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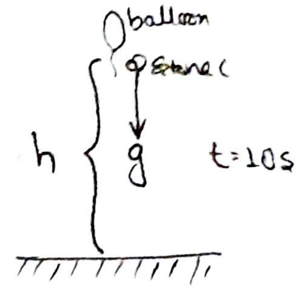
Section : C.S.E. 'K'

MECHANICS ASSIGNMENT

* Assignment - 14

Q.7.



Soln:- Given, $u = 10\,ms^{-1}$

$t = 10\,s$

Now,

$V = u + at$

or, $V = 10 - g \times 10 = 10 - 9.8 \times 10$

$\therefore V = -88\ ms^{-1}$

Again,

$v^2 = u^2 + 2as$

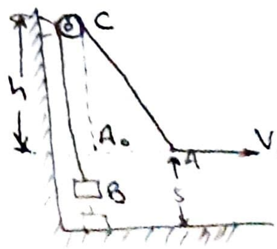
or, $\frac{v^2 - u^2}{2a} = s$

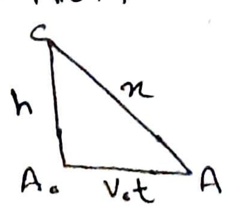
$\therefore s = \frac{(88)^2 - (10)^2}{2 \times (-9.8)} = -390\ m$

$\therefore h = 390\,m$

Q.8.

Soln:- a) Considering the displacements in ΔCA_oA





$x^2 = (V_o t)^2 + (h^2)$

$\therefore x = \sqrt{h^2 + V_o^2 t^2}$

Since, displacement in AC = displacement in block B.

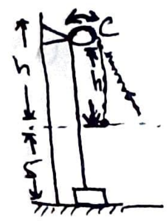
for block B, $x = \sqrt{v_0^2 t^2 + h^2}$

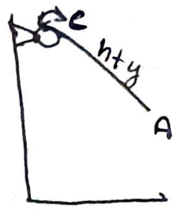
diff. w.r.t. t,

$$v = \frac{1}{2} \times \frac{2 v_0^2 t}{\sqrt{v_0^2 t^2 + h^2}}$$

$$\therefore v = \frac{v_0^2 t}{\sqrt{h^2 + v_0^2 t^2}}$$

b) Total length of string remains same,





Let, y is increase in length

So, $h + 5 + h = h + y$

$$\therefore y = 5 + h$$

from (a), $\sqrt{h^2 + v_0^2 t^2} - h = 5 + h$

for, $h = 15m$ and $v_0 = 10 m/s$

$$\sqrt{(15)^2 + (10)^2 t^2} - 15 = 5 + 15$$

or, $225 + 100t^2 = (35)^2$

or, $t^2 = \frac{1225 - 225}{100}$

$$\therefore t = 3.16 \ s$$

Q.9.

Soln: Given, $v = 10 ms^{-1}$; $u = 0$

$s = 25m$

$a = ?$

We have,

$$v^2 = u^2 + 2as$$

or, $a = \frac{v^2}{2s} = \frac{10^2}{2 \times 25} = \frac{100}{50}$

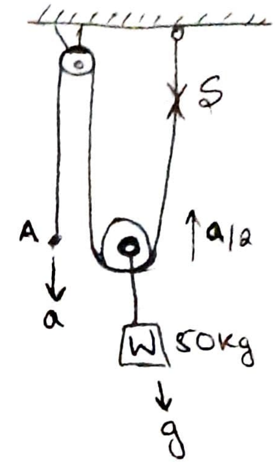
$\therefore a = \cancel{20}\ 2 \ ms^{-2}$

* Assignment - 15

Q. 7.

Soln.: We know,

$$2S - w = \frac{w}{g} \times \frac{a}{2}$$



We have, $w = 50g$ N

$a = 4$ m/s^2

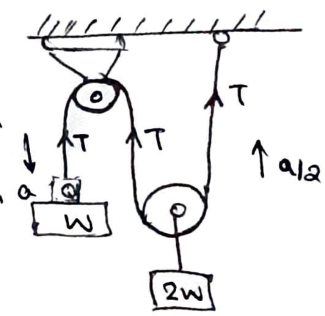
So,

$$2S - 50g = \frac{50g}{g} \times \frac{4}{2}$$

Or, $$S = \frac{100 + 50 \times 9.8}{2}$$

$\therefore$ $S = 295$ N

Q. 8.



Soln.: If the combined W and Q moves down with an acceleration a, the acceleration of the weight 2W will be $a' = -a/2$

