ML Exercise 1-1

Question 1

Hyper parameters

change before running the note book

learning_rate: 2.23

iterations: 3000

Show code

Loading Data

```
# Download dataset
!wget https://raw.githubusercontent.com/Gholamrezadar/machine-learning-exercises/main/dataset1/dataset1.csv
```

```
--2021-11-21 07:17:50--  https://raw.githubusercontent.com/Gholamrezadar/machine-learning-exercises/main/dataset1/datase
Resolving raw.githubusercontent.com (raw.githubusercontent.com)... 185.199.110.133, 185.199.111.133, 185.199.109.133, ..
Connecting to raw.githubusercontent.com (raw.githubusercontent.com)|185.199.110.133|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 12074 (12K) [text/plain]
Saving to: 'dataset1.csv.1'

dataset1.csv.1      100%[===================>]  11.79K  --.-KB/s    in 0s

2021-11-21 07:17:50 (81.5 MB/s) - 'dataset1.csv.1' saved [12074/12074]
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from tqdm.notebook import tqdm
import seaborn as sns

# use seaborn
sns.set()

# Load the data using pandas
df = pd.read_csv("dataset1.csv")
```

```
# Show a sample of the data
df.head()
```

	x	y
0	0.097627	0.626964
1	0.430379	0.846452
2	0.205527	0.756017
3	0.089766	0.427504
4	-0.152690	-1.335228

```
# Show a description of the data (might be useful later)
df.describe()
```

	x	y
count	300.000000	300.000000
mean	0.007005	0.412755
std	0.580948	1.021100
min	-0.990609	-1.547934
25%	-0.504657	-0.361192
50%	0.045096	0.316442
75%	0.460611	1.092441

Helper functions

Loss Functions

[ ] ↳ *1 cell hidden*

Gradient Descent

```
def gradientDescent(X, y, theta, lr, iteration, X_valid, y_valid, loss_fn = MSE, loss_fn_prim = MSE_prim, decay=0.0):
  # Training loss per iteration history
  train_loss_history = []
  # Validation loss per iteration history
  validation_loss_history = []
  # weights progression towards the optimal value
  theta_history = []

  # Progress bar
  with tqdm(total=iteration) as pbar:
    for itera in range(iteration):
      # TODO : Learning rate decay
      lr = lr * 1/(1 + decay * itera)

      for i in range(0, len(X.columns)):
        # partial derivative of loss function with respect to Xi
        gradient = loss_fn_prim(X, y, i, theta)

        # Actual "Gradient Descent" !
        theta[i] -= lr * gradient

      # Calculating the loss after each iteration
      # of updating the weights using Gradient Descent
      loss = loss_fn(X, y, theta)
      if X_valid is not None and y_valid is not None:
        validation_loss = loss_fn(X_valid, y_valid, theta)

      # Save the history of loss and weights
      train_loss_history.append(loss)
      if X_valid is not None and y_valid is not None:
        validation_loss_history.append(validation_loss)
      theta_history.append(theta.copy())

      # Update progress bar
      pbar.update(1)

  history = {"training_loss":train_loss_history,
             "validation_loss":validation_loss_history,
             "weights":theta_history}
  # returns loss history, latest loss, weights
  print(f"training_loss : {round(train_loss_history[-1],4)} | validation_loss : {round(validation_loss_history[-1],4)}")
  return history, loss, theta
```

```
# Add the polynomial's terms as features
# so that the univariate non-linear regression
# becomes a multivariate linear regression
```

```
# where every polynomial term is a feature for
# the linear regression
def polynomial_to_linear_regression(X, polynomial_degree):
  for i in range(2, 2 + polynomial_degree):
    X['x'+str(i)] = X['x']**i
```

Normal Equation

```
def normalEquation(X, y):
  # (X^T X)^-1 X^T Y
  XTX = np.dot(X.T,X)
  XTX_inverse = np.linalg.inv(XTX)
  XTY = np.dot(X.T,y)
  theta = np.dot(XTX_inverse, XTY)
  return theta

def regularizedNormalEquation(X, y, lambd=0.1):
  # (X^T X + lambda I)^-1 X^T Y
  XTX = np.dot(X.T,X) + np.dot(np.identity(X.shape[1]),lambd)
  XTX_inverse = np.linalg.inv(XTX)
  XTY = np.dot(X.T,y)
  theta = np.dot(XTX_inverse, XTY)
  return theta
```

