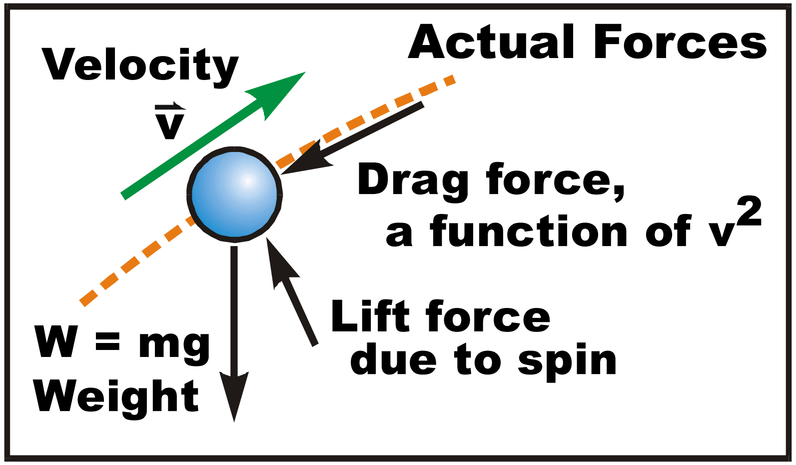
Projectile Notes

1. Definition of a Projectile: An object that is “projected” or thrown, which has no capacity for self-propulsion.

2. Actual forces on a Projectile:
   Drag, lift due to spin, weight, wind.

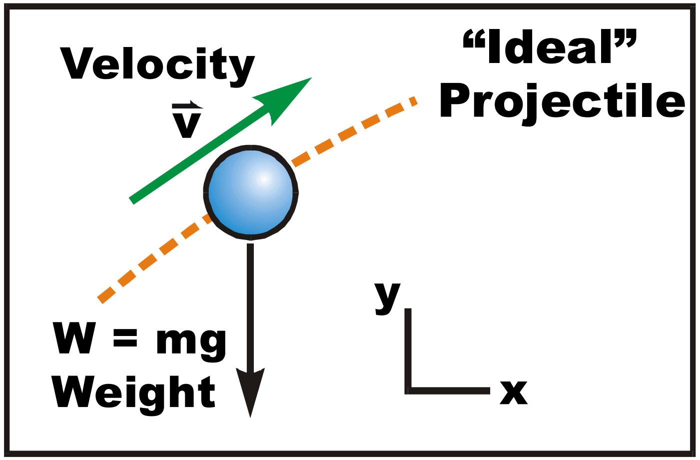
3. Are the forces on a projectile (other than weight) significant?
   In other words, does the ideal projectile model “fit” or not?



For low speed objects with reasonable mass, e.g. a shot put, or a baseball, tennis ball or golf ball tossed softly across a room, the ideal projectile model “fits” relatively well.

For high speed objects, e.g. a hit or thrown baseball, a well-hit golf ball or tennis ball, etc., drag and other forces are significant and our ideal model is not accurate. For example, a well-hit home run, by ideal theory, will travel nearly 750 ft. In reality it only travels around 450 ft—a significant difference!

Light objects, e.g. a ping pong ball, feather, foam ball, etc., do not fit the ideal model very well. A relatively small drag or spin force markedly affects the ball because the ball has such low mass.



Interesting fact: A well-hit golf ball travels farther than ideal theory predicts because of lift due to spin.