Let, T be the ~~the~~ tension in string.

for the combined weights W and Q on the left,

$$\frac{W+Q}{g} \times a = (W+Q) - T \quad \rightarrow (i)$$

for the weight 2W,

$$\frac{2W}{g} \times \left(\frac{a}{2}\right) = 2T - 2W \quad \rightarrow (ii)$$

from (i) and (ii) (adding)

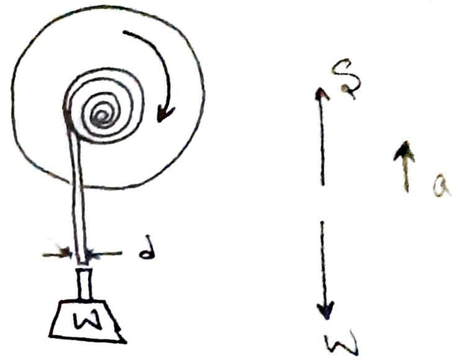
$$\frac{W+Q}{g} a + \frac{1}{2} \frac{Wa}{g} = Q$$

Or, $$Q = \frac{3}{2} \frac{Wa}{g-a} = \frac{3}{2} \times \frac{W \times 0.1g}{g - 0.1g}$$

$$\therefore Q = \frac{W}{6}$$

Q. 9.

Soln: Weight is raised vertically by winding the rope on a reel.



Reel is turned uniformly at the rate of n rps.

$$\omega = 2\pi n$$

for 1 rotation time taken $= \frac{1}{n}$ s

Now,

$$S - W = \frac{W}{g} a$$

Or, $$S = W\left(1 + \frac{a}{g}\right) \quad -(i)$$

Or $$a = \frac{V_2 - V_1}{\Delta t} = \frac{\omega(R_2 - R_1)}{1/n} = \frac{\omega d}{1/n}$$

$$\therefore a = 2\pi n^2 d \quad -(ii)$$

from (i) and (ii)

$$\therefore S = W\left(1 + \frac{2\pi n^2 d}{g}\right)$$

* Assignment - 16

Q.7.

Soln: Given, $s = 50m$

$\mu = 0.6$

$v = 0$

Here, $a = -\mu g$

we know,

$v^2 - u^2 = 2as$

or, $-u^2 = 2(-\mu g)s$

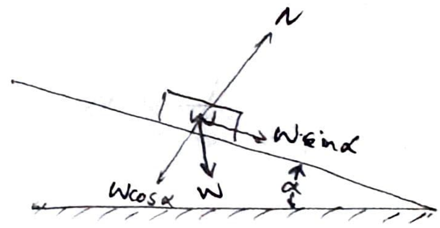
or, $u = \sqrt{2 \times 0.6 \times 9.8 \times 50}$

$\therefore u = 24.25 \ m/s$

Q.8.

Soln: for $\alpha = 30°$, friction is impeding

$\mu = \tan 30° = \frac{1}{\sqrt{3}}$



When $\alpha = 45°$,

$W\sin\alpha - f = \frac{W}{g} a$

or, $W\sin\alpha - \mu N = \frac{W}{g} a$ $\quad (N = W\cos\alpha)$

or, $\left(\sin 45° - \frac{1}{\sqrt{3}}\cos 45°\right) g = a$

or, $a = \frac{1}{\sqrt{2}}\left(1 - \frac{1}{\sqrt{3}}\right) g$

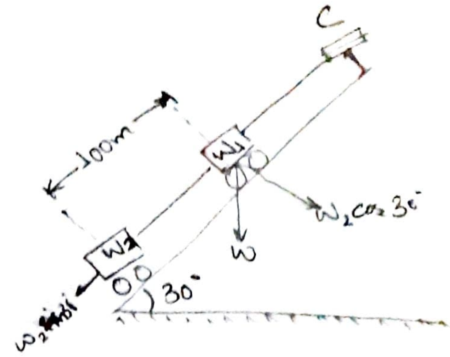
or, $a = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \times 9.8$

$\therefore a = 2.93 \ ms^{-2}$

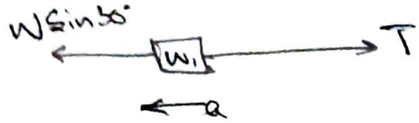
Q. 9.

