}{c} = 25 \times 10^3 \frac{eV}{c}$$

Again, $p = \frac{25 \times 10^3 \times 1.6 \times 10^{-19}}{3 \times 10^8} = 13.33 \times 10^{-24} \frac{\text{kgm}}{\text{s}}$

(c)

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{10^{-6}} \text{ J}$$

$$= \frac{6.6 \times 10^{-28} \times 3 \times 10^8}{1.6 \times 10^{-19}} \text{ eV}$$

$$\therefore E = 1.24 \text{ eV}$$

$$\therefore p = 1.24 \frac{eV}{c}$$

Also, $p = \frac{6.6 \times 10^{-34} \times c}{10^{-6} \times c} = 6.6 \times 10^{-28} \frac{\text{kg m}}{\text{s}}$

(d) $E = \frac{h\nu}{c} = \frac{6.6\times10^{-34}\times150\times10^{6}}{1.6\times10^{-19}}$

$\Rightarrow E = 6.188 \text{ eV}$

$\therefore P = 6.188 \frac{\text{eV}}{c}$

Again, $P = \frac{E}{c} = \frac{6.6\times10^{-34}\times150\times10^{6}}{3\times10^{8}}$

$\therefore P = 3.3\times10^{-34}$ Kg m/s

7. A metal surface has a photoelectric cutoff wavelength of 325.6 nm. It is illuminated with light of wavelength 259.8 nm. What is the stopping potential?

Soln:- we know $E = W_0 + eV$

And, $V = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

$= \frac{6.6\times10^{-34}\times3\times10^{8}\times65.8\times10^{-9}}{1.6\times10^{-19}\times325.6\times10^{-9}\times259.8\times10^{-9}}$

$\therefore V = 0.963$ eV , is the stopping potential

8. The surface of the sun has temperature of 6000 K. At what wavelength does the sun emit its peak intensity? How does this compare with the peak sensitivity of the human eye?

Soln:- $\lambda_{peak(sun)} = \frac{2.898\times10^{-3}}{T \text{ (in k)}} = \frac{2.898\times10^{-3}}{6000}$

$\therefore \lambda_{peak} = 4.83\times10^{-7}$ m $= 483$ nm

The peak sensitivity of human eye is 555 nm.

$\therefore \lambda_{peak(eye)} \approx 1.15\times\lambda_{peak(sun)}$

9. A proton is accelerated from rest through a potential difference of -2.36×10^{5} V. What is its de-Broglie wavelength.

Soln:- We know, $\lambda = \dfrac{h}{\sqrt{2mqV}}$

$$\Rightarrow \lambda = \frac{6.6\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.6\times10^{-19}\times2.36\times10^{5}}}$$

$$\Rightarrow \lambda = 587.7\times10^{-12}\ m$$

$$\therefore \lambda = 587.7\ pm$$

10. An electron moves in x direction direction with a speed of 3.6×10^{6} m/s. We can measure its speed to a precision of 1%. With what precision can we simultaneously measure its x coordinate?

Soln:- From Heisenberg's uncertainity principle,

$$\Delta x\,\Delta p_x = \frac{h}{4\pi}$$

or $\Delta p_x = 9.1\times10^{-31}\times3.6\times10^{6}\times\frac{1}{100}$

or, $\Delta p_x = 3.3\times10^{-26}\ \frac{kg\,m}{s}$

$$\Rightarrow \Delta x = \frac{6.6\times10^{-34}}{4\times\left(\frac{22}{7}\right)\times3.3\times10^{-26}}$$

$$\therefore \Delta x = 1.59\ nm$$