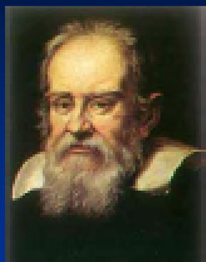
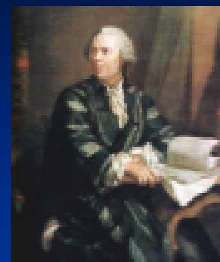
Dynamics

Simanchal Kar, PhD
Department of Mechanical Engineering
NIT Silchar

What is Dynamics?





Important contributors:

Galileo Galilei, Newton, Euler, Lagrange

Mechanics

(The study of how bodies react to forces acting on them)

Study of equilibrium of a body that is at rest/moves with constant velocity ← Statics

Dynamics → Study of accelerated motion of a body

- **Kinematics**: geometric aspects of the motion
- **Kinetics**: Analysis of forces which cause the motion

W. Wang

An Overview of Mechanics

Mechanics: The study of how bodies react to forces acting on them.

Statics: The study of bodies in equilibrium or in constant speed.

Dynamics: The study of force and torque and their effect on a accelerated moving body

1. **Kinematics** – concerned with the geometric aspects of motion
2. **Kinetics** - concerned with the forces causing the motion

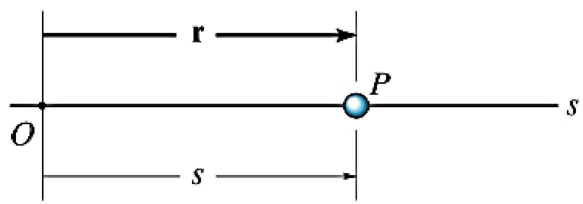
Rectilinear Kinematics: Continuous Motion:

- *Rectilinear kinematics* refers to straight line motion. The kinematics of a particle is characterized by specifying at any given time t, the particle's *position*, *velocity*, and *acceleration*.

  – **Position.** The position of the particle is represented by a position vector $\vec{r}$ starting from the origin O of the axis of the motion. If s is the algebraic scalar that represents the position coordinate of the particle, then

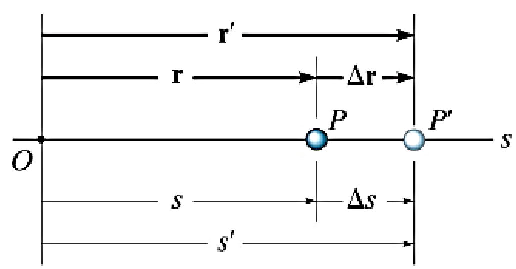
$$\vec{r} = s\,\hat{u}$$

  where $\hat{u}$ is a unit vector pointing toward the positive direction of the axis of the motion. The magnitude of $\vec{r}$ is ($r =| s |$). The direction of $\vec{r}$ is the same as $\hat{u}$ if $s > 0$ and opposite to $\hat{u}$ if $s < 0$. For analytical work it is therefore convenient to represent $\vec{r}$ by the algebraic scalar s.



  – **Displacement.** The displacement of a particle is a vector $\Delta\vec{r}$ defined as the change in he particle's position vector $\vec{r}$. Algebraically the displacement of a particle from P of coordinate s to point P' of coordinate s' is represented by

$$\Delta s = s' - s$$



- **Velocity.** The velocity of a particle is a vector
  * The average velocity is the displacement divided by time $\left(\vec{V}_{av} = \Delta\vec{r}/\Delta t\right)$ or algebraically as

$$V_{av} = \frac{\Delta s}{\Delta t}$$

  * The instantaneous velocity (or velocity) is $\left(\vec{V} = \lim_{\Delta t \to 0} \vec{V}_{av} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}\right)$ or algebraically as

$$V = \frac{ds}{dt} = \dot{s}$$

  * Instantaneous speed (or speed) is a positive scalar and refers to the magnitude of the instantaneous velocity