Soln.:- Given, $W_1 = 200\,N$

$W_2 = 100\,N$



for block A,



$$W_1 \sin 30° - T = \frac{W_1}{g} a \quad \text{— (i)}$$

for block B

$$T - W_2 \sin 30° = \frac{W_2}{g} a \quad \text{— (ii)}$$

Adding (i) and (ii),

$$\frac{1}{2}(W_1 - W_2) = \frac{a}{g}(W_1 + W_2)$$

$$\Rightarrow \frac{1}{2}(200 - 100) = \frac{a}{9.81}(200 + 100)$$

$$\Rightarrow \frac{1}{6}(9.81) = a$$

$$\therefore a = 1.635\ m/s^2$$

for W_1 and W_2 to exchange positions,

$S = 100\,m$

or, $ut + \frac{1}{2}at^2 = 100$

or, $100 = \frac{1}{2} \times 1.635 t^2$

or $t^2 = \frac{200}{1.635}$

or, $t^2 = 122.32$

$\therefore t = 11.06\,s$

* Assignment - 17

Q. 7.

Solⁿ:- $X_1 = 10N - 5s$

$X_2 = 3N - t_1 s$

To bring the body to rest,

$X_1 \cdot 5 = X_2 \cdot t$

or, $10 \times 5 = 3 \times t_1$

$\therefore t_1 = \frac{50}{3} = 16.67\ s$

Now,

Displacement in 1^{st} 5s $= \frac{1}{2} \times \frac{10}{m} \times 5^2 = \frac{250}{2m}$

After 5s,

$v = \frac{10}{m} \times 5 = \frac{50}{m}$

So, $S_2 = \frac{50}{m} \times t_2 + \frac{1}{2}\left(\frac{-3}{m}\right)t_2^2$

$\therefore S_2 = \frac{50}{m} t_2 - \frac{3}{2m} t_2^2$

We have,

$S_1 = S_2$

or, $\frac{-125}{m} = \frac{50}{m} t_2 + \frac{1}{2}\left(\frac{-3}{m}\right)t_2^2$

or, $3t_2^2 - 100t_2 - 250 = 0$

$\therefore t_2 = 35.7\ s$

Q. 8.

Solⁿ:- Under action of force $X = X_0 - kt$,

a particle starts from rest at origin,

$x_0 = 0;\ y_0 = 0$

Now,

$a = \frac{X_0}{m} - \frac{k}{m} t$

Now,

$$v = \int a\, dt$$

$$= \frac{x_0}{m} t - \frac{k}{m}\frac{t^2}{2}$$

$$\therefore x = \frac{x_0}{m}\frac{t^2}{2} - \frac{k}{m}\frac{t^3}{6}$$

Alg,

$$x = 0$$

or $t^2\left(\frac{x_0}{2m} - \frac{k}{6m}t\right) = 0$

or $t = \frac{6x_0}{2k} = \frac{6\times 12}{2\times 2}$

$$\therefore t = 18\,s$$

Q.9.

Solⁿ:-

$$X = P\cos\omega t$$

$$\Rightarrow a = \frac{P}{W} g \cos\omega t$$

Integrating w.r.t. t,

$$\int_0^t a\, dt = \int_0^t \frac{Pg}{W}\cos\omega t\, dt$$

or $[v]_0^t = \frac{Pg}{W}\left[\frac{\sin\omega t}{\omega}\right]_0^t$

or $v - 0 = \frac{Pg}{W}\left[\frac{\sin\omega t}{\omega} - 0\right]$

$$\therefore x = \frac{Pg}{W\omega}\sin\omega t$$

Integration w.r.t. t,

$$\int_0^t x\, dt = \int_0^t \frac{Pg}{W\omega}\sin\omega t$$

or $(x - 0) = \frac{Pg}{W\omega^2}[-\cos\omega t]_0^t$

$$\therefore x = \frac{Pg}{W\omega^2}[1 - \cos\omega t]$$

* ASSIGNMENT-18

Q.7.

