Testing

[ ] ↳ 3 cells hidden

ML Exercise 1-1

Question 1

Loading Data

```
# Download dataset
!wget https://raw.githubusercontent.com/Gholamrezadar/machine-learning-exercises/main/dataset1/dataset1.csv
```

```
--2021-11-20 19:14:28--  https://raw.githubusercontent.com/Gholamrezadar/machine-learning-exercises/main/dataset1/datase
Resolving raw.githubusercontent.com (raw.githubusercontent.com)... 185.199.108.133, 185.199.109.133, 185.199.110.133, ..
Connecting to raw.githubusercontent.com (raw.githubusercontent.com)|185.199.108.133|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 12074 (12K) [text/plain]
Saving to: ‘dataset1.csv.2’

dataset1.csv.2      100%[===================>]  11.79K  --.-KB/s    in 0s

2021-11-20 19:14:28 (86.2 MB/s) - ‘dataset1.csv.2’ saved [12074/12074]
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from tqdm.notebook import tqdm
import math

# Load the data using pandas
df = pd.read_csv("dataset1.csv")
```

```
# Show a sample of the data
df.head()
```

	x	y
0	0.097627	0.626964
1	0.430379	0.846452
2	0.205527	0.756017
3	0.089766	0.427504
4	-0.152690	-1.335228

```
# Show a description of the data (might be useful later)
df.describe()
```

	x	y
count	300.000000	300.000000

Helper functions

Loss Functions

```
# h(theta) =  theta transpose * X
def hypothesis(X, theta):
  y1 = theta*X
  return np.sum(y1, axis=1)

def MSE(X, y, theta):
  y_hat = hypothesis(X, theta)
  m = len(X)
  return sum((y_hat-y)**2)/(2*m)

def RMSE(X, y, theta):
  y_hat = hypothesis(X, theta)
  m = len(X)
  return np.sqrt(sum((y_hat-y)**2)/(2*m))

def MAE(X, y, theta):
  y_hat = hypothesis(X, theta)
  m = len(X)
  return sum(np.abs((y_hat-y)))/(2*m)

# Loss functions Derivatives
def MSE_prim(X, y, i, theta):
  y_hat = hypothesis(X, theta)
  Xi = X.iloc[:, i]
  m = len(X)
  return sum((y_hat-y) * Xi) / m

def RMSE_prim(X, y, i, theta):
  # src : https://math.stackexchange.com/questions/4065532/rmse-derivatives
  mse = MSE(X, y, theta)
  mse_prim = MSE_prim(X, y, i, theta)

  return mse_prim / 2 / np.sqrt(mse)

def MAE_prim(X, y, i, theta):
  # src : https://stats.stackexchange.com/questions/312737/mean-absolute-error-mae-derivative
  # src2 : https://github.com/chenxingwei/machine_learning_from_scratch/blob/master/algorithm/2.linearRegressionGradientDe
  y_hat = hypothesis(X, theta)
  # print(np.sum((X.T*(np.sign(y_hat-y)/len(X))), axis=1)[i])
  return np.sum((X.T*(np.sign(y_hat-y)/len(X))), axis=1)[i]
```

Gradient Descent

```
def gradientDescent(X, y, theta, lr, iteration, X_valid, y_valid, loss_fn = MSE, loss_fn_prim = MSE_prim, decay=0.0):
  # Training loss per iteration history
  train_loss_history = []
  # Validation loss per iteration history
  validation_loss_history = []
  # weights progression towards the optimal value
  theta_history = []

  # Progress bar
  with tqdm(total=iteration) as pbar:
    for iter in range(iteration):
      for i in range(0, len(X.columns)):
        # partial derivative of loss function with respect to Xi
        gradient = loss_fn_prim(X, y, i, theta)

        # TODO : Learning rate decay
        # lr = lr * 1/(1 + decay * iter)
```

```
      # Actual "Gradient Descent" !
      theta[i] -= lr * gradient

    # Calculating the loss after each iteration
    # of updating the weights using Gradient Descent
    loss = loss_fn(X, y, theta)
    if X_valid is not None and y_valid is not None:
      validation_loss = loss_fn(X_valid, y_valid, theta)

    # Save the history of loss and weights
    train_loss_history.append(loss)
    if X_valid is not None and y_valid is not None:
      validation_loss_history.append(validation_loss)
    theta_history.append(theta.copy())

    # Update progress bar
    pbar.update(1)

  history = {"train_loss":train_loss_history,
             "validation_loss":validation_loss_history,
             "weights":theta_history}
  # returns loss history, latest loss, weights
  return history, loss, theta
```

```
# Add the polynomial's terms as features
# so that the univariate non-linear regression
# becomes a multivariate linear regression
# where every polynomial term is a feature for
# the linear regression
def polynomial_to_linear_regression(X, polynomial_degree):
  for i in range(2, 2 + polynomial_degree):
    X['x'+str(i)] = X['x']**i
```

