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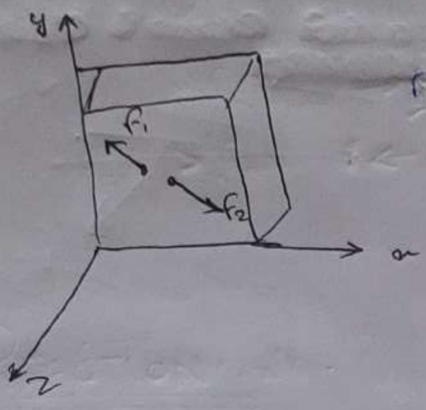
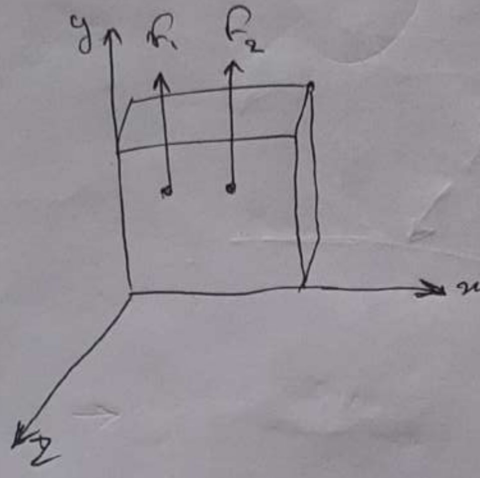
# * Forces:

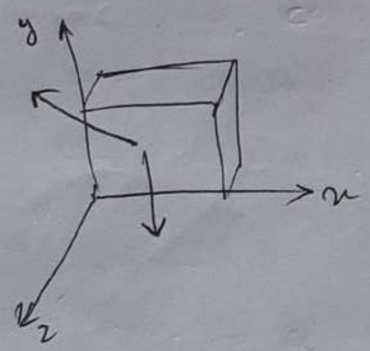
Collinear:



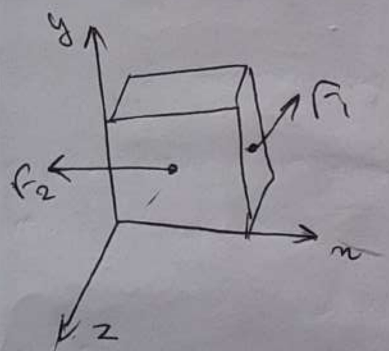

Coplanar:



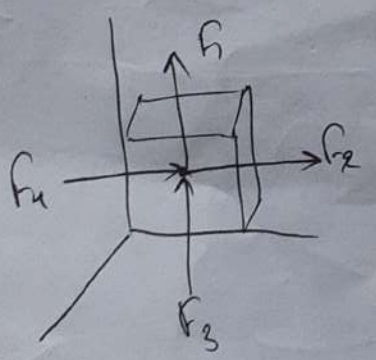

Non-collinear



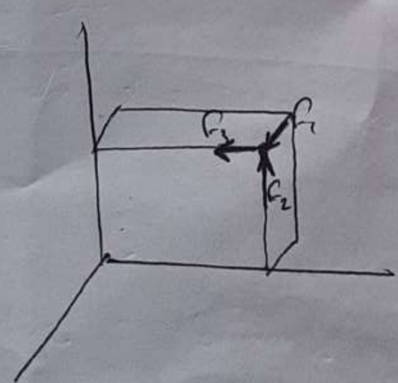

Non-coplanar



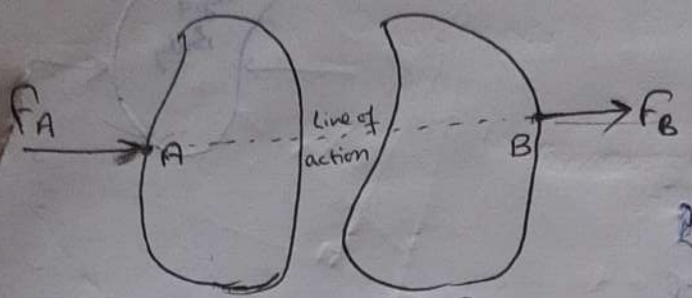

Coplanar - concurrent.




Non-coplanar, concurrent.



## * Transmissibility of Forces:

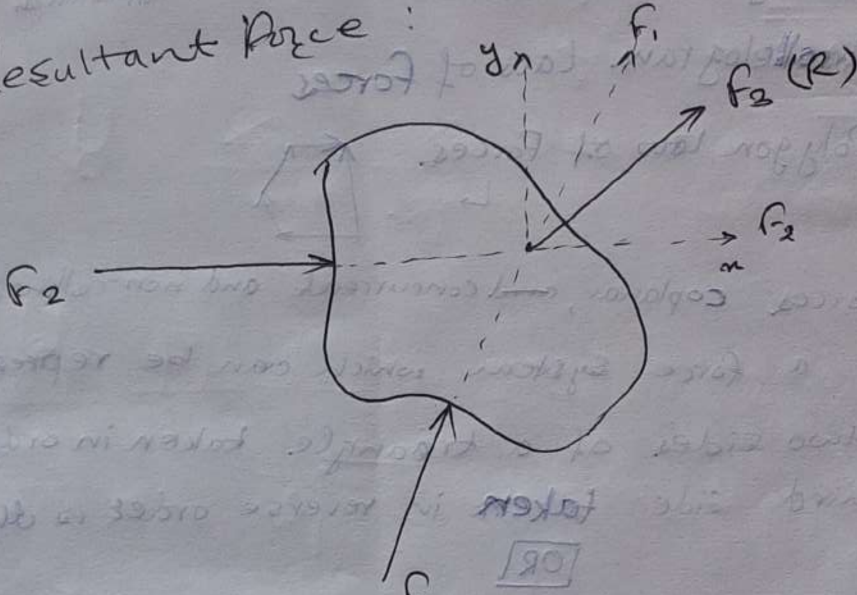



body I case I

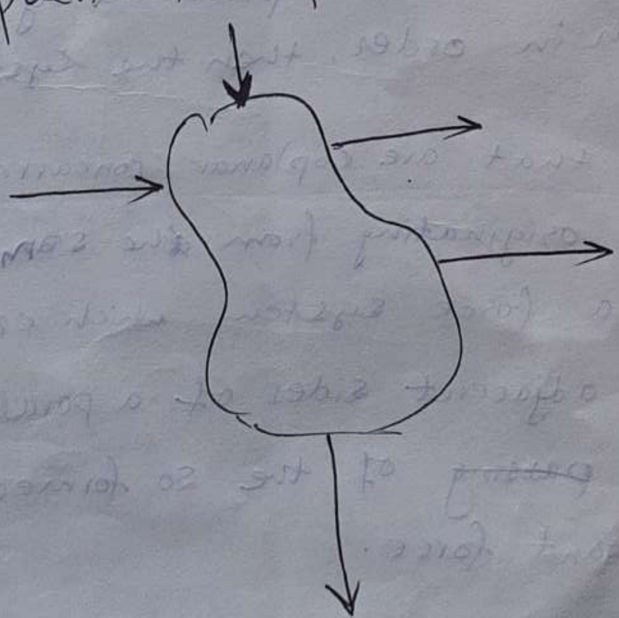
body I case II

→ Same body.
Different forces along same line of action.