$$v = |\vec{V}| = \left|\frac{ds}{dt}\right|$$

  * The average speed is a scalar and refers to the total distance divided by the total time
- **Acceleration.** The acceleration of a particle is a vector $\left(\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t} = \frac{d\vec{V}}{dt}\right)$ or algebraically as

$$a = \frac{dV}{dt} = \dot{V} = \frac{d^2 s}{dt^2} = \ddot{s}$$

  * In rectilinear kinematics, the acceleration has the same direction as the velocity when the particle is speeding and opposite when the particle is slowing down.
  * A particle can have an acceleration and yet have zero velocity
  * One additional thing is that

$$a = \frac{dV}{dt} \; or \; dt = \frac{dV}{a}$$

$$V = \frac{ds}{dt} \; or \; dt = \frac{ds}{V}$$

$$\frac{ds}{V} = \frac{dV}{a} \; or \; a \; ds = V \; dV$$

  * The last equality can be integrated to solve for the speed as an example if one can express a as a function of s.

2.1 Constant acceleration

- Let $a = a_c = constant$. Assume that $V = V_o$ and $s = s_o$ at time $t = 0$. Then
  - Velocity as a function of time

$$\int_{V_o}^{V} dV = \int_{o}^{t} a_c \, dt \; or \; V = V_o + a_c \, t$$

  - Position as a function of time

$$\int_{s_o}^{s} ds = \int_{o}^{t} (V_o + a_c \, t) \, dt \; or \; s = s_o + V_o \, t + \frac{1}{2} \, a_c \, t^2$$

  - Speed as a function of position

$$\int_{V_o}^{V} V \, dV = \int_{s_o}^{s} (a_c) \, ds \; or \; v^2 = v_o^2 + 2 \, a_c \, (s - s_o)$$

The equations of rectilinear kinematics should be applied as follow

- **Coordinate System**
  - Establish a position coordinate s along the linear path and specify its fixed origin and positive direction
  - Since the motion os along a straight line, the particle's position, velocity, and acceleration can be represented as algebraic scalars. For analytical work, the sense of s, V, and a is then determined from their algebraic signs
- **Kinematic Equations**
  - If a relationship is known between any two or the four variables a, V, s, and t, then a third variable can be obtained by using one of the kinematic equations $\left(a = \dfrac{dV}{dt}\right)$,$\left(V = \dfrac{ds}{dt}\right)$, or $(a\ ds = V\ dV)$, which relates all the variables.
  - Whenever integration is performed, it is important that the position and velocity be known at a given instant in order to evaluate either the constant of integration if an indefinite integral is used, or the limit of integration if a definite integral is used.
  - The equation developed in section 2-1 apply **only** when the acceleration is constant

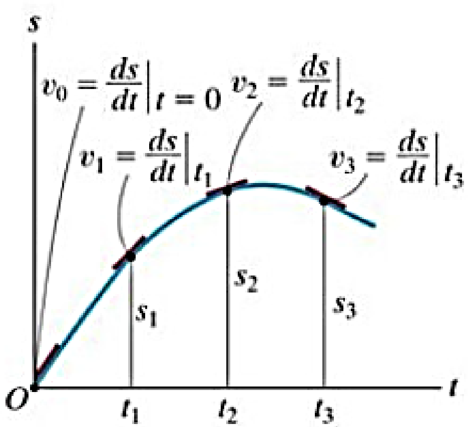
Rectilinear Kinematics: Erratic Motion

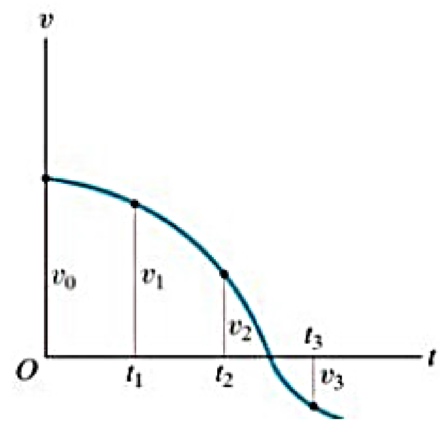
When a particle's motion during a time period is erratic or there is a discontinuity in the motion, it may be difficult to obtain a continuous function to describe its position, velocity, or acceleration. Instead, the motion may best be described graphically using a series of curves that can be generated experimentally by computer. There are several frequently occurring situations:

- **Given s-t graph, construct V-t graph.** The position of a particle can be plotted over time (s-t graph), the particle's velocity as a function of time (V-t graph)can be obtained by measuring the slope of the s-t graph.

$$\frac{ds}{dt} = V$$

slope of s-t graph = velocity

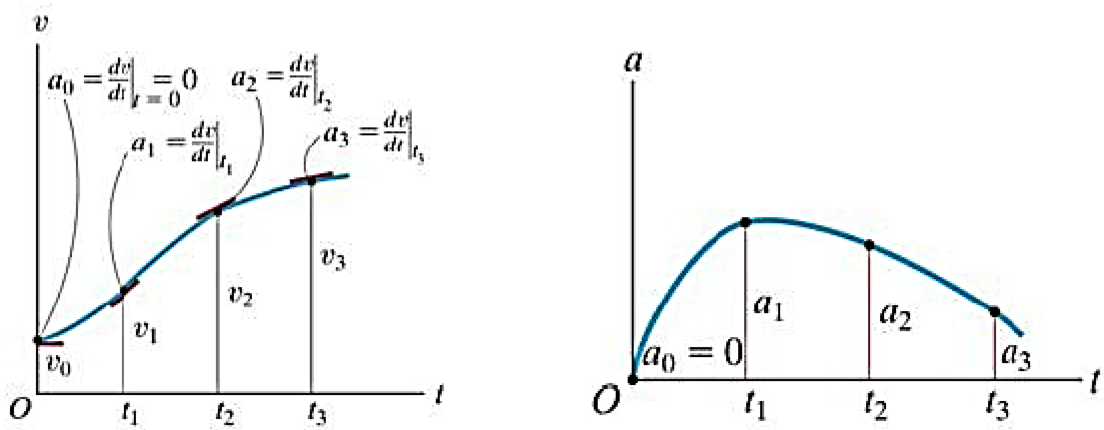




Given v-t graph, construct a-t graph. When the particle's V-t graph is known, the particle's acceleration as a function of time (a-t graph)can be obtained by measuring the slope of the V-t graph.

$$\frac{dV}{dt} = a$$

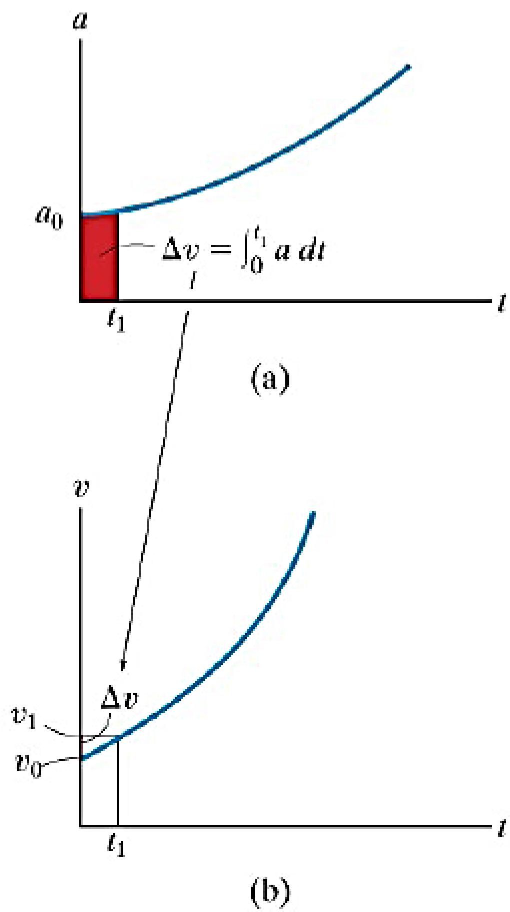
slope of V-t graph = acceleration



Given a-t graph, construct V-t graph. When the particle's a-t graph is given, the particle's velocity as a function of time (V-t graph)may be constructed by:

$$\Delta V = \int a \; dt$$

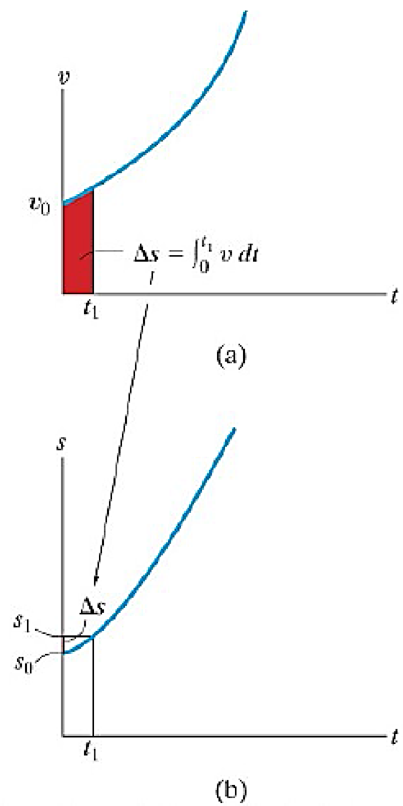
change in velocity = area under a-t graph



Given V-t graph, construct s-t graph. When the particle's V-t graph is given, the particle's position as a function of time (s-t graph)may be constructed by:

$$\Delta s = \int V\ dt$$

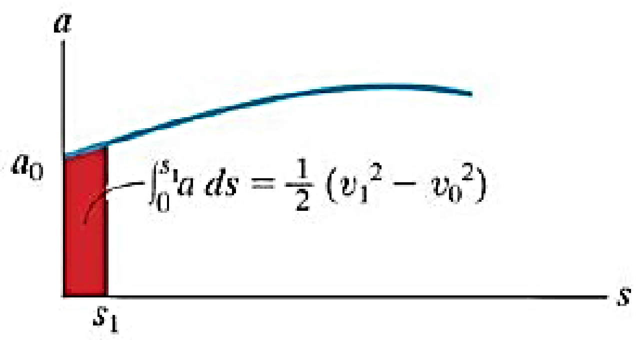
$$\text{displacement} = \text{area under V-t graph}$$

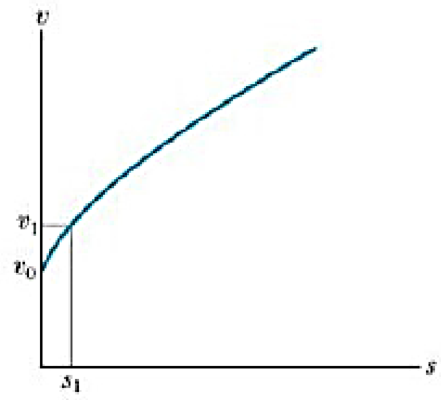


Given a-s graph, construct V-s graph. When the particle's a-s graph is given, the particle's velocity as a function of position (V-s graph)may be constructed by:

$$\int_{V_1}^{V_2} V\, dV = \frac{1}{2}\left(V_2{}^2 - V_1{}^2\right) = \int_{s_o}^{s_1} a\, ds$$