Normal Equation

```
def normalEquation(X, y):
  # (X^T X)^-1 X^T Y
  XTX = np.dot(X.T,X)
  XTX_inverse = np.linalg.inv(XTX)
  XTY = np.dot(X.T,y)
  theta = np.dot(XTX_inverse, XTY)
  return theta

def regularizedNormalEquation(X, y, lambd=0.1):
  # (X^T X + lambda I)^-1 X^T Y
  XTX = np.dot(X.T,X) + np.dot(np.identity(X.shape[1]),lambd)
  XTX_inverse = np.linalg.inv(XTX)
  XTY = np.dot(X.T,y)
  theta = np.dot(XTX_inverse, XTY)
  return theta
```

Plotting related

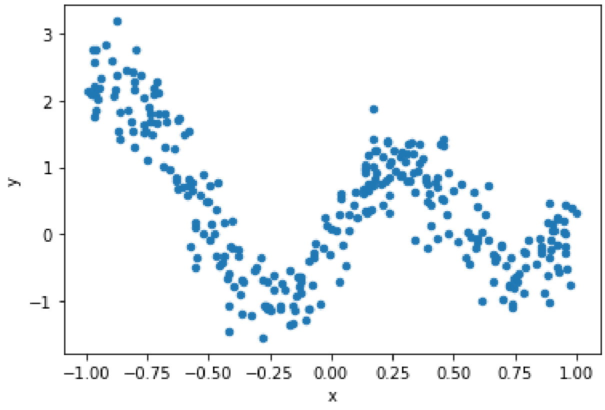
```
#  helper function used to plot a polynomial
def polyCoefficients(x, coeffs):
    o = len(coeffs)
    y = 0
    for i in range(o):
        y += coeffs[i]*x**i
    return y
```

```
# Plots a polynomial on top of the original data
def plot_curve(X, y, theta, c='r', title='', resolution=100):
  plt.figure()
  plt.title(title)
  # Plot the original data
  plt.scatter(x=X['x'],y= y)
```

```

x = np.linspace(-1, 1, resolution)
# Plot the fitted polynomial over the data
plt.plot(x, polyCoefficients(x, theta), c=c, linewidth=4)

plt.show()
```

Part 1 - Plotting the data

```
# Plot the data using matplotlib
df.plot(kind='scatter', x='x', y='y')
plt.show()
```



Part 2 - Shuffle

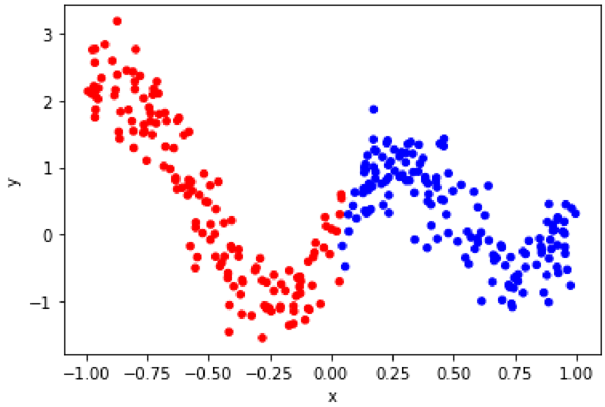
```
def plot_colorize(df):
  '''
  Assigns 'red' color to the first half of the data
  and 'blue' to the rest

  If the data is well shuffled we should see random red
  and blue circles everywhere.

  If the data is NOT well shuffled we might see a pattern between
  circles' position and their color.
  '''
  col = {True:"red", False:"blue"}
  colors = list(map(lambda x:col[x<df.shape[0]/2],df.index.tolist()))

  df.plot(kind='scatter', x='x', y='y', color=colors)
  plt.show()
```