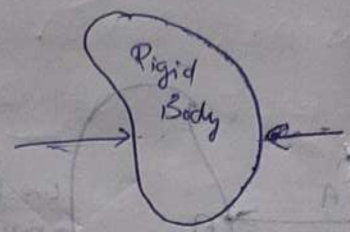
## * Resultant Force:



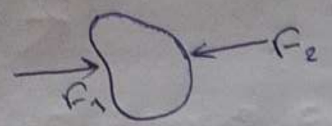

## * Superposition of Forces:



# "Resultant" of Force System:




Case I:




$|F_1| > |F_2|$ $F \rightarrow$

$|F_2| > |F_1|$ $\leftarrow F$

{i} Traiangular Law of Forces → Conditions: coplanar, concurrent, non-collinear

{ii} Parallelogram Law of forces

(iii) Polygon ~~law~~ Law of Forces
↳

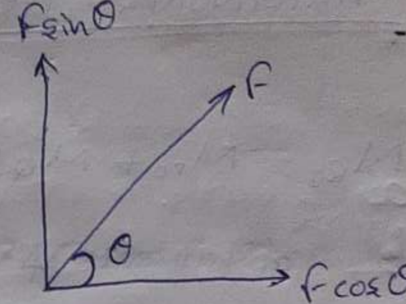
{i} → If two forces coplanar, ~~and~~ concurrent and non-collinear acting on a ~~force~~ body constitute a force system which can be represented as the two sides of a triangle taken in order, then the third side taken in reverse order is the resultant.

OR

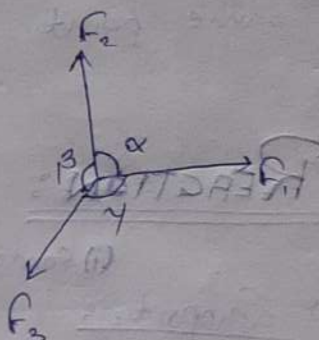
If three forces that are coplanar, concurrent and non-collinear constitute a system acting on a and if three forces can be represented by the sides of a triangle taken in order, then the system is in equilibrium.

(ii) → If two forces that are coplanar, concurrent and non-collinear originating from the same point of origin form a force system which can be represented as the two adjacent sides of a parallelogram, then the diagonal ~~passing~~ of the so formed parallelogram is the resultant force.

# Resolution of Force:




# Lami's Theorem:

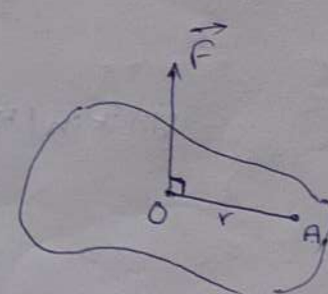



Conditions { (i) CONCURRENT
(ii) COPLANAR

imp – (iii) Entire force system should be in Equilibrium

Theorem states that,

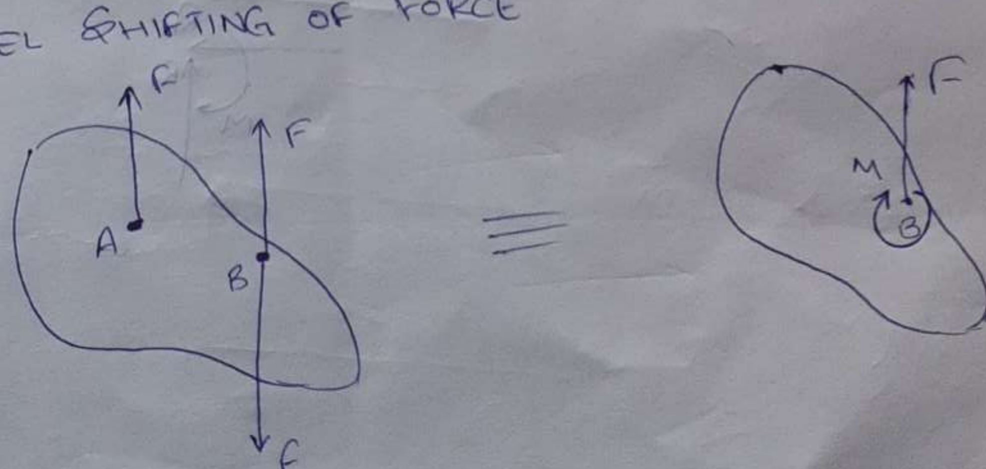
$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\gamma} = \frac{F_3}{\sin\alpha}$$



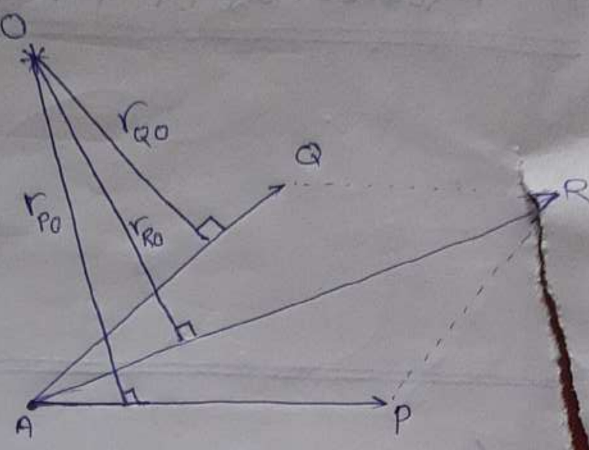

Moment,

$$\vec{M} = \vec{F} \times r$$

# Parallel Shifting of Force



# VARIGNON'S THEOREM:

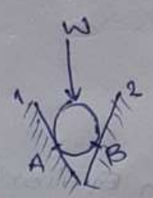



$$M_{RO} = M_{PO} + M_{QO}$$

The algebraic sum of moments of forces from a point is equal to the moment of the resultant to the same point.

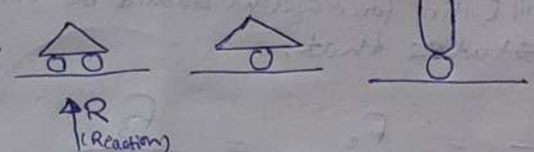
# REACTION:

(i) Contact

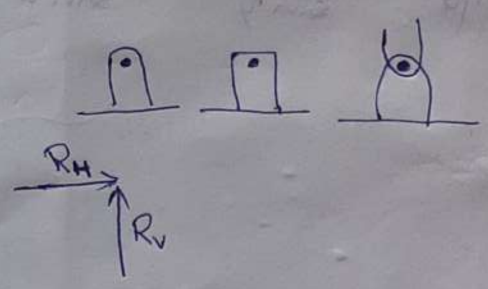



* Supports:

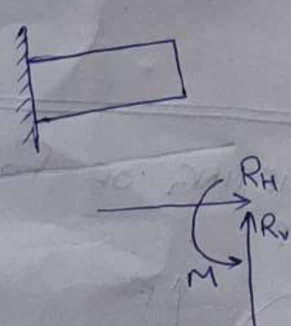
(i) Roller Support:




(ii) Pin / Hinge Support:




(iii) Fixed Support:

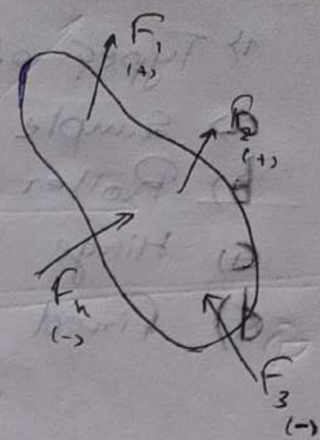


# GENERALISED EQUILIBRIUM CONDITION

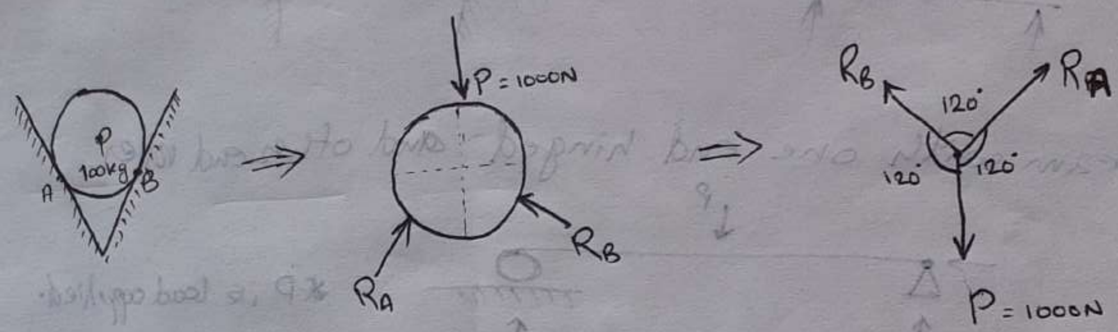
either or both conditions should be valid