Soln:- Given that, initially there is 4N tension in each Spring in the previous problem 6.

So, the differential equation will be:

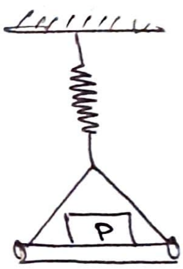
$$\frac{W}{g}\ddot{x} = -2Kx + 4 - 4$$

which remains same as earlier.

$\therefore$ There will be no change in T and x_{max}

Q.8.

Soln:- Let, weight of the Pan be W_0



Now,

$$T_0 = 2\pi\sqrt{\frac{W_0}{gk}} \quad -(i)$$

$$T_1 = 2\pi\sqrt{\frac{W_0+P}{gk}} \quad -(ii)$$

$$T_2 = 2\pi\sqrt{\frac{W_0+W}{gk}} \quad -(iii)$$

Dividing (i) and (ii),

$$\frac{T_0}{T_1} = \sqrt{\frac{W_0}{W_0+P}}$$

$$\text{or, } \left(\frac{T_0}{T_1}\right)^2 = \frac{W_0}{W_0+P}$$

$$\text{or, } (W_0+P)T_0^2 = T_1^2 W_0$$

$$\text{or, } PT_0^2 = (T_1^2 - T_0^2)W_0$$

$$\therefore W_0 = \frac{T_0^2}{T_1^2 - T_0^2} P \quad -(iv)$$

Now, Dividing (iii) and (ii),

$$\frac{T_2}{T_0} = \sqrt{\frac{W_0+W}{W_0}}$$

or $$\left(\frac{T_2}{T_0}\right)^2 = \frac{W_0+W}{W_0}$$

or, $$(T_2^2 - T_0^2)W_0 = T_0^2 W$$

or $$\frac{T_2^2 - T_0^2}{T_1^2 - T_0^2} \times T_0^2 P = T_0^2 W \quad \text{(from (iv))}$$

$$\therefore W = \frac{T_2^2 - T_0^2}{T_1^2 - T_0^2} P$$

Q.9.

Soln:- For the same arrangement as in Q.8,

$$T_1 = 2\pi\sqrt{\frac{W_0+P}{gk}} \quad \text{—(i)}$$

$$T_2 = 2\pi\sqrt{\frac{W_0+Q}{gk}} \quad \text{—(ii)}$$

Dividing (i) and (ii),

$$\frac{T_1}{T_2} = \sqrt{\frac{W_0+P}{W_0+Q}}$$

or, $$\frac{T_1^2}{T_2^2} = \frac{W_0+P}{W_0+Q}$$

or, $$(T_1^2 - T_2^2)W_0 = T_2^2 P - T_1^2 Q$$

$$\therefore W_0 = \frac{T_2^2 P - T_1^2 Q}{T_1^2 - T_2^2} \quad \text{— (iii)}$$

Squaring (i),

$$T_1^2 = 4\pi^2\left(\frac{W_0+P}{gk}\right)$$

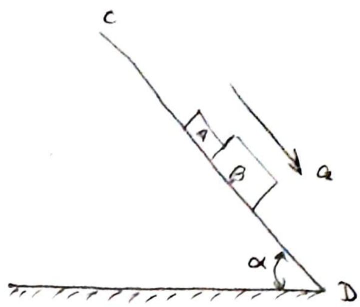
or, $$K = \frac{4\pi^2}{gT_1^2}\left(\frac{T_2^2 P - T_1^2 Q}{T_1^2 - T_2^2} + P\right) \quad \text{(from (iii))}$$

or, $$k = \frac{4\pi^2}{gT_1^2} \times \frac{(P-Q)T_1^2}{(T_1^2 - T_2^2)}$$

$$\therefore K = \frac{4\pi^2}{g} \times \frac{(P-Q)}{(T_1^2 - T_2^2)}$$

* Assignment - 19

Q.7.