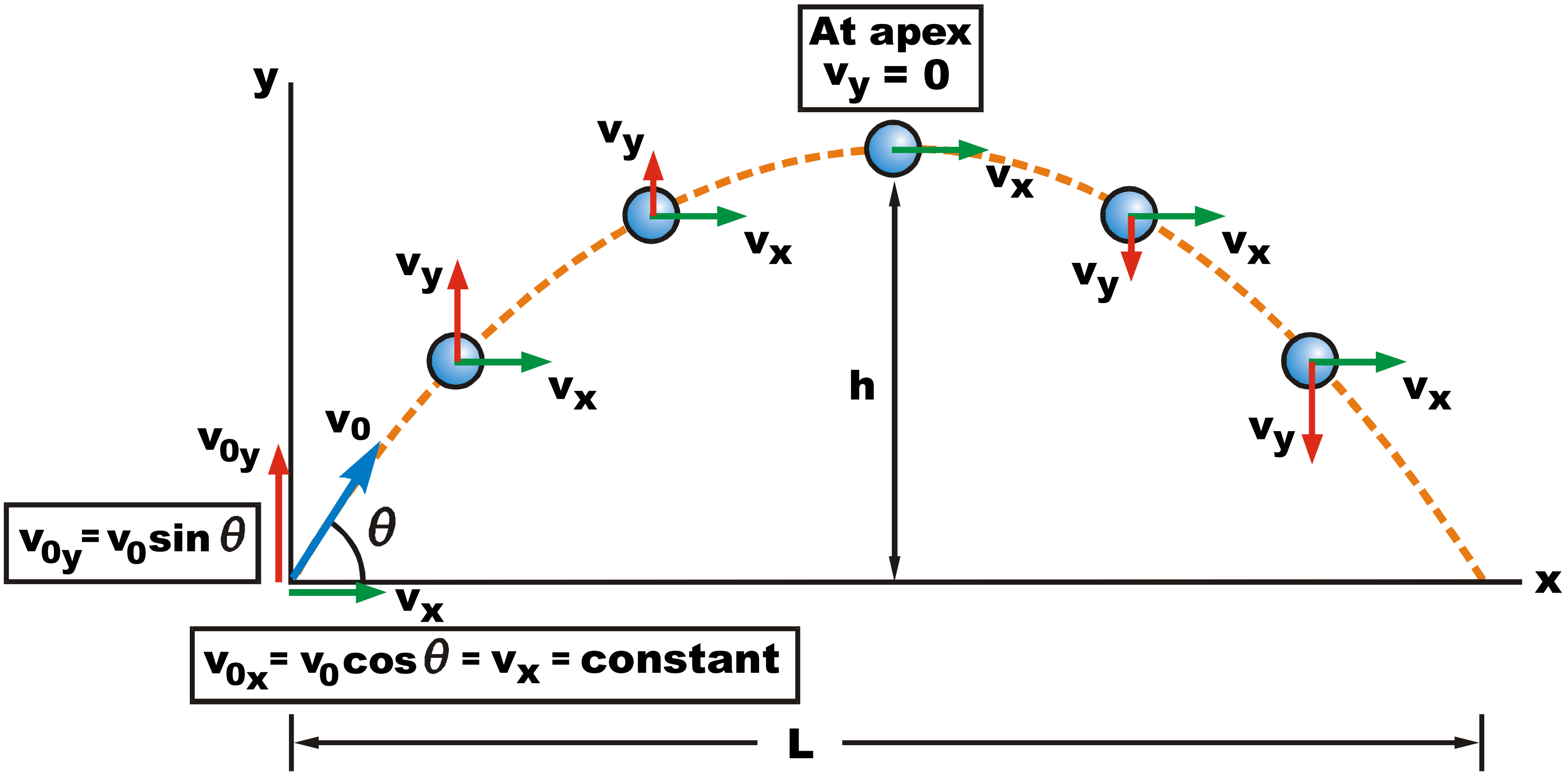
4. "Ideal" Projectile: The only force is weight.
(This is what we will cover in this class.)

5. Ideal Projectile: If the only force is weight, then the x velocity stays constant. The y velocity changes with time and position.

6. Ideal Projectile: If the only force is weight, then…

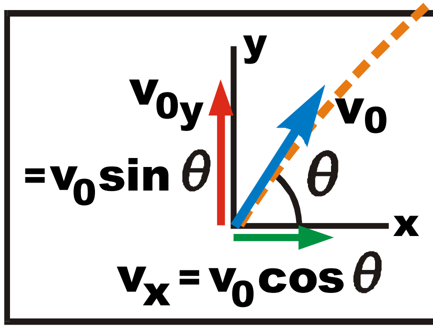
The x velocity stays constant.

The y velocity changes with time and position.



