ML Exercise 1-1

Question 1

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Nov 2021

Questions available at : Github Link

How to Run: Runtime > Run all(ctrl+f9)

Hyper Parameters

Hyper Parameters

Main :

max_iterations: 10000

train_test_split: 0.7

Basic Example :

basic_polynomial_degree: 10

basic_iterations: 1000

basic_learning_rate: 2.3

basic_decay: 5e-9

Show code

Loading Data

```
# Download dataset
!wget https://raw.githubusercontent.com/Gholamrezadar/machine-learning-exercises/main/dataset1/dataset1.csv
```

```
--2021-11-23 05:46:33--  https://raw.githubusercontent.com/Gholamrezadar/machine-learning-exercises/main/dataset1/dataset1.csv
Resolving raw.githubusercontent.com (raw.githubusercontent.com)... 185.199.108.133, 185.199.109.133, 185.199.110.133, ...
Connecting to raw.githubusercontent.com (raw.githubusercontent.com)|185.199.108.133|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 12074 (12K) [text/plain]
Saving to: 'dataset1.csv.1'

dataset1.csv.1      100%[===================>]  11.79K  --.-KB/s    in 0s

2021-11-23 05:46:33 (83.2 MB/s) - 'dataset1.csv.1' saved [12074/12074]
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from tqdm.notebook import tqdm
import seaborn as sns

# use seaborn
sns.set()

# Load the data using pandas
df = pd.read_csv("dataset1.csv")
```

```
# Show a sample of the data
df.head()
```

	x	y
0	0.097627	0.626964
1	0.430379	0.846452
2	0.205527	0.756017
3	0.089766	0.427504
4	-0.152690	-1.335228

```
# Show a description of the data (might be useful later)
df.describe()
```

	x	y
count	300.000000	300.000000
mean	0.007005	0.412755
std	0.580948	1.021100
min	-0.990609	-1.547934
25%	-0.504657	-0.361192
50%	0.045096	0.316442
75%	0.460611	1.092441
max	0.997694	3.186153

Helper functions

```
# Suppress some warnings
import warnings
warnings.simplefilter(action="ignore", category=FutureWarning)
```

Loss Functions

```
# h(theta) =  theta transpose * X
def hypothesis(X, theta):
  y1 = theta*X
  return np.sum(y1, axis=1)

def MSE(X, y, theta):
  y_hat = hypothesis(X, theta)
  m = len(X)
  return sum((y_hat-y)**2)/(2*m)

def RMSE(X, y, theta):
  y_hat = hypothesis(X, theta)
  m = len(X)
  return np.sqrt(sum((y_hat-y)**2)/(2*m))

def MAE(X, y, theta):
  y_hat = hypothesis(X, theta)
  m = len(X)
  return sum(np.abs((y_hat-y)))/(2*m)
```

```python

# Loss functions Derivatives
def MSE_prim(X, y, i, theta):
  y_hat = hypothesis(X, theta)
  Xi = X.iloc[:, i]
  m = len(X)
  return sum((y_hat-y) * Xi) / m

def RMSE_prim(X, y, i, theta):
  # src : https://math.stackexchange.com/questions/4065532/rmse-derivatives
  mse = MSE(X, y, theta)
  mse_prim = MSE_prim(X, y, i, theta)

  return mse_prim / 2 / np.sqrt(mse)

def MAE_prim(X, y, i, theta):
  # src : https://stats.stackexchange.com/questions/312737/mean-absolute-error-mae-derivative
  # src2 : https://github.com/chenxingwei/machine_learning_from_scratch/blob/master/algorithm/2.linearRegressionGradientDescent.md
  y_hat = hypothesis(X, theta)
  # print(np.sum((X.T*(np.sign(y_hat-y)/len(X))), axis=1)[i])
  return np.sum((X.T*(np.sign(y_hat-y)/len(X))), axis=1)[i]

```

Gradient Descent

```python
def gradientDescent(X, y, theta, lr, iteration, X_valid, y_valid, loss_fn = MSE, loss_fn_prim = MSE_prim, decay=0.0):
  # Training loss per iteration history
  train_loss_history = []
  # Validation loss per iteration history
  validation_loss_history = []
  # weights progression towards the optimal value
  theta_history = []

  # Progress bar
  with tqdm(total=iteration) as pbar:
    for itera in range(iteration):
      # TODO : Learning rate decay
      lr = lr * 1/(1 + decay * itera)

      for i in range(0, len(X.columns)):
        # partial derivative of loss function with respect to Xi
        gradient = loss_fn_prim(X, y, i, theta)

        # Actual "Gradient Descent" !
        theta[i] -= lr * gradient

      # Calculating the loss after each iteration
      # of updating the weights using Gradient Descent
      loss = loss_fn(X, y, theta)
      if X_valid is not None and y_valid is not None:
        validation_loss = loss_fn(X_valid, y_valid, theta)

      # Save the history of loss and weights
      train_loss_history.append(loss)
      if X_valid is not None and y_valid is not None:
        validation_loss_history.append(validation_loss)
      theta_history.append(theta.copy())

      # Update progress bar
      pbar.update(1)

  history = {"training_loss":train_loss_history,
             "validation_loss":validation_loss_history,
             "weights":theta_history}
  # returns loss history, latest loss, weights
  print(f"training_loss : {round(train_loss_history[-1],4)} | validation_loss : {round(validation_loss_history[-1],4)}")
  return history, loss, theta
```

```python
# Add the polynomial's terms as features
# so that the univariate non-linear regression
# becomes a multivariate linear regression
# where every polynomial term is a feature for
# the linear regression
def polynomial_to_linear_regression(X, polynomial_degree):
  for i in range(2, 1 + polynomial_degree):
    X['x'+str(i)] = X['x']**i
```

```python
#  helper function used to plot a polynomial
def polyCoefficients(x, coeffs):
    o = len(coeffs)
    y = 0
    for i in range(o):
        y += coeffs[i]*x**i
    return y
```

