UG Mid Sem
Semester : V^th
Date : 29/10/2021

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Branch : CSE - B
Subject : Graph Theory
Subject Code : CS305

Q. 1.
Ans

a) → Given, Line Graph L(G)
Vertices are one-one correspondence with edges of simple graph, G.
Two vertices of L(G) are adjacent if and only if the corresponding edges of G are adjacent.
So,
If G is a regular graph of degree k,
Then, every vertex is incident to k edges.

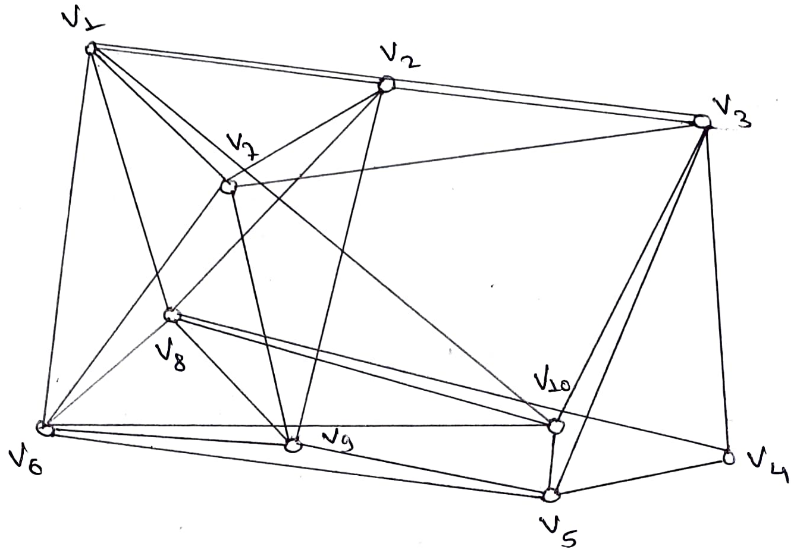
Every edge is incident to 2 vertices
At one vertex, the edge will be ~~xxxxxxx~~ adjacent to k-1 other edges,

At other vertex, the edge will be adjacent to k-1 other vertices.

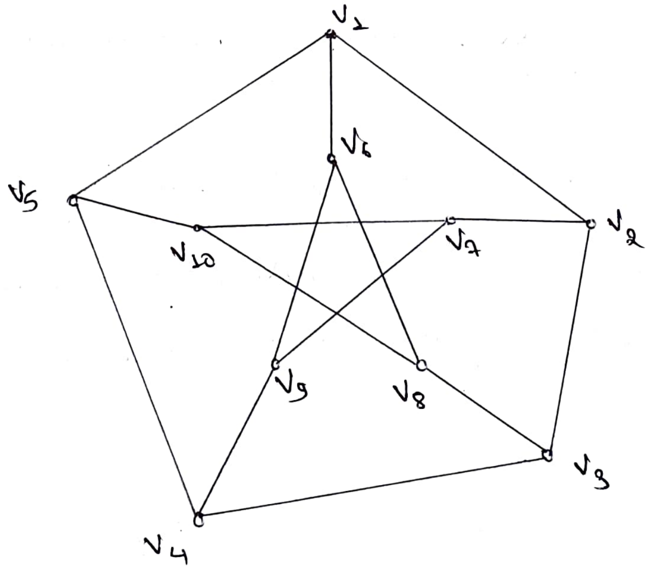
∴ L(G) will have each vertex adjacent to 2k-2 other vertices, and is regular of dgree 2k-2.

Q.1.
Ans

b) → Line Graph of K_5



Complement Graph, $\overline{L(K_5)}$



Q. 3.

Ans. To Show: $G \times H$ is Eulerian if and only if all vertices G and H have even degree or all vertices of G and H have odd degree.

Here, Any vertex (x,y) of $G \times H$ has degree $d_{G\times H}(x,y)$, such that,

$$d_{G\times H}(x,y) = d_G(x) + d_H(y)$$

i.e, every vertex of G and H has even degree

OR, if every vertex of G and H has odd degree, then, every verted $G \times H$ has even degree.

This shows that $G \times H$ is Eulerian.

Conversely, Suppose $G \times H$ is Eulerian.

So, Each ~~vertices~~ vertex (x,y) has even degree.

i.e, $d_{G\times H}(x,y)$ is even $\forall\ x \in V(G)$ and $y \in V(H)$

Let,

vertex $h \in V(H)$

So, $\deg_{G\times H}(x,h) = d_G(x) + d_H(h)$

Then, $d_{G\times H}(x) = d_{G\times H}(x,h) - d_H(h)$, $\forall\ x \in V(G)$

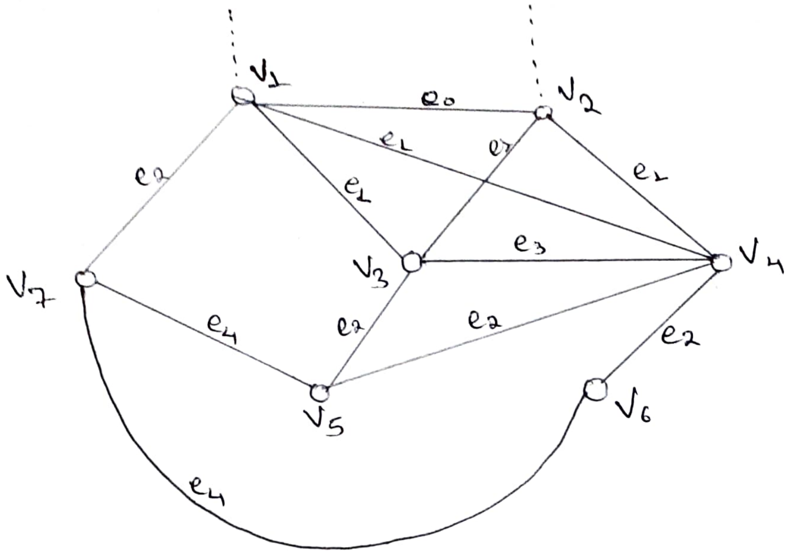
From this, we infer that the degree of every vertex of G has the same parity as $d_H(h)$.

$$d_H(y) = d_{G\times H}(g,y) - d_G(g) \quad \forall\ y \in V(H)$$

Hence, $G \times H$ is Eularian.

Q.4.

Ans



INDEX:

$V_1 \rightarrow$ Grandfather

$V_2 \rightarrow$ Grandmother

$V_7 \rightarrow$ Friends

$V_3 \rightarrow$ Parent/child

$V_4 \rightarrow$ Parent/Child

$V_5 \rightarrow$ Friends

$V_6 \rightarrow$ Friends.

$e_0 \rightarrow$ Husband/Wife

$e_1 \rightarrow$ Parent/child

$e_2 \rightarrow$ friends

$e_3 \rightarrow$ Brother/Sister

$e_4 \rightarrow$ Guardian.