Default data

```
plot_colorize(df)
print(" "*9,"Data seems to be well shuffled.")
```

Sorted data

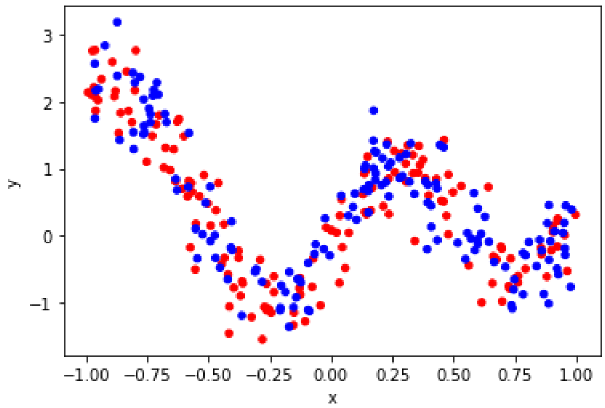
```
# Sort the data based on 'x' first, then do the
# previous part to see the result.
sorted_df = df.sort_values(by='x', ascending=True, ignore_index=True)

plot_colorize(sorted_df)
print(" "*6,"Data doesn't seem to be well shuffled.")
```




```
       Data doesn't seem to be well shuffled.
```

Shuffled data

```
# Let's shuffle the data anyways (just in case)

# pandas Doc: specifying drop=True prevents .reset_index()
# from creating a column containing the old index entries.
shuffled_df = df.sample(frac=1).reset_index(drop=True)

plot_colorize(shuffled_df)
print(" "*9,"Data seems to be well shuffled.")
```




```
          Data seems to be well shuffled.
```

Part 3 - Gradient Descent

Finding The optimal "Theta" values (weights)

Data Prepration

Add a column for bias

```
# Add a new column for simplicity of the calculations
# acts as the bias term
```

```
shuffled_df = pd.concat([pd.Series(1, index=shuffled_df.index, name='0'), shuffled_df], axis=1)
shuffled_df.head()
```

	0	x	y
0	1	0.963659	-0.527811
1	1	0.312659	1.348749
2	1	-0.125936	-0.631524
3	1	0.341276	-0.078979
4	1	0.923873	0.217419

Seperate X,y

```
# Split training data into X and y
X = shuffled_df.drop(columns="y")
y = shuffled_df.iloc[:, 2]

print(X.head(),end="\n\n")
print(y.head())
```

```
   0         x
0  1  0.963659
1  1  0.312659
2  1 -0.125936
3  1  0.341276
4  1  0.923873

0   -0.527811
1    1.348749
2   -0.631524
3   -0.078979
4    0.217419
Name: y, dtype: float64
```

```
# Split to train and valid
split = 0.7

X_train = X.iloc[ : int(len(X)*split),:].reset_index(drop=True)
X_valid = X.iloc[int(len(X)*split) : ,:].reset_index(drop=True)

y_train = y.iloc[ : int(len(X)*split)].reset_index(drop=True)
y_valid = y.iloc[int(len(X)*split) : ].reset_index(drop=True)

print(f"Train X size = {len(X_train)}")
print(f"Train y size = {len(y_train)}")
print(f"Valid X size = {len(X_valid)}")
print(f"Valid y size = {len(y_valid)}")
```

```
Train X size = 210
Train y size = 210
Valid X size = 90
Valid y size = 90
```

```
# Save a copy of X and y
# TODO might not need it
X_train_org = X_train.copy()
y_train_org = y_train.copy()
```

Polynomial Regression

a basic example

```
# polynomial degree
polynomial_degree = 12 #@param {type: "number"}
learning_rate = 0.030 #@param {type: "number"}
iterations = 1000 #@param {type: "number"}

