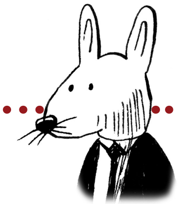
introduction to neural learning: gradient descent 4



In this chapter

- Do neural networks make accurate predictions?
- Why measure error?
- Hot and cold learning
- Calculating both direction and amount from error
- Gradient descent
- Learning is just reducing error
- Derivatives and how to use them to learn
- Divergence and alpha

> "The only relevant test of the validity of a hypothesis is comparison of its predictions with experience."
>
> —Milton Friedman, *Essays in Positive Economics* (University of Chicago Press, 1953)

Predict, compare, and learn

In chapter 3, you learned about the paradigm "predict, compare, learn," and we dove deep into the first step: *predict*. In the process, you learned a myriad of things, including the major parts of neural networks (nodes and weights), how datasets fit into networks (matching the number of datapoints coming in at one time), and how to use a neural network to make a prediction.

Perhaps this process begged the question, "How do we set weight values so the network predicts accurately?" Answering this question is the main focus of this chapter, as we cover the next two steps of the paradigm: *compare* and *learn*.

Compare

Comparing gives a measurement of how much a prediction "missed" by.

Once you've made a prediction, the next step is to evaluate how well you did. This may seem like a simple concept, but you'll find that coming up with a good way to measure error is one of the most important and complicated subjects of deep learning.

There are many properties of measuring error that you've likely been doing your whole life without realizing it. Perhaps you (or someone you know) amplify bigger errors while ignoring very small ones. In this chapter, you'll learn how to mathematically teach a network to do this. You'll also learn that error is always positive! We'll consider the analogy of an archer hitting a target: whether the shot is too low by an inch or too high by an inch, the error is still just 1 inch. In the neural network *compare* step, you need to consider these kinds of properties when measuring error.

As a heads-up, in this chapter we evaluate only one simple way of measuring error: *mean squared error*. It's but one of many ways to evaluate the accuracy of a neural network.

This step will give you a sense for how much you missed, but that isn't enough to be able to learn. The output of the *compare* logic is a "hot or cold" type signal. Given some prediction, you'll calculate an error measure that says either "a lot" or "a little." It won't tell you why you missed, what direction you missed, or what you should do to fix the error. It more or less says "big miss," "little miss," or "perfect prediction." What to do about the error is captured in the next step, *learn*.

Learn

Learning tells each weight how it can change to reduce the error.

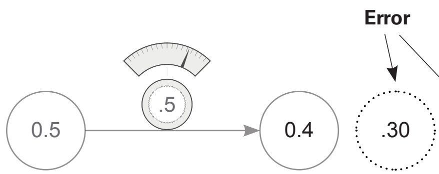
Learning is all about *error attribution*, or the art of figuring out how each weight played its part in creating error. It's the blame game of deep learning. In this chapter, we'll spend many pages looking at the most popular version of the deep learning blame game: *gradient descent*.

At the end of the day, it results in computing a number for each weight. That number represents how that weight should be higher or lower in order to reduce the error. Then you'll move the weight according to that number, and you'll be finished.

Compare: Does your network make good predictions?

Let's measure the error and find out!

Execute the following code in your Jupyter notebook. It should print `0.3025`:




```
knob_weight = 0.5
input = 0.5
goal_pred = 0.8

pred = input * knob_weight

error = (pred - goal_pred) ** 2

print(error)
```

Raw error

Forces the raw error to be positive by multiplying it by itself. Negative error wouldn't make sense.

The error is a way to measure how much you missed. There are multiple ways to calculate error, as you'll learn later. This one is *mean squared error*.

What is the goal_pred variable?

Much like `input`, `goal_pred` is a number you recorded in the real world somewhere. But it's usually something hard to observe, like "the percentage of people who *did* wear sweatsuits," given the temperature; or "whether the batter *did* hit a home run," given his batting average.

Why is the error squared?

Think about an archer hitting a target. When the shot hits 2 inches too high, how much did the archer miss by? When the shot hits 2 inches too low, how much did the archer miss by? Both times, the archer missed by only 2 inches. The primary reason to *square* "how much you missed" is that it forces the output to be *positive*. `(pred - goal_pred)` could be negative in some situations, *unlike actual error*.

Doesn't squaring make big errors (>1) bigger and small errors (<1) smaller?

Yeah … It's kind of a weird way of measuring error, but it turns out that *amplifying* big errors and *reducing* small errors is OK. Later, you'll use this error to help the network learn, and you'd rather it *pay attention* to the big errors and not worry so much about the small ones. Good parents are like this, too: they practically ignore errors if they're small enough (breaking the lead on your pencil) but may go nuclear for big errors (crashing the car). See why squaring is valuable?

Why measure error?

Measuring error simplifies the problem.

The goal of training a neural network is to make correct predictions. That's what you want. And in the most pragmatic world (as mentioned in the preceding chapter), you want the network to take input that you can easily calculate (today's stock price) and predict things that are hard to calculate (tomorrow's stock price). That's what makes a neural network useful.

It turns out that changing `knob_weight` to make the network correctly predict `goal_prediction` is *slightly* more complicated than changing `knob_weight` to make `error == 0`. There's something more concise about looking at the problem this way. Ultimately, both statements say the same thing, but trying to *get the error to 0* seems more straightforward.

Different ways of measuring error *prioritize error differently.*

If this is a bit of a stretch right now, that's OK, but think back to what I said earlier: by *squaring* the error, numbers that are less than 1 get *smaller*, whereas numbers that are greater than 1 get *bigger*. You're going to change what I call *pure error* (`pred - goal_pred`) so that bigger errors become *very* big and smaller errors quickly become irrelevant.

By measuring error this way, you can *prioritize* big errors over smaller ones. When you have somewhat large pure errors (say, 10), you'll tell yourself that you have *very* large error (10**2 == 100); and in contrast, when you have small pure errors (say, 0.01), you'll tell yourself that you have *very* small error (0.01**2 == 0.0001). See what I mean about prioritizing? It's just modifying what you *consider to be error* so that you amplify big ones and largely ignore small ones.

In contrast, if you took the *absolute value* instead of squaring the error, you wouldn't have this type of prioritization. The error would just be the positive version of the pure error—which would be fine, but different. More on this later.

Why do you want only *positive* error?

Eventually, you'll be working with millions of `input -> goal_prediction` pairs, and we'll still want to make accurate predictions. So, you'll try to take the *average error* down to 0.

This presents a problem if the error can be positive and negative. Imagine if you were trying to get the neural network to correctly predict two datapoints—two `input -> goal_prediction` pairs. If the first had an error of 1,000 and the second had an error of –1,000, then the *average error* would be *zero*! You'd fool yourself into thinking you predicted perfectly, when you missed by 1,000 each time! That would be really bad. Thus, you want the error of *each prediction* to always be *positive* so they don't accidentally cancel each other out when you average them.

What's the simplest form of neural learning?

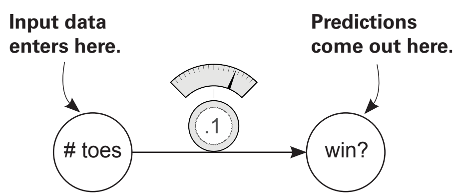
Learning using the hot and cold method.