1) $\sum_{i=0}^{n} F_i = 0$

2) $\sum M_{xx} = 0$



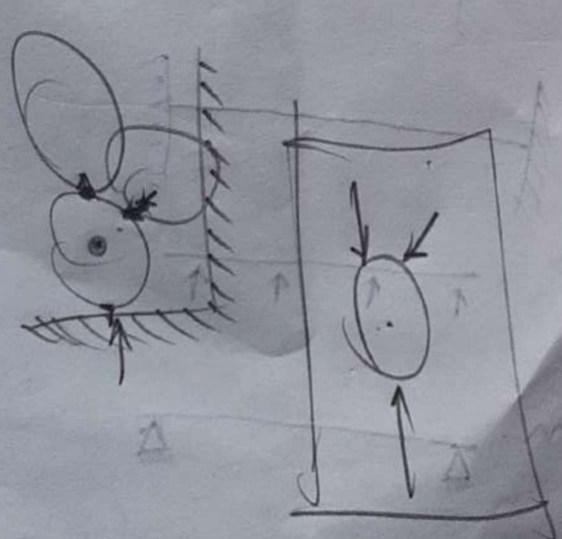

# FREE BODY DIAGRAM.




Using Lami's Theorem,

$$\frac{P}{\sin 120^\circ} = \frac{R_A}{\sin 120^\circ} = \frac{R_B}{\sin 120^\circ}$$

$$\therefore \; P = R_A = R_B = 1000N$$

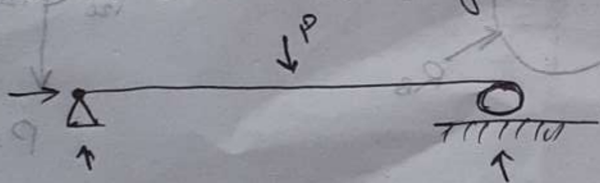


# LOADING OF BEAMS:

1) Types of Support
a) Simple Support.
b) Roller Support.
c) Hinge Support.
d) Fixed Support.

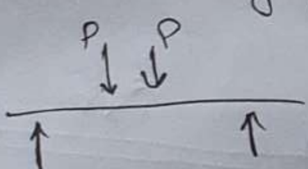
(i) Simple Supported Beam.


(ii) Beam with one end hinged and other end roller.




*P is load applied.

(iii) Over-hanging beam:




(iv) Cantilever:

(v) Propped Beam:

(vi) Fixed Beam.

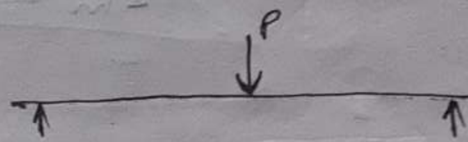
(vii) Continuous Beam:

(viii) Both end hinged beam.

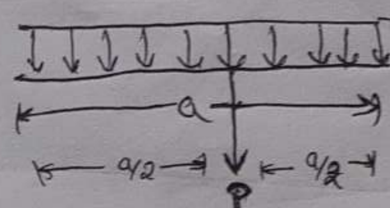
# ~~Concentrated Load:~~

# Types of Load:

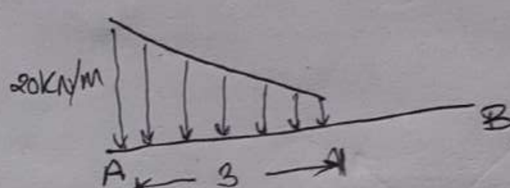
1) Concentrated load:




2) Uniformly Distributed load. :
(UDL)



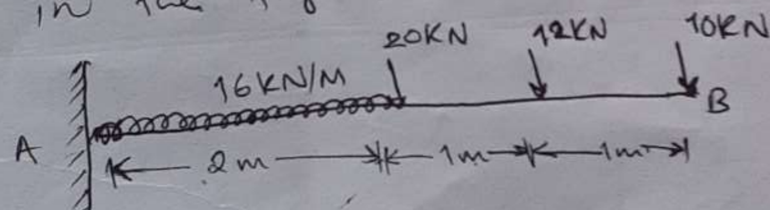

3) Uniformly Variable Load:
(UVL)

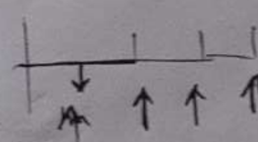



Net force $= \frac{1}{2} \times 20 \times 3 = 30$ kN. [ $\frac{h}{3}$ from base ; $\frac{2h}{3}$ from apex ]

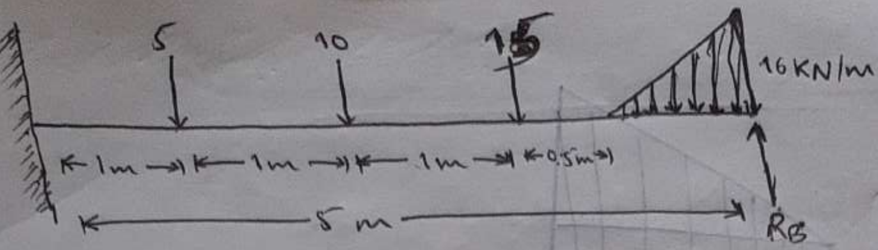
4) Point Load.
5) Moment Load.

Q. A cantilever is fixed at A and free at B. UDL is applied and 3 point loads are applied as shown in the fig. Determine all $R_x{}^n$s for the fig.



Q.




Sol$^n$:- $\Sigma H = 0$

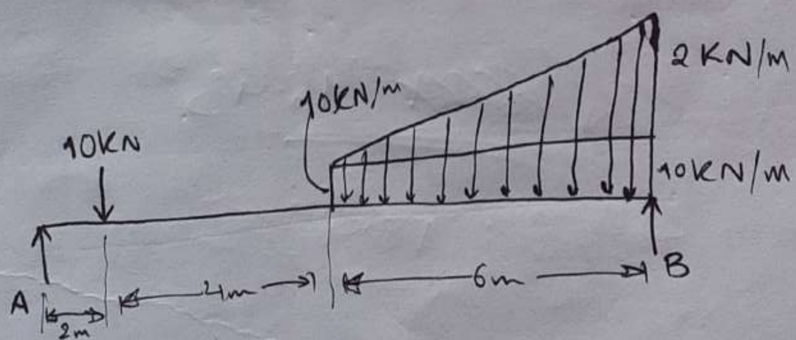
$$\Sigma V = 5 + 10 + 15 + \frac{1}{2} \times 16 \times 1.5$$
$$= 42\,KN \qquad = R_A$$

$$M_A = +R_B \times 5 - \left(\frac{1}{2} \times 16 \times 1.5\right) \times 4.5 - 15 \times 3 - 10 \times 2 - 5 \times 1$$
$$= +R_B \times 5 - 12 \times 4.5 - 15 \times 3 - 10 \times 2 - 5 \times 1$$

$$M_B = -12 \times 0.5 - 2 \times 15 - 10 \times 3 - 5 \times 4$$

---

Q.



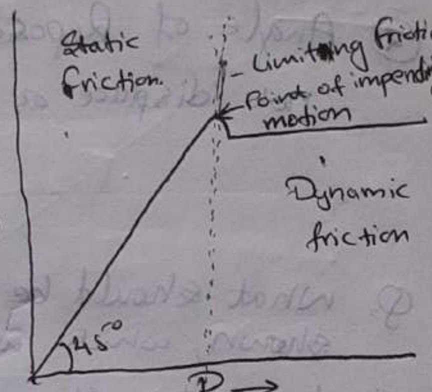

A simply supported beam ~~which has~~ with a combination load is shown in the given fig. Find out the reaction forces at A and B

Sol$^n$:- $\Sigma V = 10 + 10 \times 6 + 2 \times \frac{1}{2} \times 6 = 76\,KN$

# FRICTION: When the body moves or tends to move over another surface or over another body, an opposing force is present that will oppose/prevent the motion that develops at the contact surface.