```

polynomial_degree: 12

learning_rate: 0.030

iterations: 1000

Convert

aX + bX^2 + cX^3 + d

to

aX1 + bX2 + cX3 + d

```python
# Add the polynomial's terms as features
# so that the univariate non-linear regression
# becomes a multivariate linear regression
# where every polynomial term is a feature for
# the linear regression
polynomial_to_linear_regression(X_train, polynomial_degree)
polynomial_to_linear_regression(X_valid, polynomial_degree)

X_train.head()
```

	0	x	x2	x3	x4	x5	x6	x7	x8	x9	x10	x1
0	1	0.963659	0.928638	0.894890	0.862369	0.831029	0.800829	7.717257e-01	7.436803e-01	7.166540e-01	6.906099e-01	6.655123e-0
1	1	0.312659	0.097756	0.030564	0.009556	0.002988	0.000934	2.920776e-04	9.132075e-05	2.855227e-05	8.927129e-06	2.791149e-0
2	1	-0.125936	0.015860	-0.001997	0.000252	-0.000032	0.000004	-5.024022e-07	6.327056e-08	-7.968048e-09	1.003465e-09	-1.263724e-1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5.391851e-	1.840108e-	6.279842e-	2.143158e-	7.314078e

Training

```python
# Initialize the weights with zero
theta = np.array([0.0]*len(X_train.columns))

# Initialize the weights with random values
theta = np.random.rand(len(X_train.columns),)

print("notice : takes approximately 3 minutes for 5k iters")

# Start the training
history, loss, theta = gradientDescent(X_train,
                                        y_train,
                                        theta,
                                        learning_rate,
                                        iterations,
                                        X_valid = X_valid,
                                        y_valid = y_valid,
                                        loss_fn=MSE,
                                        loss_fn_prim=MSE_prim,
                                        decay = 0.0)
```

```
notice : takes approximately 3 minutes for 5k iters
100%                                        1000/1000 [00:46<00:00, 21.89it/s]
```

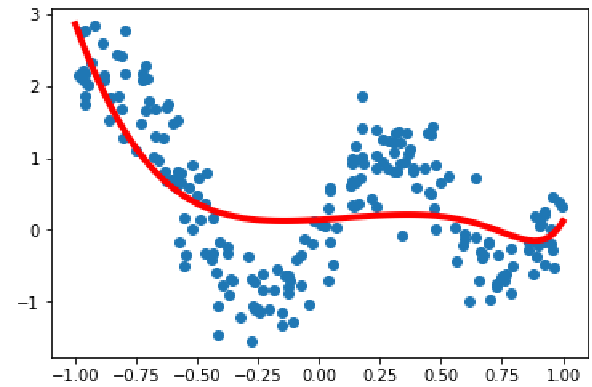
Plotting the fitted polynomials

```python
# Predicting using the learned weights(theta)
# Not used here but useful
y_hat = theta*X_valid
y_hat = np.sum(y_hat, axis=1)
print(MSE(X_valid, y_valid, theta))
```

```
0.22926660232638735
```

```python
plot_curve(X_train, y_train, theta)
```

```
print(f"Training Loss : {RMSE(X_train, y_train, theta)}")
print(f"Validation Loss : {RMSE(X_valid, y_valid, theta)}\n")
```

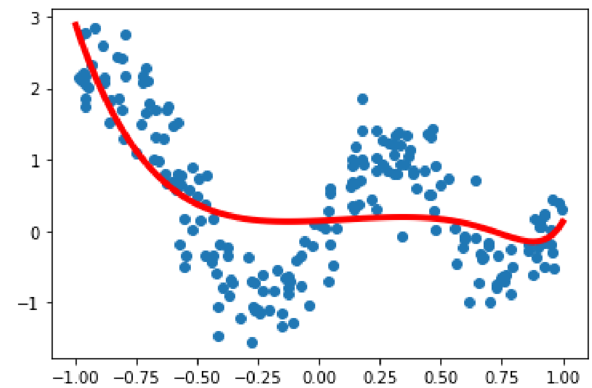



```
Training Loss : 0.5101877849255999
Validation Loss : 0.4788179218934765
```

Interactive history viewer

```
from ipywidgets import interact
import ipywidgets as widgets

@interact(iteration = widgets.IntSlider(min=0, max=iterations-1, step=100, value=0))
def plot_weight_history(iteration):
  plot_curve(X_train, y_train, history["weights"][iteration])

print("Use the slider to see the algorithms progression")
```

iteration 900




```
Use the slider to see the algorithms progression
```

Part 4 - Plotting every Case!

