ML Exercise 1-1

Question 1

Hyper parameters

change before running the note book

learning_rate: 2.23

iterations: 5000

Show code

Loading Data

```
# Download dataset
!wget https://raw.githubusercontent.com/Gholamrezadar/machine-learning-exercises/main/dataset1/dataset1.csv
```

```
--2021-11-21 09:16:49--  https://raw.githubusercontent.com/Gholamrezadar/machine-learning-exercises/main/dataset1/datase
Resolving raw.githubusercontent.com (raw.githubusercontent.com)... 185.199.110.133, 185.199.111.133, 185.199.108.133, ..
Connecting to raw.githubusercontent.com (raw.githubusercontent.com)|185.199.110.133|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 12074 (12K) [text/plain]
Saving to: ‘dataset1.csv.11’

dataset1.csv.11     100%[===================>]  11.79K  --.-KB/s    in 0s

2021-11-21 09:16:49 (110 MB/s) - ‘dataset1.csv.11’ saved [12074/12074]
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from tqdm.notebook import tqdm
import seaborn as sns

# use seaborn
sns.set()

# Load the data using pandas
df = pd.read_csv("dataset1.csv")
```

```
# Show a sample of the data
df.head()
```

	x	y
0	0.097627	0.626964
1	0.430379	0.846452
2	0.205527	0.756017
3	0.089766	0.427504
4	-0.152690	-1.335228

```
# Show a description of the data (might be useful later)
df.describe()
```

	x	y
count	300.000000	300.000000
mean	0.007005	0.412755
std	0.580948	1.021100
min	-0.990609	-1.547934
25%	-0.504657	-0.361192
50%	0.045096	0.316442
75%	0.460611	1.092441

Helper functions

Loss Functions

```
# h(theta) =  theta transpose * X
def hypothesis(X, theta):
  y1 = theta*X
  return np.sum(y1, axis=1)

def MSE(X, y, theta):
  y_hat = hypothesis(X, theta)
  m = len(X)
  return sum((y_hat-y)**2)/(2*m)

def RMSE(X, y, theta):
  y_hat = hypothesis(X, theta)
  m = len(X)
  return np.sqrt(sum((y_hat-y)**2)/(2*m))

def MAE(X, y, theta):
  y_hat = hypothesis(X, theta)
  m = len(X)
  return sum(np.abs((y_hat-y)))/(2*m)

# Loss functions Derivatives
def MSE_prim(X, y, i, theta):
  y_hat = hypothesis(X, theta)
  Xi = X.iloc[:, i]
  m = len(X)
  return sum((y_hat-y) * Xi) / m

def RMSE_prim(X, y, i, theta):
  # src : https://math.stackexchange.com/questions/4065532/rmse-derivatives
  mse = MSE(X, y, theta)
  mse_prim = MSE_prim(X, y, i, theta)

  return mse_prim / 2 / np.sqrt(mse)

def MAE_prim(X, y, i, theta):
  # src : https://stats.stackexchange.com/questions/312737/mean-absolute-error-mae-derivative
  # src2 : https://github.com/chenxingwei/machine_learning_from_scratch/blob/master/algorithm/2.linearRegressionGradientDe
  y_hat = hypothesis(X, theta)
  # print(np.sum((X.T*(np.sign(y_hat-y)/len(X))), axis=1)[i])
  return np.sum((X.T*(np.sign(y_hat-y)/len(X))), axis=1)[i]

```

Gradient Descent

```
def gradientDescent(X, y, theta, lr, iteration, X_valid, y_valid, loss_fn = MSE, loss_fn_prim = MSE_prim, decay=0.0):
  # Training loss per iteration history
  train_loss_history = []
  # Validation loss per iteration history
  validation_loss_history = []
  # weights progression towards the optimal value
```

```
    theta_history = []

    # Progress bar
    with tqdm(total=iteration) as pbar:
      for itera in range(iteration):
        # TODO : Learning rate decay
        lr = lr * 1/(1 + decay * itera)

        for i in range(0, len(X.columns)):
          # partial derivative of loss function with respect to Xi
          gradient = loss_fn_prim(X, y, i, theta)

          # Actual "Gradient Descent" !
          theta[i] -= lr * gradient

        # Calculating the loss after each iteration
        # of updating the weights using Gradient Descent
        loss = loss_fn(X, y, theta)
        if X_valid is not None and y_valid is not None:
          validation_loss = loss_fn(X_valid, y_valid, theta)

        # Save the history of loss and weights
        train_loss_history.append(loss)
        if X_valid is not None and y_valid is not None:
          validation_loss_history.append(validation_loss)
        theta_history.append(theta.copy())

        # Update progress bar
        pbar.update(1)

    history = {"training_loss":train_loss_history,
               "validation_loss":validation_loss_history,
               "weights":theta_history}
    # returns loss history, latest loss, weights
    print(f"training_loss : {round(train_loss_history[-1],4)} | validation_loss : {round(validation_loss_history[-1],4)}")
    return history, loss, theta
```

```
# Add the polynomial's terms as features
# so that the univariate non-linear regression
# becomes a multivariate linear regression
# where every polynomial term is a feature for
# the linear regression
def polynomial_to_linear_regression(X, polynomial_degree):
  for i in range(2, 1 + polynomial_degree):
    X['x'+str(i)] = X['x']**i
```

Normal Equation

```
def normalEquation(X, y):
  # (X^T X)^-1 X^T Y
  XTX = np.dot(X.T,X)
  XTX_inverse = np.linalg.inv(XTX)
  XTY = np.dot(X.T,y)
  theta = np.dot(XTX_inverse, XTY)
  return theta

def regularizedNormalEquation(X, y, lambd=0.1):
  # (X^T X + lambda I)^-1 X^T Y
  XTX = np.dot(X.T,X) + np.dot(np.identity(X.shape[1]),lambd)
  XTX_inverse = np.linalg.inv(XTX)
  XTY = np.dot(X.T,y)
  theta = np.dot(XTX_inverse, XTY)
  return theta
```

Plotting related

```
#  helper function used to plot a polynomial
def polyCoefficients(x, coeffs):
    o = len(coeffs)
```

```
    y = 0
    for i in range(o):
        y += coeffs[i]*x**i
    return y
```

```
# Plots a polynomial on top of the original data
def plot_curve(X, y, theta, c='r', title='', resolution=200):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  plt.title(title)
  # Plot the original data
  plt.scatter(x=X['x'],y= y)

  x = np.linspace(-1, 1, resolution)
  # Plot the fitted polynomial over the data
  plt.plot(x, polyCoefficients(x, theta), color=c, linewidth=4)

  plt.show()
```

```
# Plots validation and training losses per iteration
def plot_loss(history, title='', starting_iter=0):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  fig, ax = plt.subplots()
  plt.title(title)

  # X = iterations range
  x = np.linspace(0, iterations, iterations)

  # 1. Training_loss - iteration curve (Red)
  sns.lineplot(x[starting_iter:], history["training_loss"][starting_iter:], color='r')
  # 2. Validation_loss - iteration curve (green)
  sns.lineplot(x[starting_iter:], history["validation_loss"][starting_iter:], color='g')

  ax.legend(labels=["training", "validation"])

  plt.show()
```

```
def plot_lr(lr=0.1, iterations=1000, decay=None, title='learning_rate'):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  _lrs = []
  # Iterations
  _iterations = iterations
  # Initial lr
  _lr0 = lr
  _decay = 0
  # Decay
  if decay is None:
    _decay = _lr0/_iterations
  else:
    _decay = decay

  # Simulate gradient descents main loop
  _lr = _lr0
  for i in range(_iterations):
    _lr = _lr * 1/(1 + _decay * i)
    _lrs.append(_lr)

  _x = list(range(_iterations))
  _y = _lrs

  plt.figure()
  plt.title(title)
```

```python
  plt.plot(_x, _y)
  plt.show()
```

```python
def plot_every_curve(X, y, thetas, resolution=100):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  fig, axes = plt.subplots(3, 3, sharex=True, sharey=True, figsize=(16,10), constrained_layout=True)
  # fig.subplots_adjust(hspace=0.6)

  # Helper lists for accessing the current config
  fn_labels = ["MAE", "RMSE", "MSE"]
  deg_labels = ["degree 10","degree 8","degree 3"]

  # Used to plot the fitted polynomial in the range[-1,1]
  x = np.linspace(-1, 1, resolution)

  # Plotting every case in a 3 by 3 grid
  for i in range(3):
    for j in range(3):

      # 1. plot the original data (Blue)
      sns.scatterplot(x=X["x"], y=y, ax=axes[i,j])

      # 2. Plot a curve with Last iteration theta [0] (Red)
      theta = thetas[i][j][0]
      sns.lineplot(x, polyCoefficients(x, theta), color='r', ax=axes[i,j])

      # 3. Plot a curve with Middle iteration theta [1] (Green)
      theta = thetas[i][j][1]
      sns.lineplot(x, polyCoefficients(x, theta), color='g', ax=axes[i,j])

      # Legends and titles
      axes[i,j].legend(labels=[f"{iterations} iter", f"{iterations//2} iter"])
      axes[i,j].set_title(f"train_loss:{round(losses[i][j][0],3)}   valid_loss:{round(losses[i][j][1],3)}")

      # Matplotlib related code
      axes[i,j].xaxis.set_ticklabels([])
      axes[i,j].yaxis.set_ticklabels([])
      axes[i,j].set_xlabel(fn_labels[j])
      axes[i,j].set_ylabel(deg_labels[i])

  plt.show()
```

```python
def plot_every_case_loss(histories, starting_iter=0):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  plt.figure()
  fig, axes = plt.subplots(3, 3, sharex=True, sharey=False, figsize=(16,10), constrained_layout=True)

  # Helper lists for accessing the current config
  fn_labels = ["MAE", "RMSE", "MSE"]
  deg_labels = ["degree 10","degree 8","degree 3"]

  # X = iterations range
  x = np.linspace(0, iterations, iterations)

  # Plotting every case in a 3 by 3 grid
  for i in range(3):
    for j in range(3):

      # 1. Training_loss - iteration curve (Red)
      sns.lineplot(x[starting_iter:], histories[i][j][0]["training_loss"][starting_iter:], color='r', ax=axes[i,j])
      # 2. Validation_loss - iteration curve (green)
      sns.lineplot(x[starting_iter:], histories[i][j][0]["validation_loss"][starting_iter:], color='g', ax=axes[i,j])

      # Legends
      axes[i,j].legend(labels=[f"training loss", f"validation loss"])

```

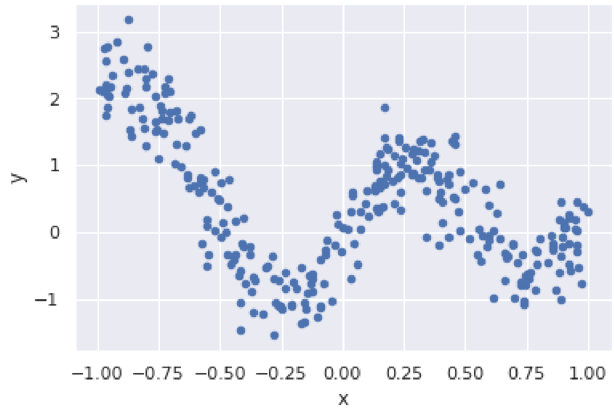
```
    # Matplotlib related code
    axes[i,j].set_xlabel(fn_labels[j])
    axes[i,j].set_ylabel(deg_labels[i])

plt.show()
```

```
# Plots a polynomial on top of the original data
def plot_normal_equations(X, y, c='r', title='', resolution=200):
  # Don't try this at home
  import warnings
  warnings.simplefilter(action="ignore", category=FutureWarning)

  fig, ax = plt.subplots()
  plt.title(title)
  fig.dpi=120

  # Plot the original data
  sns.scatterplot(x=X['x'], y=y, size=1, color='darkgray')

  x = np.linspace(-1, 1, resolution)

  # calculate theta for each method
  normal_theta = normalEquation(X_train_copy, y_train)
  reg_normal_theta1 = regularizedNormalEquation(X_train_copy, y_train, lambd=0.075)
  reg_normal_theta2 = regularizedNormalEquation(X_train_copy, y_train, lambd=0.75)
  reg_normal_theta4 = regularizedNormalEquation(X_train_copy, y_train, lambd=7.5)

  # Plot the fitted polynomial over the data
  plt.plot(x, polyCoefficients(x, normal_theta), linewidth=2)
  plt.plot(x, polyCoefficients(x, reg_normal_theta1), linewidth=2)
  plt.plot(x, polyCoefficients(x, reg_normal_theta2), linewidth=2)
  plt.plot(x, polyCoefficients(x, reg_normal_theta4), linewidth=2)

  ax.legend(labels=["No Regularization", "λ=0.075", "λ=0.75", "λ=7.5"])
  ax.xaxis.set_ticklabels([])
  ax.yaxis.set_ticklabels([])
  ax.xaxis.set_visible(False)
  ax.yaxis.set_visible(False)
  plt.show()
```

Part 1 - Plotting the data

```
# Plot the data using matplotlib
df.plot(kind='scatter', x='x', y='y')
plt.show()
```

```
*c* argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have preced
```



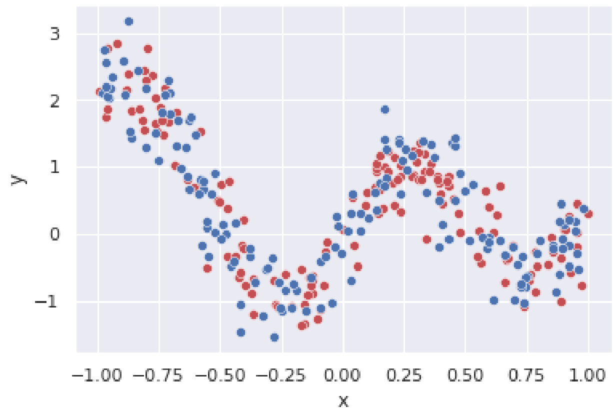
Part 2 - Shuffle

```
def plot_colorize(df):
  '''
  Assigns 'red' color to the first half of the data
```

```
and 'blue' to the rest

If the data is well shuffled we should see random red
and blue circles everywhere.

If the data is NOT well shuffled we might see a pattern between
circles' position and their color.
'''

df_red = df.loc[df.index<df.shape[0]/2]
df_blue = df.loc[df.index>=df.shape[0]/2]

sns.scatterplot(data=df_red, x='x', y='y', color='r')
sns.scatterplot(data=df_blue, x='x', y='y', color='b')

plt.show()
```

Default data

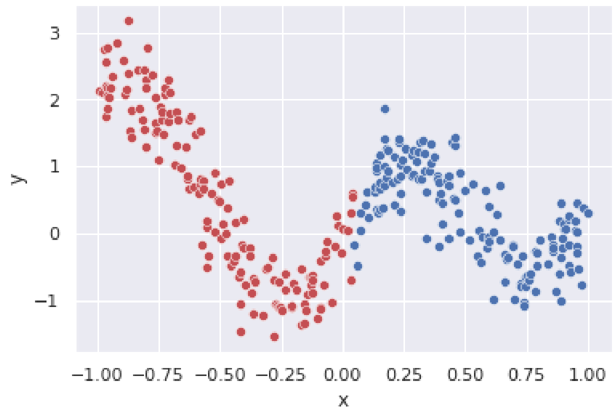
```
plot_colorize(df)
print(" "*9,"Data seems to be well shuffled.")
```




```
          Data seems to be well shuffled.
```

Sorted data

```
# Sort the data based on 'x' first, then do the
# previous part to see the result.
sorted_df = df.sort_values(by='x', ascending=True, ignore_index=True)

plot_colorize(sorted_df)
print(" "*6,"Data doesn't seem to be well shuffled.")
```

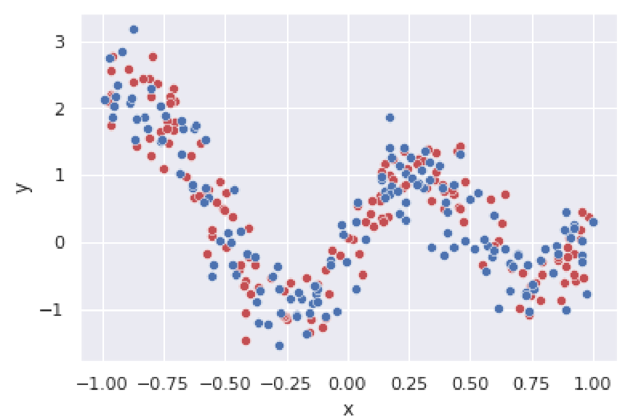



```
       Data doesn't seem to be well shuffled.
```

Shuffled data

```
# Let's shuffle the data anyways (just in case)
```

```
# pandas Doc: specifying drop=True prevents .reset_index()
# from creating a column containing the old index entries.
shuffled_df = df.sample(frac=1).reset_index(drop=True)

plot_colorize(shuffled_df)
print(" "*9,"Data seems to be well shuffled.")
```




```
Data seems to be well shuffled.
```

Part 3 - Gradient Descent

Finding The optimal "Theta" values (weights)

Data Prepration

Add a column for bias

```
# Add a new column for simplicity of the calculations
# acts as the bias term
shuffled_df = pd.concat([pd.Series(1, index=shuffled_df.index, name='0'), shuffled_df], axis=1)
shuffled_df.head()
```

	0	x	y
0	1	-0.244496	-1.151692
1	1	0.152315	0.344465
2	1	-0.492117	-0.081580
3	1	0.355633	0.936222
4	1	-0.628728	0.676142

Seperate X,y

```
# Split training data into X and y
X = shuffled_df.drop(columns="y")
y = shuffled_df.iloc[:, 2]

print(X.head(),end="\n\n")
print(y.head())
```

```
   0         x
0  1 -0.244496
1  1  0.152315
2  1 -0.492117
3  1  0.355633
4  1 -0.628728

0   -1.151692
1    0.344465
2   -0.081580
3    0.936222
```

```
4    0.676142
Name: y, dtype: float64
```

```
# Split to train and valid
split = 0.7

X_train = X.iloc[ : int(len(X)*split),:].reset_index(drop=True)
X_valid = X.iloc[int(len(X)*split) : ,:].reset_index(drop=True)

y_train = y.iloc[ : int(len(X)*split)].reset_index(drop=True)
y_valid = y.iloc[int(len(X)*split) : ].reset_index(drop=True)

print(f"Train X size = {len(X_train)}")
print(f"Train y size = {len(y_train)}")
print(f"Valid X size = {len(X_valid)}")
print(f"Valid y size = {len(y_valid)}")
```

```
Train X size = 210
Train y size = 210
Valid X size = 90
Valid y size = 90
```

```
# Save a copy of X and y
# TODO might not need it
X_train_org = X_train.copy()
y_train_org = y_train.copy()
```

Polynomial Regression

a basic example

polynomial_degree: 10

Show code

Convert

aX + bX^2 + cX^3 + d

to

aX1 + bX2 + cX3 + d

```
# Add the polynomial's terms as features
# so that the univariate non-linear regression
# becomes a multivariate linear regression
# where every polynomial term is a feature for
# the linear regression

X_train_copy = X_train.copy()
X_valid_copy = X_valid.copy()
polynomial_to_linear_regression(X_train_copy, polynomial_degree)
polynomial_to_linear_regression(X_valid_copy, polynomial_degree)


X_train_copy.head()
```

	0	x	x2	x3	x4	x5	x6	x7	x8	x9	x10
0	1	-0.244496	0.059778	-0.014616	0.003573	-0.000874	0.000214	-0.000052	1.276964e-05	-3.122130e-06	7.633493e-07
1	1	0.152315	0.023200	0.003534	0.000538	0.000082	0.000012	0.000002	2.896902e-07	4.412407e-08	6.720743e-09
2	1	-0.492117	0.242179	-0.119180	0.058651	-0.028863	0.014204	-0.006990	3.439893e-03	-1.692829e-03	8.330694e-04
3	1	0.355633	0.126475	0.044979	0.015996	0.005689	0.002023	0.000719	2.558687e-04	9.099537e-05	3.236096e-05
4	1	-0.628728	0.395299	-0.248536	0.156261	-0.098246	0.061770	-0.038837	2.441760e-02	-1.535203e-02	9.652255e-03

Training

Normal equation for comparison :

- 0.07 training loss
- 0.09 validation loss

5k iter 10th degree polynomial lr=2.3

- Training Loss : 0.07766969752936674
- Validation Loss : 0.094857479611961

```python
# Initialize the weights with zero
theta = np.array([0.0]*len(X_train_copy.columns))

# Initialize the weights with random values
theta = np.random.rand(len(X_train_copy.columns),)

print("notice : takes approximately 3 minutes for 5k iters")

# tip : nice way to find decay (https://machinelearningmastery.com/using-learning-rate-schedules-deep-learning-models-pyth
# Decay = LearningRate / Epochs
# Decay = 0.1 / 1000
# Decay = 0.0001
decay = learning_rate/iterations/11250


# For testing purposes
# iterations = 500
# learning_rate = 2.3
# decay = learning_rate/iterations/250

# Start the training
history, loss, theta = gradientDescent(X_train_copy,
                                       y_train,
                                       theta,
                                       learning_rate,
                                       iterations,
                                       X_valid = X_valid_copy,
                                       y_valid = y_valid,
                                       loss_fn=MSE,
                                       loss_fn_prim=MSE_prim,
                                       decay = 0)


```

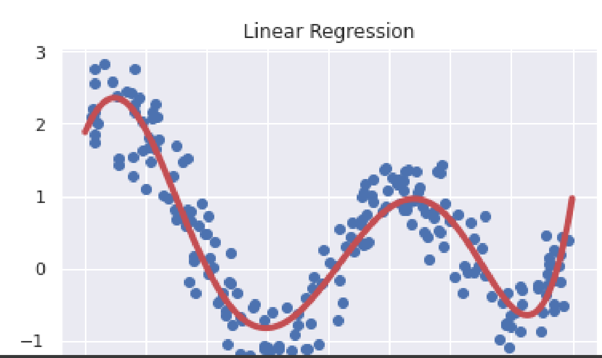
```
notice : takes approximately 3 minutes for 5k iters
100%                                        5000/5000 [02:31<00:00, 33.06it/s]
training_loss : 0.0807 | validation_loss : 0.1239
```

Plotting the fitted polynomials

```python
# Predicting using the learned weights(theta)
# Not used here but useful
y_hat = theta*X_valid_copy
y_hat = np.sum(y_hat, axis=1)
print(MSE(X_valid_copy, y_valid, theta))
```

```
0.12387766832808893
```

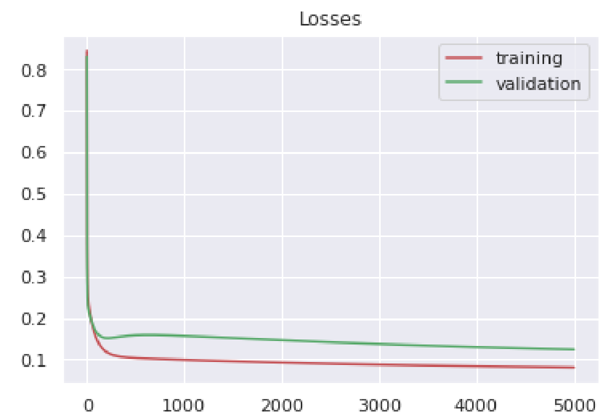
```python
plot_curve(X_train_copy, y_train, theta, title='Linear Regression')

print(f"Training Loss : {MSE(X_train_copy, y_train, theta)}")
print(f"Validation Loss : {MSE(X_valid_copy, y_valid, theta)}\n")
```



Plotting losses

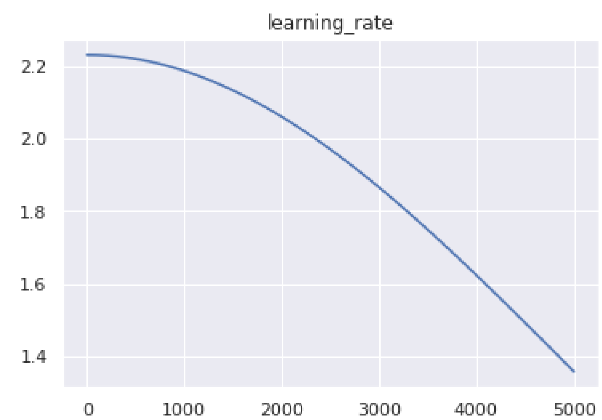
Training Loss : 0.08069536588067559

```
plot_loss(history, starting_iter=0, title='Losses')
```



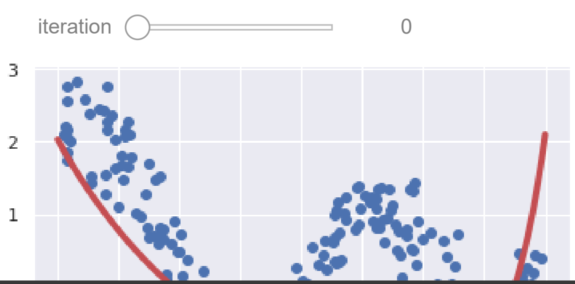
Plotting the learning rate

```
plot_lr(lr=learning_rate, iterations=iterations, decay=decay)
```



Interactive history viewer

```
from ipywidgets import interact
import ipywidgets as widgets

@interact(iteration = widgets.IntSlider(min=0, max=iterations-1, step=100, value=0))
def plot_weight_history(iteration):
  plot_curve(X_train_copy, y_train, history["weights"][iteration])

print("Use the slider to see the algorithms progression")
```



Part 4 - Plotting every Case!

Polynomial Degree :

- 5
- 8
- 10

Loss Functions :

- MSE
- RMSE
- MAE

Iterations :

- 5000
- 10000

```
# iterations = 500
polynomial_degrees = [10, 8, 3]
loss_functions = [(MAE, MAE_prim),
                  (RMSE, RMSE_prim),
                  (MSE, MSE_prim)]
fn_labels = ["MAE", "RMSE", "MSE"]
```

Training all 9 models

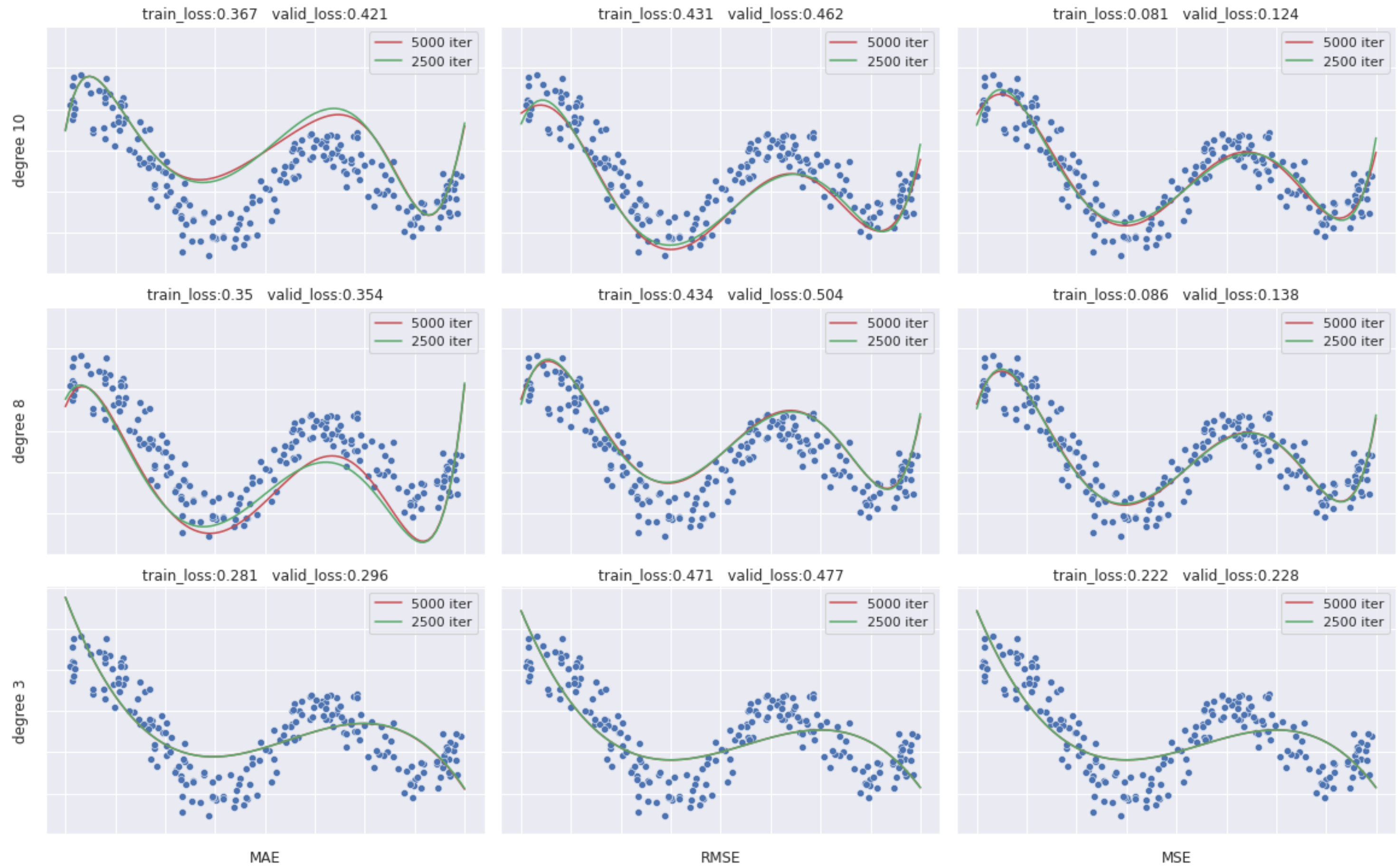
```
# Containers used for storing results about each model
# Used later by the plotting functions
thetas = [[[],[],[]],
          [[],[],[]],
          [[],[],[]]]

losses = [[[],[],[]],
          [[],[],[]],
          [[],[],[]]]

histories = [[[],[],[]],
             [[],[],[]],
             [[],[],[]]]

# start the training process for each model
for i,degree in enumerate(polynomial_degrees):
  for j, (loss_fn, loss_fn_prim) in enumerate(loss_functions):
    print(f"degree: {degree} | loss function: {fn_labels[j]}")

    # preprocess data (univariate non-linear to multivariate linear)
    X_train_copy = X_train.copy()
    X_valid_copy = X_valid.copy()
    polynomial_to_linear_regression(X_train_copy, degree)
    polynomial_to_linear_regression(X_valid_copy, degree)

    # Initialize the weights with random values
    theta = np.random.rand(len(X_train_copy.columns),)

    # Start the training
    history, loss, theta = gradientDescent(X_train_copy,
                                           y_train,
                                           theta,
```

```
                                         learning_rate,
                                         iterations,
                                         X_valid = X_valid_copy,
                                         y_valid = y_valid,
                                         loss_fn=loss_fn,
                                         loss_fn_prim=loss_fn_prim,
                                         decay = decay*0)

    # Saving latest iteration's theta for each model
    thetas[i][j].append(history["weights"][-1])
    # Saving halfway theta for each model
    thetas[i][j].append(history["weights"][int(iterations/2)-1])

    # Saving Training loss for each model
    losses[i][j].append(loss_fn(X_train_copy, y_train, theta))
    # Saving Validation loss for each model
    losses[i][j].append(loss_fn(X_valid_copy, y_valid, theta))
    # Saving Histories for each model (used for plotting loss per iteration)
    histories[i][j].append(history)

    print()
```

```
degree: 10 | loss function: MAE
100%                                    5000/5000 [02:52<00:00, 28.87it/s]
training_loss : 0.3667 | validation_loss : 0.4209

degree: 10 | loss function: RMSE
100%                                    5000/5000 [04:33<00:00, 17.76it/s]
training_loss : 0.431 | validation_loss : 0.4617

degree: 10 | loss function: MSE
100%                                    5000/5000 [02:30<00:00, 33.36it/s]
training_loss : 0.0806 | validation_loss : 0.1237

degree: 8 | loss function: MAE
100%                                    5000/5000 [02:24<00:00, 34.66it/s]
training_loss : 0.3503 | validation_loss : 0.3544

degree: 8 | loss function: RMSE
100%                                    5000/5000 [03:31<00:00, 22.96it/s]
training_loss : 0.4342 | validation_loss : 0.5045

degree: 8 | loss function: MSE
100%                                    5000/5000 [01:58<00:00, 42.39it/s]
training_loss : 0.0858 | validation_loss : 0.1384

degree: 3 | loss function: MAE
100%                                    5000/5000 [01:11<00:00, 70.39it/s]
training_loss : 0.2812 | validation_loss : 0.2957

degree: 3 | loss function: RMSE
100%                                    5000/5000 [01:18<00:00, 65.90it/s]
training_loss : 0.4708 | validation_loss : 0.4771

degree: 3 | loss function: MSE
100%                                    5000/5000 [00:47<00:00, 104.60it/s]
training_loss : 0.2216 | validation_loss : 0.2277
```

```
plot_every_curve(X_train, y_train, thetas)
```

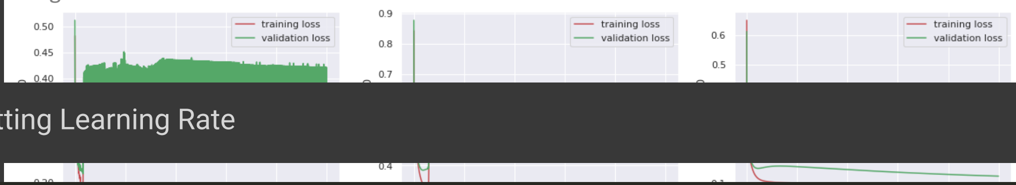
```
<Figure size 432x288 with 0 Axes>
```



Plotting train/valid loss

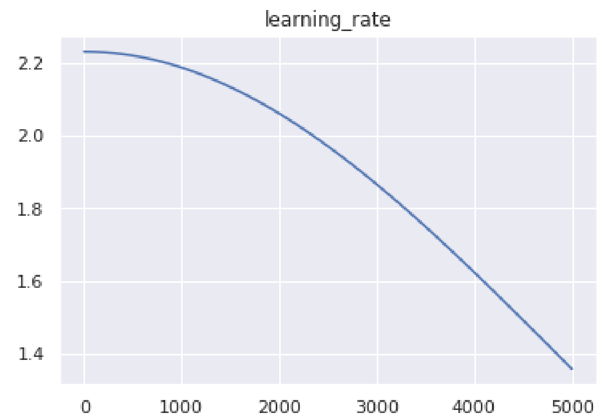
```
plot_every_case_loss(histories, starting_iter=0)
```

```
<Figure size 432x288 with 0 Axes>
```



Plotting Learning Rate

```
plot_lr(lr=learning_rate, iterations=iterations, decay=decay)
```

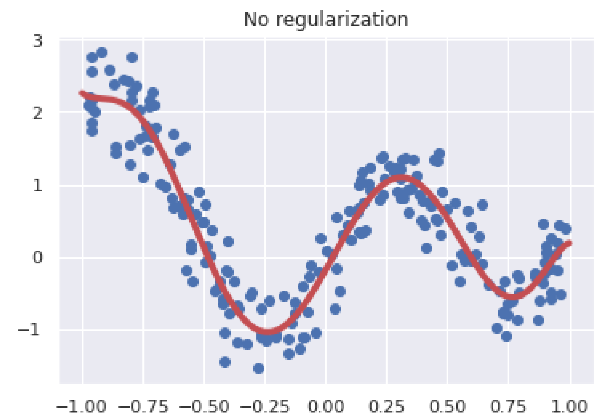


Part 5 - Normal Equation

```
# preprocess data (univariate non-linear to multivariate linear)
X_train_copy = X_train.copy()
X_valid_copy = X_valid.copy()
polynomial_to_linear_regression(X_train_copy, 8)
polynomial_to_linear_regression(X_valid_copy, 8)
```

```
normal_theta = normalEquation(X_train_copy, y_train)
```

```
# Plotting the fitted polynomial over training data
plot_curve(X_train_copy, y_train, normal_theta, title="No regularization")
print(f"Training Loss : {RMSE(X_train_copy, y_train, normal_theta)}")
```




```
Training Loss : 0.2654493437506085
```

```
# Plotting the fitted polynomial over validation data
plot_curve(X_valid_copy, y_valid, normal_theta, title="No regularization")
print(f"Validation Loss : {RMSE(X_valid_copy, y_valid, normal_theta)}\n")
```

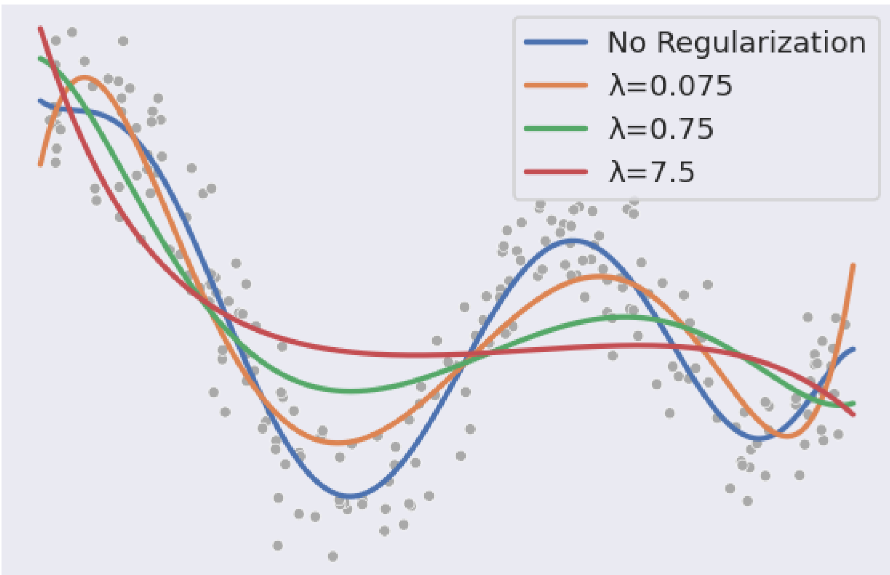
Part 6 - Regularized Normal Equation

```
loss_fn = RMSE # as asked in the question

reg_normal_theta1 = regularizedNormalEquation(X_train_copy, y_train, lambd=0.075)
# plot_curve(X_train, y_train, reg_normal_theta, title="lambda=0.1")
reg_normal_theta2 = regularizedNormalEquation(X_train_copy, y_train, lambd=0.75)
# plot_curve(X_train, y_train, reg_normal_theta, title="lambda=1")
reg_normal_theta3 = regularizedNormalEquation(X_train_copy, y_train, lambd=7.5)
# plot_curve(X_train, y_train, reg_normal_theta, title="lambda=4")
```

```
names = ["λ = 0.075", "λ = 0.75", "λ = 7.5"]

training_errors = [round(loss_fn(X_train_copy, y_train, reg_normal_theta1),5),
                   round(loss_fn(X_train_copy, y_train, reg_normal_theta2),5),
                   round(loss_fn(X_train_copy, y_train, reg_normal_theta3),5)]

validation_errors =[round(loss_fn(X_valid_copy, y_valid, reg_normal_theta1),5),
                    round(loss_fn(X_valid_copy, y_valid, reg_normal_theta2),5),
                    round(loss_fn(X_valid_copy, y_valid, reg_normal_theta3),5)]

for i in range(len(names)):
  print(f"{i} - Regularized Normal Equation ({names[i]})")
  print(f"Training Loss : {training_errors[i]}")
  print(f"Validation Loss : {validation_errors[i]}\n")
```

```
0 - Regularized Normal Equation (λ = 0.075)
Training Loss : 0.31678
Validation Loss : 0.36842

1 - Regularized Normal Equation (λ = 0.75)
Training Loss : 0.43112
Validation Loss : 0.43171

2 - Regularized Normal Equation (λ = 7.5)
Training Loss : 0.52495
Validation Loss : 0.49915
```

```
plot_normal_equations(X_train, y_train)
```



0s completed at 1:20 PM