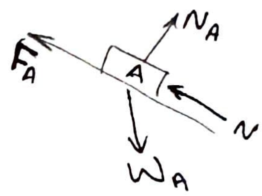


Soln: Both will have same acceleration 'a' because, both are moving together,

for 'A',

$$W_A \cos 60° - N - F_A = \frac{W_A}{g} \times a$$

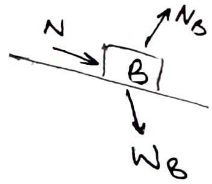
$$\therefore W_A \cos 60° - N - \mu_A W_A \sin 60° = \frac{W_A}{g} \times a \quad -(i)$$



for 'B',

$$W_B \cos 60° + N - F_B = \frac{W_B}{g} \times a$$

$$\text{or, } W_B \cos 60° + N - \mu_B W_B \sin 60° = \frac{W_B}{g} \times a \quad -(ii)$$



Dividing (i) and (ii),

$$\frac{W_A}{W_B} = \frac{W_A \cos 60° - N - \mu_A W_A \sin 60°}{W_B \cos 60° + N - \mu_B W_B \sin 60°}$$

$$\text{or, } N(W_A + W_B) = (\mu_B - \mu_A) W_A W_B \sin 60°$$

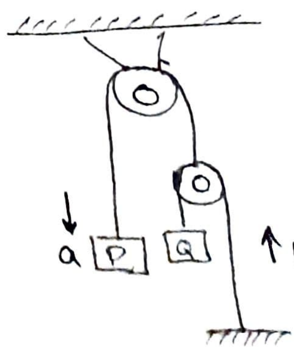
$$\text{or, } N = \frac{(0.3 - 0.15) \times 10 \times 20 \times \sqrt{3}}{2 \times 30}$$

$$\therefore N = 0.87\ N$$

$$\therefore \text{Pressure, } P = 0.87\ Pa$$

Q.8.

Soln: for Block P,



$$P - 2T = \frac{P}{g} \times a \quad -(i)$$

for Block Q,

$$T - Q = \frac{Q}{g} \times b \quad -(ii)$$

from constrained motion,

$$(a - b) + a = 0$$

$$\therefore b = 2a \quad -(iii)$$

Given, $P = Q$ — (iv)

Using (i), (ii), (iii) and (iv),

$$P - 2QP - \frac{2P}{g} b = \frac{Pb}{g \times 2}$$

$$\text{or, } -1 - \frac{2}{g} \times b = \frac{b}{2g}$$

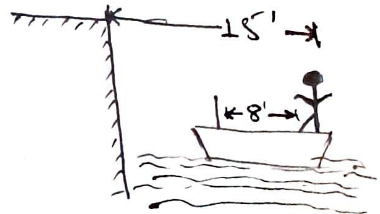
$$\therefore b = \frac{-4g}{5}$$

from (i), $\therefore a = \frac{b}{2} = \frac{-4g}{2 \times 5} = \left|\frac{-2g}{5}\right| = \frac{2g}{5}$

$\therefore$ Acceleration of block Q, $a = \frac{2g}{5}$

Q.9.

Solution: As ~~the~~ no external force is acting on the system, the center of mass will not displace.



So, Change in moment $= 0$

$$160 \times 8 = (200 + 160)x$$

$$\therefore x = \frac{32}{9} \text{ ft}$$

Distance moved $= 8 - \frac{32}{9} = \frac{40}{9}$ ft

So, distance moved by the man from the shore

$$= 15 - \frac{40}{9}$$

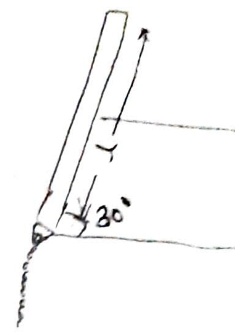
$$= \frac{95}{9} \text{ ft}$$

$$= 10.56 \text{ ft}$$

* Assignment - 20

Q. 7.

Soln: From law of Conservation of energy.