At the end of the day, learning is really about one thing: adjusting `knob_weight` either up or down so the error is reduced. If you keep doing this and the error goes to 0, you're done learning! How do you know whether to turn the knob up or down? Well, you try *both up and down* and see which one reduces the error! Whichever one reduces the error is used to update `knob_weight`. It's simple but effective. After you do this over and over again, eventually error == 0, which means the neural network is predicting with perfect accuracy.

Hot and cold learning

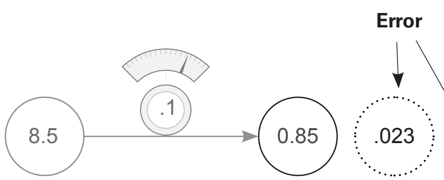
Hot and cold learning means wiggling the weights to see which direction reduces the error the most, moving the weights in that direction, and repeating until the error gets to 0.

1 An empty network




```
weight = 0.1

lr = 0.01

def neural_network(input, weight):

    prediction = input * weight

    return prediction
```

2 PREDICT: Making a prediction and evaluating error




```
number_of_toes = [8.5]
win_or_lose_binary = [1] # (won!!!)

input = number_of_toes[0]
true = win_or_lose_binary[0]

pred = neural_network(input,weight)

error = (pred - true) ** 2
print(error)
```

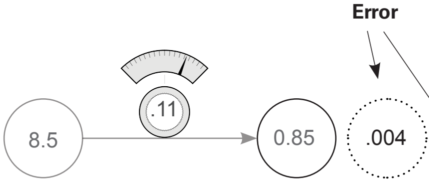
The error is a way to measure how much you missed. There are multiple ways to calculate error, as you'll learn later. This one is *mean squared error*.

Raw error

Forces the raw error to be positive by multiplying it by itself. Negative error wouldn't make sense.

❸ COMPARE: Making a prediction with a higher weight and evaluating error

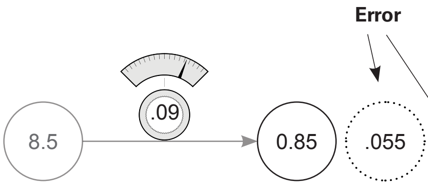
We want to move the weight so the error goes downward. Let's try moving the weight up and down using `weight+lr` and `weight-lr`, to see which one has the lowest error.



Higher

```
lr = 0.1

p_up = neural_network(input,weight+lr)

e_up = (p_up - true) ** 2
print(e_up)
```

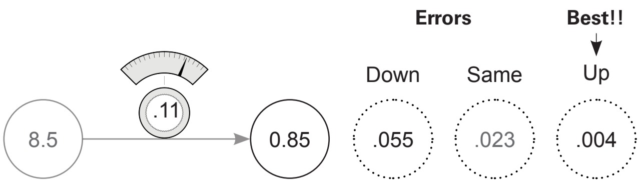
❹ COMPARE: Making a prediction with a lower weight and evaluating error



Lower

```
lr = 0.01

p_dn = neural_network(input,weight-lr)

e_dn = (p_dn - true) ** 2
print(e_dn)
```

❺ COMPARE + LEARN: Comparing the errors and setting the new weight




```
if(error > e_dn ||
  error > e_up):

   if(e_dn < e_up):
      weight -= lr

   if(e_up < e_up):
      weight += lr
```

These last five steps are one iteration of hot and cold learning. Fortunately, this iteration got us pretty close to the correct answer all by itself (the new error is only 0.004). But under normal circumstances, we'd have to repeat this process many times to find the correct weights. Some people have to train their networks for weeks or months before they find a good enough weight configuration.

This reveals what learning in neural networks really is: a *search problem*. You're *searching* for the best possible configuration of weights so the network's error falls to 0 (and predicts perfectly). As with all other forms of search, you might not find exactly what you're looking for, and even if you do, it may take some time. Next, we'll use hot and cold learning for a slightly more difficult prediction so you can see this searching in action!

Hot and cold learning

This is perhaps the simplest form of learning.

Execute the following code in your Jupyter notebook. (New neural network modifications are in **bold**.) This code attempts to correctly predict 0.8:

```
weight = 0.5
input = 0.5
goal_prediction = 0.8

step_amount = 0.001          # How much to move the weights each iteration

for iteration in range(1101):    # Repeat learning many times so the error can keep getting smaller.

    prediction = input * weight
    error = (prediction - goal_prediction) ** 2

    print("Error:" + str(error) + " Prediction:" + str(prediction))

    up_prediction = input * (weight + step_amount)     # Try up!
    up_error = (goal_prediction - up_prediction) ** 2

    down_prediction = input * (weight - step_amount)   # Try down!
    down_error = (goal_prediction - down_prediction) ** 2

    if(down_error < up_error):
        weight = weight - step_amount    # If down is better, go down!

    if(down_error > up_error):
        weight = weight + step_amount    # If up is better, go up!
```

When I run this code, I see the following output:

```
Error:0.3025 Prediction:0.25
Error:0.30195025 Prediction:0.2505
          ....
Error:2.50000000033e-07 Prediction:0.7995
Error:1.07995057925e-27 Prediction:0.8     # The last step correctly predicts 0.8!
```

Characteristics of hot and cold learning

It's simple.

Hot and cold learning is simple. After making a prediction, you predict two more times, once with a slightly higher weight and again with a slightly lower weight. You then move `weight` depending on which direction gave a smaller `error`. Repeating this enough times eventually reduces `error` to 0.

> **Why did I iterate exactly 1,101 times?**
>
> The neural network in the example reaches 0.8 after exactly that many iterations. If you go past that, it wiggles back and forth between 0.8 and just above or below 0.8, making for a less pretty error log printed at the bottom of the left page. Feel free to try it.

Problem 1: It's inefficient.

You have to predict *multiple times* to make a single `knob_weight` update. This seems very inefficient.

Problem 2: Sometimes it's impossible to predict the exact goal prediction.

With a set `step_amount`, unless the perfect `weight` is exactly `n*step_amount` away, the network will eventually overshoot by some number less than `step_amount`. When it does, it will then start alternating back and forth between each side of `goal_prediction`. Set `step_amount` to 0.2 to see this in action. If you set `step_amount` to 10, you'll really break it. When I try this, I see the following output. It never remotely comes close to 0.8!

```
Error:0.3025 Prediction:0.25
Error:19.8025 Prediction:5.25
Error:0.3025 Prediction:0.25
Error:19.8025 Prediction:5.25
Error:0.3025 Prediction:0.25
....
.... repeating infinitely...
```

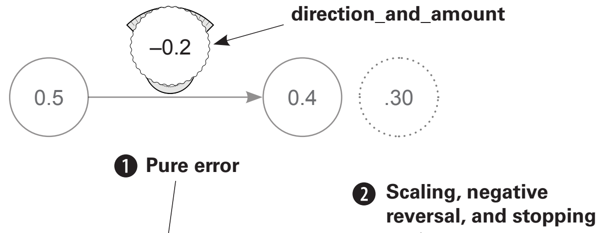
The real problem is that even though you know the correct *direction* to move `weight`, you don't know the correct *amount*. Instead, you pick a fixed one at random (`step_amount`). Furthermore, this amount has *nothing* to do with `error`. Whether `error` is big or tiny, `step_amount` is the same. So, hot and cold learning is kind of a bummer. It's inefficient because you predict three times for each `weight` update, and `step_ amount` is arbitrary, which can prevent you from learning the correct `weight` value.

What if you had a way to compute both direction and amount for each `weight` without having to repeatedly make predictions?

Calculating both direction and amount from error

Let's measure the error and find the direction and amount!

Execute this code in your Jupyter notebook:




```
weight = 0.5
goal_pred = 0.8
input = 0.5

for iteration in range(20):
    pred = input * weight
    error = (pred - goal_pred) ** 2
    direction_and_amount = (pred - goal_pred) * input
    weight = weight - direction_and_amount

    print("Error:" + str(error) + " Prediction:" + str(pred))
```

What you see here is a superior form of learning known as *gradient descent*. This method allows you to (in a single line of code, shown here in **bold**) calculate both the *direction* and the *amount* you should change `weight` to reduce `error`.

What is direction_and_amount?

`direction_and_amount` represents how you want to change `weight`. The first part ❶ is what I call *pure error*, which equals (`pred - goal_pred`). (More about this shortly.) The second part ❷ is the multiplication by the `input` that performs scaling, negative reversal, and stopping, modifying the pure error so it's ready to update `weight`.

What is the pure error?

The pure error is (`pred - goal_pred`), which indicates the raw direction and amount you missed. If this is a *positive* number, you predicted too *high*, and vice versa. If this is a *big* number, you missed by a *big* amount, and so on.

What are scaling, negative reversal, and stopping?

These three attributes have the combined effect of translating the pure error into the absolute amount you want to change `weight`. They do so by addressing three major edge cases where the pure error isn't sufficient to make a good modification to `weight`.

What is stopping?

Stopping is the first (and simplest) effect on the pure error caused by multiplying it by `input`. Imagine plugging a CD player into your stereo. If you turned the volume all the way up but the CD player was off, the volume change wouldn't matter. Stopping addresses this in a neural network. If `input` is 0, then it will force `direction_and_amount` to also be 0. You don't learn (change the volume) when `input` is 0, because there's nothing to learn. Every `weight` value has the same `error`, and moving it makes no difference because `pred` is always 0.

What is negative reversal?

This is probably the most difficult and important effect. Normally (when `input` is positive), moving `weight` upward makes the prediction move upward. But if `input` is negative, then all of a sudden `weight` changes directions! When `input` is negative, moving `weight` *up* makes the prediction go *down*. It's reversed! How do you address this? Well, multiplying the pure error by `input` will *reverse the sign* of `direction_and_amount` in the event that `input` is negative. This is *negative reversal*, ensuring that `weight` moves in the correct direction even if `input` is negative.

What is scaling?

Scaling is the third effect on the pure error caused by multiplying it by `input`. Logically, if `input` is big, your `weight` update should also be big. This is more of a side effect, because it often goes out of control. Later, you'll use *alpha* to address when that happens.

When you run the previous code, you should see the following output:

```
Error:0.3025 Prediction:0.25
Error:0.17015625 Prediction:0.3875
Error:0.095712890625 Prediction:0.490625
                    ...

Error:1.7092608064e-05 Prediction:0.79586567925
Error:9.61459203602e-06 Prediction:0.796899259437
Error:5.40820802026e-06 Prediction:0.797674444578
```

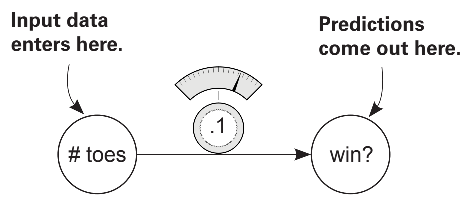
The last steps correctly approach 0.8!

In this example, you saw gradient descent in action in a bit of an oversimplified environment. Next, you'll see it in its more native environment. Some terminology will be different, but I'll code it in a way that makes it more obviously applicable to other kinds of networks (such as those with multiple inputs and outputs).

One iteration of gradient descent

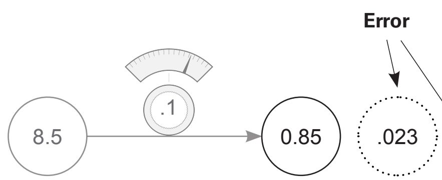
This performs a weight update on a single training example (input->true) pair.

❶ An empty network




```
weight = 0.1

alpha = 0.01

def neural_network(input, weight):

    prediction = input * weight

    return prediction
```

❷ PREDICT: Making a prediction and evaluating error



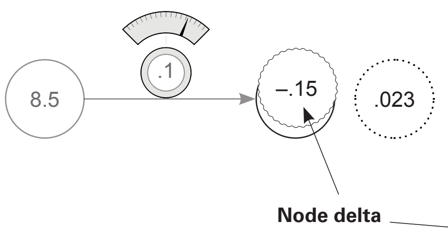
The error is a way to measure how much you missed. There are multiple ways to calculate error, as you'll learn later. This one is *mean squared error*.

```
number_of_toes = [8.5]
win_or_lose_binary = [1] # (won!!!)

input = number_of_toes[0]
goal_pred = win_or_lose_binary[0]

pred = neural_network(input,weight)

error = (pred - goal_pred) ** 2
```

Raw error

Forces the raw error to be positive by multiplying it by itself. Negative error wouldn't make sense.

❸ COMPARE: Calculating the node delta and putting it on the output node

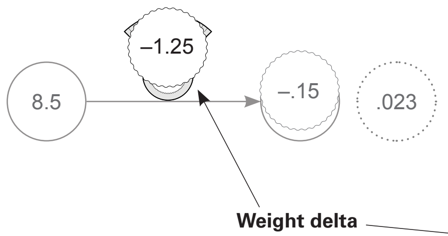



```
number_of_toes = [8.5]
win_or_lose_binary = [1] # (won!!!)

input = number_of_toes[0]
goal_pred = win_or_lose_binary[0]

pred = neural_network(input,weight)

error = (pred - goal_pred) ** 2

delta = pred - goal_pred
```

`delta` is a measurement of how much this node missed. The true prediction is 1.0, and the network's prediction was 0.85, so the network was too *low* by 0.15. Thus, `delta` is *negative* 0.15.

The primary difference between gradient descent and this implementation is the new variable `delta`. It's the raw amount that the node was too high or too low. Instead of computing `direction_and_amount` directly, you first calculate how much you want the output node to be different. Only then do you compute `direction_and_amount` to change `weight` (in step 4, now renamed `weight_delta`):

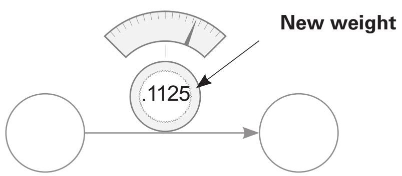
❹ LEARN: Calculating the weight delta and putting it on the weight




```
number_of_toes = [8.5]
win_or_lose_binary = [1] # (won!!!)

input = number_of_toes[0]
goal_pred = win_or_lose_binary[0]

pred = neural_network(input,weight)

error = (pred - goal_pred) ** 2

delta = pred - goal_pred

weight_delta = input * delta
```

`weight_delta` is a measure of how much a weight caused the network to miss. You calculate it by multiplying the weight's output node `delta` by the weight's `input`. Thus, you create each `weight_delta` by *scaling* its output node `delta` by the weight's `input`. This accounts for the three aforementioned properties of `direction_and_amount`: scaling, negative reversal, and stopping.

