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An Estimation of Ramberg-Osgood Constants for Materials with and without Luder's Strain Using Yield and Ultimate Strengths

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Abstract

Tensile stress-strain curves for metallic materials typically show two different behaviors *viz*. with Luder's strain and without Luder's strain. Recently, Kamaya [1] proposed a method to estimate the true stress -true strain curve using a certain plastic strain together with yield and ultimate strengths. Kamaya's method however, is not accurate enough for materials exhibiting Luder's strain in their engineering stress-strain behavior. Hence, the aim of this paper is to propose a generalization of the Kamaya's method for the materials with and without Luder's strains. This new generalized approach uses plastic strain value corresponding to the Luder's strain along with engineering yield strength and ultimate tensile strength to estimate the strain hardening exponent in the Ramberg-Osgood type of true stress - true strain relationship. The new approach was applied to 16 different materials with and without Luder's strain to validate the proposed estimation procedure. In addition, an inverse method for assessing an apparent ultimate tensile stress (stress at "an apparent" point of zero slope in an engineering stress-strain curve) for materials with low ductility due to quenching or carburizing is also suggested.

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Keywords: Ramberg-Osgood relationship; Luder's strain; strain hardening exponent; stress-strain curves; yield and ultimate strengths.

1. Introduction

The tensile test is usually represented by a graph of engineering stress vs engineering strain, as illustrated in Fig. 1 from which, important mechanical properties such as: Young' modulus, E, yield strength, S_y, ultimate tensile strength, S_u, strain hardening behavior, and stress and strain at fracture (F) can be determined. Relatively ductile materials undergo significant cross-sectional reduction due to necking of material after ultimate tensile strength (stress at point of zero slope in σ-ε curve). Hence, to study ductile materials behavior, true stress-true strain curves are often utilized.

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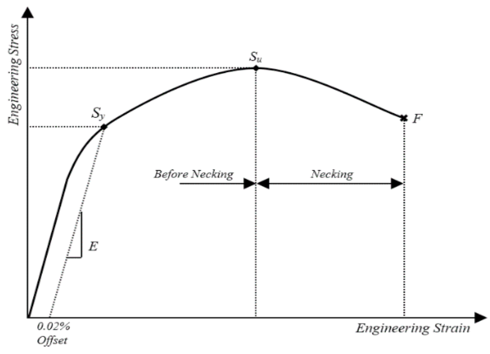

Fig. 1. Illustration of tensile test stress-strain curve.

The true stress and true strain values can be calculated using the following equations:

<u>Before necking</u>

$$True\ Strain \quad \tilde{\varepsilon} = \ln(1+\varepsilon) \tag{1}$$

$$True\ Stress \quad \tilde{\sigma} = \sigma(1+\varepsilon) \tag{2}$$

where ε and σ are engineering strain and stress, respectively.

<u>Necking region</u>

$$True\ Strain \quad \tilde{\varepsilon} = \ln(A_i/A) \tag{3}$$

$$True\ Stess \quad \tilde{\sigma} = \sigma(A_i/A) \tag{4}$$

where A_i and A are initial and actual cross-sectional areas, respectively (Eq. (4) is used with well-known Bridgman correction). Figure 2 illustrates the relation between engineering vs. true stress-strain behavior for materials (a) without and (b) with Luder's strain. A plateau behavior as shown in Fig. 2b corresponds to a constant stress whereas a strain is increasing up to a certain point. This strain is also known as Luder's strain. Beyond the Luder's strain the rest of the material stress-strain behavior is similar to that in Fig. 2a without Luder's strain.