Normal Equation

```python
def normalEquation(X, y):
  # (X^T X)^-1 X^T Y
  XTX = np.dot(X.T,X)
  XTX_inverse = np.linalg.inv(XTX)
  XTY = np.dot(X.T,y)
  theta = np.dot(XTX_inverse, XTY)
  return theta

def regularizedNormalEquation(X, y, lambd=0.1):
  # (X^T X + lambda I)^-1 X^T Y
  XTX = np.dot(X.T,X) + np.dot(np.identity(X.shape[1]),lambd)
  XTX_inverse = np.linalg.inv(XTX)
  XTY = np.dot(X.T,y)
  theta = np.dot(XTX_inverse, XTY)
  return theta
```

Plotting related

def plot_curve( )

```python
# Plots a polynomial on top of the original data
def plot_curve(X, y, theta, c='r', title='', resolution=200):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  plt.title(title)
  # Plot the original data
  plt.scatter(x=X['x'],y= y)

  x = np.linspace(-1, 1, resolution)
  # Plot the fitted polynomial over the data
  plt.plot(x, polyCoefficients(x, theta), color=c, linewidth=4)

  plt.show()
```

def plot_loss( )

```python
# Plots validation and training losses per iteration
def plot_loss(history, title='', starting_iter=0):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

```

```
  fig, ax = plt.subplots()
  plt.title(title)

  # X = iterations range
  x = np.linspace(0, iterations, iterations)

  # 1. Training_loss - iteration curve (Red)
  sns.lineplot(x[starting_iter:], history["training_loss"][starting_iter:], color='r')
  # 2. Validation_loss - iteration curve (green)
  sns.lineplot(x[starting_iter:], history["validation_loss"][starting_iter:], color='g')

  ax.legend(labels=["training", "validation"])

  plt.show()
```

def plot_lr( )

```
def plot_lr(lr=0.1, iterations=1000, decay=None, title='learning_rate'):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  _lrs = []
  # Iterations
  _iterations = iterations
  # Initial lr
  _lr0 = lr
  _decay = 0
  # Decay
  if decay is None:
    _decay = _lr0/_iterations
  else:
    _decay = decay

  # Simulate gradient descents main loop
  _lr = _lr0
  for i in range(_iterations):
    _lr = _lr * 1/(1 + _decay * i)
    _lrs.append(_lr)

  _x = list(range(_iterations))
  _y = _lrs

  plt.figure()
  plt.title(title)
  plt.plot(_x, _y)
  plt.show()
```

def plot_every_curve( )

```
def plot_every_curve(X, y, thetas,line_widths=[1, 1], resolution=100, i_max=3, j_max=3, dpi=72):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  fig, axes = plt.subplots(3, 3, sharex=True, sharey=True, figsize=(16,10), constrained_layout=True)
  fig.dpi = dpi
  # fig.subplots_adjust(hspace=0.6)

  # Helper lists for accessing the current config
  fn_labels = ["MAE", "RMSE", "MSE"]
  deg_labels = ["degree 10","degree 8","degree 5"]

  # Used to plot the fitted polynomial in the range[-1,1]
  x = np.linspace(-1, 1, resolution)

  # Plotting every case in a 3 by 3 grid
  for i in range(i_max):
    for j in range(j_max):

      # 1. plot the original data (Dots)
      sns.scatterplot(x=X["x"], y=y, ax=axes[i,j] )# ,s=70, color='darkgray')

      # 2. Plot a curve with an earlier iteration theta [1] (Red)
      theta = thetas[i][j][1]
      sns.lineplot(x, polyCoefficients(x, theta), color='r', ax=axes[i,j], linewidth=line_widths[1])

      # 3. Plot a curve with the Last iteration theta [0] (Green)
      theta = thetas[i][j][0]
      sns.lineplot(x, polyCoefficients(x, theta), color='g', ax=axes[i,j], linewidth=line_widths[0])

      # Legends and titles
      axes[i,j].legend(labels=[f"{other_iteration_to_display} iters", f"{int(iterations)} iters"])
      axes[i,j].set_title(f"train_loss:{round(losses[i][j][0],3)}   valid_loss:{round(losses[i][j][1],3)}")

      # Matplotlib related code
      axes[i,j].xaxis.set_ticklabels([])
      axes[i,j].yaxis.set_ticklabels([])
      axes[i,j].set_xlabel(fn_labels[j])
      axes[i,j].set_ylabel(deg_labels[i])

  plt.show()
```

def plot_every_case_loss( )

```
def plot_every_case_loss(histories, starting_iter=0, i_max=3, j_max=3, dpi=72):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  fig, axes = plt.subplots(3, 3, sharex=True, sharey=False, figsize=(16,10), constrained_layout=True)
  fig.dpi = dpi

  # Helper lists for accessing the current config
  fn_labels = ["MAE", "RMSE", "MSE"]
  deg_labels = ["degree 10","degree 8","degree 3"]

  # X = iterations range
  x = np.linspace(0, iterations, iterations)

  # Plotting every case in a 3 by 3 grid
  for i in range(i_max):
    for j in range(j_max):

      # 1. Training_loss - iteration curve (Red)
      sns.lineplot(x[starting_iter:], histories[i][j][0]["training_loss"][starting_iter:], color='r', ax=axes[i,j])
      # 2. Validation_loss - iteration curve (green)
      sns.lineplot(x[starting_iter:], histories[i][j][0]["validation_loss"][starting_iter:], color='g', ax=axes[i,j])

      # Legends
      axes[i,j].legend(labels=[f"training loss", f"validation loss"])

      # Matplotlib related code
      axes[i,j].set_xlabel(fn_labels[j])
      axes[i,j].set_ylabel(deg_labels[i])

  plt.show()
```