Polynomial Degree :

- 5
- 8
- 10

Loss Functions :

- MSE
- RMSE
- MAE

Iterations :

- 5000
- 10000

```
# iterations = 1500 #TODO
polynomial_degrees = [5, 8, 10]
```

```
loss_functions = [(MSE, MSE_prim),
                  (RMSE, RMSE_prim),
                  (MAE, MAE_prim)]
fn_labels = ["MAE", "RMSE", "MSE"]
iterations = 2000
```

```
thetas = [[[],[],[]],
          [[],[],[]],
          [[],[],[]]]

losses = [[[],[],[]],
          [[],[],[]],
          [[],[],[]]]

for i,degree in enumerate(polynomial_degrees):
  for j, (loss_fn, loss_fn_prim) in enumerate(loss_functions):
    print(f"degree: {degree} | loss function: {fn_labels[j]}")

    # preprocess data
    X_train_copy = X_train.copy()
    X_valid_copy = X_valid.copy()

    polynomial_to_linear_regression(X_train_copy, degree)
    polynomial_to_linear_regression(X_valid_copy, degree)

    # TODO: gradient descent
    # Initialize the weights with random values
    theta = np.random.rand(len(X_train.columns),)
    # Start the training
    history, loss, theta = gradientDescent(X_train_copy,
                                           y_train,
                                           theta,
                                           learning_rate,
                                           iterations,
                                           X_valid = X_valid_copy,
                                           y_valid = y_valid,
                                           loss_fn=loss_fn,
                                           loss_fn_prim=loss_fn_prim,
                                           decay = 0.0)

    # latest iterations theta
    thetas[i][j].append(history["weights"][-1])
    # halfway theta
    thetas[i][j].append(history["weights"][int(iterations/2)-1])

    # Training loss
    losses[i][j].append(loss_fn(X_train_copy, y_train, theta))
    # Validation loss
    losses[i][j].append(loss_fn(X_valid_copy, y_valid, theta))

    print()
```

```
degree: 5 | loss function: MAE
100%                                    2000/2000 [00:51<00:00, 38.92it/s]

degree: 5 | loss function: RMSE
100%                                    2000/2000 [01:30<00:00, 22.86it/s]

degree: 5 | loss function: MSE
100%                                    2000/2000 [01:33<00:00, 21.39it/s]
```