$$\frac{1}{2}\left(V_2{}^2 - V_1{}^2\right) = \text{area under a-s graph}$$

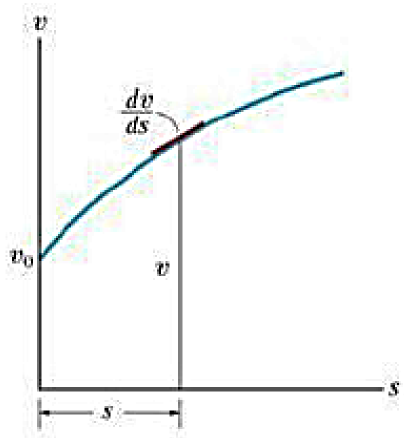


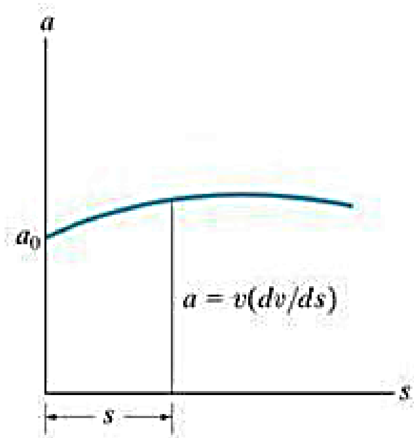


Given V-s graph, construct a-s graph. When the particle's V-s graph is given, the particle's acceleration as a function of position (a-s graph)may be constructed by:

$$a = V\ \frac{dV}{ds}$$

acceleration=Velocity $\times$ slope of V-s graph





General Curvilinear Motion

Curvilinear motion occurs when the particle moves along a curved path. Since this path is often described in three dimensions, vector analysis will be used to formulate the particle's position, velocity, and acceleration. In this section the general aspects of curvilinear motion are discussed, and in the subsequent sections we will consider three types of coordinate systems often used to analyze this motion: rectangular, Normal and Tangential, and Cylindrial.

Consider the motion of a point on a circular trajectory in the x-y plane, i.e., the r–θ plane as shown in Fig. 5.4.

The position of a point P at any time t can be specified by the x and y coordinates or the r and θ coordinates such that

$$x = r\cos\theta$$

$$y = r\sin\theta$$

or

$$r = \sqrt{x^2 + y^2} \quad \text{and} \quad \tan\theta = \frac{y}{x}$$

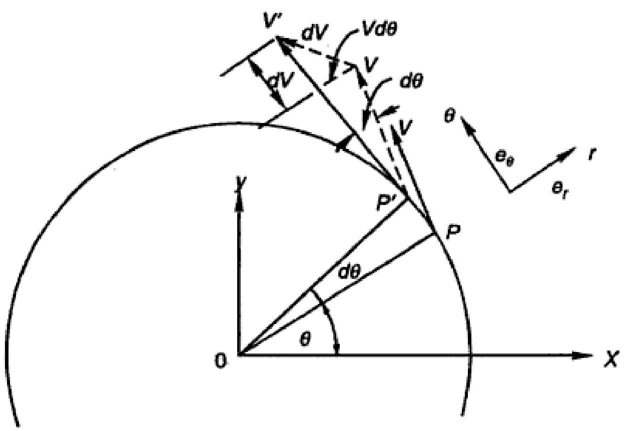


Fig. 5.4 ***Circular Motion of a Point***

Let the angular displacement of the point be $\Delta\theta$ over a differential time interval Δt. The angular velocity is, therefore,

$$\omega = \lim_{\Delta t \to 0}\left(\frac{\Delta\theta}{\Delta t}\right) = \frac{d\theta}{dt}$$

directed about an axis normal to the plane; anticlockwise positive for the right-handed triad.

The linear velocity V of the point is given by

$$V = r\omega = r\frac{d\theta}{dt} \tag{5.9}$$

directed tangentially to the circular path.

Over a differential time interval dt, the velocity of the point changes both in magnitude and in direction as shown in the sketch.