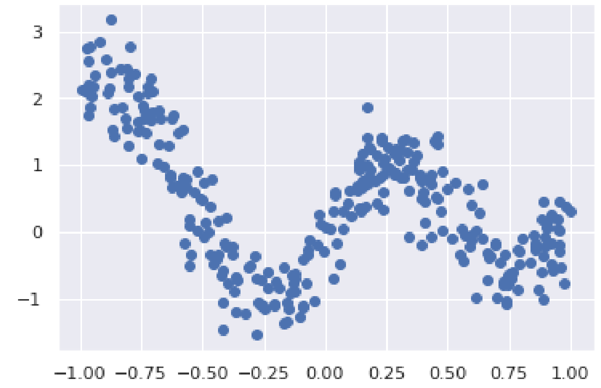
def plot_normal_equations( )

```
# Plots a polynomial on top of the original data
def plot_normal_equations(X, y, c='r', title='', resolution=200, dpi=72):
  # Don't try this at home
```

```
    import warnings
    warnings.simplefilter(action="ignore", category=FutureWarning)

    fig, ax = plt.subplots()
    plt.title(title)
    fig.dpi=dpi

    # Plot the original data
    sns.scatterplot(x=X['x'], y=y, size=1, color='darkgray')

    x = np.linspace(-1, 1, resolution)

    # calculate theta for each method
    normal_theta = normalEquation(X_train_copy, y_train)
    reg_normal_theta1 = regularizedNormalEquation(X_train_copy, y_train, lambd=0.075)
    reg_normal_theta2 = regularizedNormalEquation(X_train_copy, y_train, lambd=0.75)
    reg_normal_theta4 = regularizedNormalEquation(X_train_copy, y_train, lambd=7.5)

    # Plot the fitted polynomial over the data
    plt.plot(x, polyCoefficients(x, normal_theta), linewidth=2)
    plt.plot(x, polyCoefficients(x, reg_normal_theta1), linewidth=2)
    plt.plot(x, polyCoefficients(x, reg_normal_theta2), linewidth=2)
    plt.plot(x, polyCoefficients(x, reg_normal_theta4), linewidth=2)

    ax.legend(labels=["No Regularization", "λ=0.075", "λ=0.75", "λ=7.5"])
    ax.xaxis.set_ticklabels([])
    ax.yaxis.set_ticklabels([])
    ax.xaxis.set_visible(False)
    ax.yaxis.set_visible(False)
    plt.show()
```

Part 1 - Plotting the data

```
# Plot the data using matplotlib
plt.scatter(x=df['x'],y=df['y'])
plt.show()
```

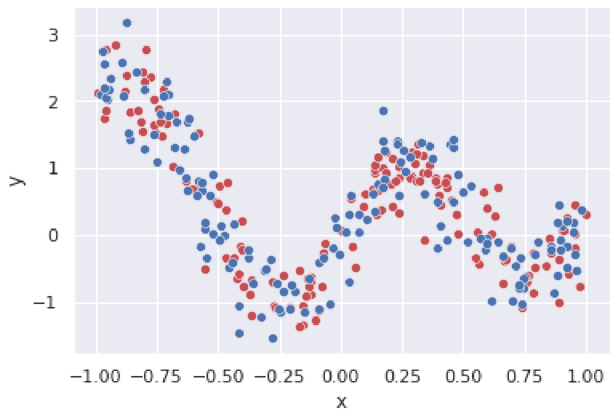


Part 2 - Shuffle

```
def plot_colorize(df):
    '''
    Assigns 'red' color to the first half of the data
    and 'blue' to the rest

    If the data is well shuffled we should see random red
    and blue circles everywhere.

    If the data is NOT well shuffled we might see a pattern between
    circles' position and their color.
    '''

    df_red = df.loc[df.index<df.shape[0]/2]
    df_blue = df.loc[df.index>=df.shape[0]/2]

    sns.scatterplot(data=df_red, x='x', y='y', color='r')
    sns.scatterplot(data=df_blue, x='x', y='y', color='b')

    plt.show()
```

Default data

```
plot_colorize(df)
print(" "*9,"Data seems to be well shuffled.")
```




```
Data seems to be well shuffled.
```

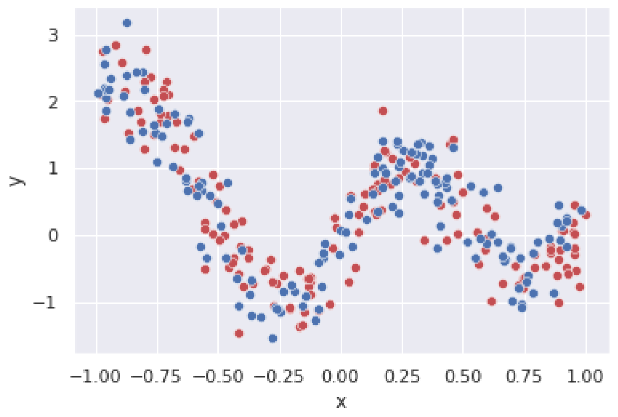
Sorted data

```
# Sort the data based on 'x' first, then do the
# previous part to see the result.
sorted_df = df.sort_values(by='x', ascending=True, ignore_index=True)

plot_colorize(sorted_df)
print(" "*6,"Data doesn't seem to be well shuffled.")
```


```
Data doesn't seem to be well shuffled.
```

Shuffled data

```
# Let's shuffle the data anyways (just in case)

# pandas Doc: specifying drop=True prevents .reset_index()
# from creating a column containing the old index entries.
shuffled_df = df.sample(frac=1).reset_index(drop=True)

plot_colorize(shuffled_df)
print(" "*9,"Data seems to be well shuffled.")
```



Data seems to be well shuffled.