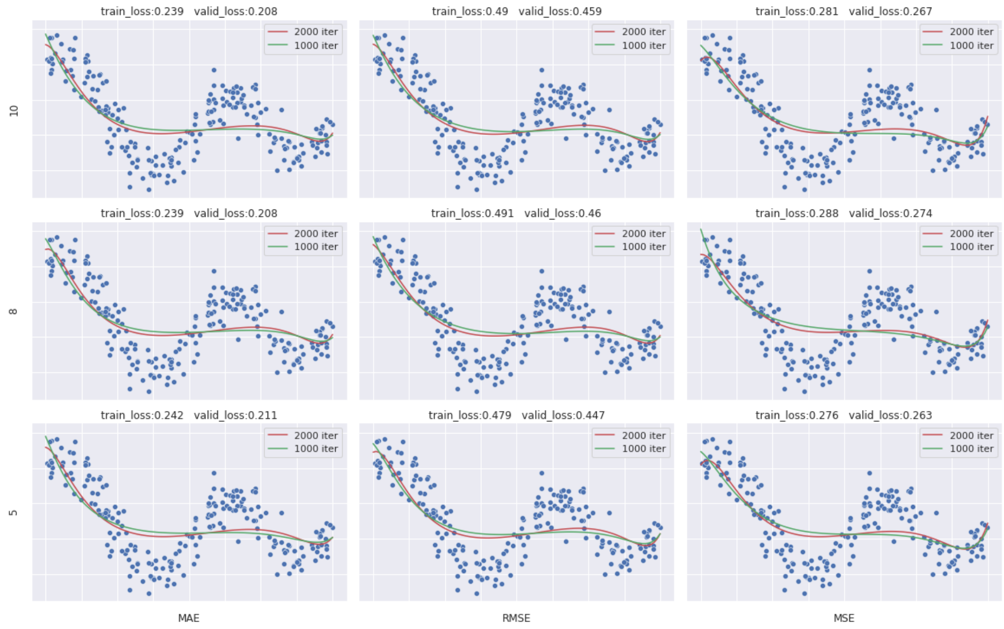
```python
# plt.subplot_tool()
```

```
100%                                    2000/2000 [00:51<00:00, 38.12it/s]
```

```python
def plot_every_curve(X, y, thetas, resolution=100):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  fig, axes = plt.subplots(3, 3, sharex=True, sharey=True, figsize=(16,10), constrained_layout=True)
  # fig.subplots_adjust(hspace=0.6)

  # Helper lists for accessing the current config
  fn_labels = ["MAE", "RMSE", "MSE"]
  deg_labels = [10,8,5]

  # Used to plot the fitted polynomial in the range[-1,1]
  x = np.linspace(-1, 1, resolution)

  # Plotting every case in a 3 by 3 grid
  for i in range(3):
    for j in range(3):

      # 1. plot the original data (Blue)
      sns.scatterplot(x=X["x"], y=y, ax=axes[i,j])

      # 2. Plot a curve with Last iteration theta [0] (Red)
      theta = thetas[i][j][0]
      sns.lineplot(x, polyCoefficients(x, theta), color='r', ax=axes[i,j])

      # 3. Plot a curve with Middle iteration theta [1] (Green)
      theta = thetas[i][j][1]
      sns.lineplot(x, polyCoefficients(x, theta), color='g', ax=axes[i,j])

      # Legends and titles
      axes[i,j].legend(labels=[f"{iterations} iter", f"{iterations//2} iter"])
      axes[i,j].set_title(f"train_loss:{round(losses[i][j][0],3)}   valid_loss:{round(losses[i][j][1],3)}")

      # Matplotlib related code
      axes[i,j].xaxis.set_ticklabels([])
      axes[i,j].yaxis.set_ticklabels([])
      axes[i,j].set_xlabel(fn_labels[j])
      axes[i,j].set_ylabel(deg_labels[i])

  plt.show()
```

```python
plot_every_curve(X_train, y_train, thetas)
```

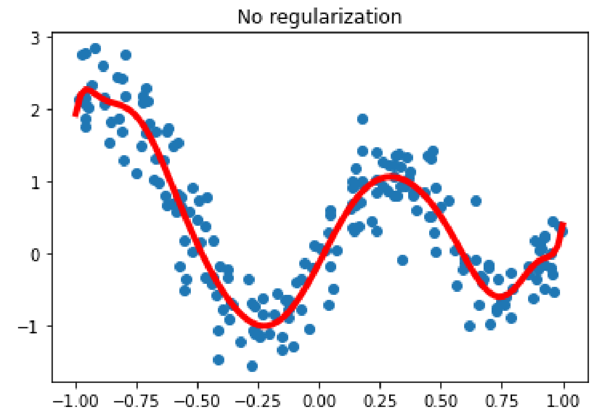
```
<Figure size 432x288 with 0 Axes>
```



Part 5 - Normal Equation

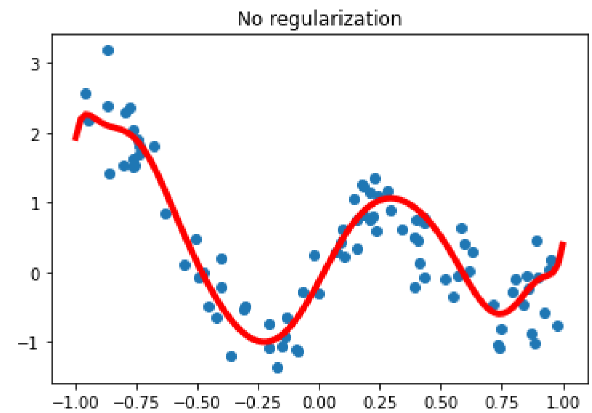
```
normal_theta = normalEquation(X_train, y_train)
```

```
# Plotting the fitted polynomial over training data
plot_curve(X_train, y_train, normal_theta, title="No regularization")
print(f"Training Loss : {RMSE(X_train, y_train, normal_theta)}")
```




```
Training Loss : 0.2750887262206953
```

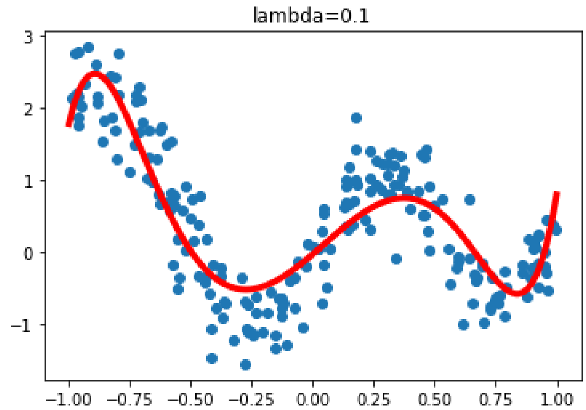
```
# Plotting the fitted polynomial over validation data
plot_curve(X_valid, y_valid, normal_theta, title="No regularization")
print(f"Validation Loss : {RMSE(X_valid, y_valid, normal_theta)}\n")
```



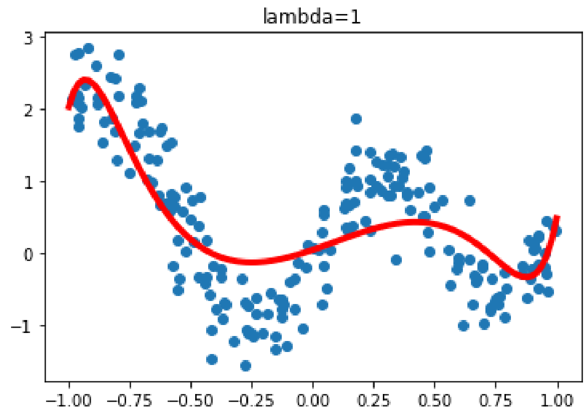

```
Validation Loss : 0.2855798993217868
```

Part 6 - Regularized Normal Equation

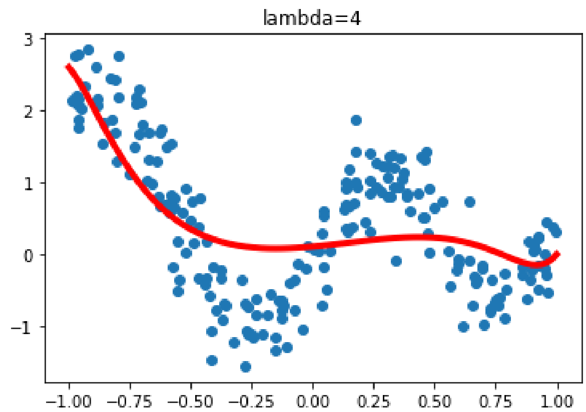
```
reg_normal_theta = regularizedNormalEquation(X_train, y_train, lambd=.1)
plot_curve(X_train, y_train, reg_normal_theta, title="lambda=0.1")
print(f"Training Loss : {RMSE(X_train, y_train, reg_normal_theta)}")
print(f"Validation Loss : {RMSE(X_valid, y_valid, reg_normal_theta)}\n")

reg_normal_theta = regularizedNormalEquation(X_train, y_train, lambd=1)
plot_curve(X_train, y_train, reg_normal_theta, title="lambda=1")
print(f"Training Loss : {RMSE(X_train, y_train, reg_normal_theta)}")
print(f"Validation Loss : {RMSE(X_valid, y_valid, reg_normal_theta)}\n")

reg_normal_theta = regularizedNormalEquation(X_train, y_train, lambd=4)
plot_curve(X_train, y_train, reg_normal_theta, title="lambda=4")
print(f"Training Loss : {RMSE(X_train, y_train, reg_normal_theta)}")
print(f"Validation Loss : {RMSE(X_valid, y_valid, reg_normal_theta)}\n")
```




```
Training Loss : 0.34234945259970995
Validation Loss : 0.33329843645565466
```




```
Training Loss : 0.43260647240880507
Validation Loss : 0.4018612985008604
```




```
Training Loss : 0.5017630147189559
Validation Loss : 0.4717228178183581
```