❺ LEARN: Updating the weight




```
number_of_toes = [8.5]
win_or_lose_binary = [1] # (won!!!)

input = number_of_toes[0]
goal_pred = win_or_lose_binary[0]

pred = neural_network(input,weight)

error = (pred - goal_pred) ** 2

delta = pred - goal_pred

weight_delta = input * delta
```

Fixed before training

```
alpha = 0.01

weight -= weight_delta * alpha
```

You multiply `weight_delta` by a small number `alpha` before using it to update `weight`. This lets you control how fast the network learns. If it learns too fast, it can update weights too aggressively and overshoot. (More on this later.) Note that the weight update made the same change (small increase) as hot and cold learning.

Learning is just reducing error

You can modify weight to reduce error.

Putting together the code from the previous pages, we now have the following:

```
weight, goal_pred, input = (0.0, 0.8, 0.5)

for iteration in range(4):

    pred = input * weight
    error = (pred - goal_pred) ** 2
    delta = pred - goal_pred
    weight_delta = delta * input
    weight = weight - weight_delta
    print("Error:" + str(error) + " Prediction:" + str(pred))
```

These lines have a secret.

> **The golden method for learning**
>
> This approach adjusts each `weight` in the correct direction and by the correct amount so that `error` reduces to 0.

All you're trying to do is figure out the right direction and amount to modify `weight` so that `error` goes down. The secret lies in the `pred` and `error` calculations. Notice that you use `pred` *inside* the `error` calculation. Let's replace the `pred` variable with the code used to generate it:

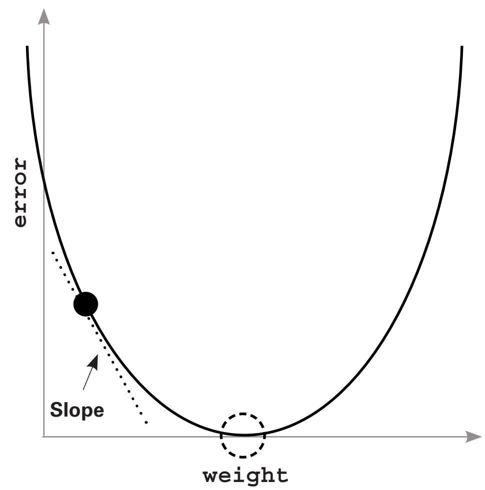
```
error = ((input * weight) - goal_pred) ** 2
```

This doesn't change the value of `error` at all! It just combines the two lines of code and computes `error` directly. Remember that `input` and `goal_prediction` are fixed at 0.5 and 0.8, respectively (you set them before the network starts training). So, if you replace their variables names with the values, the secret becomes clear:

```
error = ((0.5 * weight) - 0.8) ** 2
```

> **The secret**
>
> For any `input` and `goal_pred`, an *exact relationship* is defined between `error` and `weight`, found by combining the `prediction` and `error` formulas. In this case:
>
> ```
> error = ((0.5 * weight) - 0.8) ** 2
> ```

Let's say you increased `weight` by 0.5. If there's an exact relationship between `error` and `weight`, you should be able to calculate how much this also moves `error`. What if you wanted to move `error` in a specific direction? Could it be done?



This graph represents every value of error for every weight according to the relationship in the previous formula. Notice it makes a nice bowl shape. The black dot is at the point of *both* the current `weight` and `error`. The dotted circle is where you want to be (`error` == 0).

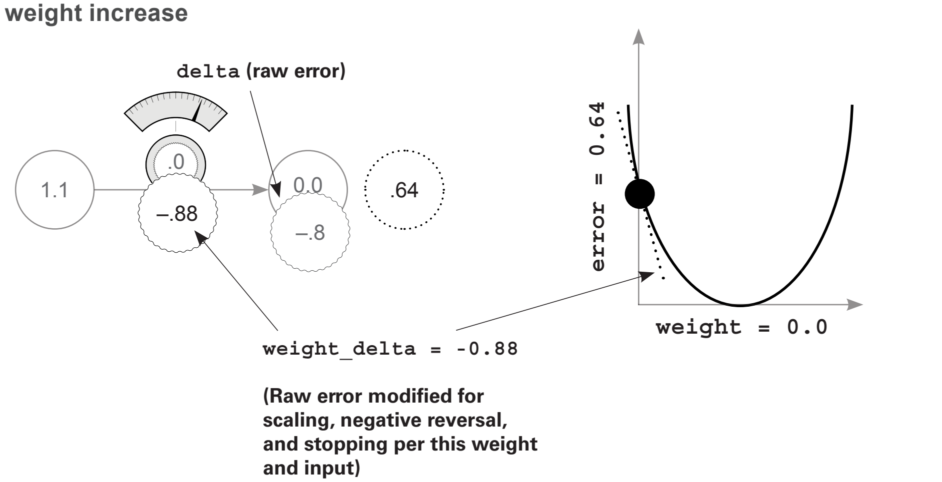
> **Key takeaway**
>
> The slope points to the *bottom* of the bowl (lowest `error`) no matter where you are in the bowl. You can use this slope to help the neural network reduce the error.

Let's watch several steps of learning

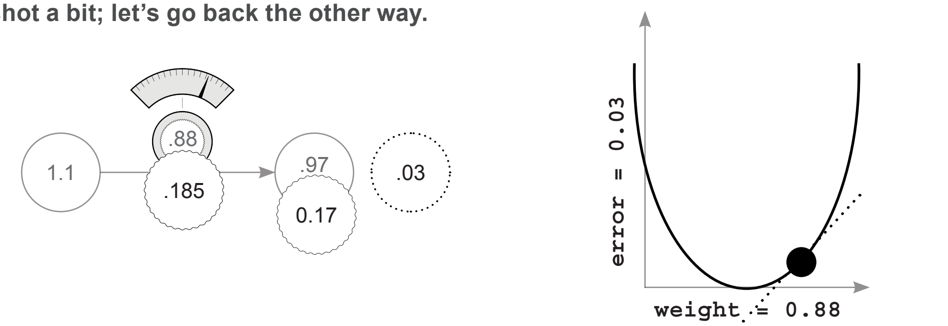
Will we eventually find the bottom of the bowl?

```
weight, goal_pred, input = (0.0, 0.8, 1.1)

for iteration in range(4):
    print("-----\nWeight:" + str(weight))
    pred = input * weight
    error = (pred - goal_pred) ** 2
    delta = pred - goal_pred
    weight_delta = delta * input
    weight = weight - weight_delta
    print("Error:" + str(error) + " Prediction:" + str(pred))
    print("Delta:" + str(delta) + " Weight Delta:" + str(weight_delta))
```

1 A big weight increase



2 Overshot a bit; let's go back the other way.



❸ Overshot again! Let's go back, but only a little.

❹ OK, we're pretty much there.

❺ Code output

```
-----
Weight:0.0
Error:0.64 Prediction:0.0
Delta:-0.8 Weight Delta:-0.88
-----
Weight:0.88
Error:0.028224 Prediction:0.968
Delta:0.168 Weight Delta:0.1848
-----
Weight:0.6952
Error:0.0012446784 Prediction:0.76472
Delta:-0.03528 Weight Delta:-0.038808
-----
Weight:0.734008
Error:5.489031744e-05 Prediction:0.8074088
Delta:0.0074088 Weight Delta:0.00814968
```

Why does this work? What is weight_delta, really?

Let's back up and talk about functions. What is a function? How do you understand one?

Consider this function:

```
def my_function(x):
    return x * 2
```

A function takes some numbers as input and gives you another number as output. As you can imagine, this means the function defines some sort of relationship between the input number(s) and the output number(s). Perhaps you can also see why the ability to learn a function is so powerful: it lets you take some numbers (say, image pixels) and convert them into other numbers (say, the probability that the image contains a cat).

Every function has what you might call *moving parts*: pieces you can tweak or change to make the output the function generates different. Consider `my_function` in the previous example. Ask yourself, "What's controlling the relationship between the input and the output of this function?" The answer is, the 2. Ask the same question about the following function:

```
error = ((input * weight) - goal_pred) ** 2
```

What's controlling the relationship between `input` and the output (`error`)? Plenty of things are—this function is a bit more complicated! `goal_pred`, `input`, `**2`, `weight`, and all the parentheses and algebraic operations (addition, subtraction, and so on) play a part in calculating the error. Tweaking any one of them would *change* the error. This is important to consider.

As a thought exercise, consider changing `goal_pred` to reduce the error. This is silly, but totally doable. In life, you might call this (setting goals to be whatever your capability is) "giving up." You're denying that you missed! That wouldn't do.

What if you changed `input` until `error` went to 0? Well, that's akin to seeing the world as you want to see it instead of as it actually is. You're changing the input data until you're predicting what you want to predict (this is loosely how *inceptionism* works).

Now consider changing the 2, or the additions, subtractions, or multiplications. This is just changing how you calculate `error` in the first place. The error calculation is meaningless if it doesn't actually give a good measure of how much you missed (with the right properties mentioned a few pages ago). This won't do, either.

What's left? The only variable remaining is `weight`. Adjusting it doesn't change your perception of the world, doesn't change your goal, and doesn't destroy your error measure. Changing `weight` means the function *conforms to the patterns in the data*. By forcing the rest of the function to be unchanging, you force the function to correctly model some pattern in the data. It's only allowed to modify how the network *predicts*.

To sum up: you modify specific parts of an error function until the `error` value goes to 0. This error function is calculated using a combination of variables, some of which you can change (weights) and some of which you can't (input data, output data, and the error logic):

```
weight = 0.5
goal_pred = 0.8
input = 0.5

for iteration in range(20):
    pred = input * weight
    error = (pred - goal_pred) ** 2
    direction_and_amount = (pred - goal_pred) * input
    weight = weight - direction_and_amount

    print("Error:" + str(error) + " Prediction:" + str(pred))
```

Key takeaway

You can modify *anything* in the `pred` calculation except `input`.

We'll spend the rest of this book (and many deep learning researchers will spend the rest of their lives) trying everything you can imagine on that `pred` calculation so that it can make good predictions. Learning is all about automatically changing the prediction function so that it makes good predictions—aka, so that the subsequent `error` goes down to 0.

Now that you know what you're allowed to change, how do you go about doing the changing? That's the good stuff. That's the machine learning, right? In the next section, we're going to talk about exactly that.

Tunnel vision on one concept

Concept: Learning is adjusting the weight to reduce the error to 0.

So far in this chapter, we've been hammering on the idea that learning is really just about adjusting `weight` to reduce `error` to 0. This is the secret sauce. Truth be told, knowing how to do this is all about understanding the *relationship* between `weight` and `error`. If you understand this relationship, you can know how to adjust `weight` to reduce `error`.

What do I mean by "understand the relationship"? Well, to understand the relationship between two variables is to understand *how changing one variable changes the other*. In this case, what you're really after is the *sensitivity* between these two variables. Sensitivity is another name for direction and amount. You want to know how sensitive `error` is to `weight`. You want to know the direction and the amount that `error` changes when you change `weight`. This is the goal. So far, you've seen two different methods that attempt to help you understand this relationship.

When you were wiggling `weight` (hot and cold learning) and studying its effect on `error`, you were experimentally studying the relationship between these two variables. It's like walking into a room with 15 different unlabeled light switches. You start flipping them on and off to learn about their relationship to various lights in the room. You did the same thing to study the relationship between `weight` and `error`: you wiggled `weight` up and down and watched for how it changed `error`. Once you knew the relationship, you could move `weight` in the right direction using two simple `if` statements:

```
if(down_error < up_error):
    weight = weight - step_amount

if(down_error > up_error):
    weight = weight + step_amount
```

Now, let's go back to the earlier formula that combined the `pred` and `error` logic. As mentioned, they quietly define an exact relationship between `error` and `weight`:

```
error = ((input * weight) - goal_pred) ** 2
```

This line of code, ladies and gentlemen, is the secret. This is a formula. This is the relationship between `error` and `weight`. This relationship is exact. It's computable. It's universal. It is and will always be.

Now, how can you use this formula to know how to change `weight` so that `error` moves in a particular direction? *That* is the right question. Stop. I beg you. Stop and appreciate this moment. This formula is the exact relationship between these two variables, and now you're going to figure out how to change one variable to move the other variable in a particular direction.

As it turns out, there's a method for doing this for *any* formula. You'll use it to reduce error.

A box with rods poking out of it

Picture yourself sitting in front of a cardboard box that has two circular rods sticking through two little holes. The blue rod is sticking out of the box by 2 inches, and the red rod is sticking out of the box by 4 inches. Imagine that I tell you these rods were connected, but I won't tell you in what way. You have to experiment to figure it out.

So, you take the blue rod and push it in 1 inch, and watch as, while you're pushing, the red rod also moves into the box by 2 inches. Then, you pull the blue rod back out 1 inch, and the red rod follows again, pulling out by 2 inches. What did you learn? Well, there seems to be a *relationship* between the red and blue rods. However much you move the blue rod, the red rod will move by twice as much. You might say the following is true:

```
red_length = blue_length * 2
```

As it turns out, there's a formal definition for "When I tug on this part, how much does this other part move?" It's called a *derivative*, and all it really means is "How much does rod X move when I tug on rod Y?"

In the case of the red and blue rods, the derivative for "How much does red move when I tug on blue?" is 2. Just 2. Why is it 2? That's the *multiplicative* relationship determined by the formula:

Derivative

```
red_length = blue_length * 2
```

Notice that you always have the derivative *between two variables*. You're always looking to know how one variable moves when you change another one. If the derivative is positive, then when you change one variable, the other will move in the *same* direction. If the derivative is *negative*, then when you change one variable, the other will move in the *opposite* direction.

Consider a few examples. Because the derivative of `red_length` compared to `blue_length` is 2, both numbers move in the same direction. More specifically, red will move twice as much as blue in the same direction. If the derivative had been –1, red would move in the opposite direction by the same amount. Thus, given a function, the derivative represents the direction and the amount that one variable changes if you change the other variable. This is exactly what we were looking for.

Derivatives: Take two

Still a little unsure about them? Let's take another perspective.

I've heard people explain derivatives two ways. One way is all about understanding how one variable in a function changes when you move another variable. The other way says that a derivative is the slope at a point on a line or curve. As it turns out, if you take a function and plot it (draw it), the slope of the line you plot is the *same thing* as "how much one variable changes when you change the other." Let me show you by plotting our favorite function:

```
error = ((input * weight) - goal_pred) ** 2
```

Remember, `goal_pred` and `input` are fixed, so you can rewrite this function:

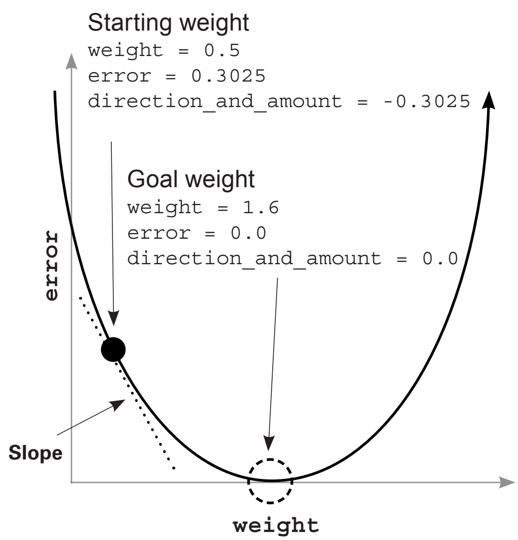
```
error = ((0.5 * weight) - 0.8) ** 2
```

Because there are only two variables left that change (all the rest of them are fixed), you can take every `weight` and compute the `error` that goes with it. Let's plot them.

As you can see, the plot looks like a big U-shaped curve. Notice that there's also a point in the middle where `error` == 0. Also notice that to the right of that point, the slope of the line is positive, and to the left of that point, the slope of the line is negative. Perhaps even more interesting, the farther away from the *goal weight* you move, the steeper the slope gets.

These are useful properties. The slope's sign gives you direction, and the slope's steepness gives you amount. You can use both of these to help find the goal `weight`.

Even now, when I look at that curve, it's easy for me to lose track of what it represents. It's similar to the hot and cold method for learning. If you tried every possible value for `weight` and plotted it out, you'd get this curve.



And what's remarkable about derivatives is that they can see past the big formula for computing `error` (at the beginning of this section) and see this curve. You can compute the slope (derivative) of the line for any value of `weight`. You can then use this slope (derivative) to figure out which direction reduces the error. Even better, based on the steepness, you can get at least some idea of how far away you are from the optimal point where the slope is zero (although not an exact answer, as you'll learn more about later).

What you really need to know

With derivatives, you can pick any two variables in any formula, and know how they interact.

Take a look at this *big whopper of a function*:

```
y = (((beta * gamma) ** 2) + (epsilon + 22 - x)) ** (1/2)
```

Here's what you need to know about derivatives. For any function (even this whopper), you can pick any two variables and understand their relationship with each other. For any function, you can pick two variables and plot them on an x-y graph as we did earlier. For any function, you can pick two variables and compute how much one changes when you change the other. Thus, for any function, you can learn how to change one variable so that you can move another variable in a direction. Sorry to harp on this point, but it's important that you know this in your bones.

Bottom line: in this book, you're going to build neural networks. A neural network is really just one thing: a bunch of weights you use to compute an error function. And for any error function (no matter how complicated), you can compute the relationship between any `weight` and the final `error` of the network. With this information, you can change each `weight` in the neural network to reduce `error` down to 0—and that's exactly what you're going to do.

What you don't really need to know

Calculus

So, it turns out that learning all the methods for taking any two variables in any function and computing their relationship takes about three semesters of college. Truth be told, if you went through all three semesters so that you could learn how to do deep learning, you'd use only a very small subset of what you learned. And really, calculus is just about memorizing and practicing every possible derivative rule for every possible function.

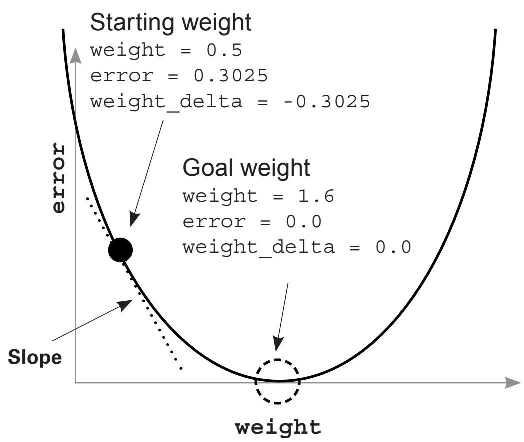
In this book, I'm going to do what I typically do in real life (cuz I'm lazy—I mean, efficient): look up the derivative in a reference table. All you need to know is what the derivative represents. It's the relationship between two variables in a function so you can know how much one changes when you change the other. It's just the sensitivity between two variables.

I know that was a lot of information to say, "It's the sensitivity between two variables," but it is. Note that this can include *positive sensitivity* (when variables move together), *negative sensitivity* (when they move in opposite directions), and zero sensitivity (when one stays fixed regardless of what you do to the other). For example, $y = 0 * x$. Move x, and y is always 0.

Enough about derivatives. Let's get back to gradient descent.

How to use a derivative to learn

weight_delta is your derivative.



What's the difference between `error` and the derivative of `error` and `weight`? `error` is a measure of how much you missed. The derivative defines the relationship between each weight and how much you missed. In other words, it tells how much changing a weight contributed to the error. So, now that you know this, how do you use it to move the error in a particular direction?

You've learned the relationship between two variables in a function, but how do you exploit that relationship? As it turns out, this is incredibly visual and intuitive. Check out the `error` curve again. The black dot is where `weight` starts out: (0.5). The dotted circle is where you want it to go: the goal weight. Do you see the dotted line attached to the black dot? That's the slope, otherwise known as the derivative. It tells you at that point in the curve how much `error` changes when you change `weight`. Notice that it's pointed downward: it's a negative slope.

The slope of a line or curve always points in the opposite direction of the lowest point of the line or curve. So, if you have a negative slope, you increase `weight` to find the minimum of `error`. Check it out.

So, how do you use the derivative to find the `error` minimum (lowest point in the `error` graph)? You move the opposite direction of the slope—the opposite direction of the derivative. You can take each `weight` value, calculate its derivative with respect to `error` (so you're comparing two variables: `weight` and `error`), and then change `weight` in the opposite direction of that slope. That will move you to the minimum.

Remember back to the goal again: you're trying to figure out the direction and the amount to change the weight so the error goes down. A derivative gives you the relationship between any two variables in a function. You use the derivative to determine the relationship between any weight and error. You then move the weight in the opposite direction of the derivative to find the lowest weight. Voilà! The neural network learns.

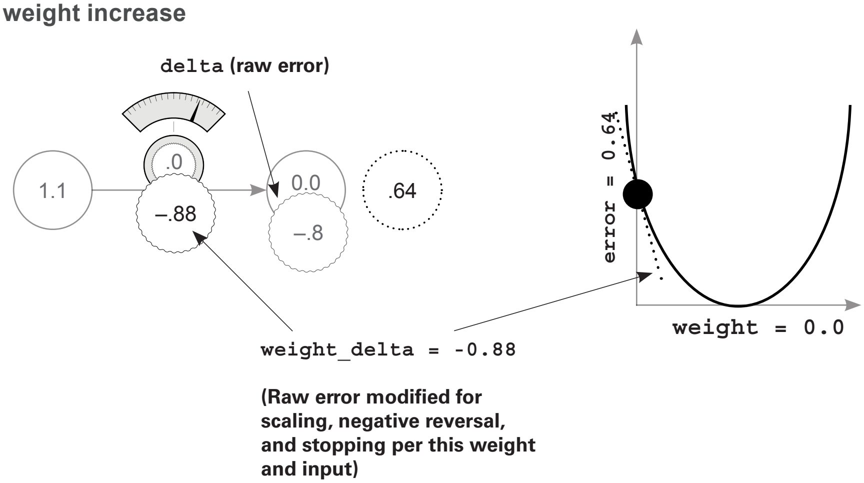
This method for learning (finding error minimums) is called *gradient descent*. This name should seem intuitive. You move the `weight` value opposite the gradient value, which reduces `error` to 0. By *opposite*, I mean you increase the weight when you have a negative gradient, and vice versa. It's like gravity.

Look familiar?

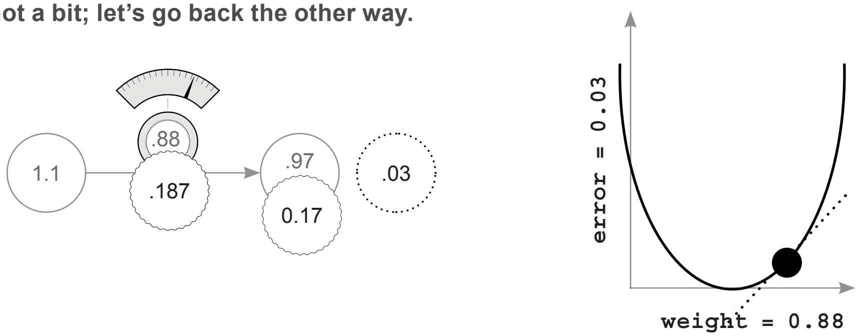
```
weight = 0.0
goal_pred = 0.8
input = 1.1

for iteration in range(4):
    pred = input * weight
    error = (pred - goal_pred) ** 2
    delta = pred - goal_pred
    weight_delta = delta * input
    weight = weight - weight_delta

    print("Error:" + str(error) + " Prediction:" + str(pred))
```

Derivative (how fast the error changes, given changes in the weight)

❶ A big weight increase



❷ Overshot a bit; let's go back the other way.



Breaking gradient descent

Just give me the code!

```
weight = 0.5
goal_pred = 0.8
input = 0.5

for iteration in range(20):
    pred = input * weight
    error = (pred - goal_pred) ** 2
    delta = pred - goal_pred
    weight_delta = input * delta
    weight = weight - weight_delta
    print("Error:" + str(error) + " Prediction:" + str(pred))
```

When I run this code, I see the following output:

```
Error:0.3025 Prediction:0.25
Error:0.17015625 Prediction:0.3875
Error:0.095712890625 Prediction:0.490625
                    ...

Error:1.7092608064e-05 Prediction:0.79586567925
Error:9.61459203602e-06 Prediction:0.796899259437
Error:5.40820802026e-06 Prediction:0.797674444578
```

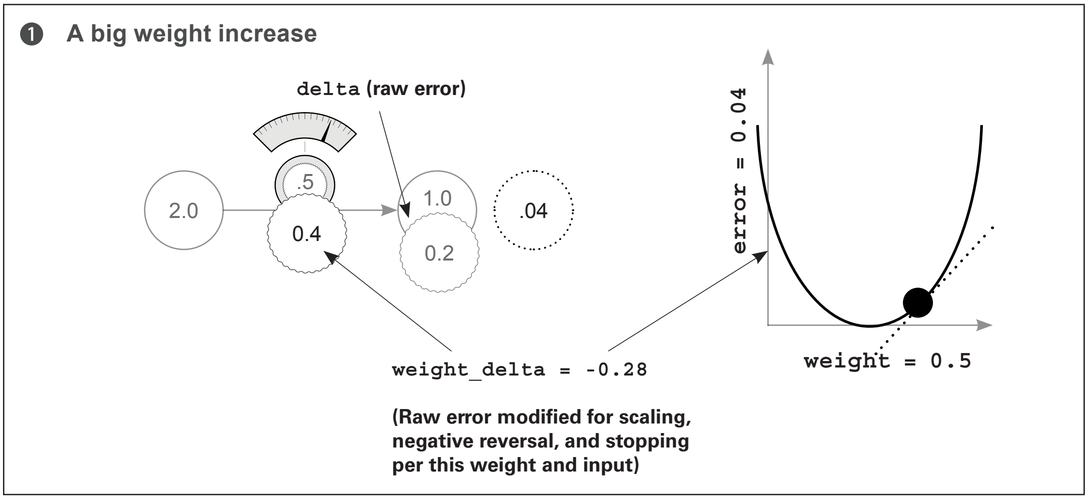
Now that it works, let's break it. Play around with the starting `weight`, `goal_pred`, and `input` numbers. You can set them all to just about anything, and the neural network will figure out how to predict the output given the input using the weight. See if you can find some combinations the neural network can't predict. I find that trying to break something is a great way to learn about it.

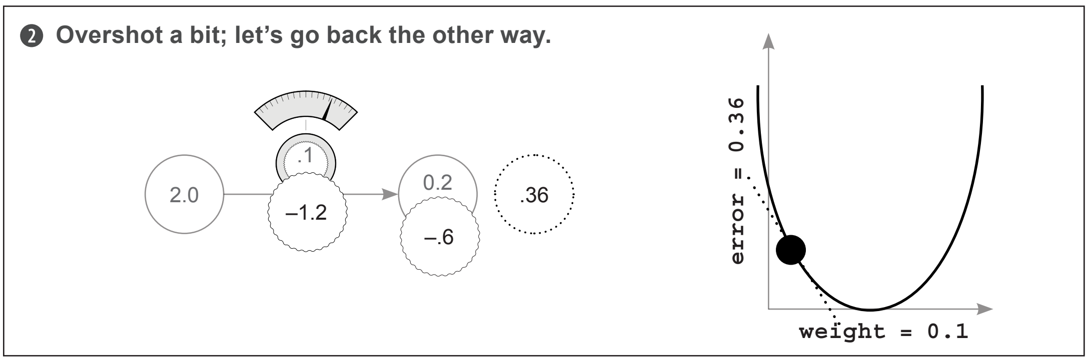
Let's try setting `input` equal to 2, but still try to get the algorithm to predict 0.8. What happens? Take a look at the output:

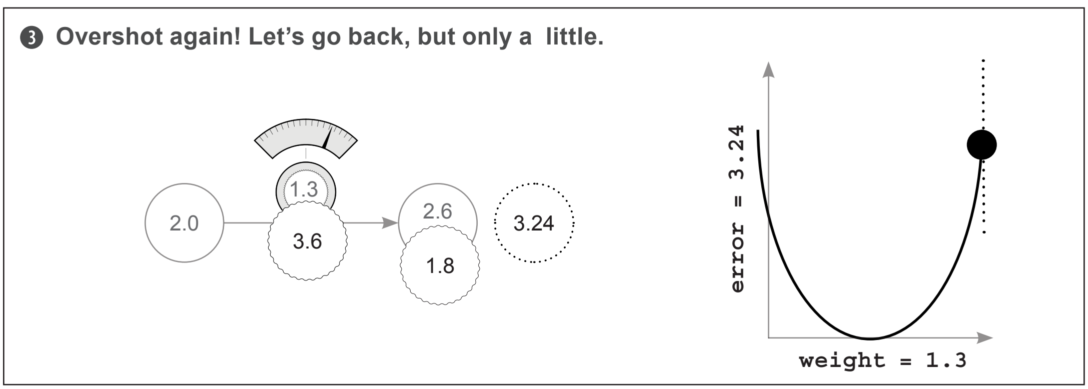
```
Error:0.04 Prediction:1.0
Error:0.36 Prediction:0.2
Error:3.24 Prediction:2.6
           ...

Error:6.67087267987e+14 Prediction:-25828031.8
Error:6.00378541188e+15 Prediction:77484098.6
Error:5.40340687069e+16 Prediction:-232452292.6
```

Whoa! That's not what you want. The predictions exploded! They alternate from negative to positive and negative to positive, getting farther away from the true answer at every step. In other words, every update to the weight overcorrects. In the next section, you'll learn more about how to combat this phenomenon.

Visualizing the overcorrections

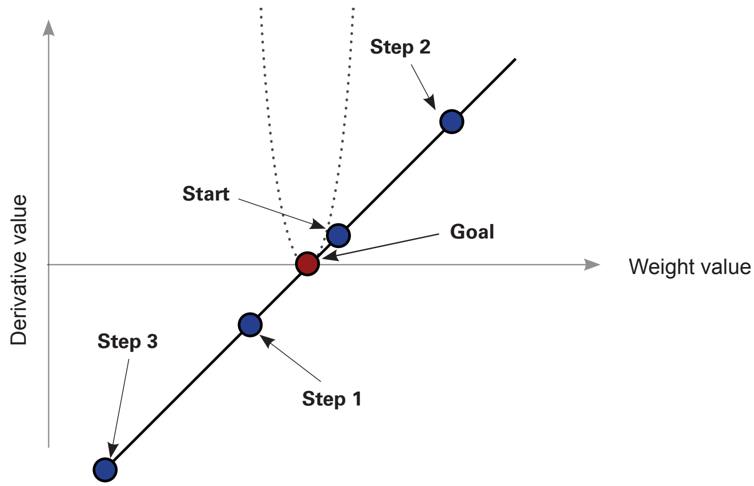






Divergence

Sometimes neural networks explode in value. Oops?



What really happened? The explosion in the error was caused by the fact that you made the input larger. Consider how you're updating the weight:

```
weight = weight - (input * (pred - goal_pred))
```

If the input is sufficiently large, this can make the weight update large even when the error is small. What happens when you have a large weight update and a small error? The network overcorrects. If the new error is even bigger, the network overcorrects even more. This causes the phenomenon you saw earlier, called *divergence*.

If you have a big input, the prediction is very sensitive to changes in the weight (because `pred = input * weight`). This can cause the network to overcorrect. In other words, even though the weight is still starting at 0.5, the derivative at that point is very steep. See how tight the U-shaped error curve is in the graph?

This is really intuitive. How do you predict? By multiplying the input by the weight. So, if the input is huge, small changes in the weight will cause changes in the prediction. The error is very sensitive to the weight. In other words, the derivative is really big. How do you make it smaller?

Introducing alpha

It's the simplest way to prevent overcorrecting weight updates.

What's the problem you're trying to solve? That if the input is too big, then the weight update can overcorrect. What's the symptom? That when you overcorrect, the new derivative is even larger in magnitude than when you started (although the sign will be the opposite).

Stop and consider this for a second. Look again at the graph in the previous section to understand the symptom. Step 2 is even farther away from the goal, which means the derivative is even greater in magnitude. This causes step 3 to be even farther from the goal than step 2, and the neural network continues like this, demonstrating divergence.

The symptom is this overshooting. The solution is to multiply the weight update by a fraction to make it smaller. In most cases, this involves multiplying the weight update by a single real-valued number between 0 and 1, known as *alpha*. Note: this has no effect on the core issue, which is that the input is larger. It will also reduce the weight updates for inputs that aren't too large.

Finding the appropriate alpha, even for state-of-the-art neural networks, is often done by guessing. You watch the error over time. If it starts diverging (going up), then the alpha is too high, and you decrease it. If learning is happening too slowly, then the alpha is too low, and you increase it. There are other methods than simple gradient descent that attempt to counter for this, but gradient descent is still very popular.

Alpha in code

Where does our "alpha" parameter come into play?

You just learned that alpha reduces the weight update so it doesn't overshoot. How does this affect the code? Well, you were updating the weights according to the following formula:

```
weight = weight - derivative
```

Accounting for alpha is a rather small change, as shown next. Notice that if alpha is small (say, 0.01), it will reduce the weight update considerably, thus preventing it from overshooting:

```
weight = weight - (alpha * derivative)
```

That was easy. Let's install alpha into the tiny implementation from the beginning of this chapter and run it where `input` = 2 (which previously didn't work):

```
weight = 0.5
goal_pred = 0.8
input = 2
alpha = 0.1

for iteration in range(20):
    pred  = input * weight
    error = (pred - goal_pred) ** 2
    derivative = input * (pred - goal_pred)
    weight = weight - (alpha * derivative)

    print("Error:" + str(error) + " Prediction:" + str(pred))
```

What happens when you make alpha crazy small or big? What about making it negative?

```
Error:0.04 Prediction:1.0
Error:0.0144 Prediction:0.92
Error:0.005184 Prediction:0.872

            ...

Error:1.14604719983e-09 Prediction:0.800033853319
Error:4.12576991939e-10 Prediction:0.800020311991
Error:1.48527717099e-10 Prediction:0.800012187195
```

Voilà! The tiniest neural network can now make good predictions again. How did I know to set alpha to 0.1? To be honest, I tried it, and it worked. And despite all the crazy advancements of deep learning in the past few years, most people just try several orders of magnitude of alpha (10, 1, 0.1, 0.01, 0.001, 0.0001) and then tweak it from there to see what works best. It's more art than science. There are more advanced ways to get to later, but for now, try various alphas until you get one that seems to work pretty well. Play with it.

Memorizing

It's time to really learn this stuff.

This may sound a bit intense, but I can't stress enough the value I've found from this exercise: see if you can build the code from the previous section in a Jupyter notebook (or a .py file, if you must) from memory. I know that might seem like overkill, but I (personally) didn't have my "click" moment with neural networks until I was able to perform this task.

Why does this work? Well, for starters, the only way to know you've gleaned all the information necessary from this chapter is to try to produce it from your head. Neural networks have lots of small moving parts, and it's easy to miss one.

Why is this important for the rest of the book? In the following chapters, I'll be referring to the concepts discussed in this chapter at a faster pace so that I can spend plenty of time on the newer material. It's vitally important that when I say something like "Add your alpha parameterization to the weight update," you immediately recognize which concepts from this chapter I'm referring to.

All that is to say, memorizing small bits of neural network code has been hugely beneficial for me personally, as well as for many individuals who have taken my advice on this subject in the past.