Part 3 - Gradient Descent

Finding The optimal "Theta" values (weights)

Data Prepration

Add a column for bias

```
# Add a new column for simplicity of the calculations
# acts as the bias term
shuffled_df = pd.concat([pd.Series(1, index=shuffled_df.index, name='0'), shuffled_df], axis=1)
shuffled_df.head()
```

	0	x	y
0	1	-0.122797	-0.700428
1	1	0.954990	-0.287307
2	1	-0.087699	-1.126259
3	1	-0.405126	0.206917
4	1	0.154457	0.754608

Seperate X,y

```
# Split training data into X and y
X = shuffled_df.drop(columns="y")
y = shuffled_df.iloc[:, 2]

print("X : \n", X.head().to_string(),end="\n\n")
print("y : \n", y.head().to_string())
```

```
X :
    0         x
0  1 -0.122797
1  1  0.954990
2  1 -0.087699
3  1 -0.405126
4  1  0.154457

y :
 0   -0.700428
1   -0.287307
2   -1.126259
3    0.206917
4    0.754608
```

```
# Split to train and valid
split = train_test_split

X_train = X.iloc[ : int(len(X)*split),:].reset_index(drop=True)
X_valid = X.iloc[int(len(X)*split) : ,:].reset_index(drop=True)

y_train = y.iloc[ : int(len(X)*split)].reset_index(drop=True)
y_valid = y.iloc[int(len(X)*split) : ].reset_index(drop=True)

print(f"Train X size = {len(X_train)}")
print(f"Train y size = {len(y_train)}")
print(f"Valid X size = {len(X_valid)}")
print(f"Valid y size = {len(y_valid)}")
```

```
Train X size = 210
Train y size = 210
Valid X size = 90
Valid y size = 90
```

```
# Save a copy of X and y
# TODO might not need it
X_train_org = X_train.copy()
y_train_org = y_train.copy()
```

Polynomial Regression

a basic example

```
# polynomial degree
polynomial_degree = basic_polynomial_degree

```

Convert

aX + bX^2 + cX^3 + d

to

aX1 + bX2 + cX3 + d

```
# Add the polynomial's terms as features
# so that the univariate non-linear regression
# becomes a multivariate linear regression
# where every polynomial term is a feature for
# the linear regression

X_train_copy = X_train.copy()
X_valid_copy = X_valid.copy()
polynomial_to_linear_regression(X_train_copy, polynomial_degree)
polynomial_to_linear_regression(X_valid_copy, polynomial_degree)


X_train_copy.head()
```

	0	x	x2	x3	x4	x5	x6	x7	x8	x9	x10
0	1	-0.122797	0.015079	-0.001852	0.000227	-0.000028	3.428672e-06	-4.210306e-07	5.170128e-08	-6.348761e-09	7.796086e-10
1	1	0.954990	0.912006	0.870957	0.831756	0.794319	7.585666e-01	7.244237e-01	6.918176e-01	6.606791e-01	6.309421e-01
2	1	-0.087699	0.007691	-0.000675	0.000059	-0.000005	4.549648e-07	-3.990011e-08	3.499213e-09	-3.068787e-10	2.691306e-11
3	1	-0.405126	0.164127	-0.066492	0.026938	-0.010913	4.421212e-03	-1.791148e-03	7.256409e-04	-2.939761e-04	1.190974e-04
4	1	0.154457	0.023857	0.003685	0.000569	0.000088	1.357840e-05	2.097281e-06	3.239401e-07	5.003488e-08	7.728246e-09

Training a basic example

Normal equation for comparison :

- 0.07 training loss
- 0.09 validation loss

5k iter 10th degree polynomial lr=2.3

- Training Loss : 0.07766969752936674
- Validation Loss : 0.094857479611961

```
# Initialize the weights with zero
theta = np.array([0.0]*len(X_train_copy.columns))

# Initialize the weights with random values
# theta = np.random.rand(len(X_train_copy.columns),)

print("notice : takes approximately 1 minute for 1k iters (MSE)")

# tip : nice way to find decay (https://machinelearningmastery.com/using-learning-rate-schedules-deep-learning-models-python-keras/)
# Decay = LearningRate / Epochs
# Decay = 0.1 / 1000
# Decay = 0.0001
# Note : didn't work well for us :(

# Hyper-Parameters
iterations = basic_iterations
learning_rate = basic_learning_rate
decay = basic_decay

# Start the training
history, loss, theta = gradientDescent(X_train_copy,
                                       y_train,
                                       theta,
                                       learning_rate,
                                       iterations,
                                       X_valid = X_valid_copy,
                                       y_valid = y_valid,
                                       loss_fn=MSE,
                                       loss_fn_prim=MSE_prim,
                                       decay = decay*0)


```

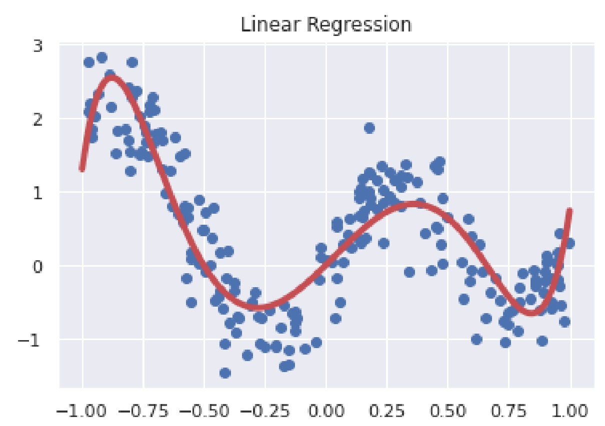
```
notice : takes approximately 1 minute for 1k iters (MSE)
100%                                        1000/1000 [00:40<00:00, 25.08it/s]
training_loss : 0.1107 | validation_loss : 0.1162
```