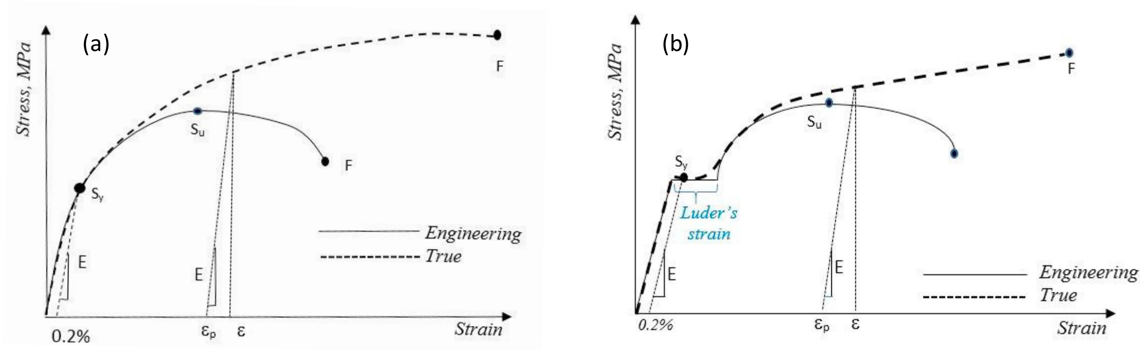

Fig. 2. Illustration of engineering vs. true stress-strain behavior in materials (a) without Luder's strain, (b) with Luder's strain, $\varepsilon_{p\text{-}L}$.

This study proposes a generalization of the Kamaya's [1] estimation of the Ramberg-Osgood constants based on yield and ultimate strengths for materials exhibiting Luder's strain behavior.
The true stress-true strain curve can be represented by Ramberg-Osgood relationship.

$$\tilde{\varepsilon} = \frac{\tilde{\sigma}}{E} + \left(\frac{\tilde{\sigma}}{H}\right)^{\frac{1}{n}} \tag{5}$$

where H is the strength coefficient and n is the strain hardening exponent.
The total true strain, $\tilde{\varepsilon}$, can be decomposed into elastic, $\tilde{\varepsilon}_e$,and plastic, $\tilde{\varepsilon}_p$, strain components.

$$\tilde{\varepsilon} = \tilde{\varepsilon}_e + \tilde{\varepsilon}_p \tag{6}$$

In the Ramberg-Osgood relationship these two strains are considered separately. The elastic strain is related to the true stress by Hooke's law whereas a power-law relationship is used for the plastic strain.

$$\tilde{\sigma} = H\tilde{\varepsilon}_p{}^n \tag{7}$$

where *H* and *n* are obtained by the best fit line in log- log coordinated of $\tilde{\sigma}$ vs. $\tilde{\varepsilon}_p$.
The *H* can be also estimated from the following relationship as

$$H = \frac{S_y}{\tilde{\varepsilon}_{yp}^n} \tag{8}$$

where $\tilde{\varepsilon}_{yp} = 0.002$ is the true offset plastic strain corresponding to the yield stress S_y.

2. Background

2.1 Non-dimensional form of the Ramberg-Osgood relationship

The Ramberg-Osgood relationship can also be represented in its non-dimensional form

$$\frac{\boldsymbol{E\tilde{\varepsilon}}}{\boldsymbol{S_y}} = \frac{\tilde{\sigma}}{\boldsymbol{S_y}} + \boldsymbol{\alpha}\left(\frac{\tilde{\sigma}}{\boldsymbol{S_y}}\right)^{\boldsymbol{N}} \tag{9}$$

where N = 1/n = strain hardening exponent, $\alpha = \frac{\boldsymbol{E}}{\boldsymbol{H^N S_y^{1-N}}}$ = non-dimensional material constant.

By comparing the plastic part of the strain given by Eqs. (5) and (9), the true plastic strain can be expressed as

$$\boldsymbol{\tilde{\varepsilon}_p} = \frac{\boldsymbol{\alpha s_y}}{\boldsymbol{E}}\left(\frac{\tilde{\sigma}}{\boldsymbol{S_y}}\right)^{\boldsymbol{N}} \tag{10}$$

2.2 Approach proposed by Kamaya [1]

Recently, Kamaya [1] proposed equations to calculate *N* values in non-dimensional Ramberg-Osgood relationship using yield and ultimate strengths and a specific value of the plastic yield strain, $\boldsymbol{\tilde{\varepsilon}_{py}}$. Using this procedure Kamaya [1] obtained the following relations for *N* as:

$\boldsymbol{N} = 3.93\times (\mathbf{ln}(\boldsymbol{S_u/S_y}))^{-0.754}$ for $\boldsymbol{\tilde{\varepsilon}_{py}} = \mathbf{0.002}$ (11)

$\boldsymbol{N} = 3.27\times (\mathbf{ln}(\boldsymbol{S_u/S_y}))^{-0.69}$ for $\boldsymbol{\tilde{\varepsilon}_{py}} = \mathbf{0.005}$ (12)