# TYPES OF FRICTIONAL FORCE:

(i) Sliding frictional force
(ii) Rolling frictional force.




# LAWS OF FRICTION:

i) The total friction that can be developed is independent of the magnitude of the area of contact.
ii) The total friction that can be developed is proportional ~~to~~ the normal force.
iii) At low velocities of sliding friction, the ~~total~~ total friction that can be developed is practically independent of the velocity.

So,
$$F \propto N$$
$$F = \mu N = \mu R$$

where, $\mu$ = Coefficient of friction

# Postulates/Axioms of the above laws:

(i) The force of friction always acts opposite to the direction of the motion.
(ii) Till the limiting value is reached for a static body, the magnitude of the frictional force is equal to the magnitude of the applied force.
(iii) The force of friction depends upon the quality of the surface.
(iv) It is independent of contact area.
(v)

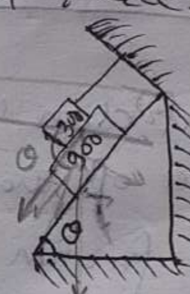
# Terminologies of the Friction:

(1) Angle of Friction: $\tan\theta = \frac{F}{N} = \mu$

(2) Angle of Repose: max. angle where the body will not displace or start moving by result of its own weight.

Q. What should be the value of $\theta$ for the figure shown, which will make the motion of 900 N block to slide down the slope at slight increase of force (P)

$\mu = 1/3$ for all contact surfaces.




Soln

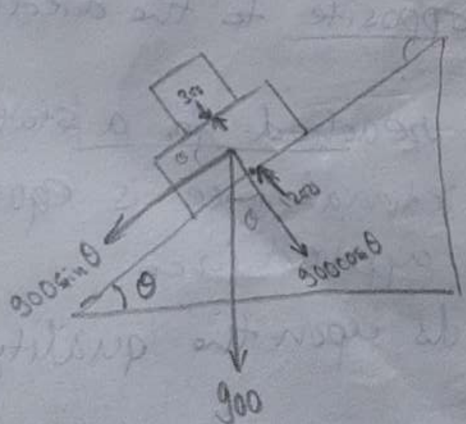
$900\sin\theta = \mu\, 1200\cos\theta - 300\cos\theta$

or, $\frac{\sin\theta}{\cos\theta} = \frac{1}{3} \times \frac{900}{900}$

$\therefore \tan\theta = \frac{1}{3}$

$\theta = \cancel{0.322}\ 18.43°$

$\therefore \theta = 18.4°$

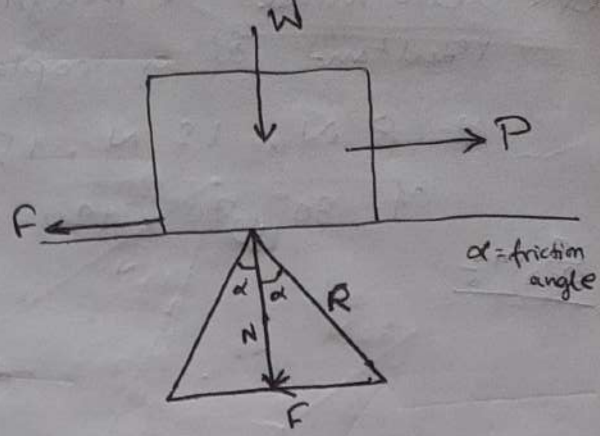



$P = \mu R$

$-900\sin\theta + 1200\cos\theta\,\mu - 300\cos\theta\,\mu = 0$

$900\sin\theta = 1200\cos\theta\,\mu - 300\cos\theta\,\mu$

# Cone of friction:

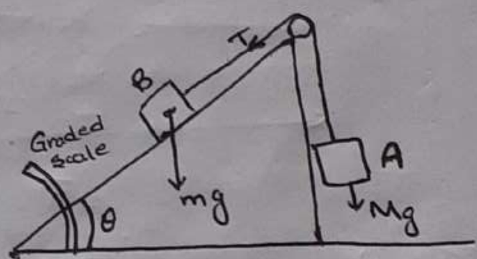



Derive an expression for coefficient of friction for the experiment as shown in the diagram.

Ans:- At block A,

$$Mg = T \quad -(i)$$

At block B,



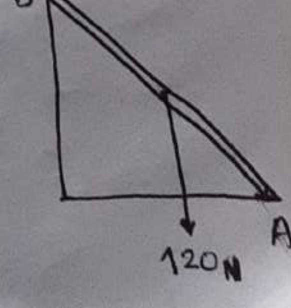

$$T - Mg\sin\theta = \mu mg\cos\theta \quad -(ii)$$

$$Mg - mg\sin\theta = \mu mg\cos\theta.$$

or, $$\frac{g(M - m\sin\theta)}{mg\sin\theta} = \mu$$

$$\therefore \mu = \left(\frac{M}{m}\sec\theta - \tan\theta\right).$$

Q. A ladder of length 'L' = 5 m and weight of 120 N is placed on a flat floor against a verticle wall as shown in the fig. If the coefficients of frictions are 0.3 and 0.2. 0.3 is at B and 0.2 is at A and the ladder is considered to be homogenous. Determine the smallest angle θ so that the ladder can be placed at equilibrium with the floor.

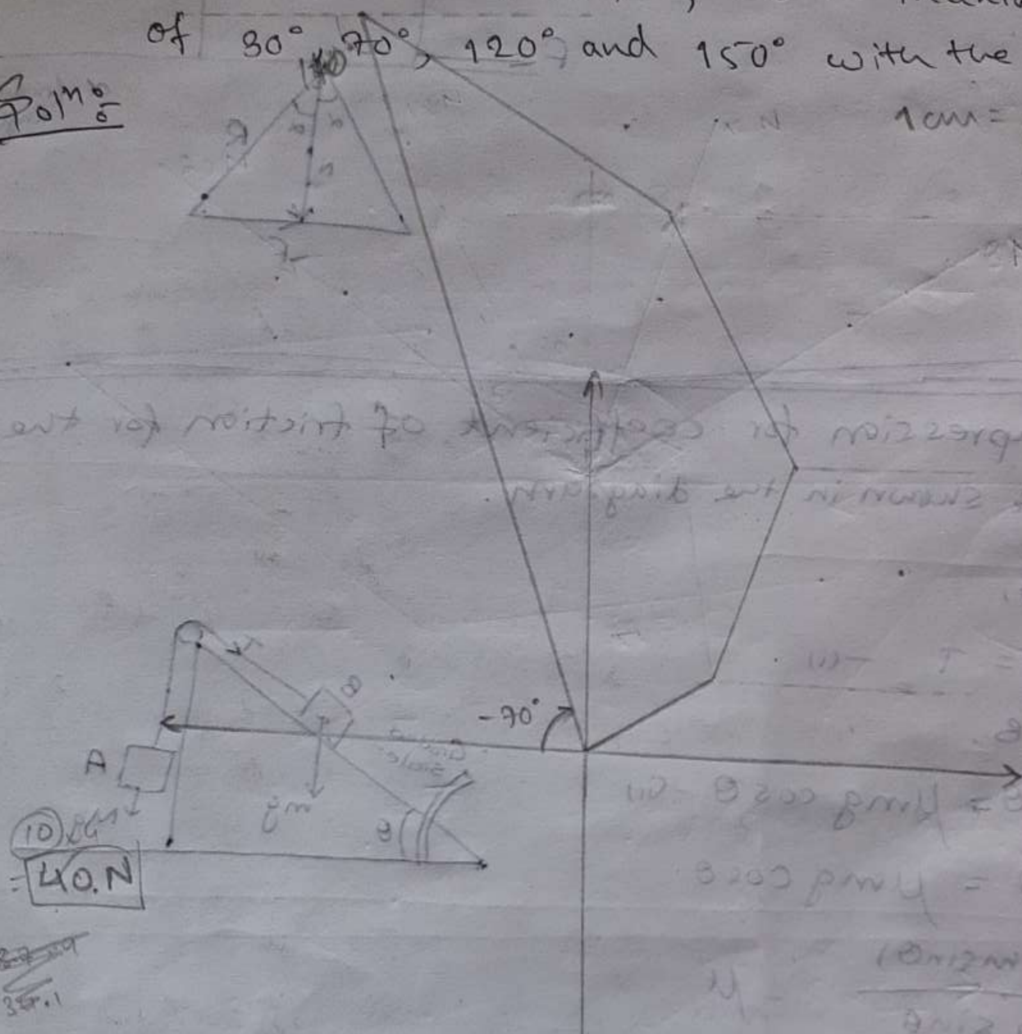



Ans:-

Q. Determine the magnitude and the direction of the resultant of a coplanar, concurrent forces of magnitude 8 N, 12 N, 15 N, 20 N making an angle of 30°, 70°, 120° and 150° with the horizontal.