Plotting the fitted polynomials

```
# Predicting using the learned weights(theta)
# Not used here but useful
y_hat = theta*X_valid_copy
y_hat = np.sum(y_hat, axis=1)
print(RMSE(X_valid_copy, y_valid, theta))
```

```
0.34090818758488145
```

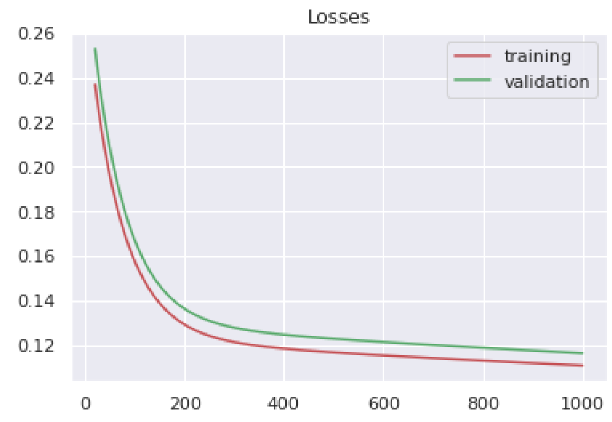
```
plot_curve(X_train_copy, y_train, theta, title='Linear Regression')

print(f"Training Loss : {RMSE(X_train_copy, y_train, theta)}")
print(f"Validation Loss : {RMSE(X_valid_copy, y_valid, theta)}\n")
```




```
Training Loss : 0.3327509590452109
Validation Loss : 0.34090818758488145
```

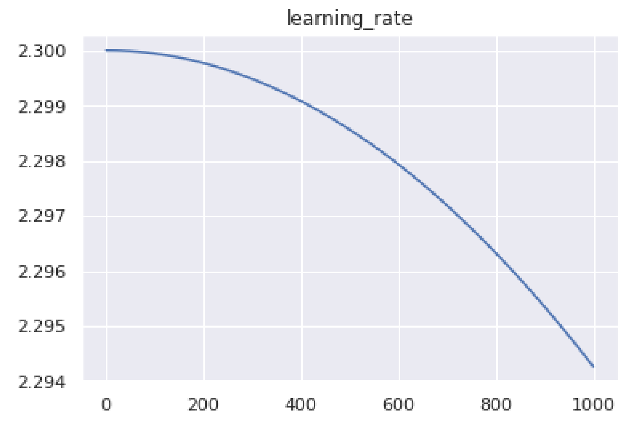
Plotting losses

```
plot_loss(history, starting_iter=20, title='Losses')
```



Plotting the learning rate

```
plot_lr(lr=learning_rate, iterations=iterations, decay=decay)
```



Interactive history viewer

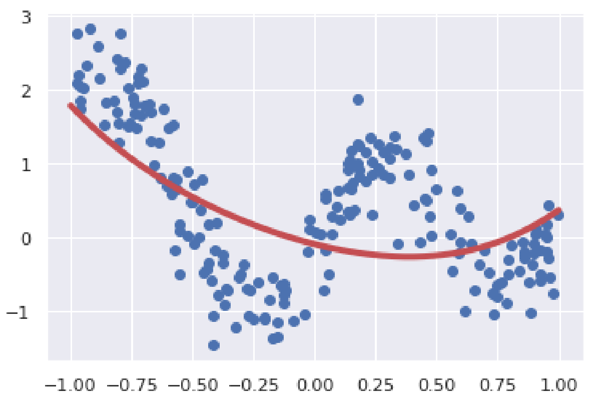
You can use the slider to see a history of thetas

which should allow you to look at the progression

of the minization algorithm!

```
from ipywidgets import interact
import ipywidgets as widgets

@interact(iteration = widgets.IntSlider(min=0, max=iterations-1, step=10, value=0))
def plot_weight_history(iteration):
  plot_curve(X_train_copy, y_train, history["weights"][iteration])

print("Use the slider to see the algorithms progression")
```

iteration 0



Use the slider to see the algorithms progression

Part 4 - Plotting every Case!

Polynomial Degree :

- 5
- 8
- 10

Loss Functions :

- MSE
- RMSE
- MAE

Iterations :

- 1000
- 10000

```
# Global Hyper-Parameters
# we plot the 'max_iteration' curve (green)
# and 'other_iteration_to_display' curve (red)
iterations = max_iterations
other_iteration_to_display = iterations//10 # The red line in the plots ()

# Grid Hyper-Parameters
polynomial_degrees = [10, 8, 5]

loss_functions = [(MAE, MAE_prim),
                  (RMSE, RMSE_prim),
                  (MSE, MSE_prim)]
fn_labels = ["MAE", "RMSE", "MSE"]

#         MAE  RMSE  MSE
lrs    = [0.26, 1.4, 2.23]
decays = [0,  0,  5e-9]
```