$\boldsymbol{N} = \mathbf{2.86}(\mathbf{ln}(\boldsymbol{S_u/S_y}))^{-0.61}$ for $\boldsymbol{\tilde{\varepsilon}_{py}} = \mathbf{0.01}$ (13)

Such estimated N values were then used for predicting true stress versus true strain curves of the Ramberg -Osgood relationship. In his analysis Kamaya didn't specify what value for $\boldsymbol{\tilde{\varepsilon}_{py}}$ should be used. Kamaya applied his estimation procedure to four ferrite steels materials using three values of $\boldsymbol{\tilde{\varepsilon}_{py}}$. Kamaya's analysis shown to be successful in relating *N* if $\boldsymbol{\tilde{\varepsilon}_{py}}$ of 0.2%, 0.5% or 1% has been properly selected. However, how this value of $\boldsymbol{\tilde{\varepsilon}_{py}}$ needs to be selected for different material was not specified.

3. Proposed method

In subsequent analysis Eqs. (11-13) are generalized to the following form:

$$N = c\left[\ln\left(\frac{S_u}{S_y}\right)\right]^{-a} \tag{14}$$

In Eq. (14), both constants *c* and *a* are functions of the true plastic yield/Luder's strain, $\tilde{\varepsilon}_{p-L}$, as it is shown in Fig. 3. Now, the *N* values for a given material can be calculated using Eq. (14) together with the corresponding true plastic yield/Luder's strain at the end of the plateau, $\tilde{\varepsilon}_{p-L}$.

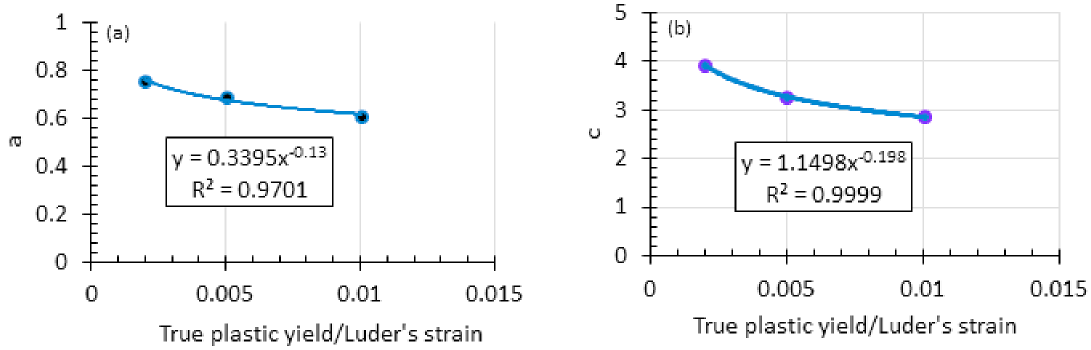

Fig. 3. Graphs of *a* and *c* in Eq. (14) for different plastic yield strain value, $\tilde{\varepsilon}_{py}$.

As can be seen from various experimental stress-strain curves depict in Fig. 4, that some materials show a plateau at yielding, where applied stress remains almost constant whereas associated Luder's strain increases. This maximum value of the plastic Luder's strain would be used in estimation of the strain hardening exponent, *N*.