7. Ideal Projectile Equations: If the only force is weight, then the x velocity stays constant ($a_x = 0$). The y velocity changes with time and position (y acceleration $a_y = -g$).

Remember to use the correct g for your units!



(Ideal) Projectile Equations

	x	y
Accel:	0	$-g$
Velocity:	$\boxed{v_x = v_0 \cos\theta}$	$\boxed{v_y = v_{0y} - gt}$
		(where, $v_{0y} = v_0 \sin\theta$)
Position:	$\boxed{x = x_0 + v_x t}$	$\boxed{y = y_0 + v_{0y} t - \frac{1}{2} g t^2}$

SI Units: $g = 9.81\ m/s^2$

US Units: $g = 32.2\ fps^2$

An additional y equation: $\boxed{v_y^2 = v_{0y}^2 - 2g(y - y_0)}$

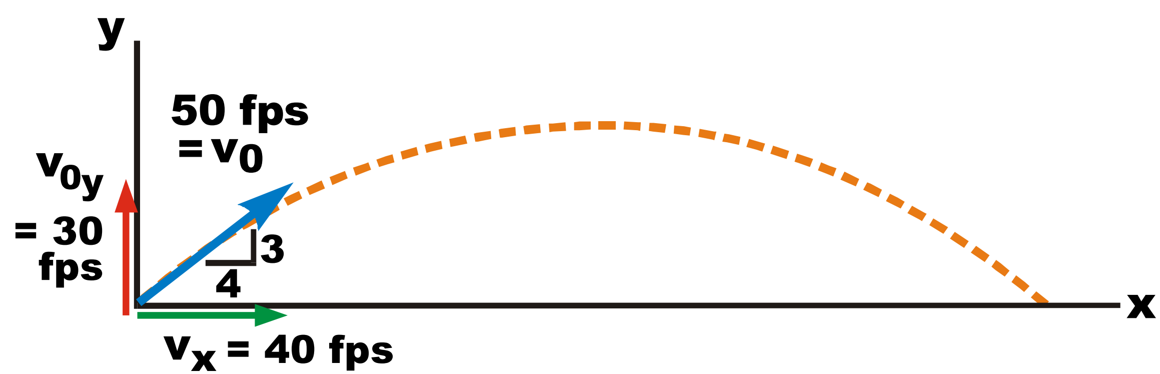
8. An ideal projectile trajectory is a *parabola*.

The position eqns are parametric eqns:

$x = f(t)$ and $y = g(t^2)$

Eliminating t from these yields a parabola: $y = f(x^2)$

A simple numerical example:



Position Eqns: $\boxed{x = x_0 + v_x t}$ $\boxed{y = y_0 + v_{0y}t - \frac{1}{2}gt^2}$

$x = 40\,t$ $y = 30\,t - 16.1\,t^2$

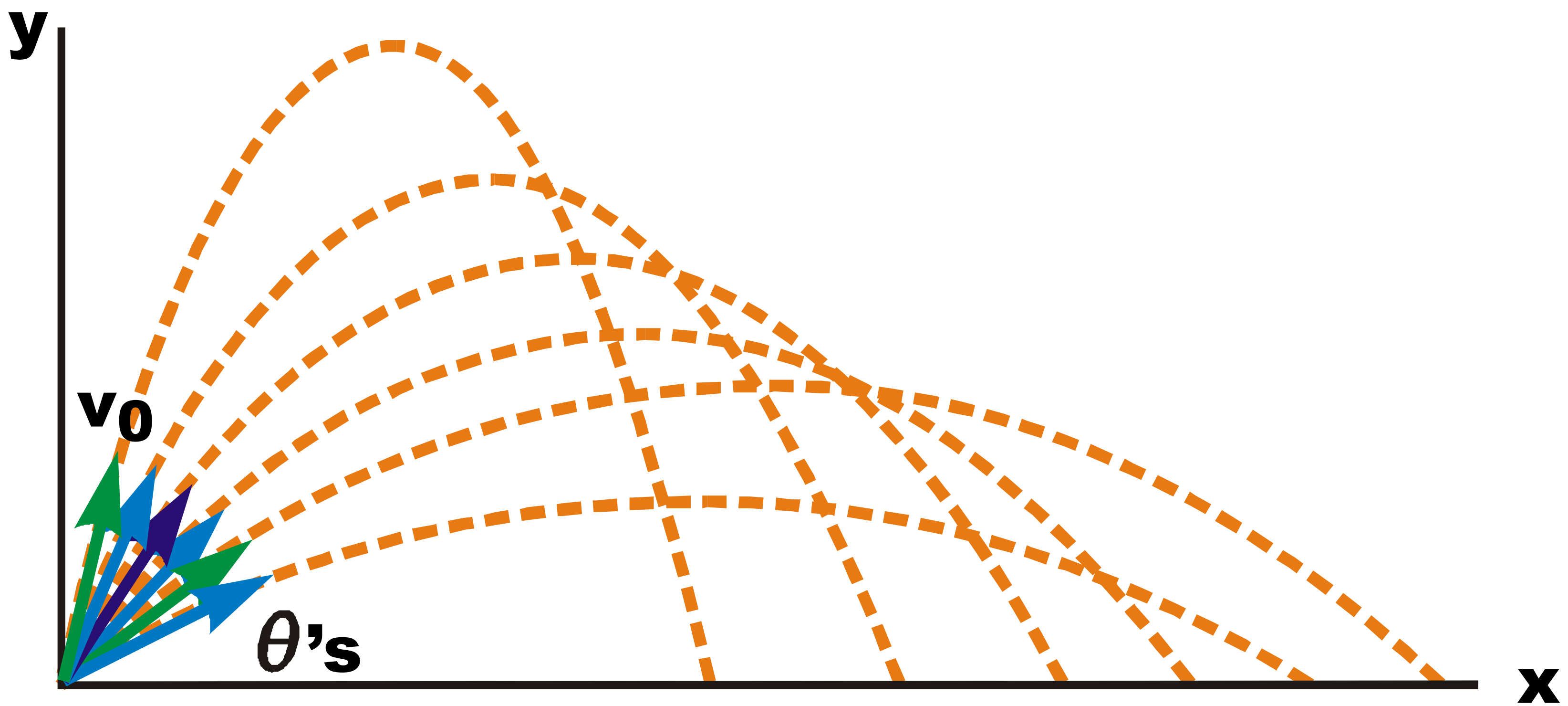
Eliminate t : $t = \dfrac{x}{40}$

$$y = \frac{30}{40}x - \frac{16.1}{40^2}x^2$$

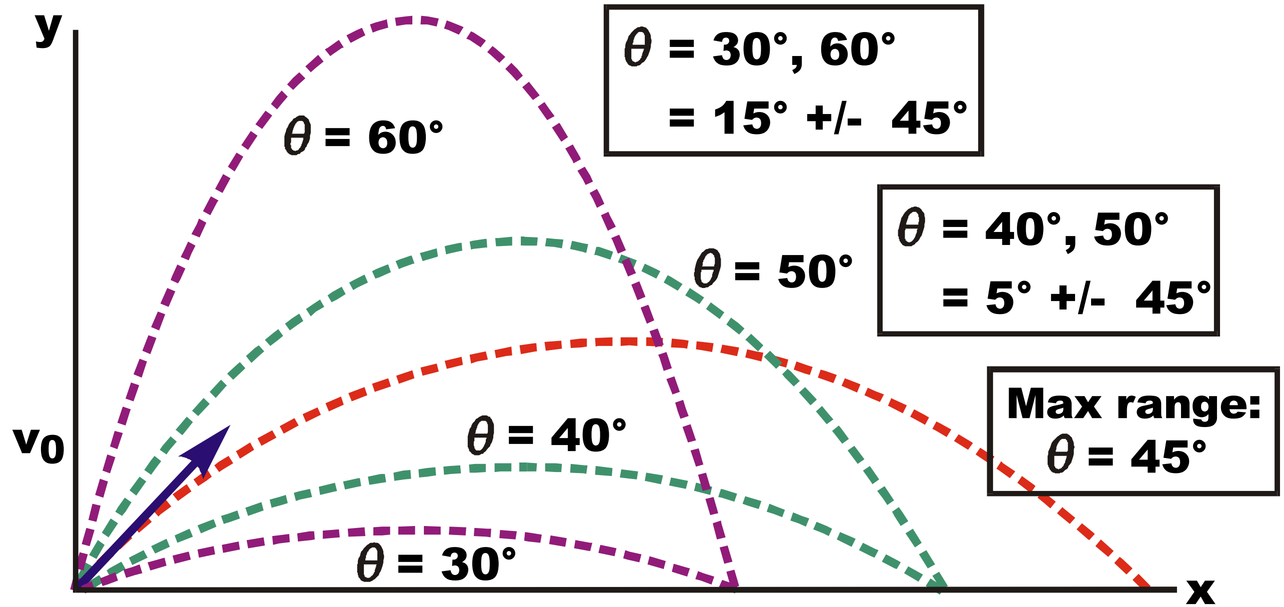
The result is a parabola : $\boxed{y = .75x - .010063\,x^2}$

We rarely use this fact to solve a problem, but you should know it.

9. **For each launch speed, v_0, and angle θ there is a different parabolic trajectory.**



10. For a given launch speed, v_0, the max range is at $\theta = 45$. For the same v_0, launch angles at equal angular increments above and below 45 give (equal) ranges shorter than the max range.



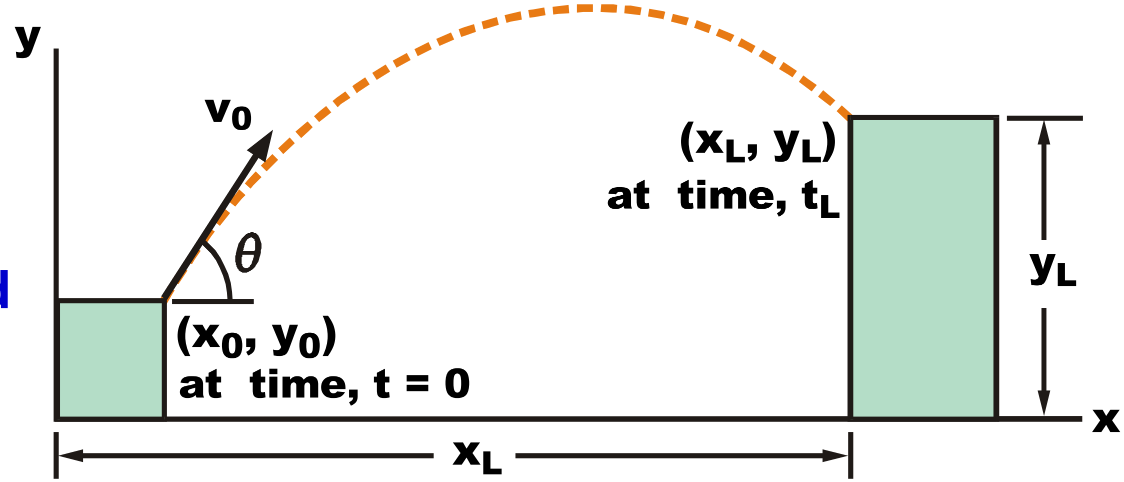
For a given v_0, max range at $\theta = 45°$ (on level ground)

For a given v_0, launch angles at equal angular increments above and below 45° give (equal) ranges shorter than the max range.

11. A general projectile motion problem involves seven "pieces" of information [x_0, y_0, θ, x_L, y_L, and t_L] .

Usually you are given five of these and asked to find the remaining two, usually applying the two position equations.

General Projectile Problem



Projectile Problem Variables: (7 pieces of info)

Launch Location: (x_0, y_0)

Launch Velocity and Angle: $(v_0$ at $\theta)$

Landing Location and Time: (x_L, y_L) at time t_L

General Problem: Given 5 out of 7 of these "pieces" of info.
Use the two position equations to solve for the remaining two:

Position: $\boxed{x = x_0 + v_x t}$ $\boxed{y = y_0 + v_{0y} t - \frac{1}{2} g t^2}$

where, $v_x = v_0 \cos\theta$ $v_{0y} = v_0 \sin\theta$