Reinforcement Learning: Basics

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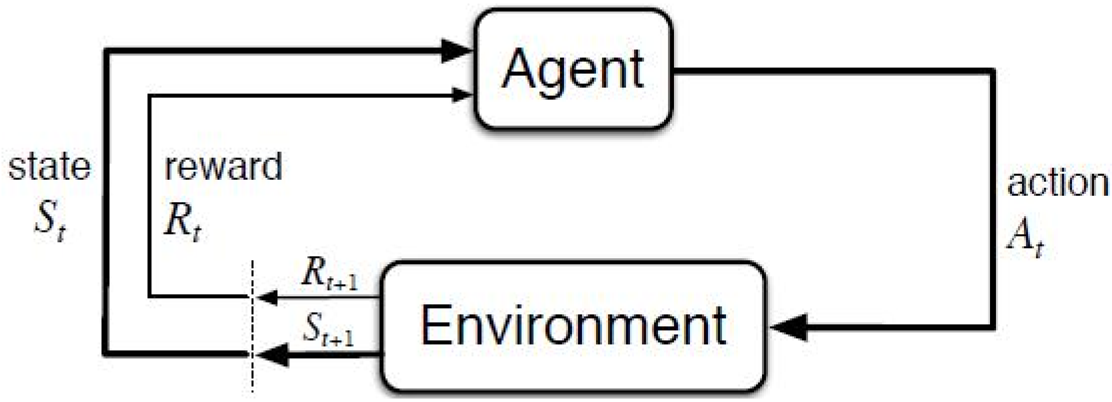
Learning Types

Supervised Learning	Unsupervised Learning	Reinforcement Learning
Labels provided for every input	No labels provided	Delayed feedback* / Numeric reward signals
Learn from labeled examples	Learn from unlabeled examples	Learn through interactions
One shot decision making	One shot decision making	Sequential decision making

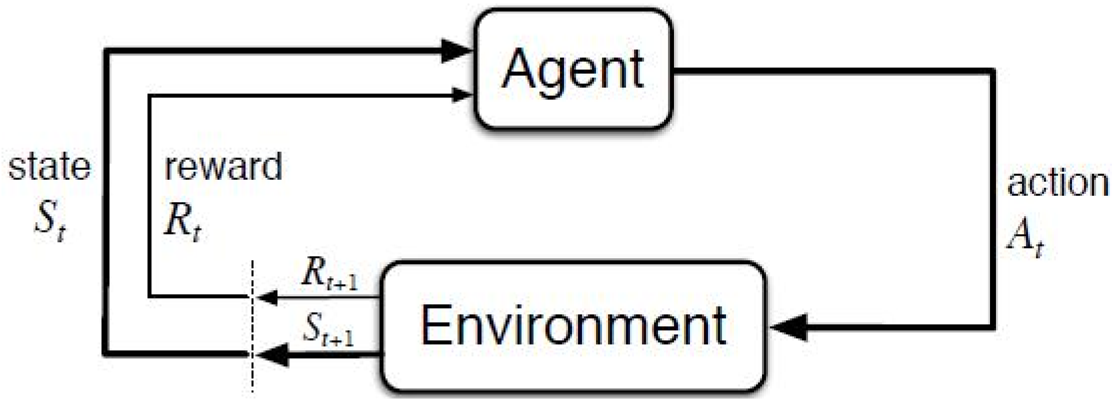
*Example: Chess game

The Big Picture



Agent and environment interact at discrete time steps: $t = 0, 1, 2, \ldots$
Agent observes state at step t: $S_t \in \mathcal{S}$
produces action at step t: $A_t \in \mathcal{A}(s)$
gets resulting reward: $R_{t+1} \in \mathcal{R} \subset \mathbb{R}$
and resulting next state: S_{t+1}

The Big Picture (cont.)



The resulting sequence or trajectory:

$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$$

Agent Learns a Policy

- Policy at step t, π_t:
  - a mapping from states to action probabilities
  - $\pi_t(s, a)$ = probability that $A_t = a$, when $S_t = s$

- Reinforcement learning methods specify how the agent changes its policy as a result of experience.

  - The agent's goal is to get as much reward as it can over the long run.

Complications

- The outcome of your actions may be uncertain
- You may not be able to perfectly sense the state of the world
- The reward may be stochastic.
- Reward is delayed (i.e. finding food in a maze)
- You may have no clue (model) about how the world responds to your actions.
- You may have no clue (model) of how rewards are being paid off.
- The world may change while you try to learn it
- How much time do you need to explore uncharted territory before you exploit what you have learned?

Returns

- Suppose the sequence of rewards after step t is: $R_{t+1}, R_{t+2}, R_{t+3}, \dots$

- We want to maximize the expected return, $\mathbb{E}(R_t)$, for each step t.

- Discounted return:

$$G_t = R_{t+1} + \gamma\, R_{t+2} + \gamma^2\, R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k\, R_{t+k+1},$$

where $\gamma, 0 \leq \gamma \leq 1$, is the discount rate.

The Markov Property

- Markov property

$$p(S_{t+1} = s', R_{t+1} = r \mid a_t, s_t, r_t, a_{t-1}, s_{t-1}, \dots, r_1, a_0, s_0) = p(S_{t+1} = s', R_{t+1} = r \mid s_t, a_t)$$

for all s', r and histories $s_t, a_t, r_t, s_{t-1}, a_{t-1}, \dots, r_1, a_0, s_0$

Markov Decision Process

- If a reinforcement learning task has the Markov Property, it is basically a Markov Decision Process (MDP).
- If state and action sets are finite, it is a finite MDP.
- To define a finite MDP, the followings need to be given:
  - State and action sets
  - Transition probabilities

$$T(\mathrm{s}, \mathrm{a}, \mathrm{s}') = p(S_{t+1} = s' | S_t = s, A_t = a)\ , \forall s, s' \in \mathcal{S}, a \in \mathcal{A}(s)$$

  - Reward probabilities

$$R(s, a, s') = \mathbb{E}[S_t = s, A_t = a, S_{t+1} = s']$$
$$\forall s, s' \in \mathcal{S}, a \in \mathcal{A}(s)$$

Value and Q Functions

- The **value of a state** is the expected return starting from that state; depends on the agent's policy:

$$v^{\pi}(s) = \mathbb{E}_{\pi}[G_t \,|\, S_t = s] = E_{\pi}[\sum_{k=0}^{\infty} \gamma^k \, R_{t+k+1} \,|S_t = s]$$

- The **value of taking an action in a state under policy** $\boldsymbol{\pi}$ is the expected return starting from that state, taking that action, and thereafter following π:

$$q^{\pi}(s,a) = \mathbb{E}[G_t \,|\, S_t = s, A_t = a) = \mathbb{E}_{\pi}[\sum_{k=0}^{\infty} \gamma^k \, R_{t+k+1} \,|S_t = s, A_t = a]$$

Bellman Equation for policy π

- The basic idea:

$$\begin{aligned} G_t &= R_{t+1} + \gamma\, R_{t+2} + \gamma^2\, R_{t+3} + \gamma^3\, R_{t+4} + \cdots \\ &= R_{t+1} + \gamma(R_{t+2} + \gamma\, R_{t+3} + \gamma^2\, R_{t+4} + \cdots) \\ &= R_{t+1} + \gamma\, G_{t+1} \end{aligned}$$

$$v^\pi(s) = \mathbb{E}_\pi[G_t |\, S_t = s) = \mathbb{E}_\pi[R_{t+1} + \gamma\, v(s_{t+1}) |\, S_t = s]$$

$$v^\pi(s) = \sum_a \pi(s,a) \sum_{s\prime} p(s'|s,a)[R(s,a,s') + \gamma v^\pi(s')]$$

$\pi_t(s,a) =$ probability that $A_t = a$, when $S_t = s$

Optimality

- There is always at least one (and possibly many) policies that is better than or equal to all the others. This is an optimal policy, denoted by π^*.

- Optimal value function:

$$v^*(s) = \max_{\pi} v^{\pi}(s), \qquad \forall\, s \in \mathcal{S}$$

- Optimal q function:

$$q^*(s,a) = \max_{\pi} q^{\pi}(s,a), \qquad \forall\, s \in \mathcal{S} \,\&\, a \in \mathcal{A}(s)$$

Optimal Value Function

- The value of a state under an optimal policy must equal the expected return for the best action from that state (Bellman equation):

$$\boxed{v^*(s) = \max_{a \in \mathcal{A}(s)} \sum_{s'} p(s'|s,a)[R(s,a,s') + \gamma v^*(s')]}$$

- Similarly for q* function:

$$q^*(s,a) = \sum_{s'} p(s'|s,a)\,[R(s,a,s') + \gamma \max_{a'} q^*(s',a')]$$

Optimal Policy

$$\boxed{\pi^*(s) = \underset{a \in \mathcal{A}(s)}{\operatorname{argmax}} \sum_{s'} p(s' \mid s, a) v^*(s')}$$

Or Equivalently:

$$\boxed{\pi^*(s) = \underset{a \in \mathcal{A}(s)}{\operatorname{argmax}}\, q^*(s, a)}$$

Recap

$$\boxed{v^*(s) \leftarrow \max_a \sum_{s'} p(s'|s,a)[R(s,a,s') + \gamma v^*(s')]}$$

OR (slightly different notation):

$$\boxed{v^*(s) \leftarrow \max_a \sum_{s',r} p(s',r|s,a)[r + \gamma v^*(s')]}$$

Value Iteration Algorithm

- Idea:
  - Start with $v_0^*(s) = 0$, which we know is right
  - Given v_i^*, calculate the values for all states for depth i+1:

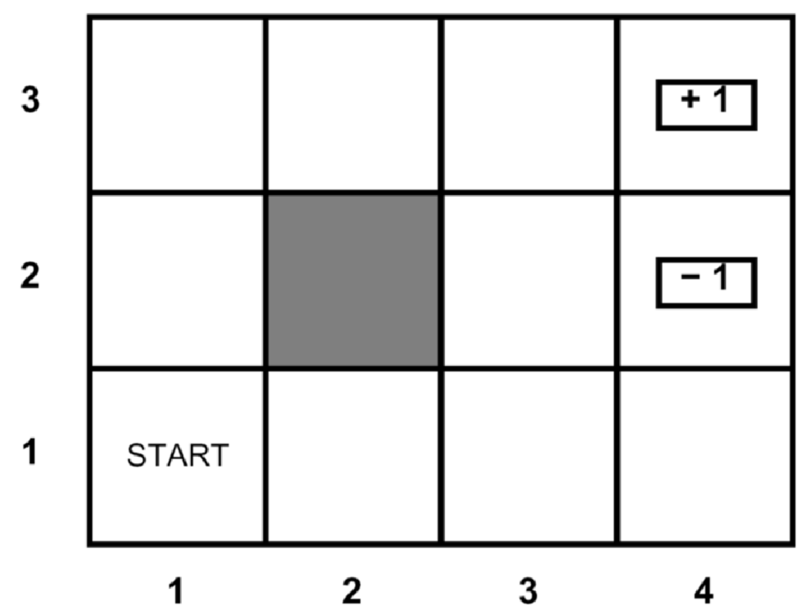
$$v_{i+1}(s) \leftarrow \max_a \sum_{s',r} p(s',r|s,a)\,[r + \gamma v_i(s')]$$

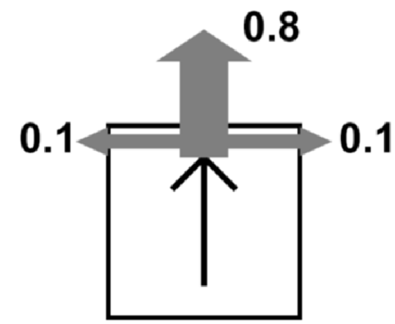
  (This is called a value update or Bellman update)
  - Repeat until convergence

- Theorem: It will converge to unique optimal values
  - Basic idea: approximations get refined towards optimal values
  - Policy may converge long before values do.

Stochastic Grid World

- The agent lives in a grid
- Walls block the agent's path
- The agent's actions do not always go as planned:
  - 80% of the time, the action North takes the agent North, if there is no wall there.
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays in the same place
- Small "living" reward each step, e.g. R(s) = –0.02
- Big rewards come at the end, i.e. R(<4,3>)=+1, R(<4,2>)=–1
- Goal: maximize sum of rewards





Dynamic Programming for Value Update

- It is assumed that $r = -0.02$ for all non-terminal states and $\gamma = 0.9$

v_0

3	0	0	0	$\boxed{+1}$
2	0		0	$\boxed{-1}$
1	0	0	0	0
	1	2	3	4

v_1

3	0	0	0.796	$\boxed{+1}$
2	0		0	$\boxed{-1}$
1	0	0	0	0
	1	2	3	4

Value Update (cont.)

- Information propagates outward from terminal states and eventually all states have correct value estimates.

v_1

3	0	0	0.796	+1
2	0		0	−1
1	0	0	0	0
	1	2	3	4

v_2

3	0	0.553	0.867	+1
2	0		0.455	−1
1	0	0	0	0
	1	2	3	4

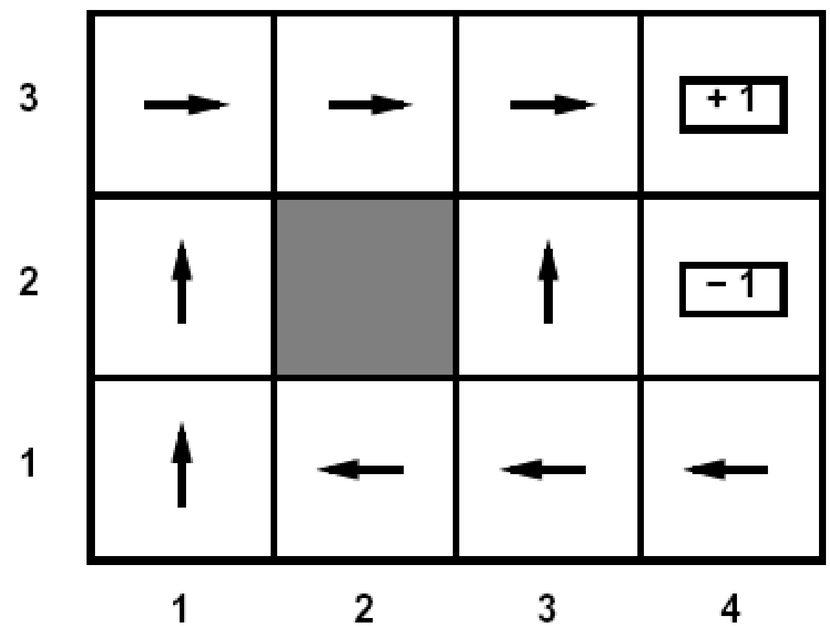
Optimal Policy

- $r = -0.02\ ,\ \gamma = 0.99$

v^*

3	0.86	0.90	0.93	+1
2	0.82		0.69	−1
1	0.78	0.75	0.71	0.49
	1	2	3	4

π^*



Policy Iteration Algorithm

- Initialize policy π randomly
- Repeat
  - Let $v \leftarrow v^{\pi}$ (solve Bellman equations)
  - Let $\pi(s) \leftarrow \underset{a}{\operatorname{argmax}} \sum_{s'} p(s'|s,a)v(s')$

- This will make $v \to v^{*}$, $\pi \to \pi^{*}$

Further Reading

- More advanced topics in Reinforcement Learning (RL)
- Inverse Reinforcement Learning
- Imitation Learning
- Deep RL

References

- R. S. Sutton and A. G. Barto: Reinforcement Learning: An Introduction, MIT Press, 1998 (1st ed.) & 2018 (2nd ed.)

- T. Mitchell, Machine Learning, Chapter 13, McGraw Hill, 1997.