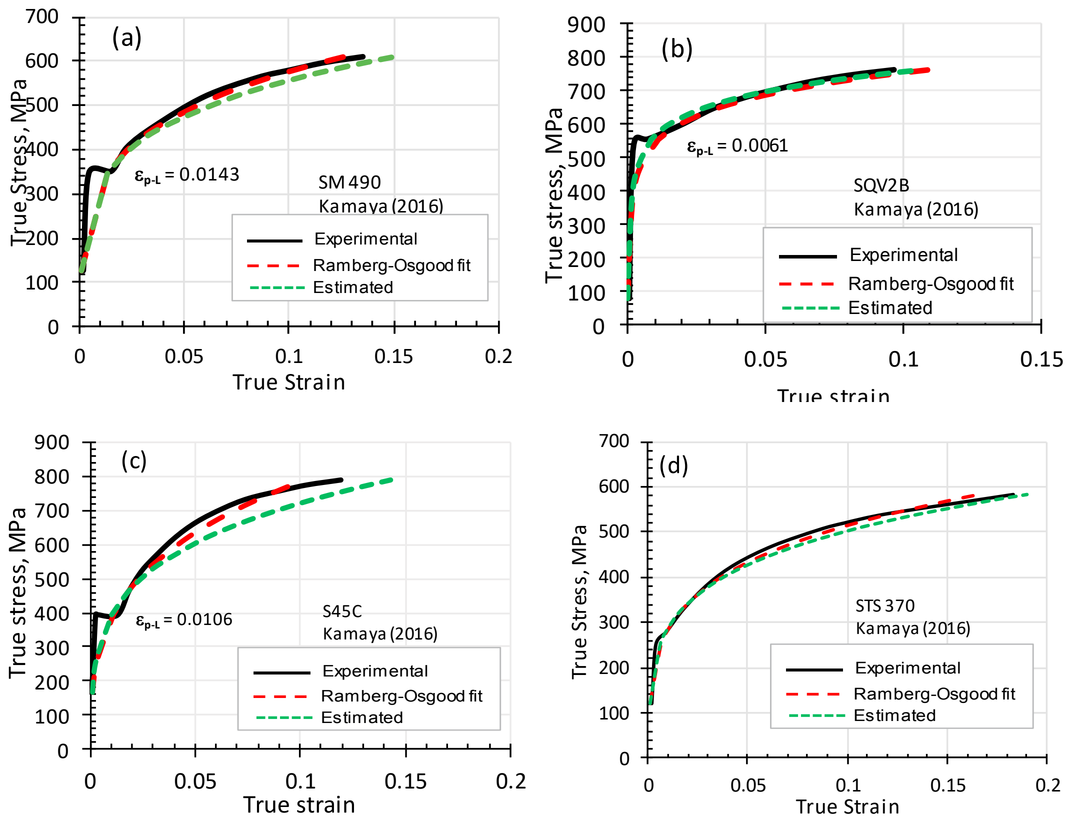

Fig. 4. (Cont.)