Plotting related

```
#  helper function used to plot a polynomial
def polyCoefficients(x, coeffs):
    o = len(coeffs)
    y = 0
    for i in range(o):
        y += coeffs[i]*x**i
    return y
```

```
# Plots a polynomial on top of the original data
def plot_curve(X, y, theta, c='r', title='', resolution=200):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  plt.title(title)
  # Plot the original data
  plt.scatter(x=X['x'],y= y)

  x = np.linspace(-1, 1, resolution)
  # Plot the fitted polynomial over the data
  plt.plot(x, polyCoefficients(x, theta), color=c, linewidth=4)

  plt.show()
```

```
# Plots validation and training losses per iteration
def plot_loss(history, title='', starting_iter=0):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  fig, ax = plt.subplots()
  plt.title(title)

  # X = iterations range
  x = np.linspace(0, iterations, iterations)

  # 1. Training_loss - iteration curve (Red)
  sns.lineplot(x[starting_iter:], history["training_loss"][starting_iter:], color='r')
  # 2. Validation_loss - iteration curve (green)
  sns.lineplot(x[starting_iter:], history["validation_loss"][starting_iter:], color='g')

  ax.legend(labels=["training", "validation"])
```

```
plt.show()
```

```
def plot_lr(lr=0.1, iterations=1000, decay=None, title='learning_rate'):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  _lrs = []
  # Iterations
  _iterations = iterations
  # Initial lr
  _lr0 = lr
  _decay = 0
  # Decay
  if decay is None:
    _decay = _lr0/_iterations
  else:
    _decay = decay

  # Simulate gradient descents main loop
  _lr = _lr0
  for i in range(_iterations):
    _lr = _lr * 1/(1 + _decay * i)
    _lrs.append(_lr)

  _x = list(range(_iterations))
  _y = _lrs

  plt.figure()
  plt.title(title)
  plt.plot(_x, _y)
  plt.show()
```

```
def plot_every_curve(X, y, thetas, resolution=100):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  fig, axes = plt.subplots(3, 3, sharex=True, sharey=True, figsize=(16,10), constrained_layout=True)
  # fig.subplots_adjust(hspace=0.6)

  # Helper lists for accessing the current config
  fn_labels = ["MAE", "RMSE", "MSE"]
  deg_labels = ["degree 10","degree 8","degree 5"]

  # Used to plot the fitted polynomial in the range[-1,1]
  x = np.linspace(-1, 1, resolution)

  # Plotting every case in a 3 by 3 grid
  for i in range(3):
    for j in range(3):

      # 1. plot the original data (Blue)
      sns.scatterplot(x=X["x"], y=y, ax=axes[i,j])

      # 2. Plot a curve with Last iteration theta [0] (Red)
      theta = thetas[i][j][0]
      sns.lineplot(x, polyCoefficients(x, theta), color='r', ax=axes[i,j])

      # 3. Plot a curve with Middle iteration theta [1] (Green)
      theta = thetas[i][j][1]
      sns.lineplot(x, polyCoefficients(x, theta), color='g', ax=axes[i,j])

      # Legends and titles
      axes[i,j].legend(labels=[f"{iterations} iter", f"{iterations//2} iter"])
      axes[i,j].set_title(f"train_loss:{round(losses[i][j][0],3)}   valid_loss:{round(losses[i][j][1],3)}")

      # Matplotlib related code
      axes[i,j].xaxis.set_ticklabels([])
```

```
    axes[i,j].yaxis.set_ticklabels([])
    axes[i,j].set_xlabel(fn_labels[j])
    axes[i,j].set_ylabel(deg_labels[i])

  plt.show()
```

