END SEM EXAM., MAY-2021, NIT SILCHAR
CLASS: B. Tech. (CSE), SEMESTER: Fourth
Sub.: Introduction to Stochastic Processes (MATH-IV) (MA-221 & MA-1251)

Full Marks: 30 **Time: 1hr 15 mins. (Uploading: Extra 5 mins.)**

Answer ***any SIX*** questions.

1. For a random process $X(t) = Y \sin wt$, Y is a uniformly distributed random variable in $(-1, 1)$. Check whether the process is wide sense stationary or not. [5]

2. Let $X(t) = A \cos wt + B \sin wt$, $Y(t) = B \cos wt - A \sin wt$ where A and B are random variables and w is a constant. Show that $X(t)$ and $Y(t)$ are wide-sense stationary if A and B are uncorrelated; zero mean random variables with the same variance. Prove that $X(t)$ and $Y(t)$ are jointly wide-sense stationary, finding the cross-correlation function. [5]

3. Let $\{X(t): -\infty < t < \infty\}$ be a zero-mean stationary, normal process with the autocorrelation function [5]

$$R_{XX}(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T} & -T \le \tau \le T \\ 0 & \text{otherwise} \end{cases}$$

Let $\{X(t_i) : i = 1, 2, \dots, n\}$ be a sequence of n samples of the process taken at the time instants

$$t_i = \frac{iT}{2}, \; i = 1, 2, \dots n$$

Find the mean and variance of the sample mean $\mu_n = \dfrac{1}{n}\sum_{i=1}^{n} X(t_i)$.

4. Define an absorbing Markov Chain. Check whether the following matrix is from absorbing Markov chain with proper explanation. [5]

$$\begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

5. Consider a simple random walk $X(n)$ with absorbing barriers at state 0 and state 3. Absorbing barrier means here $p_{00} = 1$ and $p_{33} = 1$. Find the transition probability matrix P. Find the probability of absorption into states 0 and 3. [5]

6. The following is a transition matrix for shifting from one brand to another by a customer. Being in the brand 'A' now, the probability of switching over to A, B or C next time is (0.3, 0.6, 0.1). Find the probabilities with his fourth purchase. [5]

$$P = \begin{bmatrix} 0.40 & 0.30 & 0.30 \\ 0.20 & 0.50 & 0.30 \\ 0.25 & 0.25 & 0.50 \end{bmatrix}$$

7. Define a Markov Chain. Let a fair dice is tossed repeatedly. If X_n denotes the maximum numbers occurring in the first n tosses, find the transition probability matrix P of the Markov Chain $\{X_n\}$. Find also P^2 and $P(x = 6)$. [5]

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