Training all 9 models

```
# Containers used for storing results about each model
# Used later by the plotting functions
thetas = [[[],[],[]],
          [[],[],[]],
          [[],[],[]]]

losses = [[[],[],[]],
          [[],[],[]],
          [[],[],[]]]

histories = [[[],[],[]],
             [[],[],[]],
             [[],[],[]]]

# start the training process for each model
for i,degree in enumerate(polynomial_degrees):
  for j, (loss_fn, loss_fn_prim) in enumerate(loss_functions):
    print(f"degree: {degree} | loss function: {fn_labels[j]}")

    # preprocess data (univariate non-linear to multivariate linear)
    X_train_copy = X_train.copy()
    X_valid_copy = X_valid.copy()
    polynomial_to_linear_regression(X_train_copy, degree)
    polynomial_to_linear_regression(X_valid_copy, degree)

    # Initialize the weights with random values
    theta = np.random.rand(len(X_train_copy.columns),)

    # j(loss function) specific learning rate
    _learning_rate = lrs[j]

    # j(loss function) specific decay value
    _decay = decays[j]

    # Start the training
    history, loss, theta = gradientDescent(X_train_copy,
                                          y_train,
                                          theta,
                                          _learning_rate,
                                          iterations,
                                          X_valid = X_valid_copy,
                                          y_valid = y_valid,
                                          loss_fn = loss_fn,
                                          loss_fn_prim = loss_fn_prim,
                                          decay = _decay)

    # Saving latest iteration's theta for each model
    thetas[i][j].append(history["weights"][-1])
    # Saving halfway theta for each model
    thetas[i][j].append(history["weights"][int(other_iteration_to_display)-1]) # -1 : zero-indexed

    # Saving Training loss for each model
    losses[i][j].append(loss_fn(X_train_copy, y_train, theta))
    # Saving Validation loss for each model
    losses[i][j].append(loss_fn(X_valid_copy, y_valid, theta))
    # Saving Histories for each model (used for plotting loss per iteration)
    histories[i][j].append(history)

    print()
```

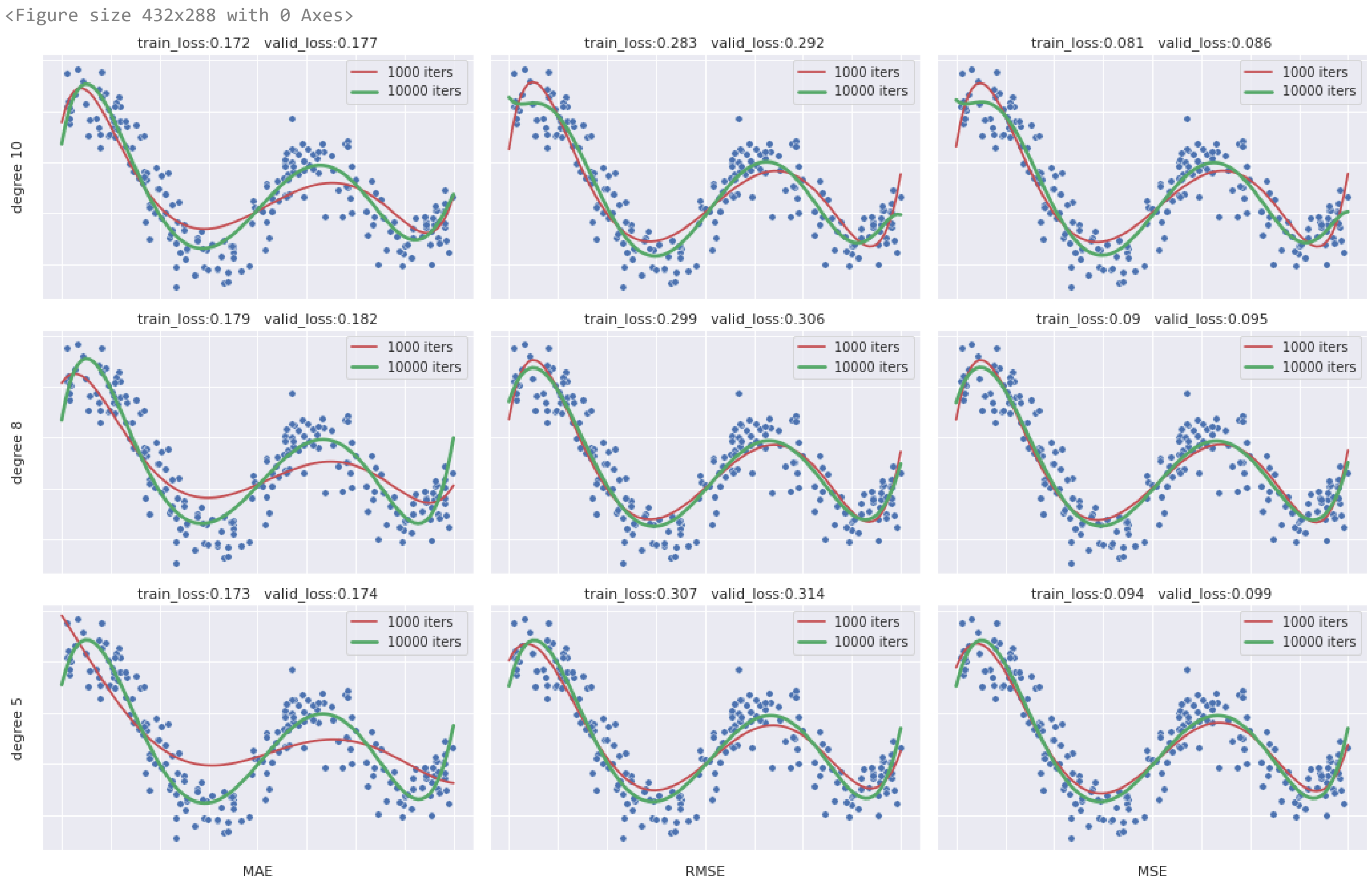
```
degree: 10 | loss function: MAE
100%                                       10000/10000 [07:09<00:00, 23.95it/s]
training_loss : 0.172 | validation_loss : 0.1766

degree: 10 | loss function: RMSE
100%                                       10000/10000 [12:15<00:00, 13.75it/s]
training_loss : 0.2825 | validation_loss : 0.2919

degree: 10 | loss function: MSE
100%                                       10000/10000 [06:42<00:00, 25.42it/s]
training_loss : 0.0807 | validation_loss : 0.0858

degree: 8 | loss function: MAE
100%                                       10000/10000 [05:59<00:00, 27.40it/s]
training_loss : 0.179 | validation_loss : 0.1819
```