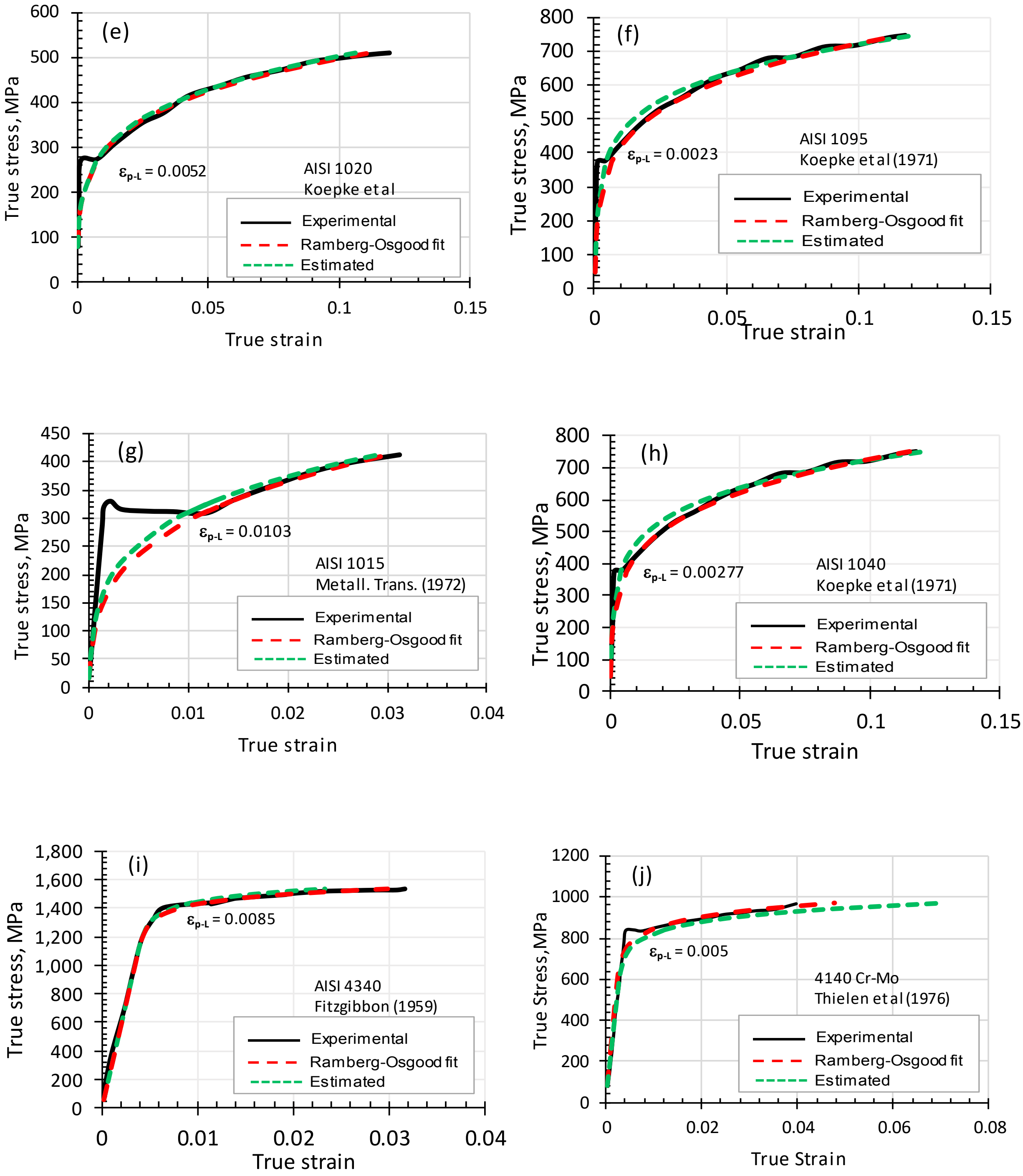
(e)
True stress, MPa
True strain
εp-L = 0.0052
AISI 1020
Koepke et al
Experimental
Ramberg-Osgood fit
Estimated
(f)
True stress, MPa
True strain
εp-L = 0.0023
AISI 1095
Koepke et al (1971)
Experimental
Ramberg-Osgood fit
Estimated
(g)
True stress, MPa
True strain
εp-L = 0.0103
AISI 1015
Metall. Trans. (1972)
Experimental
Ramberg-Osgood fit
Estimated
(h)
True stress, MPa
True strain
εp-L = 0.00277
AISI 1040
Koepke et al (1971)
Experimental
Ramberg-Osgood fit
Estimated
(i)
True stress, MPa
True strain
εp-L = 0.0085
AISI 4340
Fitzgibbon (1959)
Experimental
Ramberg-Osgood fit
Estimated
(j)
True Stress,MPa
True Strain
εp-L = 0.005
4140 Cr-Mo
Thielen et al (1976)
Experimental
Ramberg-Osgood fit
Estimated

Fig. 4. (Cont.)