Change	*Direction*	*Remarks*
dV	Tangentially forward; θ	+*ve*, if the speed increases
$Vd\theta$	Radially inwards; $-r$	− *ve* always, because r is +*ve* radially out

The acceleration of the point is the rate of change of velocity with time. The components of acceleration are

Tangential:
$$\frac{dV}{dt} = r\frac{d}{dt}\left(\frac{d\theta}{dt}\right) = r\frac{d^2\theta}{dt^2} = r\alpha \tag{5.10a}$$

Radial:
$$-\frac{Vd\theta}{dt} = -V\omega = -\frac{V^2}{r} = -\omega^2 r \tag{5.10b}$$

The tangential component of acceleration dV/dt is by virtue of a change in speed only whereas the radial component called the centripetal acceleration is by virtue of the circular trajectory of radius r traced at a speed V. Obviously,

1. For a rectilinear motion, the 'radial' component is zero because the radius r of the 'circle' is infinite.
2. For a circular motion at a constant speed V, the tangential component $dV/dt = 0$ and the point undergoes only the centripetal acceleration $\omega^2 r$ directed radially inwards towards the centre or axis of rotation. For such a case,

$$\mathbf{a} = \omega^2 r \, \mathbf{e}_r$$

In terms of angular quantities alone,

angular displacement $= d\theta$

angular velocity $= \omega = \dfrac{d\theta}{dt}$

angular acceleration $= \alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

and in the integral form,

$$\boxed{\begin{aligned} \omega &= \int \alpha \, dt + K' \\ \theta &= \int \omega \, dt + C' \end{aligned}}$$

and

If the angular acceleration α is constant,

$$\omega = \omega_0 + \alpha t \qquad (5.11)$$

where the initial angular velocity at time $t = 0$ is ω_0

and $$\theta = \int (\omega_0 + \alpha t) \, dt + C'$$

or $$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \qquad (5.12)$$

where the angular displacement θ is referred to $\theta = 0$ at $t = 0$.

Eliminating t between the equations

$$\omega^2 - \omega_0^2 = 2\alpha\theta \qquad (5.13)$$

The relations for the angular quantities are similar to those for the linear quantities derived earlier. Table 5.1 brings out a systematic comparison.

Table 5.1 Linear vs Angular Motion

	Linear Motion	*Angular Motion*	*Remarks*
Displacement	s	θ	$ds = r\,d\theta$
Velocity	$v = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$	$V = r\omega$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$	$a = r\alpha$
Initial velocity	u	ω_0	
Expressions relating the displacement, velocity, accelera-tion and time	$V = u + at$	$\omega = \omega_0 + \alpha t$	valid only for constant accele-ration
	$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
	$V^2 - u^2 = 2as$	$\omega - \omega_0 = 2\alpha\theta$	
	$s = \int V\,dt + C$	$\theta = \int \omega\,dt + C'$	
	$V = \int a\,dt + K$	$\omega = \int \alpha\,dt + K'$	

Table 5.2 Expressions for Velocity and Acceleration in Different Coordinate Systems

Entity	*Cartesian Coordinates*	*Cylindrical Coordinates*
$\mathbf{r}$	$x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$	$r\,\mathbf{e}_r + z\,\mathbf{e}_z$
$\mathbf{V}$	$u\,\mathbf{i} + v\,\mathbf{j} + \omega\,\mathbf{k}$	$V_r\,\mathbf{e}_r + V_\theta\mathbf{e}_\theta + V_z\,\mathbf{e}_z$
a_x	$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial z}$	$\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_z\frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}$
a_y	$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial z}$	$\frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_z\frac{\partial V_\theta}{\partial z} + \frac{V_r V_\theta}{r}$
a_z	$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} + \omega\frac{\partial \omega}{\partial z}$	$\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}$

Assignments submit by 30-3-2020

In the following situations, determine whether the speed or velocity is changing, both are changing, or neither is changing. A car's cruise control is set to a constant 50 mph and ...