Solⁿ:

1 cm = $\frac{1}{4}$ N

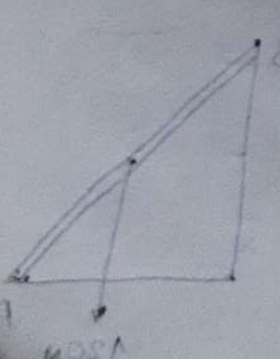



$$f_x = 8\cos 30^\circ + 12\cos 70^\circ - 15\cos 60^\circ - 20\cos 30^\circ$$
$$= -13.78\ N$$

$$f_y = 12\cos 20 + 8\cos 60 + 19\cos + 20\cos 60^\circ$$
$$= 38.26\ N$$

Hence, $R = \sqrt{f_x^2 + f_y^2} = 40.66\ N$

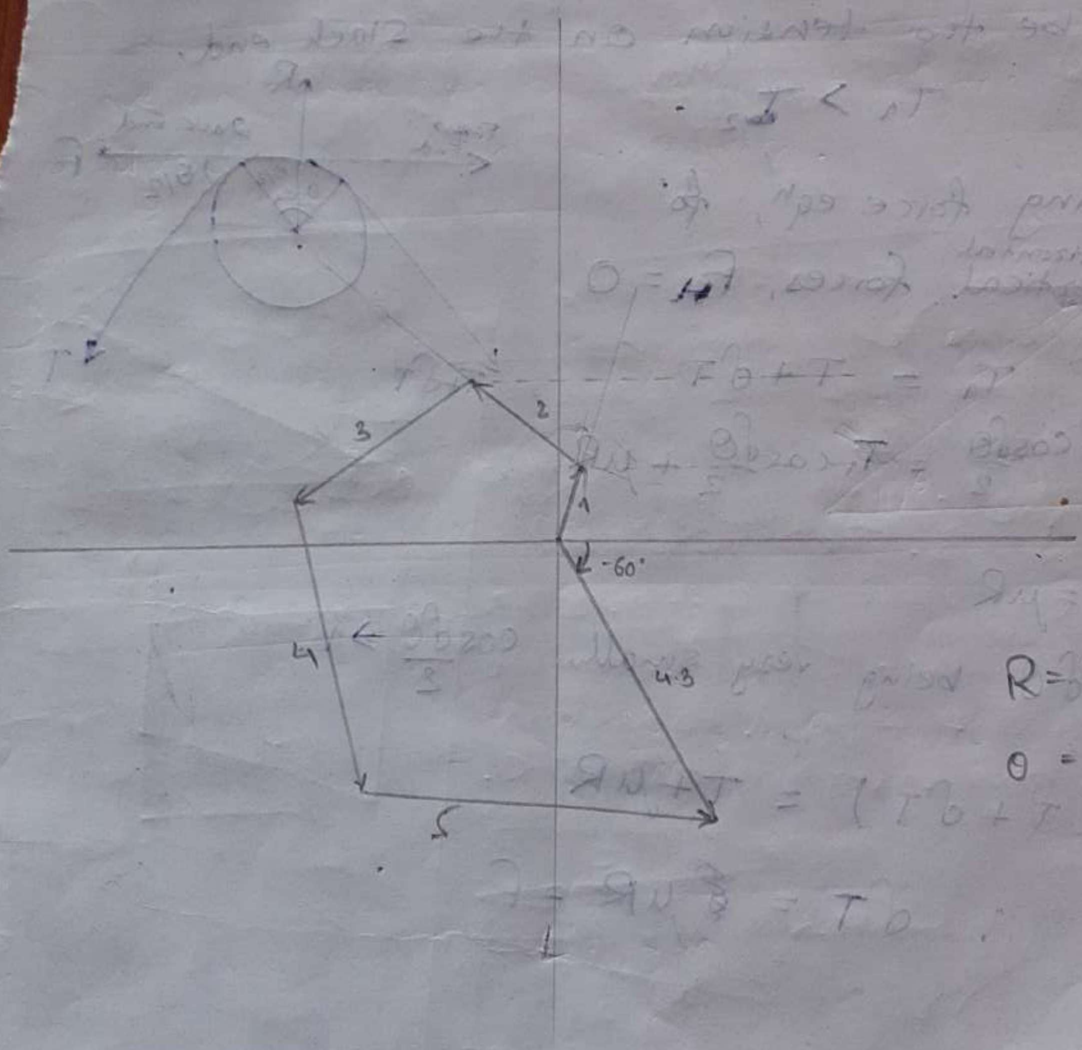
$$\tan\theta = \frac{f_y}{f_x} = \frac{38.26}{13.78} \Rightarrow \theta = \tan^{-1}\left(\frac{38.26}{13.78}\right) = 70.19^\circ$$



Q. ~~Five~~ forces 10N, 20N, 30N, 40N and 50N are acting at a point such that the angle between them is equal. If 10N is on horizontal, find out resultant of the system.

1cm = 10N

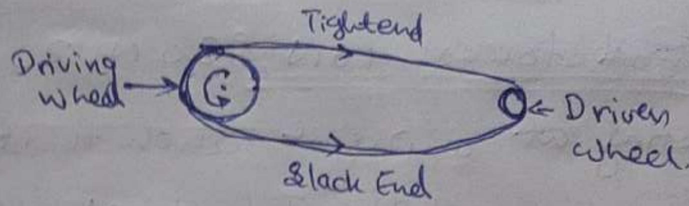
72°




R = 43 N

θ =

# Belt Drive:

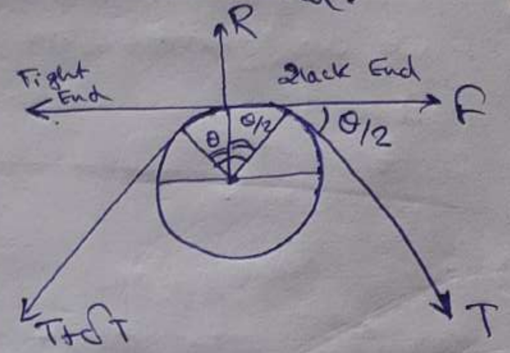



Using force equilibrium,

$$F_H = 0$$

Let us consider a small angle $\delta\theta$ as an included angle which is part of our actual contact angle $\theta$.