Plotting fitted curves

```
training_loss : 0.2991 | validation_loss : 0.3059
```

```python
plot_every_curve(X_train,
                 y_train,
                 thetas,
                 line_widths=[3,2],
                 i_max=len(polynomial_degrees),
                 j_max=len(loss_functions),
                 dpi=64)
```

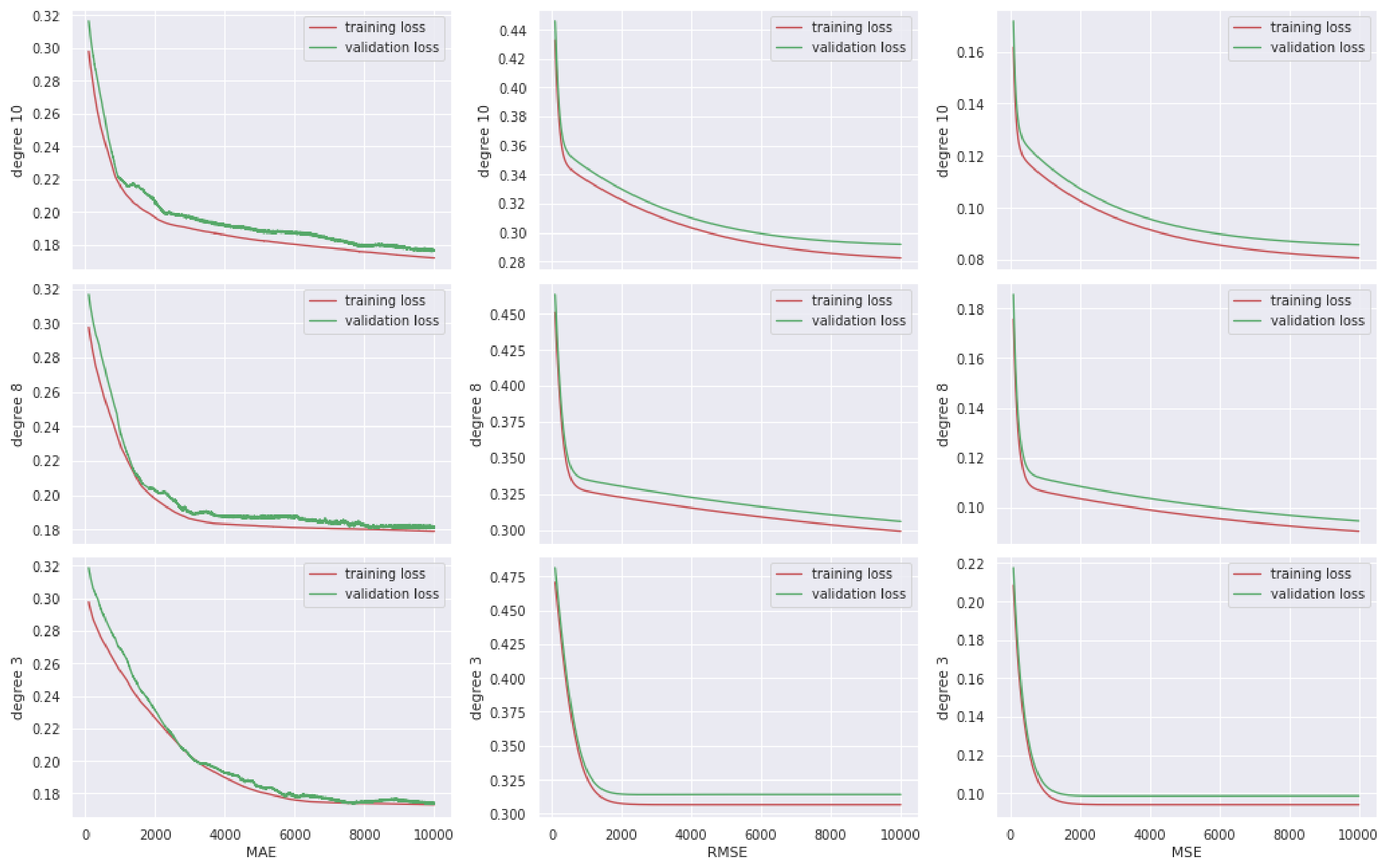
```
<Figure size 432x288 with 0 Axes>
```



Plotting train/valid loss

```python
plot_every_case_loss(histories,
                     starting_iter=100,
                     i_max=len(polynomial_degrees),
                     j_max=len(loss_functions),
                     dpi=64)
```

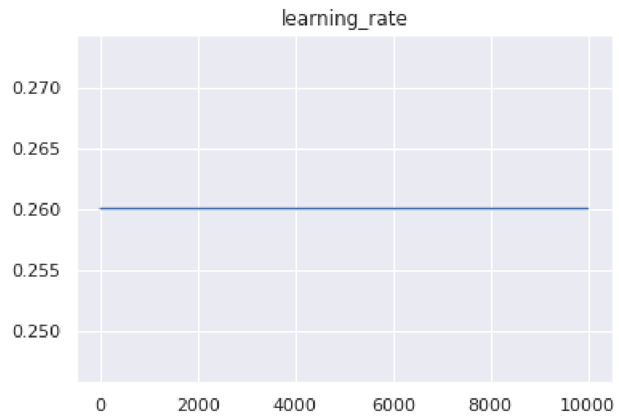
```
<Figure size 432x288 with 0 Axes>
```