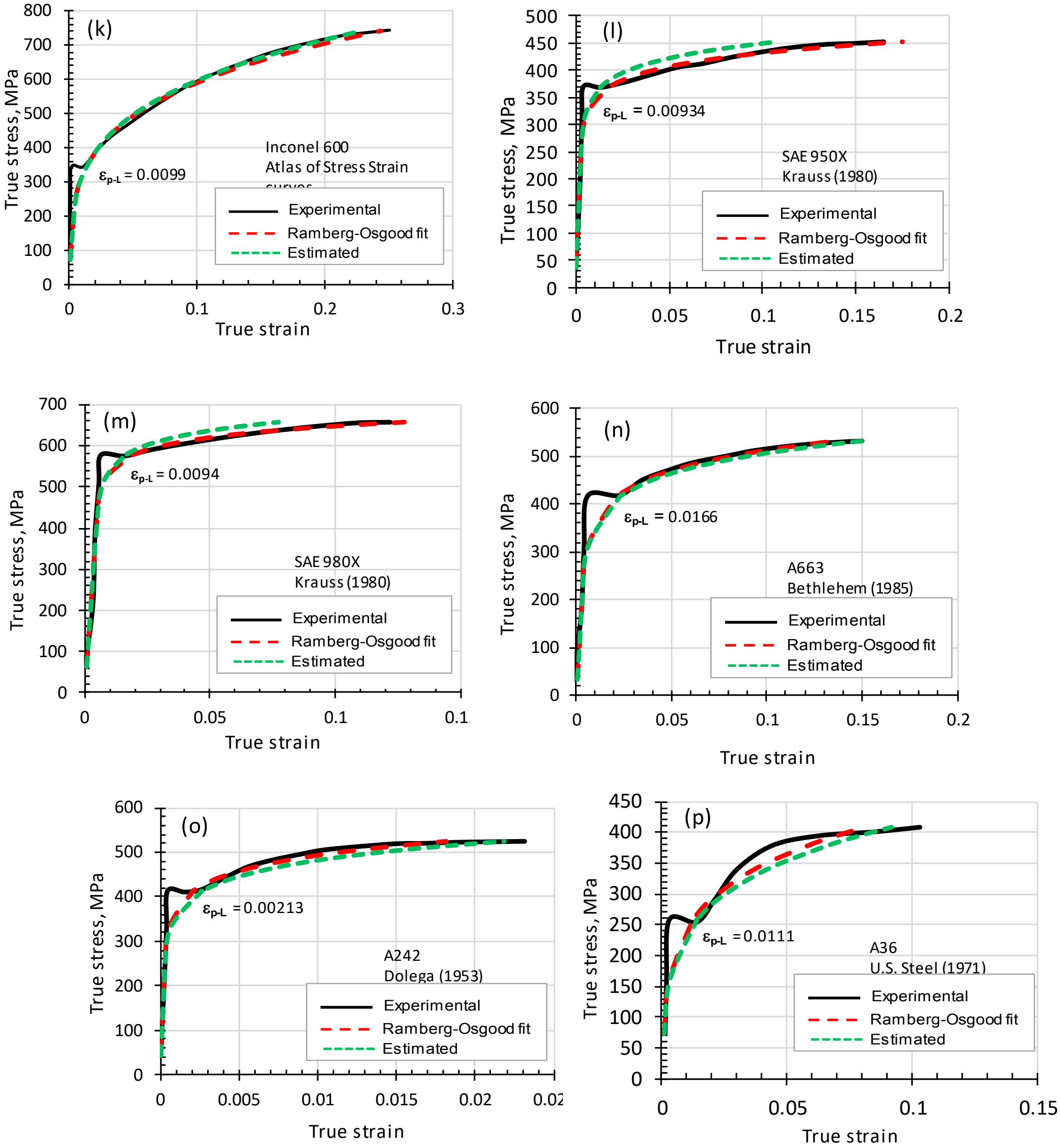

Fig. 4 a-p. Comparison of true stress-strain curves from experimental data, fitted by the Ramberg-Osgood relationship and estimated using Luder's plastic strain at plateau, $\varepsilon_{p\text{-}L}$.

As it is seen from Fig. 4 a-p, that by using true plastic yield strain at end of yield stress plateau (plastic Luder's strain) gives fairly good estimation of stress-strain curves, and thus strain hardening exponent *N*.

Figure 5 compares the Ramberg-Osgood constants N=1/n and H calculated from experimental curve and estimated by the proposed method.

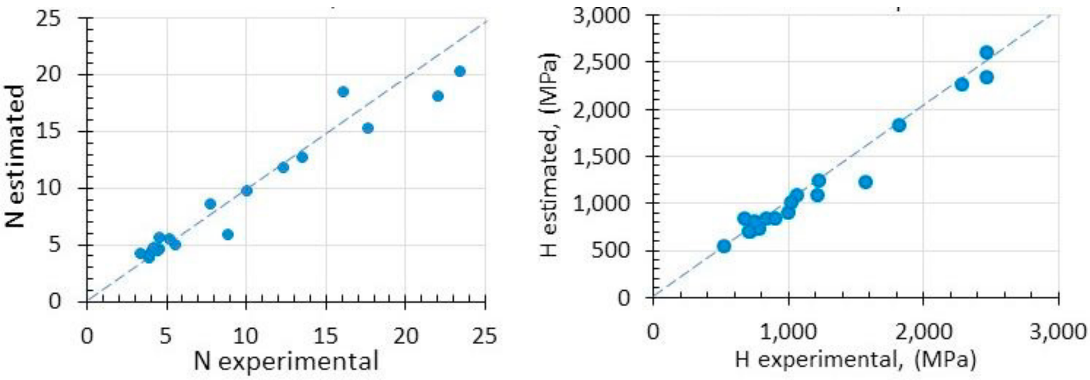

Fig. 5. Comparison of Ramberg-Osgood constants calculated from experimental data vs. estimated values.