$$mgh_1 = \frac{1}{2}mv^2 - mgh_2$$

$$\text{And, } h_1 = \frac{l \sin 30^\circ}{2}$$

$$h_2 = l \sin 30^\circ$$

$$\text{Or, } \frac{1}{2}mv^2 = mg \sin 30^\circ \left(\frac{l}{2} + l\right)$$

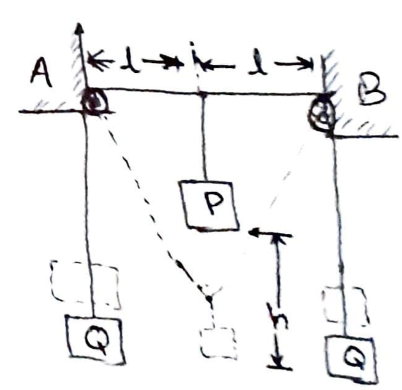
$$\text{Or } \frac{1}{2}v^2 = \frac{g}{2}\left(\frac{l}{2} + l\right)$$

$$\text{Or } v^2 = \frac{3l}{2}g$$

$$\therefore v = \sqrt{\frac{3lg}{2}}$$

Q. 8.

Soln: Using work-energy theorem,



$$0 = Ph - 2Q\left(\sqrt{l^2 + h^2} - l\right) + \frac{1}{2} \times \frac{P}{g} V_P^2 + \frac{Q}{g} V_Q^2$$

for h to be maximum,

$$4Q^2 l^2 + 4Q^2 h^2 = 4Q^2 l^2 + P^2 h^2 + 4QPlh$$

$$\text{Or, } (4Q^2 - P^2)h^2 - 4QPlh = 0$$

$$\therefore h = \frac{4QPl}{(4Q^2 - P^2)}$$

Q.9.

Sol^n:- There are only internal forces,

$\therefore$ Momentum is conserved.

So,

$$m_M (V_m)_2 + m_B (V_B)_2 = m_m (V_m)_1$$

$$(V_m)_1 = 10 \text{ fps}$$

$$m_M = 150 \text{ lb}$$

$$m_B = 200 \text{ lb}$$

$(V_M)_2 = (V_B)_2 = V$ = final speed will be same

or, $150 \times V + 200V = 150 \times 10$

$\therefore V = 4.3$ fps

* Assignment - 21

Q. 7.

Soln:- Lateral thrust $= \frac{WV^2}{gR}$

$V = 45$ mph $= 66$ ft/s

$R = 1000$ ft

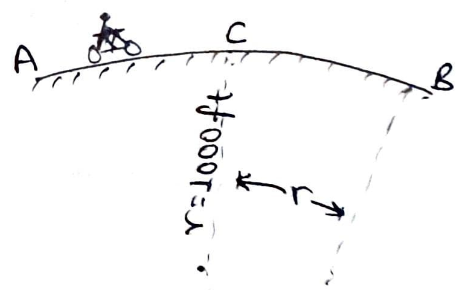
$g = 32.17$ ft/s^2

$W = 60$ tonns $= 120000$ lb

$\therefore$ Lateral thrust $= \frac{120000 \times (66)^2}{32.17 \times 1000} = 16,250$ lb

Q. 8.

Soln: At point C,



$W = N + \frac{WV^2}{gR}$

Given, $W = 500$ lb

$V = 45$ mph $= 66$ ft/s

$R = 1000$ ft

$g = 32.17$ ft/s^2

So, $N = 500 - \frac{500 \times (66)^2}{32.17 \times 1000}$

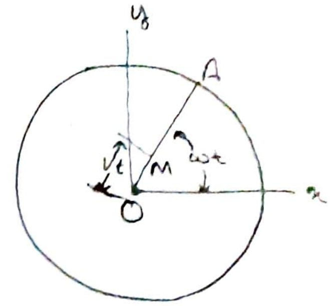
$\therefore N = 432$ lb

Q.9.

Sol^n:

$x = vt\cos\omega t$

$y = vt\sin\omega t$



acceleration in x-direction:

$$\frac{dx}{dt} = V\cos\omega t - \frac{Vt}{\omega}\sin\omega t$$

$$\frac{d^2x}{dt^2} = \frac{-V}{\omega}\sin\omega t - \frac{V}{\omega}\sin\omega t - \frac{V}{\omega^2}t\cos\omega t$$

Here, $t = \frac{2\pi}{\omega}$

$$\therefore \frac{d^2x}{dt^2} = -6\pi$$

Acceleration in y-direction,

$$\frac{dy}{dt} = V\sin\omega t + \frac{V}{\omega}t\cos\omega t$$

$$\frac{d^2y}{dt} = \frac{V}{\omega}\cos\omega t + \frac{V}{\omega}\cos\omega t - \frac{V}{\omega^2}t\sin\omega t$$

At $t = \frac{2\pi}{\omega}$

$$\frac{d^2y}{dt^2} = 6$$

So, Total Acceleration, $a = \sqrt{\left(\frac{d^2y}{dt^2}\right)^2 + \left(\frac{d^2x}{dt^2}\right)^2}$

$= \sqrt{36 + 36\pi^2}$

$\therefore a = 19.8 \ ft/s^2$

$\therefore$ Total friction, $F = ma = \frac{160}{32.2} \times 19.8 = 98.5$ lb

* ASSIGNMENT - 22

Q.7.

Soln:- Using equation of trajectory:

$$y = x\tan\theta - \frac{gx^2}{2v^2} \times \sec^2\theta$$

Here, $y = 0$

$x = 15000\,ft$

$v = 1000\,fps$

So,
$$0 = 15000\tan\theta - \frac{(32.17)(15000)(15000)}{2(1000)(1000)}(1+\tan^2\theta)$$

or, $3619.125\tan^2\theta - 15000\tan\theta + 3619.125 = 0$

or, $\tan\theta = 3.548$ and 0.282

$\therefore \theta = 74.26°$ or $15.75°$

$$t_2 = \frac{2V_0\sin\theta_2}{g} = \frac{2000\times\sin(74.26°)}{32.17} = 59.84\,s$$

$$t_1 = \frac{2V_0\sin\theta_1}{g} = \frac{2000\times\sin(15.75°)}{32.17} = 15.34\,s.$$

$\therefore t_2 - t_1 = 44.5\,s$

Q.8.

Soln:- Using third equation of motion,

$$v^2 = u^2 + 2as$$

Here, $u = 0$

$$a = \frac{w}{m}(\cos 60° - \mu\sin 60°) = 32.2(0.5 - 0.2\times0.87)$$

$\therefore a = 10.5\,ft/s^2$

$s = 20ft$

$$\therefore V = \sqrt{2\times10.5\times20} = 20.5\,ft\,s^{-1}$$

Using second eqn. of motion,

for x-direction, $s = ut + \frac{1}{2}at^2$

Here, $s = 15$ ft

$u = 205 \times \sin 30^\circ$ ft/s

$a = 32.2$ ft/s^2

$\therefore t = 0.7$ s

for y-direction,

$x - 2 = v \cos 30^\circ t$

~~Here~~, $v = 20.5$ ft/s, $t = 0.7$ s

$\therefore x = 14.4$ ft

$\therefore$ Distance x to the point D is 14.4 ft.

Q.9

Soln:- for x-direction,

$10 = V_0 \cos 15 t$

or $t = \dfrac{10}{V_0 \cos 15}$

$\therefore t = \dfrac{10.352}{V_0}$ — (i)

for y-direction,

$y = vt + \frac{1}{2}at^2$

Here, $y = -5$ ft

$v = V_0 \sin 15^\circ$

$a = -32.2$ ft/s^2

So, $-5 = V_0 \sin 15^\circ \times \dfrac{10.352}{V_0} - \dfrac{1}{2} \times 32.2 \times \left(\dfrac{10.352}{V_0}\right)^2$

or, $V_0^2 = \dfrac{16.1 \times (10.352)^2}{10.352 \times \sin 15^\circ + 5}$

or, $V_0 = \sqrt{224.67}$

$\therefore V_0 = 15$ fps