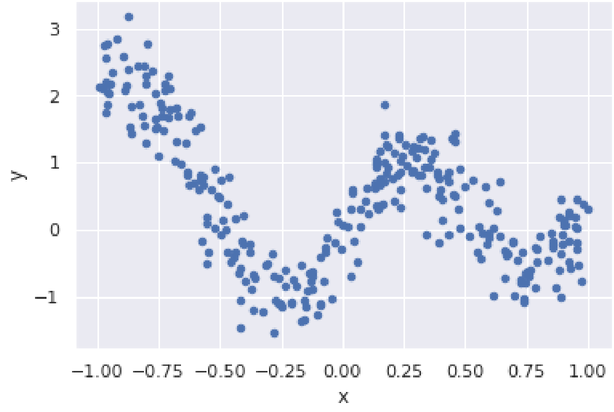
```
def plot_every_case_loss(histories, starting_iter=0):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  fig, axes = plt.subplots(3, 3, sharex=True, sharey=False, figsize=(16,10), constrained_layout=True)

  # Helper lists for accessing the current config
  fn_labels = ["MAE", "RMSE", "MSE"]
  deg_labels = ["degree 10","degree 8","degree 5"]

  # X = iterations range
  x = np.linspace(0, iterations, iterations)

  # Plotting every case in a 3 by 3 grid
  for i in range(3):
    for j in range(3):

      # 1. Training_loss - iteration curve (Red)
      sns.lineplot(x[starting_iter:], histories[i][j][0]["training_loss"][starting_iter:], color='r', ax=axes[i,j])
      # 2. Validation_loss - iteration curve (green)
      sns.lineplot(x[starting_iter:], histories[i][j][0]["validation_loss"][starting_iter:], color='g', ax=axes[i,j])

      # Legends
      axes[i,j].legend(labels=[f"training loss", f"validation loss"])

      # Matplotlib related code
      axes[i,j].set_xlabel(fn_labels[j])
      axes[i,j].set_ylabel(deg_labels[i])

  plt.show()
```

```
  # Plots a polynomial on top of the original data
  def plot_normal_equations(X, y, c='r', title='', resolution=200):
    # Don't try this at home
    import warnings
    warnings.simplefilter(action="ignore", category=FutureWarning)

    fig, ax = plt.subplots()
    plt.title(title)
    fig.dpi=120

    # Plot the original data
    sns.scatterplot(x=X['x'], y=y, size=1, color='darkgray')

    x = np.linspace(-1, 1, resolution)

    # calculate theta for each method
    normal_theta = normalEquation(X_train, y_train)
    reg_normal_theta1 = regularizedNormalEquation(X_train, y_train, lambd=0.075)
    reg_normal_theta2 = regularizedNormalEquation(X_train, y_train, lambd=0.75)
    reg_normal_theta4 = regularizedNormalEquation(X_train, y_train, lambd=7.5)

    # Plot the fitted polynomial over the data
    plt.plot(x, polyCoefficients(x, normal_theta), linewidth=2)
    plt.plot(x, polyCoefficients(x, reg_normal_theta1), linewidth=2)
    plt.plot(x, polyCoefficients(x, reg_normal_theta2), linewidth=2)
    plt.plot(x, polyCoefficients(x, reg_normal_theta4), linewidth=2)

    ax.legend(labels=["No Regularization", "λ=0.075", "λ=0.75", "λ=7.5"])
    ax.xaxis.set_ticklabels([])
    ax.yaxis.set_ticklabels([])
    ax.xaxis.set_visible(False)
    ax.yaxis.set_visible(False)
    plt.show()
```

Part 1 - Plotting the data

```
# Plot the data using matplotlib
df.plot(kind='scatter', x='x', y='y')
plt.show()
```

```
*c* argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have preced
```



Part 2 - Shuffle

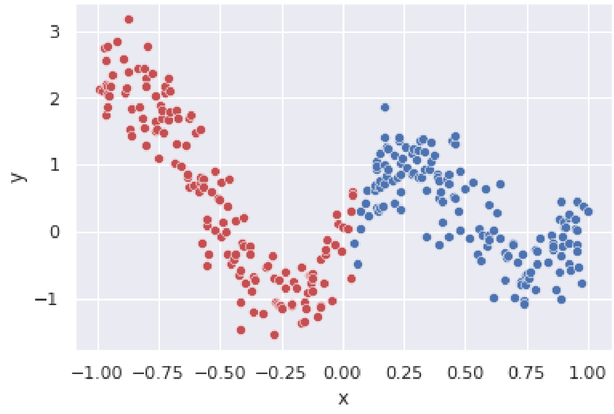
```
def plot_colorize(df):
    '''
    Assigns 'red' color to the first half of the data
    and 'blue' to the rest

    If the data is well shuffled we should see random red
    and blue circles everywhere.

    If the data is NOT well shuffled we might see a pattern between
    circles' position and their color.
    '''

    df_red = df.loc[df.index<df.shape[0]/2]
    df_blue = df.loc[df.index>=df.shape[0]/2]

    sns.scatterplot(data=df_red, x='x', y='y', color='r')
    sns.scatterplot(data=df_blue, x='x', y='y', color='b')

    plt.show()
```

Default data

```
plot_colorize(df)
print(" "*9,"Data seems to be well shuffled.")
```

Sorted data

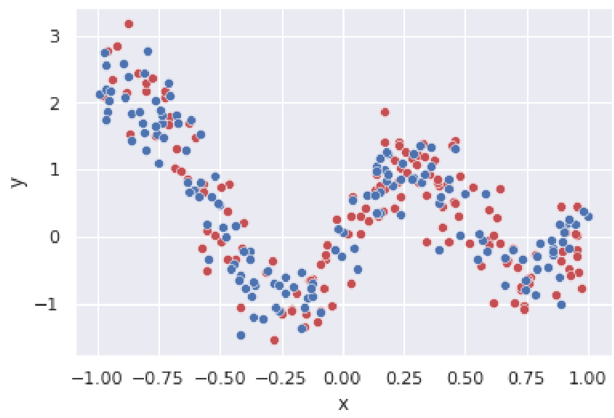
```
# Sort the data based on 'x' first, then do the
# previous part to see the result.
sorted_df = df.sort_values(by='x', ascending=True, ignore_index=True)

plot_colorize(sorted_df)
print(" "*6,"Data doesn't seem to be well shuffled.")
```




```
       Data doesn't seem to be well shuffled.
```

Shuffled data

```
# Let's shuffle the data anyways (just in case)

# pandas Doc: specifying drop=True prevents .reset_index()
# from creating a column containing the old index entries.
shuffled_df = df.sample(frac=1).reset_index(drop=True)

plot_colorize(shuffled_df)
print(" "*9,"Data seems to be well shuffled.")
```




```
          Data seems to be well shuffled.
```

Part 3 - Gradient Descent

Finding The optimal "Theta" values (weights)

Data Prepration

Add a column for bias

```
# Add a new column for simplicity of the calculations
# acts as the bias term
shuffled_df = pd.concat([pd.Series(1, index=shuffled_df.index, name='0'), shuffled_df], axis=1)
shuffled_df.head()
```

	0	x	y
0	1	-0.582246	0.725637
1	1	-0.726199	2.085352
2	1	0.561058	-0.445023
3	1	0.530651	0.739235
4	1	0.395262	0.760835

Seperate X,y

```
# Split training data into X and y
X = shuffled_df.drop(columns="y")
y = shuffled_df.iloc[:, 2]

print(X.head(),end="\n\n")
print(y.head())
```

```
   0         x
0  1 -0.582246
1  1 -0.726199
2  1  0.561058
3  1  0.530651
4  1  0.395262

0    0.725637
1    2.085352
2   -0.445023
3    0.739235
4    0.760835
Name: y, dtype: float64
```

```
# Split to train and valid
split = 0.7

X_train = X.iloc[ : int(len(X)*split),:].reset_index(drop=True)
X_valid = X.iloc[int(len(X)*split) : ,:].reset_index(drop=True)

y_train = y.iloc[ : int(len(X)*split)].reset_index(drop=True)
y_valid = y.iloc[int(len(X)*split) : ].reset_index(drop=True)

print(f"Train X size = {len(X_train)}")
print(f"Train y size = {len(y_train)}")
print(f"Valid X size = {len(X_valid)}")
print(f"Valid y size = {len(y_valid)}")
```

```
Train X size = 210
Train y size = 210
Valid X size = 90
Valid y size = 90
```

```
# Save a copy of X and y
# TODO might not need it
X_train_org = X_train.copy()
y_train_org = y_train.copy()
```

Polynomial Regression

a basic example

polynomial_degree: 10

Show code

Convert

aX + bX^2 + cX^3 + d

to

aX1 + bX2 + cX3 + d

```
# Add the polynomial's terms as features
# so that the univariate non-linear regression
# becomes a multivariate linear regression
# where every polynomial term is a feature for
# the linear regression
polynomial_to_linear_regression(X_train, polynomial_degree)
polynomial_to_linear_regression(X_valid, polynomial_degree)

X_train.head()
```

	0	x	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11
0	1	-0.582246	0.339011	-0.197388	0.114928	-0.066917	0.038962	-0.022685	0.013209	-0.007691	0.004478	-0.002607
1	1	-0.726199	0.527366	-0.382973	0.278115	-0.201967	0.146668	-0.106510	0.077348	-0.056170	0.040791	-0.029622
2	1	0.561058	0.314786	0.176614	0.099091	0.055596	0.031192	0.017501	0.009819	0.005509	0.003091	0.001734
3	1	0.530651	0.281590	0.149426	0.079293	0.042077	0.022328	0.011848	0.006287	0.003336	0.001770	0.000939
4	1	0.395262	0.156232	0.061753	0.024409	0.009648	0.003813	0.001507	0.000596	0.000235	0.000093	0.000037

Training

Normal equation for comparison :

- 0.07 training loss
- 0.09 validation loss

5k iter 10th degree polynomial lr=2.3

- Training Loss : 0.07766969752936674
- Validation Loss : 0.094857479611961

```
# Initialize the weights with zero
theta = np.array([0.0]*len(X_train.columns))

# Initialize the weights with random values
theta = np.random.rand(len(X_train.columns),)

print("notice : takes approximately 3 minutes for 5k iters")

# tip : nice way to find decay (https://machinelearningmastery.com/using-learning-rate-schedules-deep-learning-models-pyth
# Decay = LearningRate / Epochs
# Decay = 0.1 / 1000
# Decay = 0.0001
decay = learning_rate/iterations/11250


# For testing purposes
# iterations = 500
# learning_rate = 2.3
# decay = learning_rate/iterations/250

# Start the training
history, loss, theta = gradientDescent(X_train,
                                       y_train,
                                       theta,
                                       learning_rate,
                                       iterations,
                                       X_valid = X_valid,
                                       y_valid = y_valid,
                                       loss_fn=MSE,
                                       loss_fn_prim=MSE_prim,
                                       decay = decay)


```

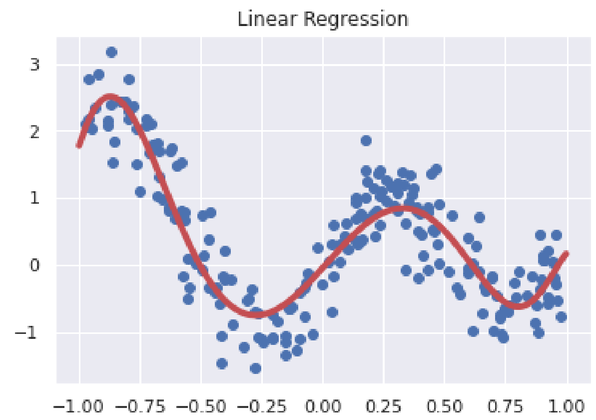
```
notice : takes approximately 3 minutes for 5k iters
100%                                              3000/3000 [01:42<00:00, 28.61it/s]
training_loss : 0.0974 | validation_loss : 0.0851
```

Plotting the fitted polynomials

```python
# Predicting using the learned weights(theta)
# Not used here but useful
y_hat = theta*X_valid
y_hat = np.sum(y_hat, axis=1)
print(MSE(X_valid, y_valid, theta))
```

```
0.0851166629303032
```

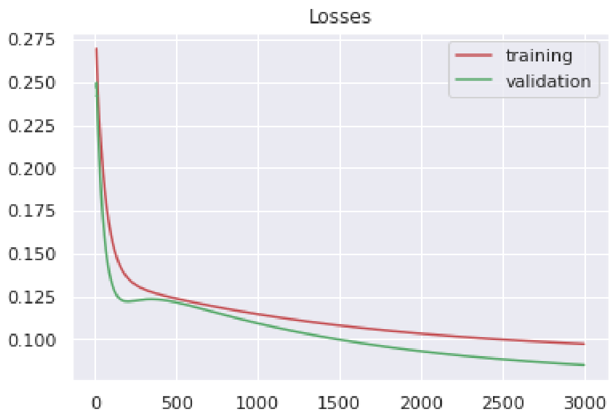
```python
plot_curve(X_train, y_train, theta, title='Linear Regression')

print(f"Training Loss : {MSE(X_train, y_train, theta)}")
print(f"Validation Loss : {MSE(X_valid, y_valid, theta)}\n")
```




```
Training Loss : 0.09736143022197676
Validation Loss : 0.0851166629303032
```

Plotting losses

```python
plot_loss(history, starting_iter=10, title='Losses')
```



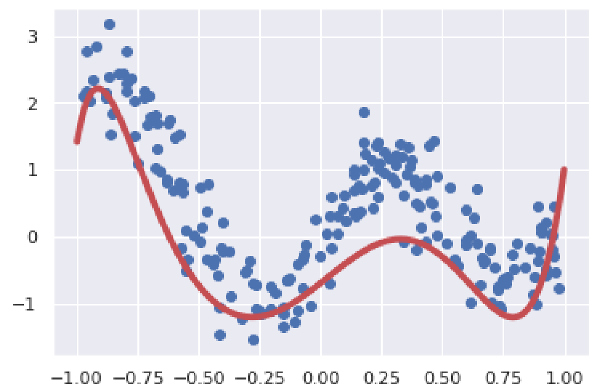
Plotting the learning rate

```python
plot_lr(lr=learning_rate, iterations=iterations, decay=decay)
```

Interactive history viewer

```
from ipywidgets import interact
import ipywidgets as widgets

@interact(iteration = widgets.IntSlider(min=0, max=iterations-1, step=100, value=0))
def plot_weight_history(iteration):
  plot_curve(X_train, y_train, history["weights"][iteration])

print("Use the slider to see the algorithms progression")
```

iteration 600




```
Use the slider to see the algorithms progression
```

Part 4 - Plotting every Case!

Polynomial Degree :

- 5
- 8
- 10

Loss Functions :

- MSE
- RMSE
- MAE

Iterations :

- 5000
- 10000

```
# iterations = 100
polynomial_degrees = [5, 8, 10]
loss_functions = [(MSE, MSE_prim),
                  (RMSE, RMSE_prim),
                  (MAE, MAE_prim)]
fn_labels = ["MAE", "RMSE", "MSE"]
```

Training all 9 models

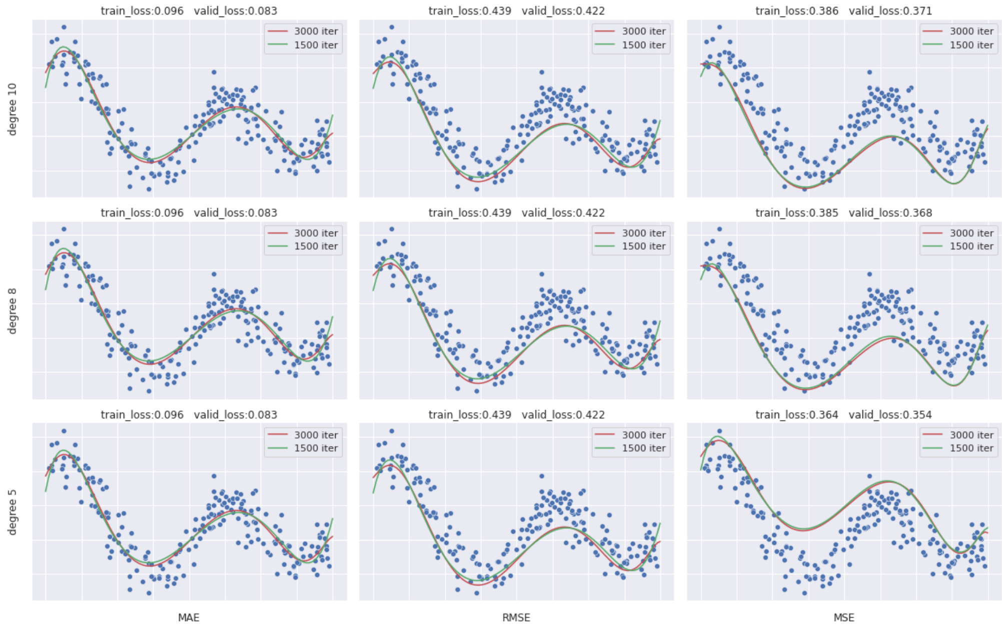
```
# Containers used for storing results about each model
# Used later by the plotting functions
thetas = [[[],[],[]],
```

```
          [[],[],[]],
          [[],[],[]]]

losses = [[[],[],[]],
          [[],[],[]],
          [[],[],[]]]

histories = [[[],[],[]],
             [[],[],[]],
             [[],[],[]]]

# start the training process for each model
for i,degree in enumerate(polynomial_degrees):
  for j, (loss_fn, loss_fn_prim) in enumerate(loss_functions):
    print(f"degree: {degree} | loss function: {fn_labels[j]}")

    # preprocess data (univariate non-linear to multivariate linear)
    X_train_copy = X_train.copy()
    X_valid_copy = X_valid.copy()
    polynomial_to_linear_regression(X_train_copy, degree)
    polynomial_to_linear_regression(X_valid_copy, degree)

    # Initialize the weights with random values
    theta = np.random.rand(len(X_train.columns),)

    # Start the training
    history, loss, theta = gradientDescent(X_train_copy,
                                           y_train,
                                           theta,
                                           learning_rate,
                                           iterations,
                                           X_valid = X_valid_copy,
                                           y_valid = y_valid,
                                           loss_fn=loss_fn,
                                           loss_fn_prim=loss_fn_prim,
                                           decay = 0.0)

    # Saving latest iteration's theta for each model
    thetas[i][j].append(history["weights"][-1])
    # Saving halfway theta for each model
    thetas[i][j].append(history["weights"][int(iterations/2)-1])

    # Saving Training loss for each model
    losses[i][j].append(loss_fn(X_train_copy, y_train, theta))
    # Saving Validation loss for each model
    losses[i][j].append(loss_fn(X_valid_copy, y_valid, theta))
    # Saving Histories for each model (used for plotting loss per iteration)
    histories[i][j].append(history)

    print()
```

```
degree: 5 | loss function: MAE
```
100% 3000/3000 [01:43<00:00, 29.71it/s]
```
training_loss : 0.0957 | validation_loss : 0.083

degree: 5 | loss function: RMSE
```
100% 3000/3000 [03:05<00:00, 16.16it/s]
```
training_loss : 0.4388 | validation_loss : 0.422

degree: 5 | loss function: MSE
```
100% 3000/3000 [01:53<00:00, 26.02it/s]
```
training_loss : 0.3856 | validation_loss : 0.3707

degree: 8 | loss function: MAE
```
100% 3000/3000 [01:41<00:00, 30.32it/s]
```
training_loss : 0.0959 | validation_loss : 0.0832
```

```
plot_every_curve(X_train, y_train, thetas)
```

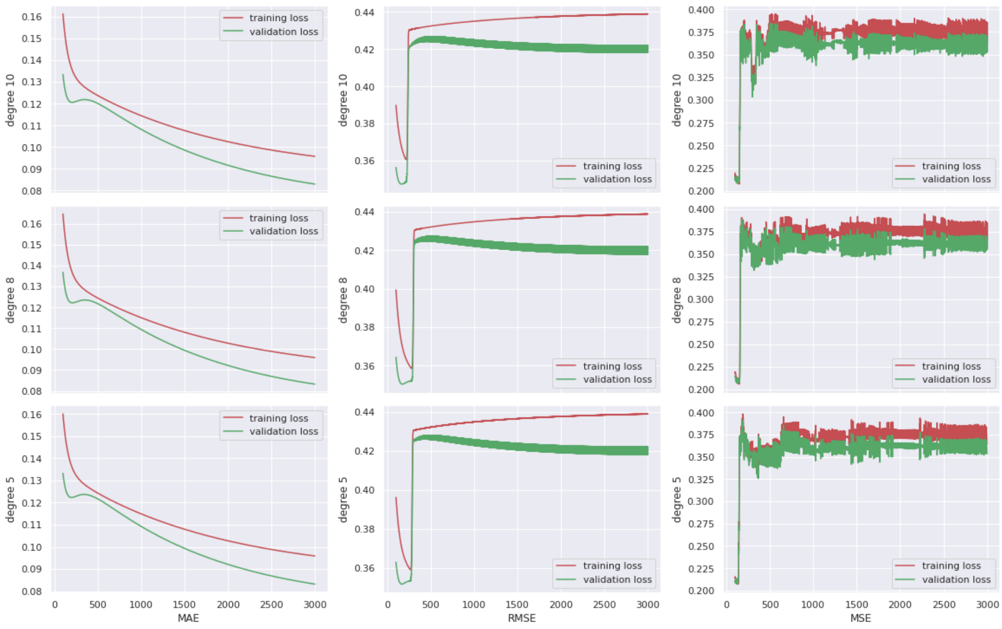
```
<Figure size 432x288 with 0 Axes>
```



Plotting train/valid loss

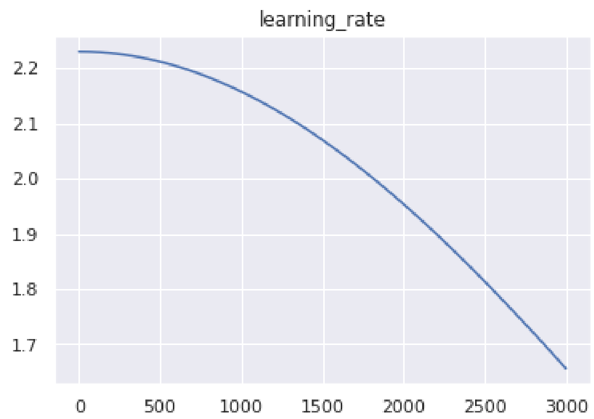
```
plot_every_case_loss(histories, starting_iter=100)
```

```
<Figure size 432x288 with 0 Axes>
```



Plotting Learning Rate

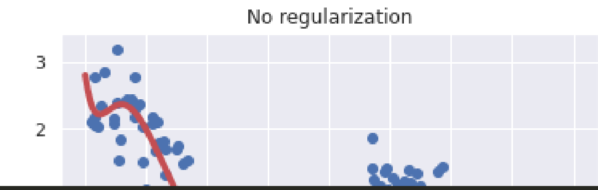
```
plot_lr(lr=learning_rate, iterations=iterations, decay=decay)
```



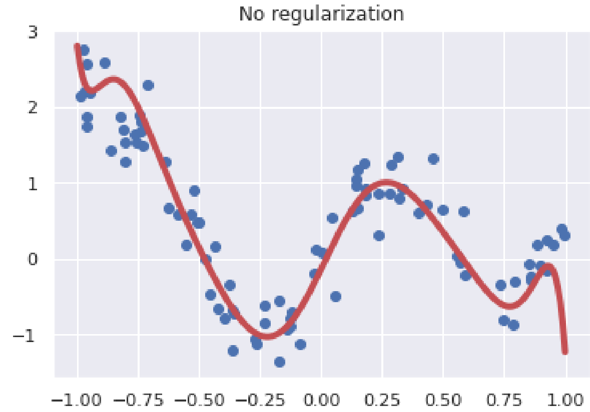
Part 5 - Normal Equation

```
normal_theta = normalEquation(X_train, y_train)
```

```
# Plotting the fitted polynomial over training data
plot_curve(X_train, y_train, normal_theta, title="No regularization")
print(f"Training Loss : {RMSE(X_train, y_train, normal_theta)}")
```




```python
# Plotting the fitted polynomial over validation data
plot_curve(X_valid, y_valid, normal_theta, title="No regularization")
print(f"Validation Loss : {RMSE(X_valid, y_valid, normal_theta)}\n")
```




```
Validation Loss : 0.28624132493466486
```

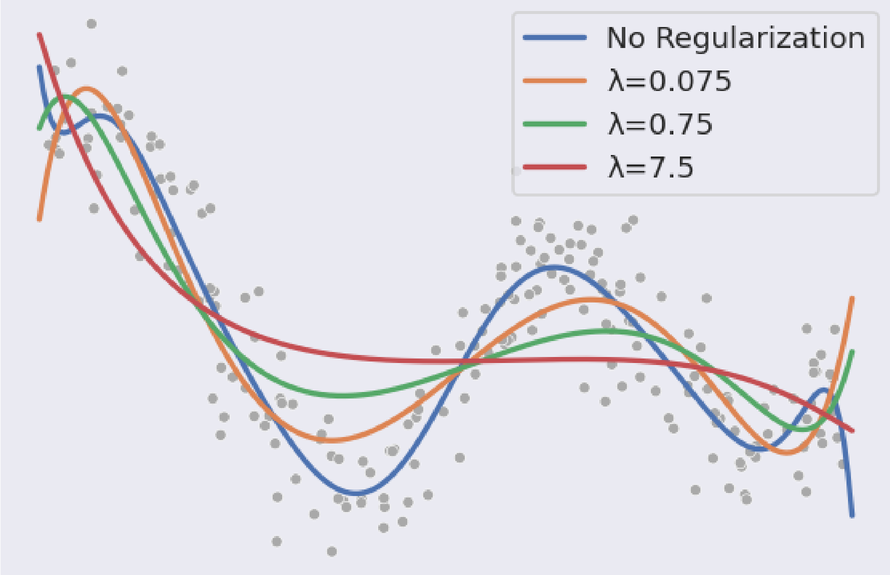
Part 6 - Regularized Normal Equation

```python
loss_fn = RMSE # as asked in the question

reg_normal_theta1 = regularizedNormalEquation(X_train, y_train, lambd=0.075)
# plot_curve(X_train, y_train, reg_normal_theta, title="lambda=0.1")
reg_normal_theta2 = regularizedNormalEquation(X_train, y_train, lambd=0.75)
# plot_curve(X_train, y_train, reg_normal_theta, title="lambda=1")
reg_normal_theta3 = regularizedNormalEquation(X_train, y_train, lambd=7.5)
# plot_curve(X_train, y_train, reg_normal_theta, title="lambda=4")
```

```python
names = ["λ = 0.075", "λ = 0.75", "λ = 7.5"]

training_errors = [round(loss_fn(X_train, y_train, reg_normal_theta1),5),
                   round(loss_fn(X_train, y_train, reg_normal_theta2),5),
                   round(loss_fn(X_train, y_train, reg_normal_theta3),5)]

validation_errors =[round(loss_fn(X_valid, y_valid, reg_normal_theta1),5),
                    round(loss_fn(X_valid, y_valid, reg_normal_theta2),5),
                    round(loss_fn(X_valid, y_valid, reg_normal_theta3),5)]

for i in range(len(names)):
  print(f"{i} - Regularized Normal Equation ({names[i]})")
  print(f"Training Loss : {training_errors[i]}")
  print(f"Validation Loss : {validation_errors[i]}\n")
```

```
0 - Regularized Normal Equation (λ = 0.075)
Training Loss : 0.34547
Validation Loss : 0.32648

1 - Regularized Normal Equation (λ = 0.75)
Training Loss : 0.42381
Validation Loss : 0.38665

2 - Regularized Normal Equation (λ = 7.5)
Training Loss : 0.52674
Validation Loss : 0.49811
```

```python
plot_normal_equations(X_train, y_train)
```



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