3.1 Inverse analysis

In the previous section, N values were estimated using S_u and S_y and $\tilde{\varepsilon}_{p-L}$. Some materials with low ductility show a monotonically increasing engineering stress-strain curve till failure. These materials fail at stress S_F before reaching the peak stress with zero slope which conventionally defies the ultimate tensile strength value, S_u. For such materials the maximum ultimate strength, S_u, coincides with failure stress, S_F, i.e. $S_u = S_F$. In such cases, it is difficult to predict the true stress strain curve without knowing the ultimate stress at point of zero slope value. Hence, in this section an inverse approach for the apparent ultimate stress, $S_{u\ app}$, estimation is suggested. The apparent $S_{u\ app}$ is estimated from Eq. (11) using experimental values of N, S_y and corresponding 0.2% plastic offset strain. Figure 6 shows three materials, A, B and C, with low ductility that has been chosen to demonstrate this inverse approach [12].

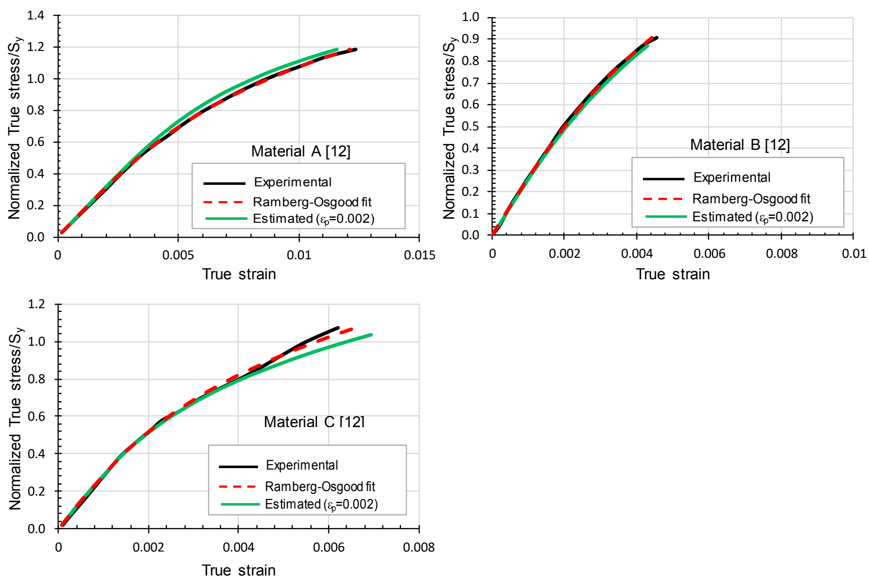

Fig. 6. Comparison of Ramberg-Osgood constants calculated from experimental data vs. estimated values.

Table 1 summarizes the inverse analysis of estimated normalized apparent ultimate strength, $S_{u\ app}/S_y$, for these steels.

Table 1: Estimated normalized apparent ultimate strength.

Material	N (experimental)	Normalized S_F/Sy (experimental)	Normalized $Su\ app/Sy$ (estimated)
A	14.90	1.19	2.44
B	4.56	0.91(estimated)	2.32
C	23.62	1.1	3.35

The applicability of the inverse approach to estimate the Ramberg-Osgood curves is shown in Fig. 7. It is seen from Fig. 7 that the predicted curves have fairly good agreement with the experimental ones. Our preliminary results indicate that $S_{u\ app}$ can be used to estimate fatigue limit for steels with Brinell hardness HB > 600 where they follow a similar relation between fatigue limit vs. S_u as it is often observed for more ductile steels with 200 < HB < 600.

4. Conclusions

The new estimation procedure for strain hardening exponent, N=1/n, in the true stress-true strain curve for materials with and without Luder's strain is developed and discussed. The following conclusions can be drawn:

- For materials without Luder's strain the Ramberg-Osgood constant are estimated using yield strength, ultimate strength and the plastic offset strain of 0.2%.
- For a material which experimental stress-strain curve shows Luder's strain (yield plateau), the true plastic Luder's strain value is used for the determination of the Ramberg Osgood constants.
- In addition, an inverse procedure for estimation of an apparent ultimate strength, $S_{u\ app}$, for low ductility materials is also suggested.
- Among basic mechanical properties obtained from a tensile test such as yield and ultimate strengths, elongation, and area reduction also the Luder's strain (if present) should be reported.

Acknowledgments

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