Plotting Learning Rate

```python
for lr, decay, fn_name in zip(lrs, decays, fn_labels):
    print(f"{fn_name} | initial learning rate = {lr} | decay={decay}")
    plot_lr(lr=lr, iterations=iterations, decay=decay)
    print()
```

```
MAE | initial learning rate = 0.26 | decay=0
```



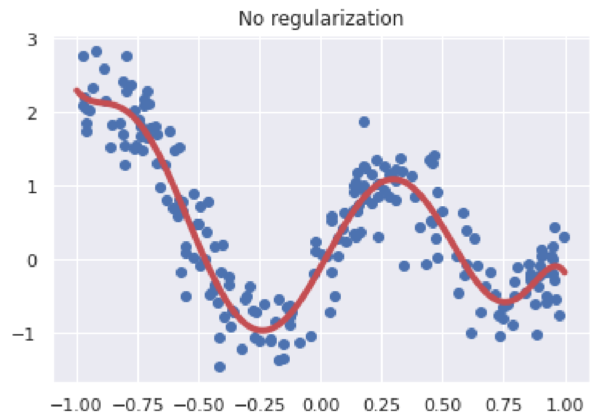

```
RMSE | initial learning rate = 1.4 | decay=0
```

Part 5 - Normal Equation

```python
# preprocess data (univariate non-linear to multivariate linear)
X_train_copy = X_train.copy()
X_valid_copy = X_valid.copy()
polynomial_to_linear_regression(X_train_copy, 8)
polynomial_to_linear_regression(X_valid_copy, 8)
```

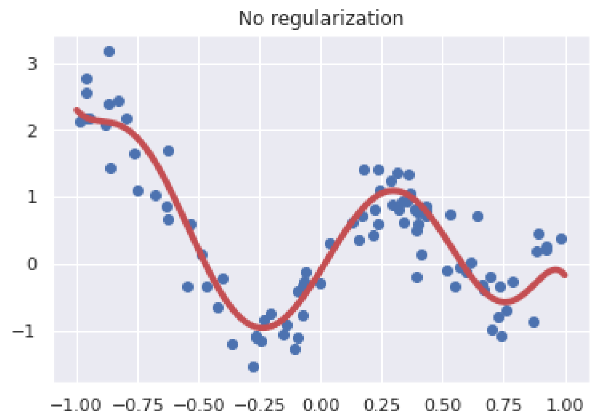
```python
normal_theta = normalEquation(X_train_copy, y_train)
```

```python
# Plotting the fitted polynomial over training data
plot_curve(X_train_copy, y_train, normal_theta, title="No regularization")
print(f"Training Loss(RMSE) : {RMSE(X_train_copy, y_train, normal_theta)}")
```




```
Training Loss(RMSE) : 0.27755996509703385
```

```python
# Plotting the fitted polynomial over validation data
plot_curve(X_valid_copy, y_valid, normal_theta, title="No regularization")
print(f"Validation Loss : {RMSE(X_valid_copy, y_valid, normal_theta)}\n")
```




```
Validation Loss : 0.2878978107333649
```

Part 6 - Regularized Normal Equation

```python
loss_fn = RMSE # as asked in the question

reg_normal_theta1 = regularizedNormalEquation(X_train_copy, y_train, lambd=0.075)
# plot_curve(X_train, y_train, reg_normal_theta, title="lambda=0.1")
reg_normal_theta2 = regularizedNormalEquation(X_train_copy, y_train, lambd=0.75)
# plot_curve(X_train, y_train, reg_normal_theta, title="lambda=1")
reg_normal_theta3 = regularizedNormalEquation(X_train_copy, y_train, lambd=7.5)
# plot_curve(X_train, y_train, reg_normal_theta, title="lambda=4")
```

```python
names = ["λ = 0.075", "λ = 0.75", "λ = 7.5"]

training_errors = [round(loss_fn(X_train_copy, y_train, reg_normal_theta1),5),
                   round(loss_fn(X_train_copy, y_train, reg_normal_theta2),5),
                   round(loss_fn(X_train_copy, y_train, reg_normal_theta3),5)]

validation_errors =[round(loss_fn(X_valid_copy, y_valid, reg_normal_theta1),5),
                    round(loss_fn(X_valid_copy, y_valid, reg_normal_theta2),5),
                    round(loss_fn(X_valid_copy, y_valid, reg_normal_theta3),5)]

for i in range(len(names)):
  print(f"{i} - Regularized Normal Equation ({names[i]})")
  print(f"Training Loss : {training_errors[i]}")
  print(f"Validation Loss : {validation_errors[i]}\n")
```

```
0 - Regularized Normal Equation (λ = 0.075)
Training Loss : 0.33571
Validation Loss : 0.34334

1 - Regularized Normal Equation (λ = 0.75)
Training Loss : 0.43287
Validation Loss : 0.44485

2 - Regularized Normal Equation (λ = 7.5)
Training Loss : 0.51534
Validation Loss : 0.53339
```

```python
plot_normal_equations(X_train, y_train, dpi=105)
```

Thanks for reading. if you have any questions:

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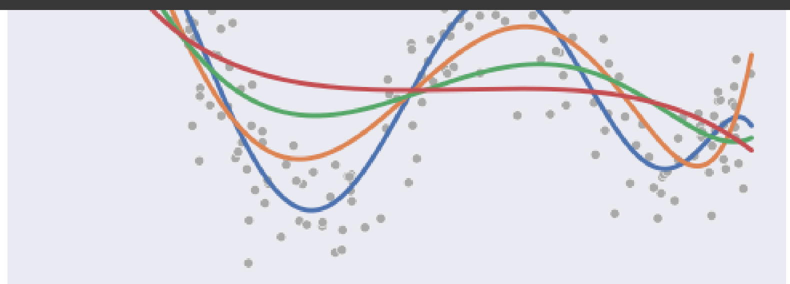
- telegram : @gholamrezadar



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