Let $T_1$ be the tension on the ~~end~~ tight end.
Let $T_2$ be the tension on the slack end.
Such that, $T_1 > T_2$




Considering force eq^n, ~~fo~~
for ~~vertical~~ horizontal forces, $F_H = 0$

$$T_1 = T + \delta T$$

$$(T+\delta T)\cos\frac{\delta\theta}{2} = T_1 \cos\frac{\delta\theta}{2} + \mu R$$

$$F = \mu R$$

and, $\delta\theta$ being very small, $\cos\frac{\delta\theta}{2} \to 1$.

or, $(T + \delta T) = T + \mu R$

$$\therefore \delta T = \mu R = F$$

Now,
for vertical forces,

$$R = (T+\delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

or, $R = 2T\sin\frac{\delta\theta}{2} + \delta T\sin\frac{\delta\theta}{2}$

Since, $\delta\theta$ is very small, $\sin\frac{\delta\theta}{2} \to \frac{\delta\theta}{2}$

So, $R = 2 \cdot T \cdot \frac{\delta\theta}{2} + \delta T \frac{\delta\theta}{2}$

or, $R = T\delta\theta + \delta T \frac{\delta\theta}{2}$

$$\delta T = \mu R$$

$$R = \frac{\delta T}{\mu} = T\delta\theta$$

$$R = T\delta\theta$$

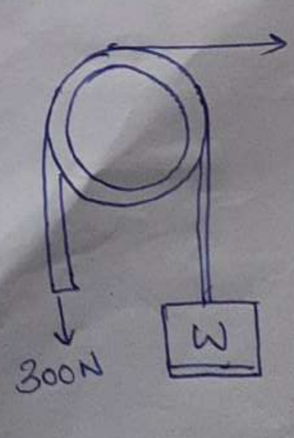
$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \int_0^{\theta} \mu \delta\theta$$

$$\therefore T_1 = T_2 e^{\mu\theta}$$

A body is hanging on a rope exerting the downward force of 900 N. The rope is wound over the pulley $1\frac{1}{4}$ th time over the pulley. Find out how much tension is required on the free end, so that 900 N body is balanced?

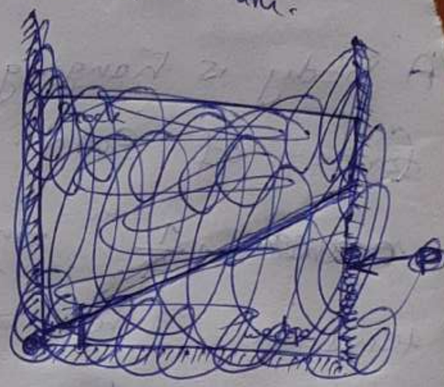
(Hint: $1\frac{1}{4}$ in radian gives point of contact. $1\frac{1}{4} = 1.25 \times 2\pi$ )

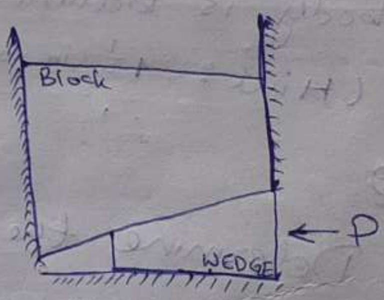
Q. Determine the max. weight that can be lowered by a person who can exert a 300 N pull on the rope if the rope is wrapped $2\frac{1}{2}$ times along a horizontal spur as shown in fig. The coefficient of friction betn the spur is 0.3.



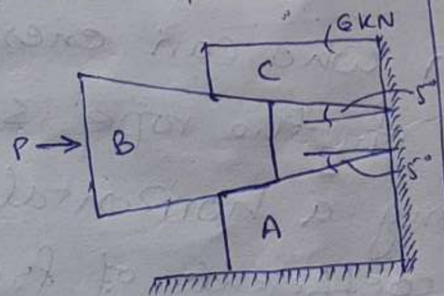
Q. ~~Draw~~ A block weighing 900 N is rested between two walls. A wedge is inserted such that the ~~wall ends~~ block hangs on the inclined surface of the wedge. The weight of the block is 900 N. The coefficient of friction between block and wall is 0.3. Coeff. of friction between block and wegde is 0.22. The wedge weighs 250 N. Calculate all the forces acting on the body. Draw FBD to validate the answer. Calculate P for the system to be in equilibrium.

Soln:-



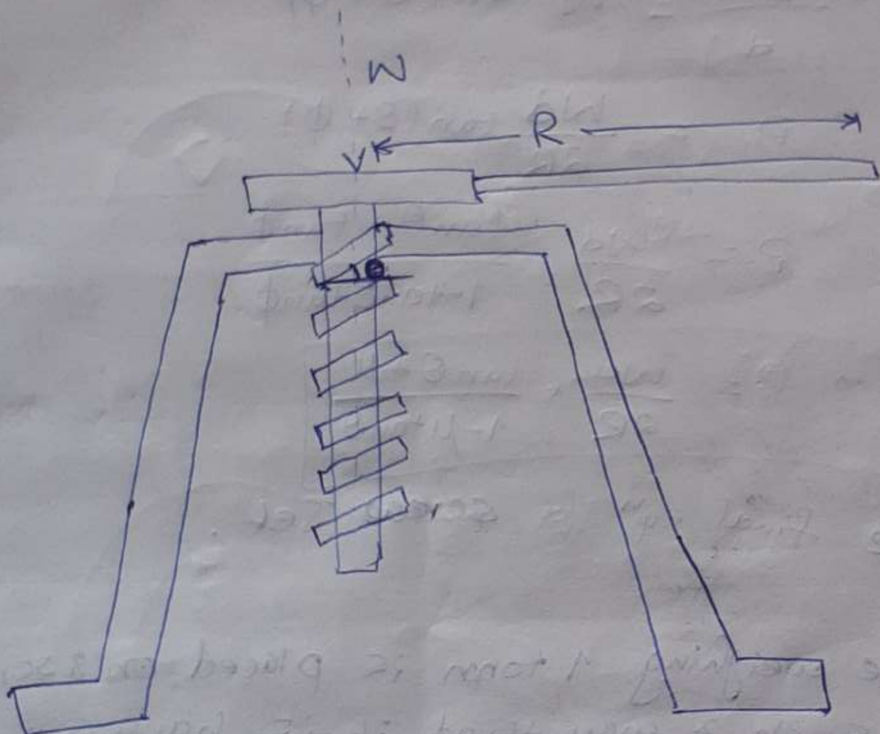



Q. In the fig. shown, C is a stone block weighing 6 KN. It is being rested slightly by means of two wooden wedges A and B, weight of A and B are negligible, with a force P on the wedge B. The angle is given. If the coefficient of friction is 0.3 for all the contact surfaces, compute the value of P required to impend the upward motion of block C.



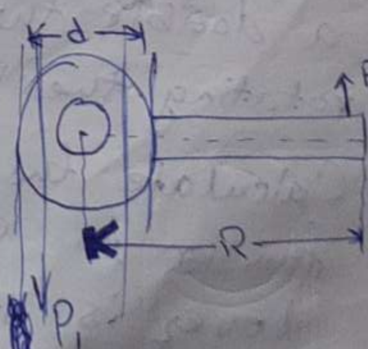
# Working of a Screw Jet:

- Lead of the screw, $l$
- Applied Load, $p$ (Effort)
- Distance between lever end point to centre, $R$
- Mean Diameter of screw, $d$

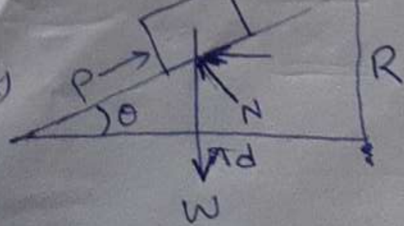

Let $P_1$ be the effort such that if we substitute this $P_1$ at the mean diameter, it will produce the same ~~force/torque~~ moment of force using $p$ at distance $R$.

$$P_1 = \frac{2PR}{d}$$




Let $\mu$ be the coefficient of friction betn sliding faces and $N$ is the normal reaction force present at the

$P > W$ (Assuming screw jet is raising the weight)




$R_1$ is the resultant of $f$ and $N$.

$$P_1 = R_1 \sin(\theta + \phi) \quad \text{-(i)}$$

$$W = R_1 \cos(\theta + \phi) \quad -(ii)$$

Dividing (i) by (ii),

$$\frac{P_1}{W} = \frac{R_1 \sin(\theta+\phi)}{R_1 \cos(\theta+\phi)}$$

$$\therefore P_1 = W \tan(\theta+\phi)$$

Substituting $P_1 = \frac{2PR}{d}$

we have, $\frac{2PR}{d} = W \tan(\theta+\phi)$

$$\therefore P = \frac{Wd}{2R} \tan(\theta+\phi)$$

$$\text{or, } P = \frac{Wd}{2R} \frac{\tan\theta + \tan\phi}{1 - \tan\theta \tan\phi}$$

$$\boxed{\Rightarrow P = \frac{Wd}{2R} \times \frac{\tan\theta + \mu}{1 - \mu \tan\theta}}$$

This is the final eqⁿ for screw Jet.