1. the car is driving on a level straight-away. [Speed is changing] [Velocity is changing] [Both are changing] [Neither are changing]

2. the car rounds a curve. [Speed is changing] [Velocity is changing] [Both are changing] [Neither are changing]

3. the car drives over a hill. [Speed is changing] [Velocity is changing] [Both are changing] [Neither are changing]

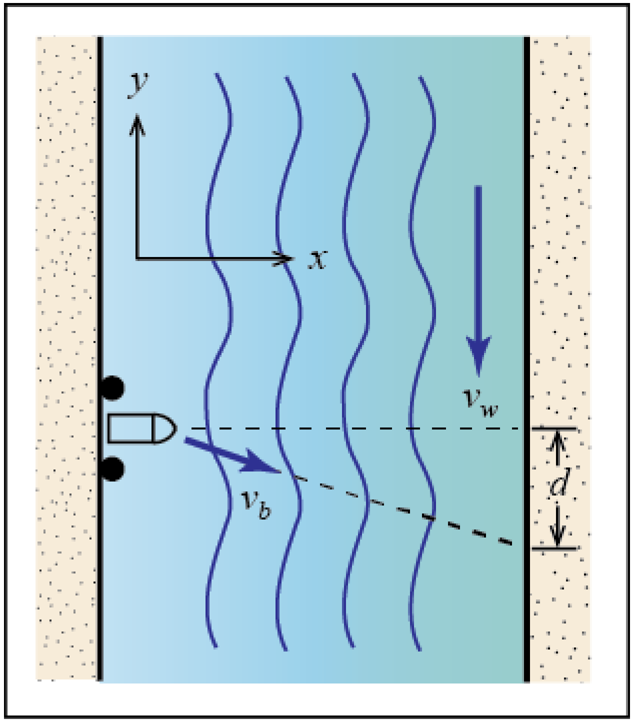
4. the car is driving up a hill of constant grade. [Speed is changing] [Velocity is changing] [Both are changing] [Neither are changing]

5. the car is driving on a level straight-away when the driver applies the brakes. [Speed is changing] [Velocity is changing] [Both are changing] [Neither are changing]

A fisherman sits in a motor boat at the shore of a river flowing downstream at 4 m/s. The fisherman wishes to reach the other side of the river. If the river is 60 meters wide and the motor boat has a maximum speed of 3 m/s with respect to the water, what is the shortest amount of time it could take the fisherman to perform the crossing? Note that the flow of the river is parallel to the shore on both sides of the river.

a) 20 seconds
b) 15 seconds
c) 60/5 seconds
d) 60/7 seconds

A fishing boat wishes to reach the other side of a briskly flowing river. The boat attempts to travel straight across the river, but the strength of the current pushes the boat downstream during the crossing. As a result of the river's current, the velocity of the boat has components in the y- as well as the x-direction. If the velocity of the boat relative to the shore is a constant $\mathbf{v}_b = (5\mathbf{i} - 2\mathbf{j})$ m/s and the river is 100 meters wide, what is the distance d that the boat is pushed downstream during its crossing?



3. If the crank of the engine shown in Fig. A rotates $\omega = 4\pi$ radians/sec and the crank radius $r = 10$ in., find

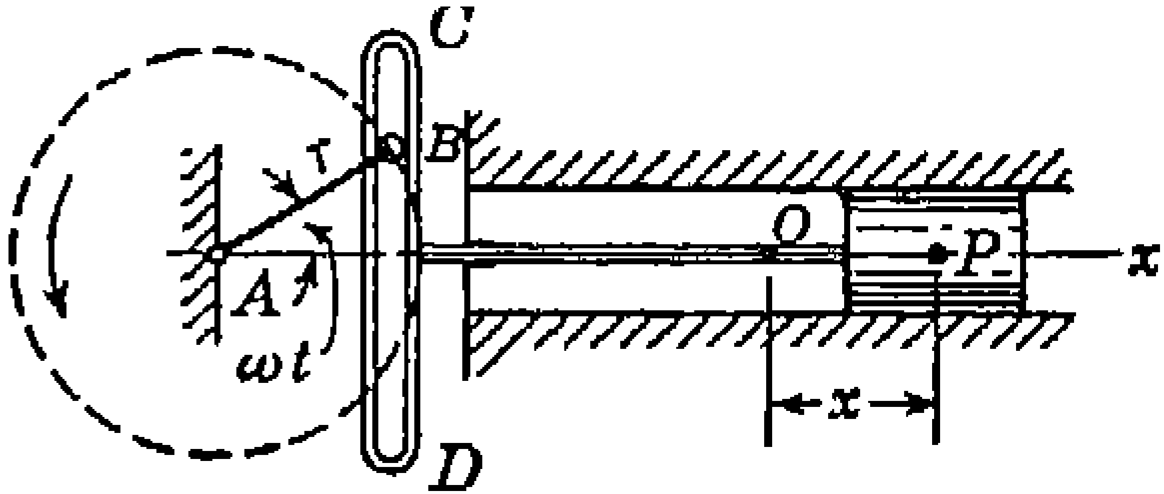


FIG. A

the maximum velocity and maximum acceleration of the piston.

Example 5.5 A wheel of radius 0.5 m is turned to advance up on a right-handed screw of pitch 1 cm as shown in Fig. Ex. 5.5. At an instant when the wheel is turned at a rotational speed of 2 rad/s, determine the velocity and acceleration of the hand held at *A*. If the wheel was accelerated rotationally at 0.6 rad/s^2, what would be the velocity and acceleration of the hand?

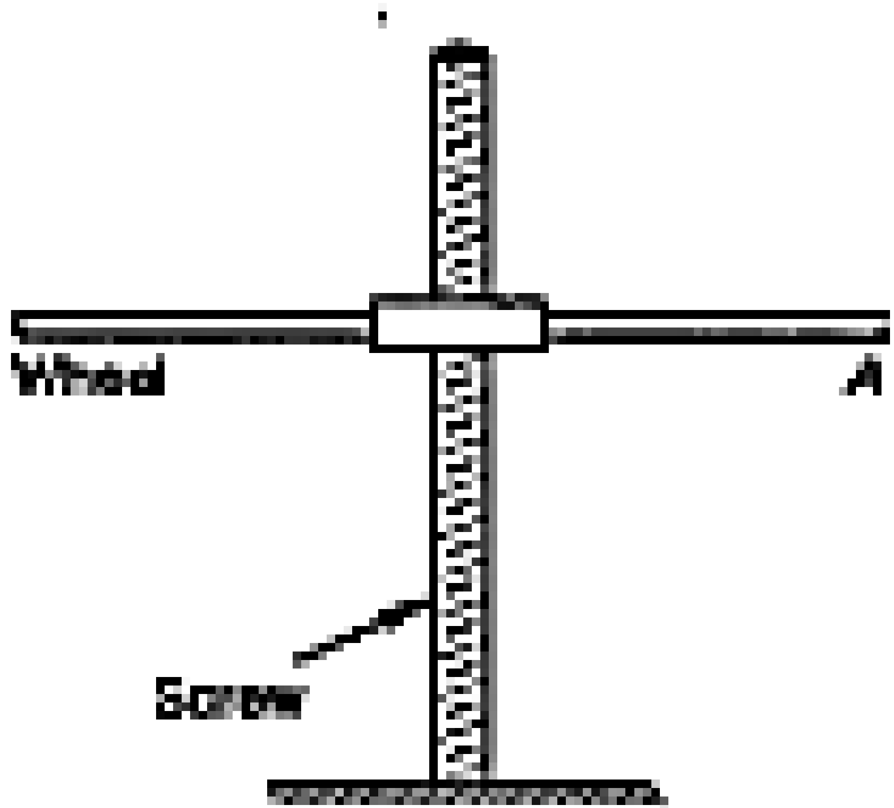


Fig. Ex. 5.5

references

- Washington university materials'
- Conceptual dynamics
- Mechanics –K L kumar
- Mechanics - Timoshenko