

calculate forward and inverse kinematics of a robot with 3rd DOF

Forward kinematics

Question:

Given:

- **Link lengths: L_1, L_2, L_3**
- **Joint angles: $\theta_1, \theta_2, \theta_3$**

Unknown:

- **The position of the end effector in Cartesian coordinates (x, y, z) .**

Answer:

$$x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$z = 0$$

Inverse kinematics

Question:

Given:

- **The position of the end effector in Cartesian coordinates: (x, y, z)**
- **Link lengths: L_1, L_2, L_3**

Unknown:

- **The joint angles: $\theta_1, \theta_2, \theta_3$**

Answer:

$$\theta_1 = \arctan(y/x)$$

$$\theta_2 = \arccos((r^2 - L_1^2 - L_2^2) / (2L_1L_2))$$

$$\theta_3 = \phi - \theta_2$$

$$\text{where } r = \sqrt{x^2 + y^2} \text{ and } \phi = \arctan(y/x) - \arctan((L_2 \sin(\theta_2)) / (L_1 + L_2 \cos(\theta_2)))$$