Q. A vehicle weighing 1 tonn is placed on a screw jet in such a way that it is hanging in the jet itself. The vehicle needs to be lifted 0.5 m upwards such that it can be fulfilled by 1 rotation of the lever. The screw jet is a double threaded screw. The person who is rotating the lever can exert 1200 N max. Calculate the helix angle if coefficient of friction ~~is~~ 0.33 throughout. Length of the lever is 1.5 m. Mean diameter is 0.25 m.

# Centroid and Moment of Inertia :

* Centre of Gravity: It is a point through which the resultant of forces of gravity ~~~~ of the body acts.

* Centroid (Centre of Area): It is defined as a point in a plane area such that the moment of area about any axis through that point is zero.

* Centre of gravity $= \frac{\Sigma W_i X_i}{\Sigma W_i}$ (for x-axis)

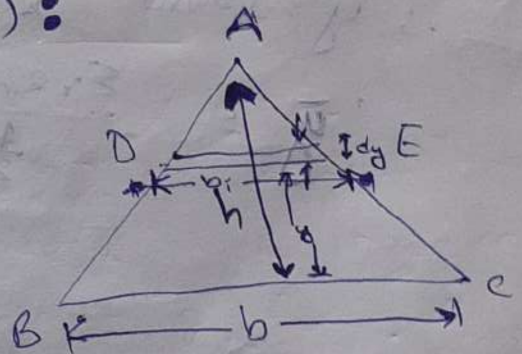
  W is weight of the body
  x is position from the reference point.

* Centroid $= \frac{\Sigma A_i x_i}{\Sigma A_i}$ (for x-axis)

  A is point in a plane area.

* Centroid of simple figures using axis of symmetry.

* Centroid of a triangle (scalar):




Considering $\triangle ADE \sim \triangle ABC$

$$\frac{b}{b_i} = \frac{h}{h-y}$$

Let the strip DE be rectangular,

Then, Area of ~~this~~ strip DE $= b_i \times dy$   $b_i$ is instantaneous length.

Centroid about Y-axis $= \frac{b(h-y)}{h}\, dy$

$$\bar{Y} = \int \frac{y\, dA}{A}$$

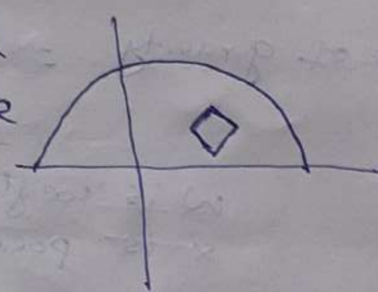
$$\bar{X} = \frac{\Sigma A_i x_i}{A_i}$$

$$\bar{Y} = \frac{\Sigma A_i Y_i}{A_i}$$

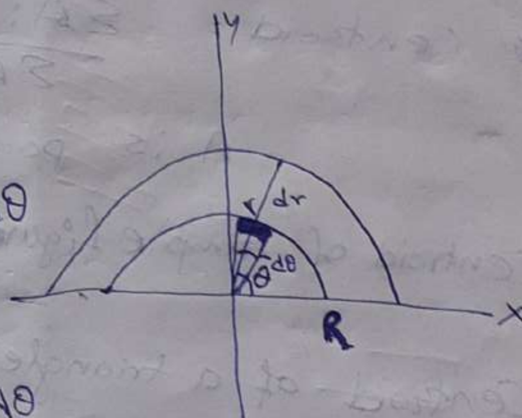
This is true for all the simple figures.
For finding centroid of small figures using symmetry following formulas are valid: $\bar{Y} = \frac{\Sigma y \cdot dA}{A}$ and $\bar{X} = \frac{\Sigma x\, dA}{A}$

# Centroid of a Semicircle :

Let semi-circle has the small strip dr lying at the angle $d\theta$ from the base.



Moment about



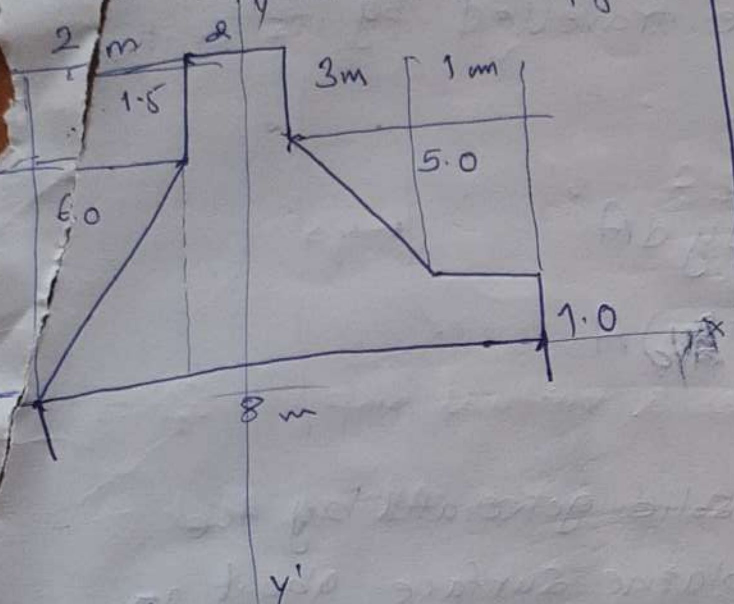

Area of small elemental strip $= dr \cdot r\,d\theta$

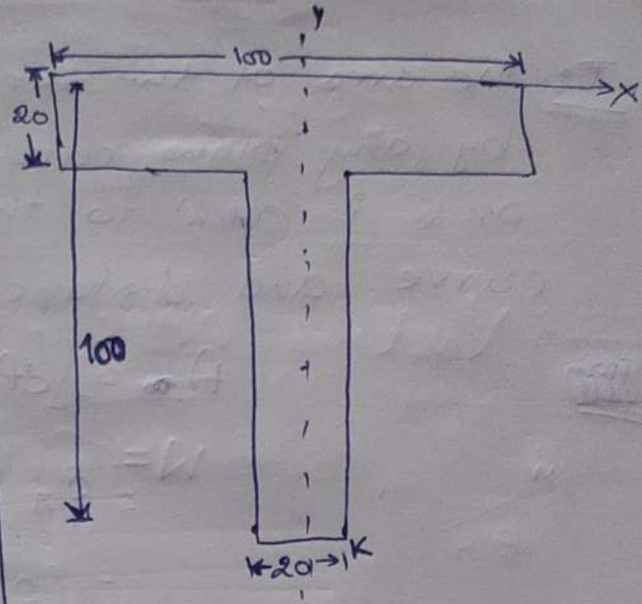
$\therefore\ y = r\sin\theta \qquad x = r\cos\theta$

$$\therefore\ \bar{y} = \frac{\Sigma\, r\sin\theta\, dr \times r\,d\theta}{\frac{1}{2}\pi R^2}$$

Q. find centroid of T-section as shown in the figure:

Q. Determine the centroid of a section of a concrete dam as shown in the figure.




# Analytical Expression of Centroid:

① 1-D line, $x_c = \dfrac{\Sigma l_i x_i}{\Sigma l_i} = \int_L x \, dL$

② 2-D surface, $\left\{ \cancel{x_c = \dfrac{\Sigma l_i x_i}{\Sigma l_i} = \int_L x \, dL} \quad \cancel{y_c = \dfrac{\Sigma l_i y_i}{\Sigma l_i} = \int_L y \, dL} \right\}$

