Math 237: Algorithms for Finding the Echelon Form and Reduced Echelon Form

I Algorithm for obtaining the Echelon Form.

Let M be an $m \times n$ matrix with entries m_{ij}.

Step 1: Find the first (from the left) nonzero column.

Step 2: If the first nonzero column is the jth column use row operations to make $m_{1j} \neq 0$.
The entry m_{1j} will be a **pivot**.
Recommendation: Make $m_{1j} = 1$.

Step 3: Use row operations to make all entries in the column below the **pivot** equal to zero, i.e. make $m_{2j} = m_{3j} = ... = 0$.
Recommendation: Use only row operations of the type $R_i \leftrightarrow R_i + c \cdot R_1$, with $i > 1$.

Step 4: Let **"new M"** be the $(m-1) \times (n-j)$ matrix obtained from **"old M"** by deleting the first row and the first j columns.

Repeat **Steps 1** through **4** for **"new M"** until the original matrix M is in Echelon Form.

II Algorithm for obtaining the Reduced Echelon Form or Row Canonical Form.

Let M be an $m \times n$ matrix with entries m_{ij} **that is in Echelon Form**.

Step 1: Use row operations to make all **pivots** equal to one.

Step 2: Identify the **lowest pivot**, the pivot closest to the bottom right corner of the matrix.

Step 3: If the **lowest pivot** is the entry m_{ij}, use row operations to make all entries in the column above the **lowest pivot** equal to zero, i.e. make $m_{1j} = m_{2j} = ...m_{i-1j} = 0$.
Recommendation: Use only row operations of the type $R_k \leftrightarrow R_k + c \cdot R_i$, with $k < i$.

Step 4: Let **"new M"** be the $(i-1) \times (j-1)$ consisting of the first $i-1$ rows and $j-1$ columns of **"old M"** .

Repeat **Steps 1** through **4** for **"new M"** until the original matrix M is in Reduced Echelon Form.