Area, $x_c = \dfrac{\Sigma A_i x_i}{\Sigma A_i} = \dfrac{\int_A x \, dA}{\int_A dA}$

Also called as, first moment of area

$y_c = \dfrac{\Sigma A_i y_i}{\Sigma A_i} = \dfrac{\int_A y \, dA}{\int_A dA}$

③ 3-D object, Volume $x_c = \dfrac{\Sigma V_i x_i}{\Sigma V_i} = \dfrac{\int_V x \, dV}{\int_V dV}$

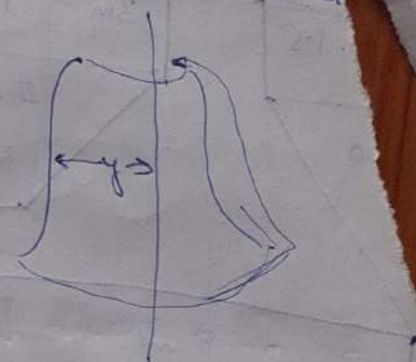
$y_c = \dfrac{\Sigma V_i y_i}{\Sigma V_i} = \dfrac{\int_V y \, dV}{\int_V dV}$

$z_c = \dfrac{\Sigma V_i z_i}{\Sigma V_i} = \dfrac{\int_V z \, dV}{\int_V dV}$

# Pappus Guldinus Theorem:

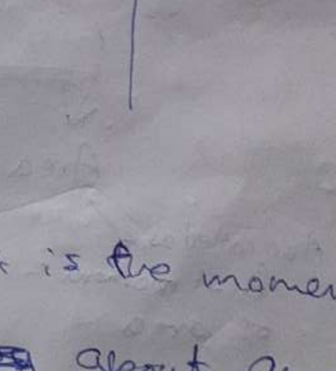
I: The area of the surface generated by the revolution by any plane curve about a ~~known~~ non-intersecting axis is equal to the product of length of the curve and distance travelled by its centroid.

$$A = \int dA$$
$$= 2\pi \int y \, dA$$
$$= (2\pi y_c) A$$




II: The volume of the solid generated by the revolution of any plane surface about a non-intersecting axis is equal to the product of the area of the surface and its distance travelled by the surface.

$$V = \int_V dV$$
$$= 2\pi \int y \, dA$$
$$= (2\pi y_c) A$$



# Second moment of ~~inertia~~ Area: It is the moment of first moment of ~~inertia~~ area about any reference axis.

$$I_y = \int_A y^2 dA$$
$$I_x = \int_A x^2 dA$$

NOTE: Second moment of Area $\neq$ Moment of Inertia.

as, moment of inertia deals with mass.

# Radius of Gyration:

It is defined as the root mean square distance of the object's parts from either its Centre of Gravity or its axes.

$$r_x = \sqrt{\frac{I_x}{A}} \quad ; \quad r_y = \sqrt{\frac{I_y}{A}}$$

# Perpendicular Axis Theorem:

$$I_y + I_x = I_z$$

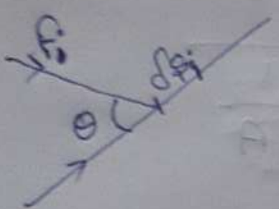
# Parallel Axis Theorem:

$$I_{xB} = I_{xc} + Ah^2$$

$$I_x = I_{GV} + Ah^2$$

h = distance betn the centroid and the point of reference

# Virtual Work :



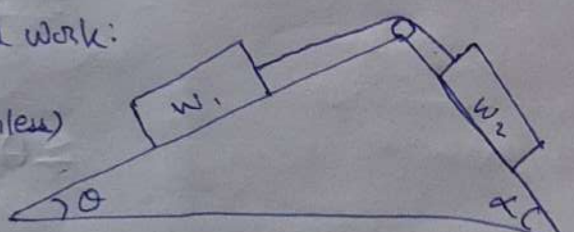

$$\delta W = f\cos\theta \; '\delta s'$$

If a force $f_i$ produces a virtual displacement of $\delta s_i$ (where, $i$ is instantenous) then the virtual work of

$$\delta W = \delta U_i = f\cos\theta \cdot \delta s_i$$

<u>Q.</u> Two ~~pulleys~~ ~~of~~ weights of $w_1$ and $w_2$ are hanging over a pulley on an inclined surface. The inclination angles are $\theta$ and $\alpha$. Find out relation between $w_1$ and $w_2$ in terms of inclination angle.

<u>Soln:-</u> Assumptions for Virtual work:




(i) System is ideal (frictionless)
(ii) Reaction forces are not taken/considered
(iii) Virtual work is produced by a displacement, so the ~~surface~~ forces should be taken such that the body is able to produce displacement & work.

Note

i) The increment of work is considered to be +ve if projection of force and displacement have the same sense

(ii) The body is in equilibrium if the net work done by each virtual displacement due to the active forces combined to give zero effect.

$$\Sigma f_i \cdot \delta x_i = 0$$

Q.

~~Set 2.3~~ : Q. ~~2.22 - 2.28, 2.31, 2.33, 2.36, 2.39, 2.40~~ 3.65 3.66 3.63 3.99

~~Set 2.4~~ : ~~Q. 2.45 - 2.51, 2.53, 2.56, 2.57~~

~~Set 2.5 : Q. 2.64, 2.61, 2.60, 2.66, 2.67, 2.69,~~

~~figure f, i, j, 2.77~~

2.3 → ~~2.22 - 2.28, 2.31, 2.33~~, 2.36, 2.39, 2.40

2.4 → ~~2.48 - 2.51, 2~~

4.4 → Q. 4.36

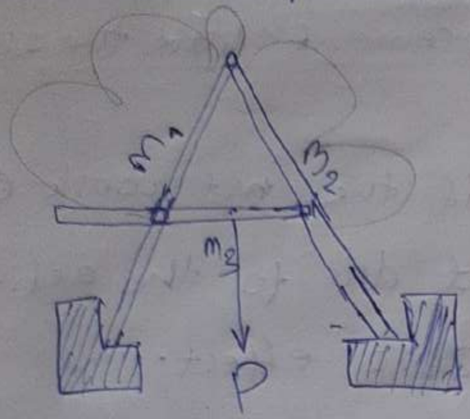
Appendix : AAC

(fig c, b, h, j, i) & 4.3

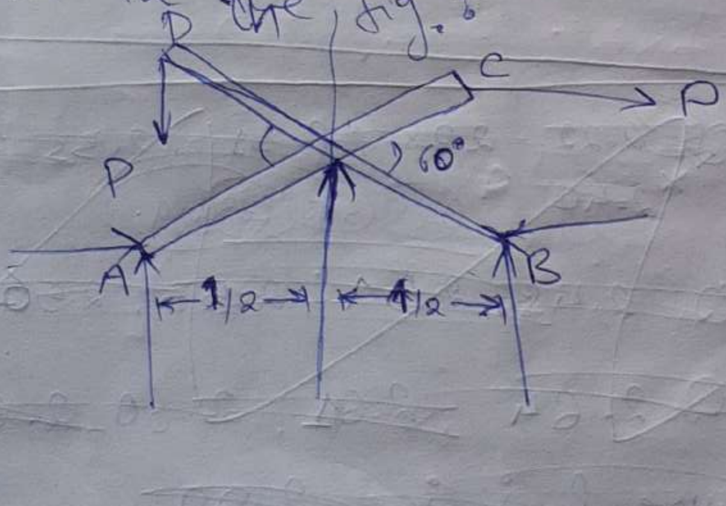
4.57 WIP

(fig. 4.57. g, i, f, j)

# Method of Members:




**Q.** Determine horizontal and vertical component for ~~A~~ ~~and~~ ~~B~~ ~~for~~ the X frames shown in the fig.:




Sol^n: