THE PROPERTIES OF TUNGSTEN AND THE CHARACTERISTICS OF TUNGSTEN LAMPS

By W. E. FORSYTHE and A. G. WORTHING

ABSTRACT

Radiational, electrical, thermal, and mechanical properties of tungsten.—For the temperatures for which data are available these properties are shown in figures and tables, for 100° intervals, for a temperature range from 300° K. to the melting-point (3655° K.). Many interrelations between the properties are pointed out.

Characteristics of tungsten lamps.—This includes a discussion with data of changes with temperature for (1) voltage, current, wattage, candle-power, efficiency, and life of vacuum lamps; (2) dependence of current, voltage, etc., upon filament dimensions; (3) photographic effects; and (4) gas losses.

End losses.—Losses due to heat conduction from a filament to the leads and supports are discussed and methods are given to correct for them. There is given a table showing filament lengths necessary in order that the central portions of different-sized filaments, operating at various temperatures, shall be free from effects due to end losses.

Temperature, brightness, and efficiency of some commercial lamps.—Tables are given showing data for commercial lamps.

INTRODUCTION

Probably no other substance has ever been so widely studied as tungsten, either in the number of properties considered or in the range of temperature covered. Considerable data have been accumulated, parts of which have been published from time to time. Results on some of the properties of tungsten have been given by Langmuir,[1] but due to the vast amount of work which has since been done and to the different temperature scales that have been used in its presentation, this work no longer satisfies the present needs. In this paper, data obtained in the Nela Research Laboratory, together with the results of others, are set forth in a compact, consistent form on a temperature scale determined by the fundamental constants proposed for the forthcoming International Critical Tables.

This paper is divided into two parts. One deals with the properties of tungsten, the other with the characteristics of tungsten lamps. In general, properties and characteristics are given as functions of the true temperature for well-aged filaments. The accuracy of the data is indicated by the number of significant figures given. In general, the last one given is the first uncertain figure. In some cases,

[1] *Physical Review*, 7, 302, 1916.

as will be apparent, these have been rounded off to 0 or 5. Where extrapolations have been made, it has been indicated. The subdivisions are as follows:

Properties of tungsten: temperature scale (brightness temperature); spectral emissivity; average luminous emissivity; color emissivity; total emissivity; color temperature; radiation temperature; melting-point; resistivity; brightness; radiation intensity; luminous efficiency; spectra; thermionic emission; vaporization; thermal expansion; atomic heats; thermal conductivity; thermoelectric effects; deviation from Lambert's cosine law; polarization of light emitted; mechanical properties—density, lattice spacing, simple rigidity, Young's modulus, compressibility, tensile strength and reduction of area at break, hardness.

Characteristics of tungsten lamps: temperature, voltage, candle-power relations for vacuum lamps; dependence of current, voltage, etc., upon filament dimensions; wire weight; photographic effects; life of lamps; gas losses; end losses; temperature, brightness, and efficiency of some commercial lamps.

PROPERTIES OF TUNGSTEN

Temperature scale.—The temperature scale used is one recently adopted by the General Electric Company's research laboratories. On this scale the gold point is the Day and Sosman[1] value of 1336° K., and the c_2 of Wien's equation is taken as 1.433 cm deg., the value adopted for the forthcoming International Critical Tables. On this scale the palladium point is 1829° K.[2] For convenience a black body held at this temperature has been used in the calibrations.

There are several properties of tungsten that may be used in measuring temperature once the relation between the property chosen and the temperature is known. For the most part, the brightness of the tungsten surface as measured with a disappearing filament optical pyrometer with a particular standard red-glass pyrometer filter[3] has been used. Such a brightness measurement

[1] *American Journal of Science*, **29**, 93, 1910.

[2] Hyde and Forsythe, *Astrophysical Journal*, **51**, 244, 1920.

[3] Hyde, Cady, and Forsythe, *ibid.*, **42**, 294, 1915.

does not give the true temperature directly but a temperature that is somewhat lower, called the "brightness temperature."[1] The relation between this temperature and the true temperature as derived from Wien's equation is

$$\frac{1}{T}-\frac{1}{S_\lambda}=\frac{\lambda \cdot 2.303 \log e_\lambda}{c_2}, \tag{1}$$

where T is the true temperature, S_λ the brightness temperature, and e_λ the spectral emissivity, corresponding to the wave-length λ. Using this method the accuracy of the true temperature scale in its experimental realization depends upon the accuracy attained in determining the following three factors: (1) the brightness temperature, (2) the wave-length[2] used, and (3) the spectral emissivity.[3]

Spectral emissivity.—An attempt to determine the spectral emissivity of tungsten for various temperatures was made some years ago by Mendenhall and Forsythe.[4] A black body was made of tungsten strips mounted in the form of a narrow, hollow wedge. With this device an attempt was made to measure the true temperature by observations on the inside of the wedge, and the emissivity by the ratio of the brightness of the tungsten as measured on the outside to that of the inside of the wedge. Due to inability to obtain good tungsten ribbon, poor black-body conditions prevailed, and too high emissivities resulted. The problem was later attacked by Worthing[5] who used tungsten tubes of about 1.3 mm external diameter and 0.8 mm internal diameter, with small radial holes about 0.1–0.2 mm in diameter (see Fig. 1). With such a

[1] Hyde, "Pyrometer Symposium," *Amer. Inst. Mining Met. Eng.*, p. 288, 1920; Forsythe, *Transactions Faraday Society*, **15**, 14, 1920.

[2] Forsythe, *Journal of the Optical Society of America*, **5**, 89, 1921.

[3] *Physical Review*, **10**, 377, 1917; *Zeitschrift für Physik*, **22**, 9, 1924; *General Electric Review*, 1924. See, also, Burgess and Waltenberg, *Bulletin Bureau of Standards*, **11**, 591, 1915; Pirani, *Zeitschrift für Physik*, **13**, 753, 1912; Pirani and Meyer, *Elektrotech. u. Maschinenbau*, **33**, 397, 444, 1915; Pirani, *Verh. d. deut. physik. Gesell.*, **13**, 19, 1911; Langmuir, *Physical Review*, **6**, 138, 1915; Langmuir, *ibid.*, **7**, 302, 1916; Shackelford, *ibid.*, **8**, 470, 1916; Henning and House, *Zeitschrift für Physik*, **16**, 63, 1923; Pirani and Meyer, *Verh. d. deut. physik. Gesell.*, **14**, 681, 1912; Pirani, *Physikalische Zeitschrift*, **13**, 753, 1912; Lax and Pirani, *Zeitschrift für Physik*, **22**, 273, 1924.

[4] *Astrophysical Journal*, **37**, 38, 1915.

[5] *Physical Review*, **10**, 377, 1917.

tungsten-tube black body, the true temperature was determined from observations of the radiation coming from the small radial holes, and the emissivity from the ratio of the brightness of the tungsten measured on the outer surface of the tube to the brightness of the black body measured through the small radial holes. Calculations using measured values of the heat conductivity showed

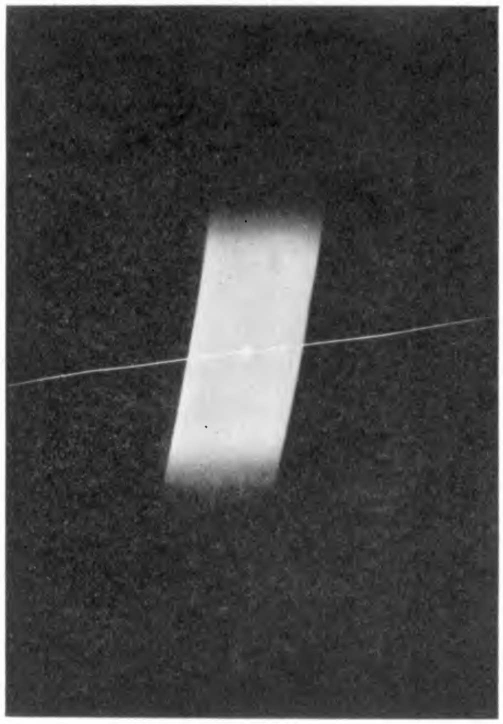

FIG. 1.—Photograph showing tungsten tube black body with pyrometer filament projected against small radial hole.

that the difference in temperature between the inside and outside of the tungsten tube was small, amounting to not more than 3° at a temperature of 2600° K. Corrections were made for the calculated differences.

Measurements of normal emissivity for red ($\lambda = 0.665\ \mu$) and for blue ($\lambda = 0.467\ \mu$) were made over a range of temperatures from about 1000° K. to 3200° K. (Table I). These values of emissivity have been checked by reflectivity measurements at room temperature and at 1300° K. In Figure 2 are given the results that were obtained. Variations of emissivity with wave-length for a wide range of wave-lengths, including infra-red[1,2] and ultra-violet regions,[3] are shown in Figure 3 for several temperatures.

Average luminous emissivity.—The ratio of the total normal brightness of tungsten to that of a black body at the same temperature is the average normal luminous emissivity. This may be obtained either by direct comparison of the tungsten brightness with that of a black body or by computation, making use of the crova wave-length.[4] Both methods have been used in obtaining the values given in Figure 2 and Table I.

Color emissivity.—The ratio of the normal brightness of tungsten at a temperature T to that of a black body at a temperature equal to its color temperature (see below) is called the "color emissivity."[5] The following equation gives the color emissivity e_c (Fig. 2 and Table I) as a function of T_c, the color temperature; S, the brightness temperature; and the wave-length, λ.

$$\log e_c = \frac{c_2}{2.303\lambda} \cdot \left(\frac{1}{T_c} - \frac{1}{S}\right). \tag{2}$$

In a study of the radiation laws Hyde[6] stated his criterion I for incandescent metals. For its fulfilment, this criterion required that the ratio of the brightness of a source for two different color temperatures shall be the same as for a black body for the same two temperatures. This would mean constancy of color emissivity. Hyde showed that the ratio of brightnesses for tungsten was less than for the black body. This means that he found the color emissivity of tungsten to decrease with increase in temperature. This is in agreement with values given in Table I.

1 Weniger and Pfund, *Physical Review*, **14**, 427, 1919.

2 Coblentz, *Bulletin of the Bureau of Standards*, **5**, 312, 1918.

3 Hulburt, *Astrophysical Journal*, **45**, 149, 1917.

4 Forsythe, *Journal of Franklin Institute*, **197**, 517, 1924.

5 Worthing, "Pyrometer Symposium," *Amer. Inst. Mining Met. Eng.*, p. 397, 1920.

6 *Astrophysical Journal*, **36**, 89, 1913.

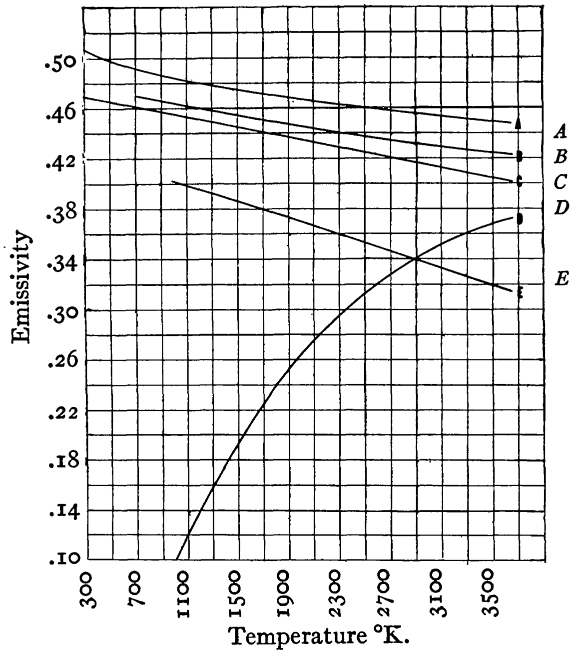

FIG. 2.—Various emissivities of tungsten as functions of temperature. Curve A, normal spectral emissivity for blue ($\lambda = 0.467\ \mu$). Curve C, normal spectral emissivity for red ($\lambda = 0.665\ \mu$). Curve B, average normal luminous emissivity. Curve E, normal color emissivity. Curve D, total emissivity.

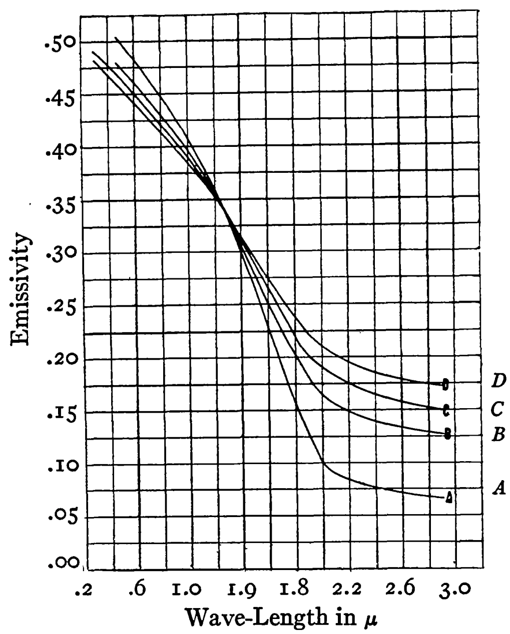

FIG. 3.—Spectral emissivities for different temperatures as a function of the wave-length over a wide range. Curve A, $T = 300°$ K. Curve B, $T = 1300°$ K. Curve C, $T = 1700°$ K. Curve D, $T = 2100°$ K.

TABLE I—A†

PROPERTIES OF TUNGSTEN

Temperature Degrees K. T	Spectral Emissivity $e_{0.665\,\mu}$	Spectral Emissivity $e_{0.467\,\mu}$	Average Luminous Emissivity e_v	Color Emissivity e_c	Total Emissivity e_t	Brightness Temperature $S_{0.665\,\mu}$	Color Temperature T_c	Radiation Temperature T_R	Relative Lengths at Different Temperatures L/L_0	Thermal Conductivity watts/cm deg. K.	Atomic Heats cal./g atom deg. C_p	Thompson Effect Microvolts degree
300	.470	.505	...	...	.032	...	...	...	1.0000	...	...	...
400	.468	.501	...	...	.042	...	...	...	1.0005	...	...	...
500	.466	.498	...	...	.053	...	...	...	1.0010	...	...	...
600	.464	.495	...	...	.064	...	...	...	1.0014	...	...	...
700	.462	.492	...	...	.076	...	...	...	1.0018	...	...	...
800	.460	.490	...	...	.088	...	...	...	1.0023	...	...	...
900	.458	.488	...	...	.101	...	...	...	1.0028	...	...	...
1000	.456	.486	.464	.396	.114	966	1006	581	1.0032	.84*	6.20	...
1100	.454	.484	.463	.393	.128	1058	1108	659	1.0036	.87*	6.35	...
1200	.452	.482	.462	.391	.143	1149	1210	738	1.0041	.90*	6.50	...
1300	.450	.480	.460	.388	.158	1240	1312	819	1.0046	.93	6.65	...
1400	.448	.478	.459	.386	.175	1330	1414	905	1.0052	.96	6.80	...
1500	.445	.476	.457	.383	.192	1420	1517	991	1.0057	.99	6.95	...
1600	.443	.475	.456	.381	.207	1509	1619	1080	1.0063	1.02	7.10	...
1700	.441	.473	.455	.378	.222	1597	1722	1167	1.0069	1.04	7.25	...
1800	.439	.472	.454	.376	.236	1684	1825	1254	1.0075	1.07	7.40	−18
1900	.437	.470	.453	.373	.249	1771	1929	1342	1.0081	1.09	7.55	−20
2000	.435	.469	.452	.370	.260	1857	2033	1428	1.0088	1.11	7.70	−22
2100	.433	.467	.450	.367	.270	1943	2137	1514	1.0094	1.13	7.85	−24
2200	.431	.466	.449	.364	.279	2026	2242	1601	1.0101	1.15	8.00	−26
2300	.429	.464	.448	.362	.288	2109	2347	1688	1.0108	1.17	8.15	−28
2400	.427	.463	.447	.359	.296	2192	2452	1775	1.0116	1.19	8.30	−30
2500	.425	.462	.446	.356	.303	2274	2557	1859	1.0124	1.21	8.45	...
2600	.423	.460	.444	.353	.311	2356	2663	1945	1.0132	1.23*	...	...
2700	.421	.459	.443	.350	.318	2437	2770	2031	1.0140	1.25*	...	...
2800	.419	.458	.442	.347	.323	2516	2878	2116	1.0149*	1.27*	...	...
2900	.417	.456	.441	.345	.329	2595	2986	2202	1.016*	...	...	...
3000	.415	.455	.440	.343	.334	2673	3094	2286	1.017*	...	...	...
3100	.413	.454	.438	.341	.337	2750	3202	2371	1.018*	...	...	...
3200	.411	.452	.437	.338*	.341*	2827	3311*	2455*	1.019*	...	...	...
3300	.409	.451	.436	.335*	.344*	2903	3422*	2538*	1.020*	...	...	...
3400	.407*	.450*	.435*	.332*	.348*	2978	3533*	2621*	1.021*	...	...	...
3500	.405*	.449*	.434*	.329*	.351*	3053	3646*	2704*	1.023*	...	...	...
3655‡	.402*	.447*	.433*	.324*	.354*	3165	3817*	...	...	...	...	...

* These values are extrapolated.
† Data given in this table apply to aged tungsten filaments.
‡ Melting-point.

TABLE I—B*

PROPERTIES OF TUNGSTEN

Temperature Degrees K. T	Resistivity Micro-ohms cm ρ‡	$\frac{T}{\rho}\frac{d\rho}{dT}$ §‡	Normal Brightness Candles per cm² B_n	$\frac{T}{B_n}\frac{dB_n}{dT}$	Crova Wave-Length λ_c‖	Total Radiation Intensity watts/cm² η	$\frac{T}{\eta}\frac{d\eta}{dT}$	Luminous Efficiency Lumens per watt eff.	$\frac{T}{(eff.)}\frac{d(eff.)}{dT}$	Electron Emission amp./cm² i	$\frac{T}{i}\frac{di}{dT}$	Rate of Vaporization Grams per cm²/sec v	$\frac{T}{v}\frac{dv}{dT}$
300	5.64	...	...	...	...	.0015	...	...	...	...	...	...	...
400	8.06	...	...	...	...	.006	...	...	...	...	...	...	...
500	10.74	...	...	...	...	.019	...	...	...	...	...	...	...
600	13.54	...	...	...	...	.048	...	...	...	...	...	...	...
700	16.46	...	...	...	...	.105	...	...	...	...	...	...	...
800	19.47	...	...	...	...	.206	...	...	...	...	...	...	...
900	22.58	...	...	...	...	.379	...	...	...	...	...	...	...
1000	25.70	1.21	.00012	22.	.6077	.654	5.35	...	...	...	...	...	...
1100	28.85	1.20	.0010	21.	.6038	1.072	5.35	...	...	...	...	...	...
1200	32.02	1.20	.006	20.	.6004	1.691	5.35	...	...	...	...	...	...
1300	35.24	1.20	.029	18.6	.5971	2.576	5.35	...	...	...	...	...	...
1400	38.52	1.20	.11	17.2	.5034	3.82	5.29	0.09	11.8	5.75×10^{-9}	39.6	...	...
1500	41.85	1.20	.33	16.1	.5902	5.55	5.23	0.20	10.8	7.58×10^{-8}	37.1	...	...
1600	45.22	1.20	.92	15.2	.5874	7.77	5.15	0.40	10.0	8.05×10^{-7}	34.9	3.7×10^{-20}	67
1700	48.63	1.20	2.26	14.4	.5850	10.59	5.07	0.71	9.3	6.31×10^{-6}	33.0	1.92×10^{-18}	63
1800	52.08	1.20	5.05	13.7	.5826	14.22	4.99	1.16	8.7	3.92×10^{-5}	31.3	6.22×10^{-17}	59
1900	55.57	1.20	10.40	13.0	.5806	18.55	4.91	1.88	8.1	2.04×10^{-4}	29.7	1.41×10^{-15}	56
2000	59.10	1.20	20.0	12.3	.5785	23.72	4.85	2.78	7.5	8.92×10^{-4}	28.3	2.32×10^{-14}	53
2100	62.65	1.20	35.6	11.7	.5769	29.82	4.79	3.95	6.9	3.46×10^{-3}	27.1	2.91×10^{-13}	50
2200	66.25	1.20	61.3	11.2	.5753	37.18	4.74	5.47	6.4	1.14×10^{-2}	25.9	2.90×10^{-12}	48
2300	69.90	1.20	100.5	10.8	.5737	45.9	4.69	7.25	6.0	3.63×10^{-2}	24.9	2.34×10^{-11}	46
2400	73.55	1.20	157.0	10.3	.5724	55.8	4.64	9.37	5.6	1.02×10^{-1}	24.0	1.58×10^{-10}	44
2500	77.25	1.20	237.5	9.9	.5711	67.6	4.59	11.67	5.3	2.67×10^{-1}	23.1	9.18×10^{-10}	42
2600	81.0	1.20	347.0	9.6	.5701	80.8	4.55	14.28	5.0	6.48×10^{-1}	22.3	4.64×10^{-9}	40
2700	84.7	1.20	498.0	9.2	.5691	96.2	4.51	17.26	4.7	1.47	21.5	2.06×10^{-8}	38
2800	88.5	1.20	694.	8.9	.5682	112.9	4.47	20.43	4.4	3.21	20.8	8.28×10^{-8}	36
2900	92.3	1.20	949.	8.6	.5674	132.1	4.43	23.80	4.1	...	...	2.99×10^{-7}	35
3000	96.2	1.20	1257.	8.3	.5666	153.9	4.40	27.10	3.8	...	...	9.92×10^{-7}	34
3100	100.0	1.20	1647.	8.0	.5659	177.5	4.37	31.0	3.6	...	...	3.01×10^{-6}	33
3200	103.8	1.20	2110.	7.8	.5652	203.	4.34	34.6	3.4	...	...	8.79×10^{-6}	32
3300	107.8	1.20	2685.	7.7	.5645	232.	4.31	38.5	3.3	...	...	2.29×10^{-5}	31
3400	111.7	1.20	3370.	7.6	.5638	264.†	4.29†	42.6†	3.2†	...	...	5.74×10^{-5}	30
3500	115.7	1.20	4220.	7.4	.5631	300.†	4.27†	45.9†	3.1†	...	...	1.36×10^{-4}	30
3655¶	121.8†	1.20†	5740.	7.3	.5621	360.†	...	53.1†	3.0†	...	...	4.70×10^{-4}	28

* Data given in this table apply to well-aged tungsten.

† These values are extrapolated.

‡ These values depend on the dimensions at room temperature. In intercomparing values for the various properties, account should be taken of the thermal expansion as well as the variation from Lambert's cosine law.

§ The constancy of this quantity at high temperatures for a particular specimen makes temperature determinations from resistance measurements simple, since the temperature corresponding to any resistance can then be computed from resistances at two known temperatures.

‖ The values of the Crova wave-lengths given are the limiting values. The Crova wave-length between two temperatures is the average of the two corresponding limiting Crova wave-lengths.

¶ Melting-point.

TABLE I—C

SOME ELECTRICAL AND RADIATIONAL CHARACTERISTICS OF A TUNGSTEN WIRE 1 CM LONG AND 1 CM IN DIAMETER MOUNTED IN A VACUUM

The function $V'\sqrt[3]{I'}$ is independent of the filament diameter. For the dependence of current, voltage, etc., upon filament dimensions see equations 18–25. See also Table VI.

Temperature Degrees K. T	Resistance Ohms R'	Potential Drop Volts V'	Current Amperes I'	Power Watts W'	$V'\sqrt[3]{I'}$	Mean Horizontal Candle-Power C'*†	Luminous Flux Lumens*
300....	7.18×10^{-6}	$.057\times10^{-3}$	$.008\times10^{+3}$	.00045	$.11\times10^{-3}$	....	....
400....	10.26	.38	.037	.014	1.3	....	....
500....	13.66	.85	.062	.053	3.4	....	....
600....	17.23	1.57	.091	.143	7.1	....	....
700....	20.96	2.60	.124	.323	13.0	....	....
800....	24.80	4.00	.161	.644	21.8	....	....
900....	28.75	5.87	.204	1.200	34.6	....	....
1000....	32.73	8.23	.251	2.068	51.9	.00012	.0012
1100....	36.84	11.19	.304	3.392	75.2	.0010	.010
1200....	40.80	14.77	.362	5.35	105.3	.006	.06
1300....	44.90	19.15	.427	8.17	144.2	.029	.29
1400....	49.10	24.5	.494	12.12	194.0	.11	1.1
1500....	53.35	30.7	.575	17.62	255.	.33	3.3
1600....	57.60	37.7	.655	24.72	328.	.95	9.5
1700....	61.95	45.7	.738	33.72	413.	2.34	23.5
1800....	66.35	54.9	.827	45.35	515.	5.22	52.4
1900....	70.75	64.2	.908	58.30	622.	10.78	108.3
2000....	75.20	75.5	1.004	75.8	756.	20.75	208.5
2100....	79.80	87.2	1.093	95.4	899.	37.0	372.
2200....	84.40	100.4	1.190	119.5	1064.	63.8	641.
2300....	89.00	114.6	1.287	147.5	1247.	104.7	1052.
2400....	93.65	129.4	1.383	179.0	1442.	164.0	1648.
2500....	98.40	146.3	1.487	217.5	1670.	248.0	2490.
2600....	103.2	164.1	1.590	260.8	1915.	363.0	3650.
2700....	107.8	182.9	1.700	310.5	2180.	522.0	5240.
2800....	112.7	203.0	1.800	365.5	2470.	730.	7330.
2900....	117.5	224.2	1.910	428.0	2780.	1000.	10050.
3000....	122.5	247.7	2.020	501.0	3130.	1325.	13300.
3100....	127.4	272.	2.130	578.5	3495.	1740.	17500.
3200....	132.2	296.	2.24	663.	3875.	2235.	22450.
3300....	137.3	323.	2.35	760.	4295.	2850.	28650.
3400....	142.2	351.	2.47	866.	4740.	3580.	36000.
3500....	147.2	381.	2.59	985.	5230.	4500.	45200.
3655....	155.0	428.	2.76	1180.	5950.	6150.	61800.

* As measured through clear glass bulbs.

† Differences between these values and the brightnesses in Table I—*B* are due to Lambert's cosine-law variations, thermal expansion, and 1 per cent bulb absorption.

Total emissivity.—Total emissivity[1] is defined by the following:

$$\eta = e_t \sigma T^4 , \tag{3}$$

where η is the rate of total energy radiation per unit area, σ the Stefan-Boltzmann constant, T the temperature, and e_t the total emissivity. Since, for a filament at high temperatures mounted *in*

[1] Worthing and Forsythe, *Physical Review*, **18**, 144, 1921.

vacuo the wattage input corrected for end losses (see below) divided by the area of the filament surface gives η, total emissivities (Fig. 2 and Table X) are thus easily obtainable once the temperature and the thermal expansion are known. Values below 1200° K. were furnished by Dr. H. A. Jones,[1] of the Research Laboratory at Schenectady.

Color temperature.—A light-source the integral color of which may be matched with that of a black body at some definite temperature is said to have a color temperature. As defined, color temperature[2] is the temperature of a black body which has the same integral color as the source studied. Color matchings[3] for such determinations of color temperature can be most satisfactorily made with a regular photometer set-up, using a contrast photometer. The color temperatures of tungsten were determined[4] by direct comparisons of tungsten lamps with a black body by this method for a temperature range of from about 1700° to 2600° K. If the black body and the source studied, as with tungsten, have the same spectral-energy distribution when color matched, such color temperatures can be obtained by comparisons of the relative spectral brightnesses for two wave-lengths. The best color match with tungsten and a black body as determined by spectral brightness ratios at 0.467 μ and 0.665 μ shows a deviation[5] at 0.567 μ varying from about 0.8 per cent at 1600° K. to about 1.6 per cent at 2600° K., the tungsten being the brighter. The color temperatures of tungsten as a function of the brightness temperature for $\lambda=0.665\ \mu$ have thus been determined by measuring the ratio of the red ($\lambda=0.665\ \mu$) to the blue brightness ($\lambda=0.467\ \mu$), called the "red-blue ratio," for temperatures between 1200° and 3000° K.[6] Using the red-and-blue emissivities, the color temperature and brightness-temperature relations were also calculated. Results obtained by

[1] To be published.

[2] Hyde, Cady, and Forsythe, *Physical Review*, **10**, 395, 1917.

[3] Forsythe, *Journal of the Optical Society of America*, **6**, 476, 1922; Hyde, Cady, and Forsythe, *Physical Review*, **10**, 396, 1917.

[4] *Ibid.*, p. 401, 1917.

[5] Hyde, *Astrophysical Journal*, **36**, 114, 1912; Forsythe, *Journal of the Optical Society of America*, **7**, 1115, 1923.

[6] Forsythe, *ibid.*, p. 1118, 1923.

these two methods differed slightly. A weighted average of the two sets has been adopted as the best approximation (Table I and Fig. 4). Such values are in good agreement with values found by the direct photometric comparison except at very low and very high temperatures.

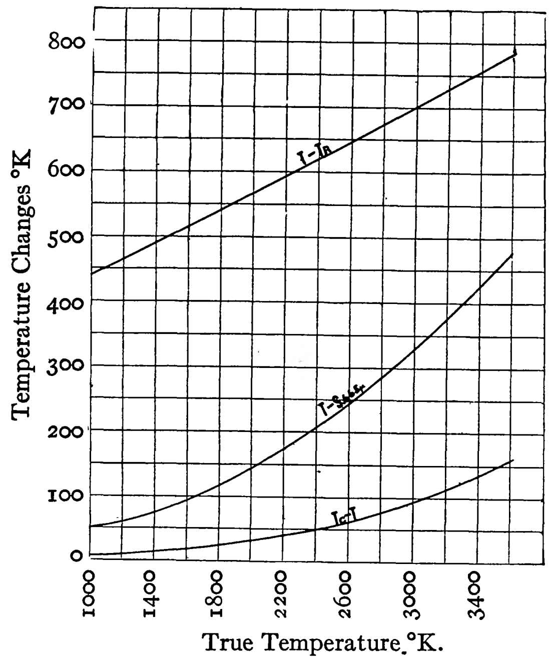

FIG. 4.—Curves showing the difference between true temperature (T) and color temperature (T_c), curve (T_c-T); and brightness temperature (S), curve ($T-S$ 0.665μ); and radiation temperature (T_R), curve ($T-T_R$).

Radiation temperature.—If the temperature of a tungsten filament is determined from the rate of total energy radiation as though it were a black body, a value, lower than the true temperature, called the "radiation temperature"[1] (T_R) is obtained. Thus, as in equation (3),

$$\eta=\sigma T_R^4 . \tag{4}$$

From equations 3 and 4 it follows that the true temperature and the radiation temperature are related as follows (Fig. 2 and Table I):

$$T_R=\sqrt[4]{e_t}\cdot T . \tag{5}$$

[1] Hyde, "Pyrometer Symposium," *Amer. Inst. Mining Met. Eng.*, p. 288, 1920.

Melting-point.—Many experimenters have sought the melting-point of tungsten. Excluding cases where there were very questionable, extended extrapolations, the methods used have depended upon the disappearing filament pyrometer for the measurement of spectral-brightness temperature either of tungsten or of a black body made of tungsten at the melting-point. In the former case, the spectral emissivity of tungsten at the melting-point must be employed as in equation 1 to obtain the true temperature; in the latter case, the result obtained is the true temperature.

In order to melt the tungsten it was electrically heated either in wire or rod form, or as arc terminals. When the true temperature has been measured directly, filaments or terminals with small radially drilled holes have been used.

The results of the various workers have been summed up by one of the authors.[1] Taking into consideration only the results for which the metallurgy of tungsten and the art of pyrometry have been on a satisfactorily sound basis, 3655° K. was obtained as the best weighted average for the melting-point of tungsten. This, with the possible exception of carbon, is probably the highest of all melting-points.

Resistivity.—The resistivity of tungsten[2] was obtained from the same readings as was the total emissivity. It was found that the resistance and temperature are related by the following equation:

$$\frac{R}{R_0}=\left(\frac{T}{T_0}\right)^{\beta}, \tag{6}$$

where β is a constant having a value of about 1.20. While β varies slightly from sample to sample, the relation between log R and log T was always found to be linear—to within the errors of measurements—for the range 1000°–3200° K. There is but a small departure from the linear relation even down to room temperature (Table I). Values below 1000° K. were furnished by Dr. Jones, of Schenectady.

[1] Worthing, *ibid.*, p. 374, 1920; *Zeitschrift für Physik*, **22**, 9, 1924; *General Electric Review*, 1924.

[2] Worthing, *Physical Review*, **12**, 207, 1918; *ibid.*, **18**, 144, 1921.

Variation of resistivity with pressure, P,[1] for the range $O<P<$ 12,000 kg wt./cm^2 is given by

$$\frac{1}{\rho}\frac{d\rho}{dp}=-1.35\times10^{-6}\frac{1}{\text{kg wt./cm}^2}\,. \tag{7}$$

The variation of this function with temperature between 0° C. and 100° C. amounts to only 2 or 3 per cent.

Brightness.—Normal brightness of a material is defined as the candle-power per square centimeter of a flat piece of material measured normal to the surface. In this paper the brightnesses given are independent of the characteristics of a particular observer's eye, being based on the Middlekauff and Skogland[2] candle-power scale. Three methods have been used. The first method consisted[3] in making candle-power determinations of a known length of uniformly heated filaments with known cross-sections. The second method consisted in measuring the brightness of the surface with an optical pyrometer calibrated as a brightness photometer.[4] The third method consisted in calculating the brightness from the color emissivity and the brightness of a black body. The values given in Table I are the weighted averages of normal brightnesses found by the three methods mentioned above. If the average brightness in all directions is desired, these values must be increased by about 5 per cent to correct for the variation of tungsten from Lambert's cosine law (see below).

Radiation intensity.—Radiation intensity, η (Table I),[5] is the ratio of the rate of total energy output of a source to its surface. For tungsten at high temperatures the rate of total energy output may be taken as the wattage input of a filament mounted *in vacuo* when end losses are either eliminated or corrected for by one of the methods outlined below. It is to be noted that the values of η given in Table I refer to a unit area at the temperatures given.

[1] Bridgman, *Proceedings of the National Academy of Science*, **6**, 505, 1920.

[2] *Bulletin of the Bureau of Standards*, **2**, 484, 1914.

[3] Forsythe, *Physical Review*, **19**, 436, 1922.

[4] Forsythe, *Journal of the Franklin Institute*, **197**, 517, 1924.

[5] Worthing, *Physical Review*, **10**, 393, 1917.

Efficiency.—The efficiency of a tungsten lamp is the ratio of the output in lumens to the input in watts,[1] that is, in lumens per watt. The luminous efficiency of tungsten as a material (Table I) is the efficiency of the lamp corrected for various lamp losses (see below).

The efficiency of tungsten as a light-source increases with the temperature due to the shift of the point of maximum intensity toward the wave-length of maximum visibility. There is at the same time a decrease in the efficiency due to an increase in the total emissivity and a decrease in the average visible emissivity with an increase in temperature as can be seen from the curves in Figure 2. This means that the luminous efficiency of tungsten approaches that of a black body at high temperatures.

Efficiencies computed from observed B_n, η, L/L_0, and Lambert's cosine-law deviation were found to be about $1\frac{1}{2}$ per cent less than the directly measured efficiencies of ordinary 40-watt straight filament lamps after correcting for bulb absorption and end losses. This difference was ascribed to uncertainties in end-loss corrections, Lambert's cosine-law correction, deviations from actual color match with a black body, and in determination of lamp-filament temperatures. Values given in the table have been adjusted on these assumptions to eliminate differences.

A number of the electrical and radiational characteristics of a cylindrical tungsten wire of one-centimeter length and one-centimeter diameter have been found useful. In Table I, C, a number of such characteristics are given as a function of the temperature. The function $V'\sqrt[3]{I'}$ (equation 25) given in column 6 of this table is useful in estimating the temperature of a straight filament mounted in a vacuum, providing the length is known. It is also useful in maintaining at a constant temperature a filament whose diameter varies during burning, as for instance in a study of the rate of evaporation of a filament. The use of this function will give correct results only when the resistivity and surface condition do not change.

Spectra.—The ordinary arc and spark spectra of tungsten contain several thousand lines. Of these only a few important ones

[1] Hyde, Cady, and Forsythe, *ibid.*, p. 401, 1917.

have been selected for Table II. Data on wave-lengths,[1] relative intensities,[2] lines that reverse in the arc,[3] and ultimate rays[4] have been taken from different sources.

TABLE II

STRONGEST ARC AND SPARK LINES OF TUNGSTEN

λ in A	Intensity Arc Spark	λ in A	Intensity Arc Spark
2397.110.........	10 U	4484.197	10
2702.127.........	10	4843.829	10
3215.578.........	5 U		
3572.477.........	10	5006.169	10 10
3592.426.........	10	5015.334	10
3613.793.........	10 U		
3641.419.........	10	5053.300	15 15 R
3736.220.........	10 .	5224.680	20 20
4008.769.........	10 10 U	5492.331	15 15
....	....	5514.712	20 20
....	....	5648.391	10 10
4294.623.........	10 10	5735.093	15
4302.123.........	8 5 U	5804.844	10

R. These lines may appear reversed in the arc.
U. These lines are ultimate rays.

The long-wave photo-electric limit for tungsten, i.e., the longest wave-length of radiation which can, by incidence on the surface of tungsten, cause the emission of electrons, is 2615° A.[5]

The orbital energy levels, I/N, for X-rays shown in Figure 5 are those given by Bohr and Coster,[6] corrected, however, to agree with a calcite spacing of 3.028 A as adopted for the forthcoming International Critical Tables. The wave-lengths given are the average of several determinations.[7] The quantum notation is the recent

[1] Belke, *Zeitschrift für wiss. Photographie*, **17**, 132, 1917.

[2] Exner and Haschek, *Die Spektren der Elemente*, 1911.

[3] Hagenbach and Konen, *Atlas of Emission Spectra.*

[4] Gramont, *Comptes Rendus*, **171**, 1106, 1920.

[5] Hammer, *Physical Review*, **20**, 198, 1922.

[6] *Zeitschrift für Physik*, **12**, 342, 1923.

[7] A. H. Compton, *Physical Review*, **7**, 646, 1916; **8**, 753, 1916; Siegbahn, *Philosophical Magazine*, **38**, 639, 1919; Stenstrom, Thesis, Lund, 1919; Duane and Shimizu, *Physical Review*, **16**, 526, 1920; Duane and Patterson, *Proceedings of the National Academy of Science*, **6**, 518, 1920; Duane and Stenstorm, *ibid.*, p. 607, 1920; Hjalmar, *Zeitschrift für Physik*, **15**, 65, 1923.

one adopted by Bohr and Coster. There are certain additional lines which cannot be incorporated in this figure.

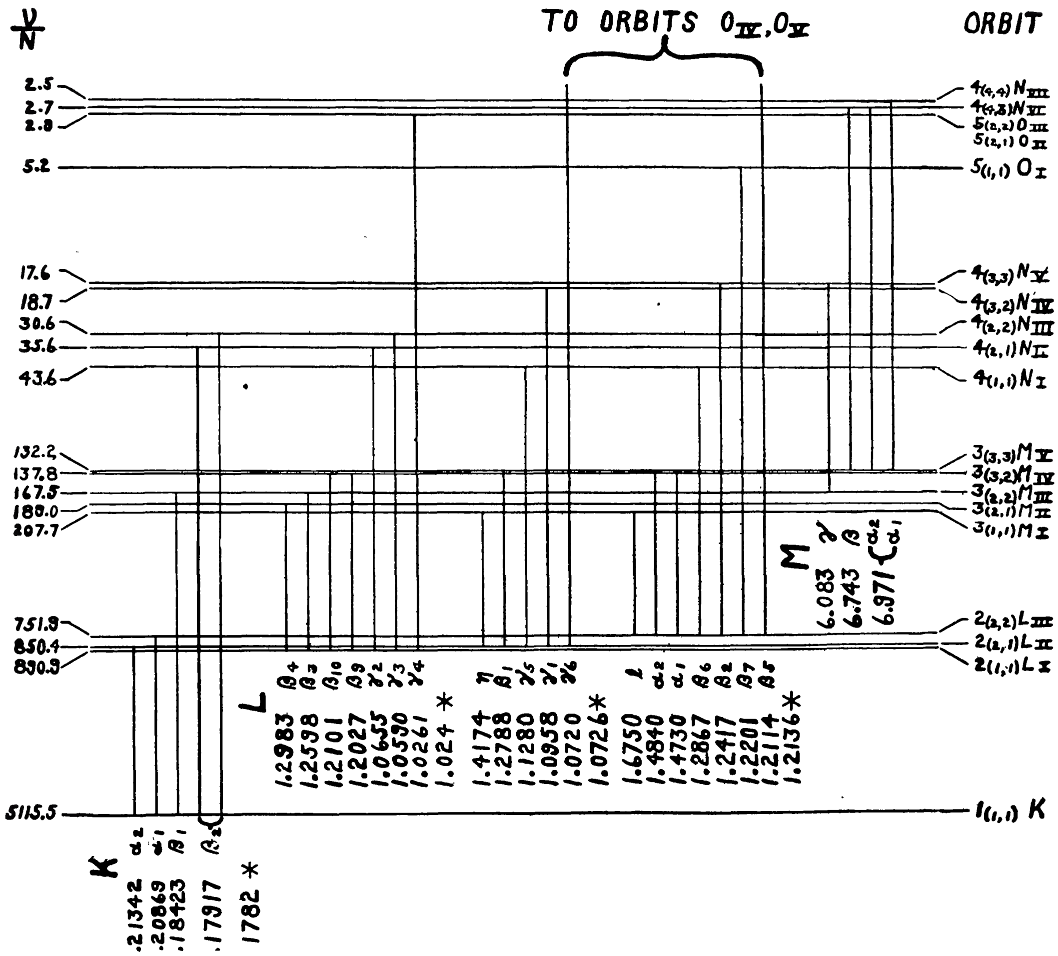

FIG. 5.—Chart showing X-ray spectra for the normal tungsten atom (calcite spacing equals 3.028 A). Orbital energy levels for the *K*, *L*, *M*, *N*, and *O* orbits are expressed in units equivalent to the energy level of the innermost orbit of the hydrogen atom when corrected for the finite mass of its electron. The scale is logarithmic. Both energy levels and quantum designations are taken from a paper by Bohr and Coster (see p. 15, n. 6). The wave-lengths are averages taken from several papers (see p. 15, n. 7). Values starred are experimental absorption limits.

Thermionic emission.—The values of electron emission given in Table I have been furnished by Dr. Dushman,[1] of the General

[1] *General Electric Review*, **26**, 154, 1923; Davisson and Germer, *Physical Review*, **20**, 300, 1922.

Electric Research Laboratory at Schenectady. His results are given in equation form by

$$i=AT^2e^{-\frac{b_0}{T}}, \tag{8}$$

where i is the thermionic current density, T the temperature, e the natural logarithm base, A the constant 60.2 amp./cm², and b_0 a specific constant for the material having the value 52,600°.

Vaporization.—The rates of vaporization of tungsten given in Table I are taken from data by Dr. Langmuir,[1] corrected for temperature by Dr. H. A. Jones, of the General Electric Research Laboratory at Schenectady.

Thermal expansion.—Expansion[2] of tungsten between 300° and 900° K. was determined in the usual manner. For higher temperatures, use was made of hairpin tungsten filaments mounted in special lamp bulbs, permitting expansion measurements on 18 cm of the filament which was uniformly heated. Results (Table I) are well represented by

$$\frac{L-L_0}{L_0}=4.44\times10^{-6}(T-300)+4.5\times10^{-11}(T-300)^2 +2.20\times10^{-13}(T-300)^3, \tag{9}$$

where L_0 and L refer respectively to filament lengths at 300° K. and at the temperature T. The coefficient of expansion at 300°, 1300°, and 2300° K. are, respectively, 4.44×10^{-6}, 5.19×10^{-6}, and 7.26×10^{-6} per degree.

Atomic heats.—The performance of a filament *in vacuo*, while passing from one steady state to another in consequence of a change in the heating current, depends in part upon the atomic heat.[3] With a set-up by which the current through a tungsten filament could be measured at different time-intervals after the applied voltage had changed, data were obtained which yielded atomic heat (Table I).

[1] *Physical Review*, *ibid.*, **2**, 329, 1913.

[2] Worthing, *ibid.*, **10**, 638, 1917.

[3] Worthing, *ibid.*, **12**, 199, 1918. See also, Corbino, *Zeitschrift für Physik*, **13**, 375, 1912; Pirani, *Verh. d. deut. physik. Gesell.*, **14**, 1037, 1912; Gaehr, *Physical Review*, **12**, 396, 1918; Smith and Bigler, *ibid.*, **19**, 268, 1922.

Thermal conductivity.—Variation of the temperature of a filament near a lead-in wire is caused by conduction of heat from the filament to the lead. From observed temperature variations and other known properties of tungsten, heat-conductivity[1] determinations of tungsten were made (Table I) for the range 1500° to 2500° K. For this range the relation between temperature and heat conductivity is linear.

Thermo-electric effect.—Variation of temperature near a lead-in wire depends upon the direction of the heating current. This variation is due to the Thompson effect.[2] By studying this variation with change in current direction, determinations of the Thompson e.m.f. (Table I) were made for the range 1500° to 2200° K.

Thermo-electric properties of tungsten for temperatures between 0° and 100° C. and pressures between atmospheric and 12,000 kg wt./cm^2 have been determined by Bridgman.[3] For the variations in the thermal e.m.f., E, and in Peltier e.m.f., P, in a tungsten-lead circuit and for the coefficient of the Thompson effect in a tungsten wire, he has given the following equations for atmospheric pressure and temperatures between 0° and 100° C.:

$$E=(1.594t+0.01705t^2) \text{ microvolts} \quad (10)$$

$$P=(1.594+0.0341t)(t+273) \text{ microvolts} \quad (11)$$

$$\sigma=0.0341(t+273) \frac{\text{microvolts}}{\text{degree}} \quad (12)$$

Variations of E, P, and σ with pressure P for $O<P<12{,}000$ kg wt./cm^2 are shown in Table III for three temperatures.

Deviation from Lambert's cosine law.—Fulfilment of Lambert's cosine law for luminous radiation requires that an element of the light-source shall appear equally bright from whatever angle viewed. A black body fulfils this requirement. Tungsten deviates,[4] however, in that as the angle of viewing the surface changes from normal incidence to grazing incidence, the brightness, at first slowly and then more rapidly, increases from the normal brightness to a

[1] Worthing, *ibid.*, **4**, 535, 1914.

[2] Worthing, *ibid.*, **5**, 445, 1915.

[3] *Proceedings of the American Academy of Arts and Science*, **53**, 269, 1918.

[4] Worthing, *Astrophysical Journal*, **36**, 345, 1912.

maximum brightness of about 115 per cent of normal at a 75° angle, after which it decreases rapidly and probably reaches zero at 90°

TABLE III

EFFECT OF PRESSURE ON THERMO-ELECTRIC PROPERTIES OF TUNGSTEN

PRESSURE	t=0° C.		t=60° C.			t=100° C.		
kg wt./cm²	P μ-volts	σ $\frac{\mu\text{-volts}}{\text{deg.}}$	E μ-volts	P μ-volts	σ $\frac{\mu\text{-volts}}{\text{deg.}}$	E μ-volts	P μ-volts	σ $\frac{\mu\text{-volts}}{\text{deg.}}$
2000.........	+ 5.9	+.020	+1.415	+ 8.7	+.025	+ 2.53	+10.4	+.028
4000.........	11.7	.045	2.87	17.1	.037	4.99	20.3	.026
6000.........	17.2	.085	4.30	27.1	.073	7.60	32.0	.057
8000.........	24.1	.057	5.72	34.4	.106	10.12	43.4	.146
10,000.........	30.4	.035	7.10	42.3	.143	12.61	55.4	.235
12,000.........	+35.7	+.093	+8.58	+51.8	+.151	+15.14	+64.8	+.198

E is the thermal e.m.f. of a couple composed of a branch of uncompressed metal and a branch of compressed metal, one junction being at 0° C., the other at temperatures indicated. A positive e.m.f. indicates tendency for current to pass from uncompressed to compressed metal at the hot junction.

P is the Peltier effect at a junction between the compressed and uncompressed branches. The plus sign indicates heat absorption at junction with passage of current from the uncompressed to the compressed branch.

σ is the excess of the Thompson coefficient in compressed over uncompressed metal.

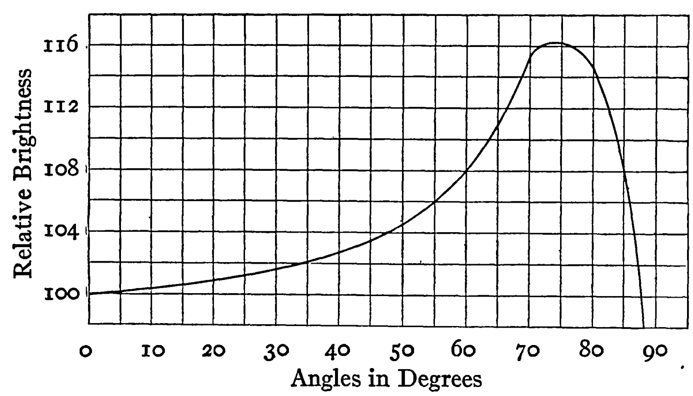

FIG. 6.—The deviation from Lambert's cosine law of the emission from tungsten at normal operating temperature.

(Fig. 6). The variation with wave-length is small but real, and whether or not there is a variation with temperature is not definitely determined.

Computation shows that the average brightness of a cylindrical tungsten-wire filament viewed normal to its length is about 3 per cent greater than the normal brightness, and that the average brightness, taking account of all direction of emission, as is done when the brightness is computed from the total light-output and filament dimensions, is about 5 per cent greater than the normal brightness. This correction must be taken into account when intercomparing the values for B_n, η, and efficiency values shown in Table I.

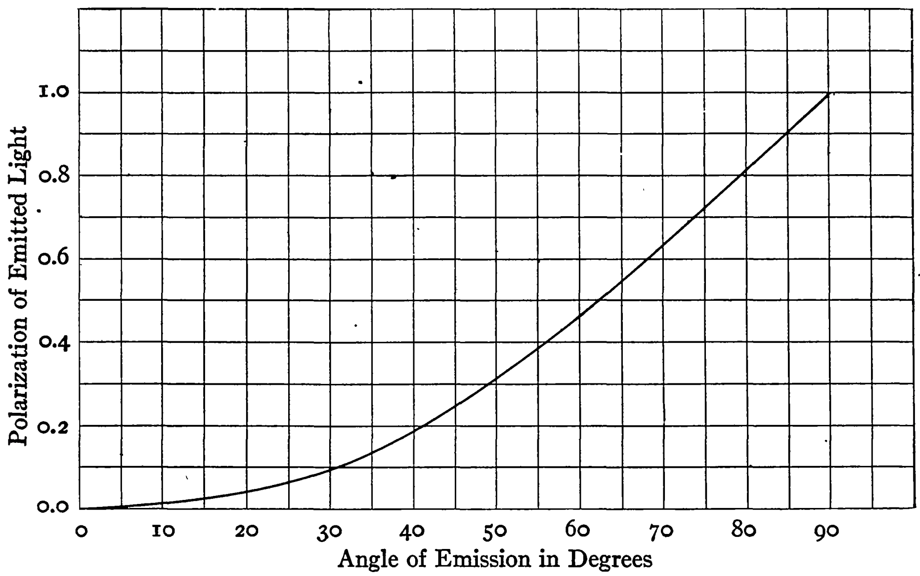

FIG. 7.—Polarization of the light emitted from tungsten at different angles of emission.

Polarization of light emitted.—The light emitted by tungsten in directions other than normal is polarized. The variation with angle of emission is shown in Figure 7. The direction of the electric vector of the stronger component of an emitted ray falls in the plane of emission, that is, the plane determined by the ray considered and the normal to the surface at the point of emission. For a straight tungsten filament of circular cross-section, as may be shown, the light emitted in a direction normal to its length is polarized by about 19 or 20 per cent. The direction of the electric vector of the stronger component is normal to the length of the filament.

Density.—The density of tungsten in wire form as ordinarily prepared is quite closely 19.3 g/cm^3 at room temperature; as determined by X-Ray analyses[1] in ingot form it is somewhat less. Whether or not a variation occurs with continued drawing is uncertain.

Lattice spacing.—Tungsten crystals have a body-centered, cubic lattice. The edge of the elementary cube as determined by X-ray analysis[1] is 3.155 A. This is consistent with the above-given density, 184 (o = 16) as the atomic weight, and 1.663×10^{-24} g as the mass of the hydrogen atom.

Simple rigidity.—The rigidity[2] of tungsten varies greatly with wire size, grain structure, and temperature. Our knowledge regarding these variations is quite limited. Results that have been obtained indicate tungsten at room temperature to have the greatest rigidity of all substances recorded in the literature. Depending upon wire size and grain structure, variations in the modulus of rigidity between 0.9 and 2.2×10^{12} dynes/cm^2 (i.e., between about 9000 and 22,000 kg wt/mm^2) have been found at room temperature. How much of the variation is to be ascribed to wire size, to grain structure, or to impurities is uncertain.

The variation with wire size consists of an increase in the modulus with a decrease in the diameter. The results at room temperature obtained with variations in grain structure seem confused. The temperature variations, however, are more definite. Starting at room temperature, there seems first to be a relatively slow decrease in the rigidity modulus amounting only to about 5 per cent in going to 1000° K. and then gradually a much higher rate of decrease so that at 2000° K. it is only about 15 per cent of the room-temperature value (Fig. 8). The three curves represent successive runs on the same specimen of wire. They show unexplained variations, a part of which are probably due to changes in grain structure.

Young's modulus.—Very little data on Young's modulus[3] for

[1] Hull, *Physical Review*, **17**, 576, 1921; Davey, letter.

[2] Sieg, *Physical Review*, **9**, 337, 1917; Worthing, *ibid.*, **12**, 219, 1918; Schriever, *ibid.*, **20**, 96, 1922; W. Giess, *Physica*, **3**, 322, 1923.

[3] Fink, *Transactions of the Electrochemical Society*, **22**, 503, 1912; Dodge, *Physical Review*, **11**, 311, 1918; Schonborn, *Zeitschrift für Physik*, **8**, 377, 1922; Worthing, *Physical Review*, **12**, 219, 1918; W. Giess, *Physica*, **3**, 322, 1923.

tungsten are available. Results published, however, indicate that tungsten at room temperature has the highest modulus of all substances recorded in the literature. Values varying from 3.40 to 4.15×10^{12} dynes/cm² (i.e., from about 35,000 to about 42,000 kg wt./mm²) have been found. Probably the variations with grain structure and wire diameter are of the same order as with the rigidity modulus. The variation of Young's modulus with temperature has not been carried as far as it has been in the measurements of the rigidity modulus; but, in going from room temperature to 1300° K., the upper limit at which experiments were made, a

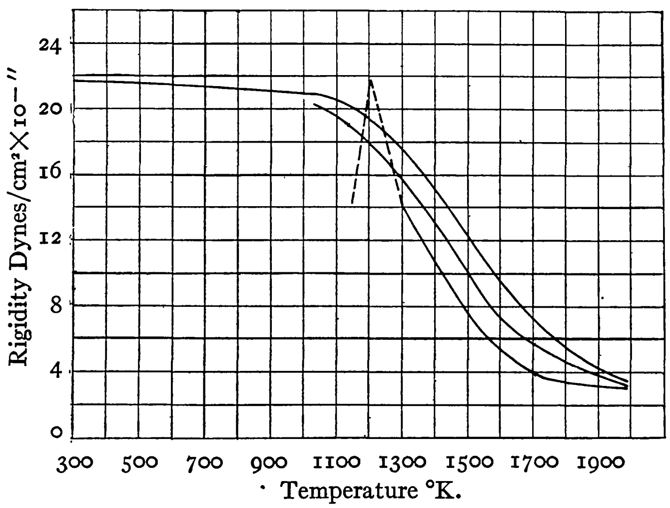

FIG. 8.—Rigidity of tungsten as a function of temperature

similar slow decrease (amounting in this case to a little less than 10 per cent) was found.

Compressibility.—Values for the compressibility of tungsten are taken from a paper by Bridgman.[1] He used two specimens, A, swaged but not drawn, with a diameter of 0.48 cm; and B, swaged and drawn with a diameter of 0.051 cm. Tests at 30° C. and 75° C. were made. The temperature variation was very small. For 30° C. he gives

$$\frac{\Delta v}{v_0}=-10^{-7}(2.93-1.5\times10^{-5}p)p \tag{13}$$

$$\frac{\Delta v}{v_0}=-10^{-7}(3.15-1.6\times10^{-5}p)p, \tag{14}$$

[1] *Proceedings of the American Academy of Arts and Science*, **58**, 165, 1923.

where $\frac{\Delta v}{v_0}$ represents the fractional change in volume and p the pressure in kg wt./cm².

For the compressibility $-\frac{1}{v}\frac{dv}{dp}$, the foregoing give at room temperature 2.93×10^{-7} and 3.15×10^{-7} $\frac{1}{\text{kg wt./cm}^2}$. The bulk moduli, $-v\frac{dp}{dv}$, which follow are 3.34×10^{12} dynes/cm² and 3.11×10^{12} dynes/cm². For comparison two other values 2.77×10^{12} dynes/cm²

TABLE IV

TENSILE STRENGTH OF TUNGSTEN

Kind of Material	Diameter		Tensile Strength	
	Mils	Millimeters	lb. wt./in.²	kg wt./mm²
Sintered tungsten ingot	200 by 250	....	18,000	13
Swaged rod	216	5.49	50,600	36
Swaged rod	125	3.18	107,000	75
Swaged rod	80	2.03	176,000	124
Swaged rod	26	.660	215,000	151
Drawn wire	18.0	.457	264,000	186
Drawn wire	7.23	.184	340,000	239
Drawn wire	5.78	.147	366,000	257
Drawn wire	5.50	.139	378,000	266
Drawn wire	3.96	.101	483,000	340
Drawn wire	1.14	.029	590,000	415

at 300° K. and 3.55×10^{12} dynes/cm² are taken from a paper by one of the present authors.[1]

Tensile strength and reduction of area at break.—Other elastic constants of particular interest in the metallurgy of tungsten are the tensile strength and the reduction of area at the point of break. Results here given are taken from papers by Dr. Zay Jeffries.[2] Tensile strengths at room temperature given in Table IV refer only to swaged rods and drawn wire obtained from one particular ingot, and for these reasons are believed to show purely the effects of variation in wire size. Variations in tensile strength and reduction of area at break for various temperatures are shown in Figures 9 and 10 for four wire specimens having approximately the same diameters but different heat treatments.

[1] Worthing, *Physical Review*, **12**, 219, 1918.

[2] *Amer. Inst. Mining Met. Eng.*, **138**, 1037, 1918; *ibid.*, **146**, 575, 1919.

Hardness.—The data on hardness given in Table V were furnished by Dr. Samuel L. Hoyt, of the Research Laboratory of the General Electric Company at Schenectady.

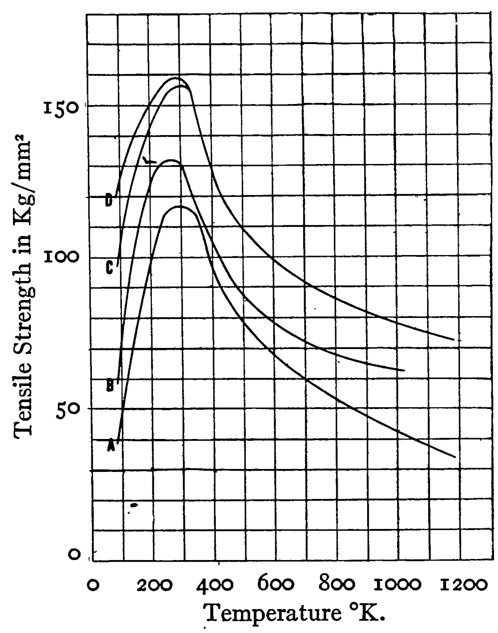

FIG. 9.—Tensile strength of tungsten as a function of temperature. (*A*) 4.2 mm rod swaged to 0.76 mm and drawn to 0.63 mm at temperatures changing gradually from 1580° K. to 1280° K. and then equiaxed. (*B*) 1.07-mm equiaxed rod drawn at 1280° K. to 0.71 mm. (*C*) 3.18-mm equiaxed rod swaged at 1280° K. to 0.76 mm and then drawn at same temperature to 0.64 mm. (*D*) 4.2-mm rod swaged to 0.76 mm and drawn to 0.63 mm at temperatures changing gradually from 1580° K. to 1280° K. but not equiaxed.

TABLE V

HARDNESS OF TUNGSTEN

Sample Number	Sample	Rockwell, "C"	Scleroscope
1.........	1-in. ingot	20	45
2.........	Same as No. 1 swaged to $\frac{3}{4}$ in.:		
	a = surface	39	55–56
	b = center	23	35 ?
3.........	Swaged tungsten	45	83
4.........	Tool-steel comparison	61	95

Sample No. 2 gave a Brinell number of 350, but the impression showed fine cracks around it, so the figure may be a little in error.

CHARACTERISTICS OF TUNGSTEN LAMPS

Temperature, voltage, candle-power relations for vacuum lamps.—The variation of the candle-power and wattage of vacuum tungsten lamps has been studied[1] over a wide range of voltage and results given in percentages of the normal values. From the known

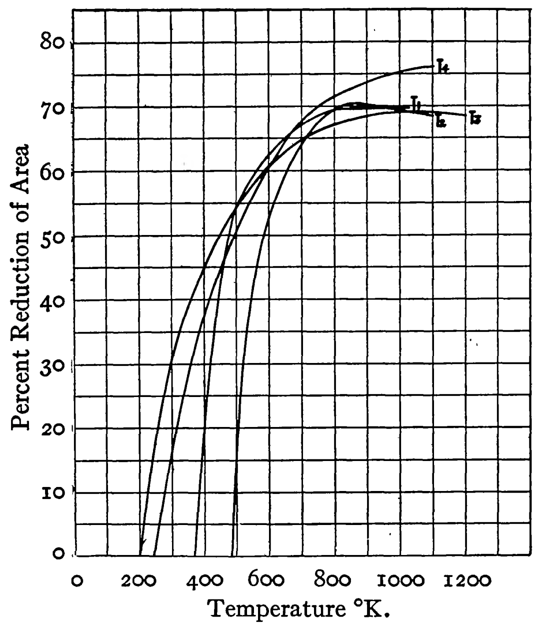

FIG. 10.—Reduction of area of tungsten wires at break as a function of temperature. T_1, 1.07-mm equiaxed rod drawn at 1280° K. to 0.71 mm. T_2, 4.2-mm rod swaged to 0.76 mm and drawn to 0.63 mm at temperatures changing gradually from 1580° K. to 1280° K. and then equiaxed. T_3, 3.18-mm equiaxed rod swaged at 1280° K. to 0.76 mm and then drawn at same temperature to 0.64 mm. T_4, 4.2-mm rod swaged to 0.76 mm and then drawn to 0.63 mm at temperatures changing gradually from 1580° K. to 1280° K. but not equiaxed.

relation between luminous efficiency and temperature,[2] these values have been arranged to show their dependence upon temperature. The values given in Table VI were obtained from ordinary lamps, and have not been corrected for end losses.

[1] Cady, *Electrical Review and Western Electrician*, **59**, 1092, 1911; Middlekauff and Skogland, *Bulletin of the Bureau of Standards*, **11**, 484, 1914; Shackelford, "Pyrometer Symposium," *Amer. Inst. Mining Met. Eng.*, p. 627, 1920.

[2] Hyde, Cady, and Forsythe, *Physical Review*, **10**, 401, 1917.

A factor, the approximate value of which is very useful for predicting variations due to small changes in operating conditions, is

TABLE VI

CHARACTERISTICS OF VACUUM TUNGSTEN LAMPS AS A FUNCTION OF TEMPERATURE

Temperature Maximum, K.	Color-Temperature Average, K.	Luminous Efficiency Lumens per Watt	Brightness Candles per cm²	Percentage Volts	Percentage Watts	Percentage Current	Percentage Candle-Power	$\frac{V}{cp}\frac{dcp}{dV}$	Percentage Life L	$\frac{T}{L}\frac{dL}{dT}$
2000°.	2024°	2.50	20.0	55.0	38.7	70.3	10.4	4.1	1,090,000.	−53
2100..	2128	3.65	35.6	63.6	48.7	76.5	18.6	3.9	100,100.	50
2200..	2231	5.00	61.3	73.3	61.1	83.3	32.2	3.8	11,400.	48
2300..	2335	6.70	100.5	83.5	75.2	90.0	52.2	3.6	1490.	46
2400..	2440	8.50	157.0	95.0	92.1	97.0	82.9	3.5	246.	44
2450..	2493	9.8	193.0	100.0	100.0	100.	100.0	3.4	100.	43
2500..	2546	10.8	237.5	105.9	109.6	103.5	122.0	3.4	45.	42
2600..	2652	13.2	347.	118.5	130.6	110.3	179.3	3.3	9.4	40
2700..	2758	15.8	498.	132.3	155.6	117.6	257.0	3.2	2.3	−38

that represented by the ratio of a percentage change in current to the accompanying percentage change in voltage:

$$\frac{V}{I}\frac{dI}{dV}=D\,. \tag{15}$$

For tungsten D may be taken as roughly constant and equal to 0.6. Other corresponding percentage changes follow:

$$\frac{V}{W}\cdot\frac{dW}{dV}=1+D=1.6 \tag{16}$$

$$\frac{V}{R}\cdot\frac{dR}{dV}=1-\mathrm{D}=0.4\,. \tag{17}$$

To illustrate: If a 110-volt lamp is operated at a 1 per cent increase in voltage, the consequent increases in current, wattage, and resistance are roughly 0.6, 1.6, and 0.4 per cent.

Dependence of current, voltage, etc., upon filament dimensions.—For the comparison of performances of different-sized filaments and for the planning of special lamps or experimental set-ups using tungsten filaments, the following considerations which apply to vacuum conditions only are given. End-loss effects are ignored.

The relations sought depend upon the elementary equations,

$$W=2\pi r\eta L \tag{18}$$

$$R=\frac{\rho L}{\pi r^2} \tag{19}$$

$$V=\rho\frac{I}{\pi r^2}\cdot L\ , \tag{20}$$

where W, R, I, and V have their usual significance of wattage, resistance, current, and voltage, and η, ρ, r, and L refer, respectively, to radiation intensity, resistivity, filament radius, and filament length. For various conditions of comparison of two filaments designated by the subscript following the brackets, these equations lead to several simple relations of which the following are generally the most useful:

$$\left[\frac{I_1}{I_2}=\left(\frac{r_1}{r_2}\right)^{\frac{3}{2}}\right]_{T_1=T_2} \tag{21}$$

$$\left[\frac{V_1}{V_2}=\left(\frac{r_2}{r_1}\right)^{\frac{1}{2}}\frac{L_1}{L_2}\right]_{T_1=T_2} \tag{22}$$

$$\left[\frac{W_1}{W_2}=\frac{r_1L_1}{r_2L_2}\right]_{T_1=T_2} \tag{23}$$

$$\left[\frac{r_1}{r_2}=\left(\frac{L_1}{L_2}\right)^2=\left(\frac{W_1}{W_2}\right)^{\frac{2}{3}}=\left(\frac{I_1}{I_2}\right)^{\frac{2}{3}}\right]_{\substack{T_1=T_2\\ V_1=V_2}} \tag{24}$$

$$[I_1V_1^3=I_2V_2^3]_{\substack{T_1=T_2\\ L_1=L_2}}\ . \tag{25}$$

Knowing the current required to heat a certain-sized wire to a specified temperature, equation 21 shows how to obtain the current required to heat any other sized wire to the same temperature. Similarly, equation 24 shows the variations in the dimensions of filaments which must occur in order that the filaments shall operate at the same voltage and temperature, while equation 25 shows the conditions for operating a given filament so that, as it vaporizes, its temperature shall remain constant, provided that the character of a filament surface does not change materially during continued operation.

Simple-design equations may be obtained from equations 18 and 19. They are:

$$L=\left(\frac{IV^3}{4\pi\rho\,\eta^2}\right)^{\frac{1}{3}} \tag{26}$$

$$r=\frac{IV}{2\pi\eta\,L}\,. \tag{27}$$

To determine, for instance, the dimensions of a filament, supported by its leads only, which shall operate at 10 volts and 1 amp. and yield a temperature of 2400° K., it is necessary, first, to find from Table I values of η and ρ corresponding to 2400° K., and then to substitute these values in the equations. In this instance the length is 6.9 cm and the radius is 0.040 mm. To take account of the end-loss corrections, as shown by equation 29, the total length should be increased by 2 ΔL_B, in this case by 0.47 cm. The foregoing refers only to operation after the initial seasoning.

In Table VII are given the currents necessary to heat various-sized filaments to different temperatures and also the voltage gradients for filaments of different radii at different temperatures. The values given apply to uniformly heated parts of the filaments. If an attempt is made to calculate lengths for various voltages or the voltage for a definite length from these data, corrections must be made for the end cooling (see equation 29).

Wire weight.—When speaking of the size of tungsten wire, it is customary to give the weight in milligrams of a 200-mm length rather than the diameter. Table VIII has been prepared to show the relation between the wire weight and the radius in both centimeters and mils. The value, 19.3, has been used as the density.

Photographic effects.—The photographic effects[1] given apply only to orthochromatic plates. The values given in Figure 11 are for the relative photographic effectiveness at various temperatures for constant illumination through lead glass. This is equivalent to the relative plate speeds, i.e., the reciprocal of the time required to produce equal blackening of the plate.

Life of lamps.—The life of a vacuum tungsten lamp is a function of two factors: the temperature of operation and the wire size. The

[1] Luckiesh, Holliday, and Taylor, *Journal of Franklin Institute*, **196**, 495, 1923.

TABLE VII*

CURRENT. I, IN AMPERES AND POTENTIAL GRADIENT, dV/dL, IN VOLTS PER CENTIMETER FOR VARIOUS-SIZED TUNGSTEN FILAMENTS AT DIFFERENT TEMPERATURES

Temperature, ° K.	Radius in Centi-meters	.001	.002	.003	.005	.008	.010	.015	.020	.030	.050	.080	.100
1000	$I=$	.022	.63	.116	.250	.51	.70	1.32	2.00	3.80	8.0	16.0	22.0
	$dV/dL=$	.184	.130	.107	.083	.065	.058	.047	.041	.034	.026	.021	.018
1200	$I=$	.032	.092	.167	.365	.73	1.03	1.88	2.90	5.3	11.4	23.0	32.0
	$dV/dL=$	.325	.230	.188	.148	.115	.103	.084	.073	.060	.046	.036	.033
1400	$I=$	.045	.127	.235	.50	1.02	1.42	2.60	4.00	7.4	15.9	32.0	45.0
	$dV/dL=$	.55	.385	.315	.245	.194	.173	.141	.122	.100	.078	.061	.055
1600	$I=$	.059	.167	.305	.66	1.33	1.86	3.40	5.3	9.7	21.0	42.0	59.
	$dV/dL=$	.85	.60	.485	.374	.300	.265	.215	.188	.153	.119	.094	.084
1800	$I=$	.074	.207	.385	.82	1.66	2.35	4.25	6.6	12.1	26.0	53.	74.
	$dV/dL=$	1.23	.87	.71	.55	.435	.385	.315	.275	.225	.174	.130	.123
2000	$I=$	.090	.255	.465	1.00	2.02	2.85	5.2	8.0	14.7	31.5	65.	90.
	$dV/dL=$	1.70	1.20	.98	.76	.60	.54	.440	.380	.310	.240	.190	.170
2100	$I=$	.098	.275	.51	1.09	2.20	3.10	5.7	8.7	16.1	34.5	70.	98.
	$dV/dL=$	1.96	1.39	1.14	.88	.69	.62	.50	440	.360	.280	.220	.196
2200	$I=$	.106	.300	.55	1.19	2.40	3.35	6.2	9.5	17.4	37.5	76.	106.
	$dV/dL=$	2.25	1.59	1.30	1.01	.80	.71	.58	.50	.410	.320	.250	.225
2300	$I=$	.115	.325	.60	1.29	2.61	3.65	6.7	10.3	18.9	41.0	82.	115.
	$dV/dL=$	2.56	1.80	1.47	1.15	.90	.81	.66	.57	.465	.360	.285	.255
2400	$I=$	.124	.350	.65	1.39	2.80	3.90	7.2	11.1	20.5	44.0	89.	124.
	$dV/dL=$	2.90	2.05	1.68	1.30	1.03	.92	.75	.65	.53	.410	.325	.290
2500	$I=$	.133	.375	.69	1.49	3.00	4.20	7.7	11.9	21.8	47.0	95.	133.
	$dV/dL=$	3.28	2.32	1.90	1.47	1.16	1.04	.85	.73	.60	.465	.365	.330
2600	$I=$	.143	.400	.74	1.59	3.20	4.50	8.2	12.7	23.5	50.0	102.	143.
	$dV/dL=$	3.65	2.60	2.10	1.64	1.30	1.16	.95	.82	.67	.52	.410	.370
2700	$I=$	.152	.430	.79	1.70	3.45	4.80	8.8	13.6	25.0	54.	109.	152.
	$dV/dL=$	4.109	2.90	2.35	1.84	1.45	1.29	1.06	.92	.75	.58	.460	.410
2800	$I=$	.162	.460	.84	1.81	3.65	5.1	9.4	14.5	26.5	57.	116.	162.
	$dV/dL=$	4.55	3.20	2.65	2.05	1.61	1.44	1.18	1.02	.83	.64	.51	.455
2900	$I=$	.172	.485	.89	1.92	3.90	5.4	10.0	15.4	28.0	61.	123.	172.
	$dV/dL=$	5.0	3.55	2.90	2.25	1.78	1.59	1.30	1.12	.92	.71	.56	.50
3000	$I=$	.182	.52	.95	2.05	4.10	5.8	10.6	16.4	30.0	64.	130.	182.
	$dV/dL=$	5.6	3.95	3.20	2.50	1.97	1.76	1.43	1.24	1.01	.78	.62	.56

* These values of the voltage gradients apply to the uniformly heated parts of the filaments. See Table I, *C*. To take account of the cooling due to two leads, a computed length should be increased by $2\ \Delta L_B$ (see equation 29).

temperature through determining the rate of vaporization is by far the more important factor. Empirically the average life,

TABLE VIII

WEIGHT IN MILLIGRAMS OF 200-MILLIMETER LENGTHS AND RESISTANCE IN OHMS OF 1-CENTIMETER LENGTHS OF TUNGSTEN WIRE FOR VARIOUS RADII GIVEN IN CENTIMETERS AND MILS. THE VALUE, 19.3 G/CM³, HAS BEEN USED AS THE DENSITY IN THE CALCULATIONS

Radius cm	Weight of 200-mm Length mg	Resistance of 1 cm of Wire Ohms	Radius mils	Weight of 200-mm Length mg	Resistance of 1 cm of Wire Ohms
.001........	1.21	1.745*	0.5	1.96	1.080*
.002........	4.85	.436	1.	7.82	.2705
.003........	10.91	.1935	2.	31.3	.0675
.004........	19.40	.1085	3.	70.4	.0301
.005........	30.3	.0698	4.	125.2	.01690
.006........	43.7	.0485	5.	195.5	.01075
.007........	59.2	.0356	6.	287.	.00751
.008........	77.6	.0273	7.	383.	.00552
.009........	98.2	.0215	8.	501.	.00423
.010........	121.2	.01745	9.	634.	.00334
.015........	273.	.00775	10.	782.	.002705
.020........	485.	.00436	15.	1760.	.001205
.050........	3030.	.000698	20.	3130.	.000675

* These values apply to well-seasoned wire at 20°C. For wire that is not seasoned, values as much as 50 per cent higher may be obtained, depending upon the method of production.

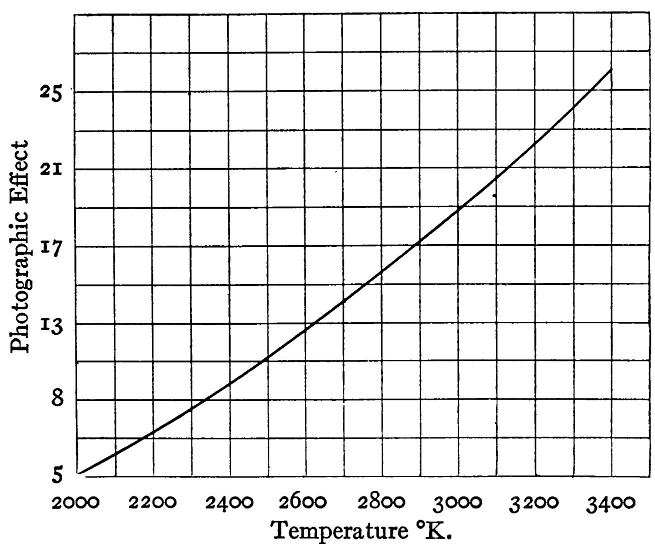

FIG. 11.—Photographic effect for tungsten at equal illumination for different temperatures.

whatever the diameter, has been found to coincide with the period of burning required for the reduction by vaporization of the original diameter by a given percentage. Since, with an increased diameter, there is an increased thickness of layer to be vaporized during life, there is also an increased life, if the temperature of operation is not changed, and hence the dependency on filament diameter. Values given in Table VI obtained from the vaporization rate[1] are relative, and show only the variation with temperature. The unit of life is taken as 1000 hours at 2450° K. This corresponds approximately to the case of the commercial 25-watt, 115-volt

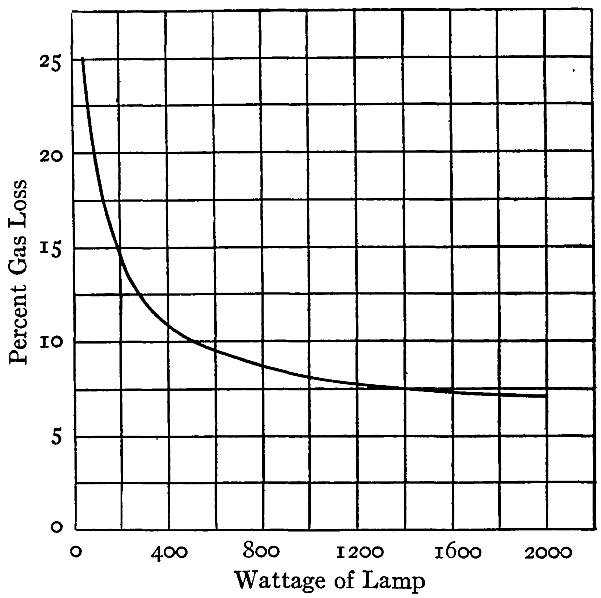

FIG. 12.—Percentage gas loss for different wattage gas-filled lamps

vacuum lamp. Values in Table XI show results for many commercial lamps, taking account of both factors.

Gas losses.—By operating the filament of a tungsten lamp in an atmosphere of an inert gas, the rate of vaporization is enormously reduced, thus permitting the filament to be operated at a much higher temperature for the same life. There is a loss of energy due to the conduction and convection of heat by the gas. This loss of heat has been found to depend upon the size of the filament, being much smaller for large filaments. If the filament is coiled into a helix,[2] the energy losses depend upon the diameter of the

[1] Langmuir, *Physical Review*, 2, 329, 1913.

[2] *Ibid.*, 34, 401, 1912.

helix. The gas loss[1] as a function of the wattage of the lamp containing a mixture of argon and nitrogen as used in ordinary gas-filled lamps is shown in Figure 12.

End losses.—Incandescent-lamp filaments are cooled by the lead and support wires. As a result, the input of energy as well as the outputs of radiant energy, luminous flux, and thermionic emission are diminished. These reductions are summed up as end losses.[2]

TABLE IX

VALUES OF Q, $\sqrt{2\rho\eta}/Q$, AND VARIOUS EFFECTIVE VOLTAGE CORRECTIONS FOR VARIOUS MAXIMUM TEMPERATURES, T_m, FOR USE WITH EQUATIONS 28–37 IN COMPUTING END LOSSES. THE JUNCTION TEMPERATURE IS ASSUMED TO BE $T_m/4$

Temperature, K.	Q cm$^{-\frac{1}{2}}$	$\sqrt{2\rho\eta}/Q = \Delta V_B$ Volts	ΔV_C Volts	ΔV_D Volts	ΔV_E Volts
1000°	.079	.066	.15	.20	.26
1200	.116	.089	.20	.27	.34
1400	.156	.112	.25	.34	.43
1600	.196	.137	.31	.41	.54
1800	.238	.162	.36	.49	.62
2000	.284	.189	.42	.57	.73
2200	.328	.217	.49	.65	.84
2400	.375	.246	.55	.74	.95
2600	.422	.276	.62	.83	1.06
2800	.470	.308	.69	.92	1.18
3000	.519	.340	.76	1.02	1.31
3200	.569	.374	.84	1.12	1.44

There are three methods of eliminating or correcting for end losses. The first employs very fine-wire volt connections to points on the filament at a known distance apart, but far enough removed from the cooling leads or supports so that the part included will be at a uniform temperature. In the second method two filaments of the same diameter but of different known lengths are mounted on

[1] Van Horn, "Pyrometer Symposium," *Amer. Inst. Mining Met. Eng.*, p. 638, 1920.

[2] Worthing, *Journal of Franklin Institute*, **194**, 597, 1922; Hyde, Cady, and Worthing, *Trans. Illuminating Engineering Society*, **6**, 238, 1911. See also, Amrine, *ibid.*, **8**, 385, 1913; Stead, *Journal of the Institution of Electrical Engineers* (England), **58**, 107, 1920.

the same stem, the short one being long enough so that the temperature of its central part is not affected by end losses. The differences in voltage, light-output, etc., for the two filaments for the same current are the voltage drop, light-output, etc., for a uniformly heated filament, the length of which is the difference in lengths of the two filaments. The third method depends upon measuring the variations in brightness of the filament from the uniformly heated part to the lead, and, from the temperature distribution thus obtained, computing the end-loss effects. All three methods have been used in determining the results reported in Tables IX

TABLE X*

Temperature, K.	Radius in Centimeters												
	.001	.002	.003	.005	.008	.010	.015	.020	.030	.050	.080	.100	.500
1000°	3.	4.	5.	6.	8.	9.	11.	12.	15.	20.	25.	28.	65.
1200	2.	3.	3.5	4.0	5.5	6.0	7.5	8.5	10.	13.	17.	19.	42.
1400	1.5	2.0	2.5	3.0	4.0	4.5	5.5	6.5	8.0	10.	13.	14.	32.
1600	1.1	1.6	2.0	2.5	3.2	3.6	4.4	5.1	6.2	8.0	10.	11.	25.
1800	.9	1.3	1.6	2.1	2.7	2.96	3.6	4.2	5.1	6.6	8.4	9.5	21.
2000	.8	1.1	1.4	1.7	2.2	2.47	3.0	3.5	4.3	5.5	7.0	8.0	17.
2200	.68	.96	1.2	1.5	1.9	2.15	2.6	3.0	3.7	4.8	6.1	6.8	15.
2400	.59	.84	1.0	1.3	1.7	1.88	2.3	2.7	3.2	4.2	5.3	5.9	13.
2600	.52	.74	.90	1.17	1.48	1.66	2.0	2.3	2.9	3.7	4.7	5.3	12.
2800	.47	.67	.82	1.06	1.33	1.50	1.8	2.1	2.6	3.3	4.2	4.7	11.
3000	.42	.60	.74	.95	1.21	1.35	1.7	1.9	2.3	3.0	3.8	4.3	10.
3200	.39	.55	.67	.87	1.10	1.23	1.5	1.7	2.1	2.8	3.5	3.9	9.

* Distance in centimeters from leads for various-sized long tungsten filaments, and various maximum temperatures, to points where the temperatures are only 0.1 per cent less than the maximum temperature ($7.0\sqrt{r}/Q$). For temperatures 0.01 per cent less the distances are 4/3 those given; for temperatures 1 per cent lower than the maximum the distances are 2/3 those given. For short filaments the filament half-lengths are about 13 per cent greater.

and X, but, for the most part, the end-loss corrections shown were determined by the third method.

Figure 13 shows the variations occurring in temperature A, resistance B, radiation intensity C, brightness D, and thermionic emission E in a long, straight tungsten filament ($r=0.01$ cm) near a cooling lead in a vacuum lamp when the central maximum temperature is 2400° K. Distribution curves corresponding to any other maximum filament temperature and filament radius may be obtained from these curves by increasing all abscissae by the ratio of $[Q/\sqrt{r}]_0$ to $Q/\sqrt{r}$, where r is the radius and Q is a function of the maximum temperature and of certain properties of tungsten at this temperature. The fraction $Q\sqrt{r}$ (see Table IX) refers to the conditions sought and $[Q\sqrt{r}]_0$ to the condition diagrammed in Figure 13.

In Figure 13 the areas between the curves, the y-axis, and the line $y=1.0$ represent end losses. In the case of curve B, suppose a line drawn at about $L=0.27$ cm; then the area below the curve and to the left of this line will be found equal to that part of the end-loss area above this curve and to the right of this line. Thus there is an effective filament shortening, ΔL_B, in this case which

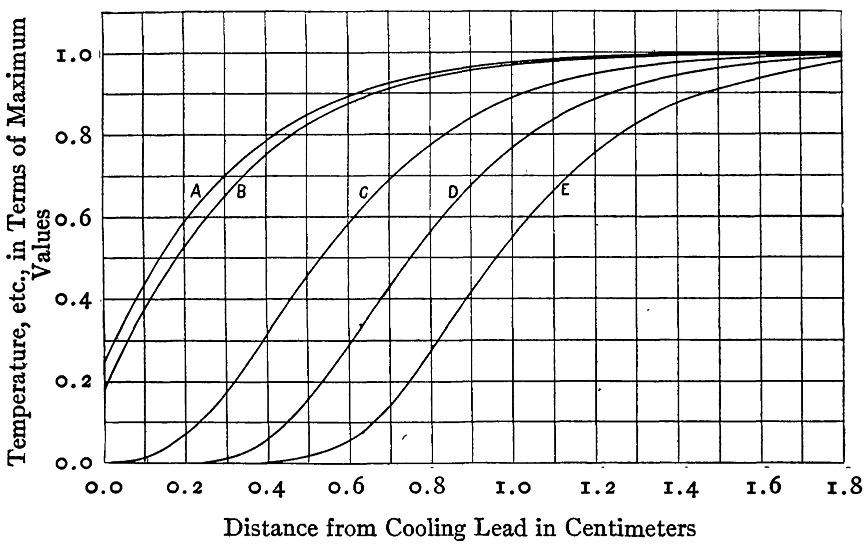

FIG. 13.—Various distribution curves near a cooling junction for a long tungsten wire ($r=0.01$ cm.) electrically heated *in vacuo* to a central maximum temperature of 2400° K. (A) temperature; (B) resistance; (C) radiation intensity; (D) brightness; (E) thermionic emission intensity.

amounts to 0.27 cm. This means that the resistance of the filament for the condition specified with the cooling leads is equal to the resistance of a uniformly heated filament which is 0.27 cm shorter for each junction of filament to lead wires. For curves C, D, and E, the effective filament shortenings, ΔL_C, ΔL_D, and ΔL_E, are about 0.60, 0.80, and 1.04 cm. Effective filament shortenings vary with filament radius, maximum filament temperature, and the property (e.g., energy input, etc.) studied.

The difference between the supply rate and the radiation rate near a cooled support or lead is the rate of conduction of heat to it.

This heat-conduction loss is represented in Figure 13 by the area between curves B and C. If the ordinates are expressed in watts per cm of filament length, the heat-conduction loss is given directly in watts. For the case represented, it is the wattage of a portion of the uniformly heated filament whose length is 0.33 cm, the difference between the effective shortenings, 0.60 cm and 0.27 cm. In general, this loss is given relatively by $\frac{\Delta L_C - \Delta L_B}{L}$, where L is the filament half-length. Equations that follow may be used for its evaluation.

The effective filament shortenings from the various standpoints are given by

$$\Delta L_A = 0.88 \sqrt{r}/Q \quad \text{(heat content)} \tag{28}$$

$$\Delta L_B = 1.00 \sqrt{r}/Q \quad \text{(energy input, voltage, or resistance)} \tag{29}$$

$$\Delta L_C = 2.25 \sqrt{r}/Q \quad \text{(radiation output)} \tag{30}$$

$$\Delta L_D = 3.00 \sqrt{r}/Q \quad \text{(light-emission)} \tag{31}$$

$$\Delta L_E = 3.85 \sqrt{r}/Q \quad \text{(thermionic emission).} \tag{32}$$

These equations show that all shortenings vary directly as $\sqrt{r}$ and inversely as Q (Table IX), a function of the maximum temperature. These equations give end-loss corrections directly when the filament's length, radius, and maximum temperature are known.

When the filament length and radius have not been measured directly, the end-loss corrections may be obtained from a consideration of the effective voltage corrections,[1] ΔV_A, etc., corresponding to the various standpoints. Limiting consideration to the case of a filament with only two leads, and representing by L, V, etc., the filament half-length, half the observed voltage, etc., these effective voltage corrections become the difference between the voltage drop over L, were it uniformly heated throughout, and the voltage drop over a shortened length of uniformly heated filament yielding the observed radiation, or luminous or thermionic output. These shortened lengths are $(L - \Delta L_A)$, etc. The voltage corrections sought are the products of ΔL_A, ΔL_B, etc., by dV/dL for a uni-

[1] Langmuir, *Transactions Faraday Society*, 17, 621, 1922.

formly heated filament. Since dV/dL equals $\sqrt{2\rho\eta/r}$, where ρ is the resistivity and η the total radiation intensity, from equations 28 to 32 there follow:

$$\Delta V_A = 0.88\sqrt{2\rho\eta/Q} \quad \text{(heat content)} \tag{33}$$

$$\Delta V_B = 1.00\sqrt{2\rho\eta/Q} \quad \text{(energy input, voltage, or resistance)} \tag{34}$$

$$\Delta V_C = 2.25\sqrt{2\rho\eta/Q} \quad \text{(radiation output)} \tag{35}$$

$$\Delta V_D = 3.00\sqrt{2\rho\eta/Q} \quad \text{(light-emission)} \tag{36}$$

$$\Delta V_E = 3.85\sqrt{2\rho\eta/Q} \quad \text{(thermionic emission).} \tag{37}$$

These corrections (Table IX) depend upon the maximum temperature T, but not upon dimensions of the filament.

Empirically the variation of ΔV_B with T_m is closely given by

$$\Delta V_B = \tfrac{1}{2}(0.00028\,T_n - 0.17)\,. \tag{38}$$

Other ΔV's can be represented by similar empirical equations. To illustrate the use of these equations and the table, suppose the thermionic-emission correction is desired for a filament with two leads between which there is a 10-volt drop and a central temperature of 2800° K. The factor "f" by which the observed thermionic emission must be multiplied to obtain the emission were the filament uniformly heated throughout its length is given by

$$\text{“}f\text{”} = \frac{L}{L-\Delta L_E} = \frac{L\frac{dV}{dL}}{(L-\Delta L_E)\frac{dV}{dL}} = \frac{V+\Delta V_B}{V+\Delta V_B-\Delta V_E}\,. \tag{39}$$

Since for this case $V=5$ volts, $\Delta V_B=0.31$ volts, and $\Delta V_E=1.18$ volts, the correction factor is 1.28.

From the value of "f" given by the first part of equation 39, i.e., $\frac{L}{L-\Delta L_E}$, it can be shown that the percentage end-loss corrections for different temperatures for filaments of the same size in similar lamps vary inversely as the values of Q given in Table IX. For different-sized filaments in similar lamps the percentage correction varies as $\sqrt{r}$. Thus, if the end-loss correction is known for 1 maximum temperature for a particular filament, such correction can be easily computed for other maximum temperatures and for filaments of different radii for lamps similarly constructed.

Distances along a filament from a cooling lead to points where the temperatures are within 0.1 per cent of those of the uniformly heated portions are given in Table X. If temperatures within 0.01 per cent are desired, take distances four-thirds times those given in the table. If temperatures within 1 per cent only are desired, take lengths two-thirds times those here listed. Since the temperature at the center of short filaments is lowered by the cooling of both leads, their half-lengths must be about 13 per cent greater than lengths given in the table, in order that their central temperatures shall not be more than these same percentages lower than the central temperatures of long filaments when heated by the same currents. Thus, for a filament with a 0.01-cm radius and a central temperature of 1800° K., a total filament length of twice 3.3 cm or 6.6 cm must be used in order that its central temperature shall not be more than 1°8 lower than that of a long filament of the same cross-section heated with the same current. Distances for other properties may be obtained from their temperature relations.

Some indications of the energy input and light-output end losses for commercial lamps are of interest.[1] The ordinary 40-watt, 115-volt vacuum tungsten lamp with a filament of 51 cm long and 0.038 mm in diameter has two large copper leads and eleven small molybdenum hook supports. The temperature is about 2450° K. While the lead-junction temperature is only a little greater than $T_m/4$, the support temperatures are considerably greater, about 1750° K. Because of this high value, equations 28–38 cannot be applied directly. By graphical means employing the curves of Figure 13, their contribution to the total end losses may be computed. In this way, total effective shortenings from the energy-input and light-output standpoints of 1.1 and 5.2 cm have been found. These correspond to 2.2 and 10 per cent, whence there is a decrease in efficiency of 7.8 per cent.

Temperature, brightness, and efficiency of some commercial lamps. —In Table XI the temperature, efficiency, and brightness of some

[1] Worthing, *Journal of Franklin Institute*, **194**, 608, 1922.

vacuum tungsten lamps[1] for 1000-hour life are given. In Table XII some similar data, together with average color temperatures, are

TABLE XI

TEMPERATURE, EFFICIENCY, AND BRIGHTNESS OF VACUUM LAMPS

Lamp	Lumens per Watt	Maximum Temperature, K.	Maximum Brightness Candles/cm²
50-watt carbon	3.3	2115°	55
50-watt gem	4.0	2180	78
50-watt tantalum	4.9	2160	53
10-watt tungsten	7.7	2355	128
25-watt tungsten	9.8	2450	193
40-watt tungsten	10.0	2460	206
60-watt tungsten	10.1	2465	211

TABLE XII

TEMPERATURE, EFFICIENCY, AND BRIGHTNESS OF GAS-FILLED TUNGSTEN LAMPS

Lamp	Lumens per Watt	Maximum Temperature, K.	Average Color Temperature, K.	Maximum Brightness of Filament Candles/cm²
Regular gas-filled lamps:				
50-watt	10.0	2685°	2670°	469
75-watt	11.8	2735	2705	563
100-watt	12.9	2760	2740	605
200-watt	15.2	2840	2810	781
300-watt	16.3	2870	2840	862
500-watt	18.1	2930	2920	1015
1000-watt	20.0	2990	2980	1225
2000-watt	21.2	3020	3000	1350
Special lamps:				
1000-watt stereopticon	24.2	3185	3175	2065
900-watt movie	27.3	3290	3220	2660
10-kw	31.0	3350	3300	3050
30-kw	31.0	3350	3300	3050
Daylight lamps:				
200-watt	10.0	2860	....	....
500-watt	11.2	2960	....	....
Photographic:				
750-watt	....	3065	....	....
1500-watt	....	3105	....	....

given for present-day, gas-filled lamps. Efficiency as functions of wattage for regular 115-volt tungsten lamps, is also shown

[1] Forsythe, *General Electric Review*, **26**, 830, 1923.

in Figure 14. In the vacuum lamp, the variation is due to the increase in temperature permissible in the larger filaments for the same percentage of loss of filament during life by vaporization. In the gas-filled lamp, there is the additional variation resulting from percentage decrease in gas loss with increase in filament size (also coil diameter) as well as that due to an increase in operating temperature.

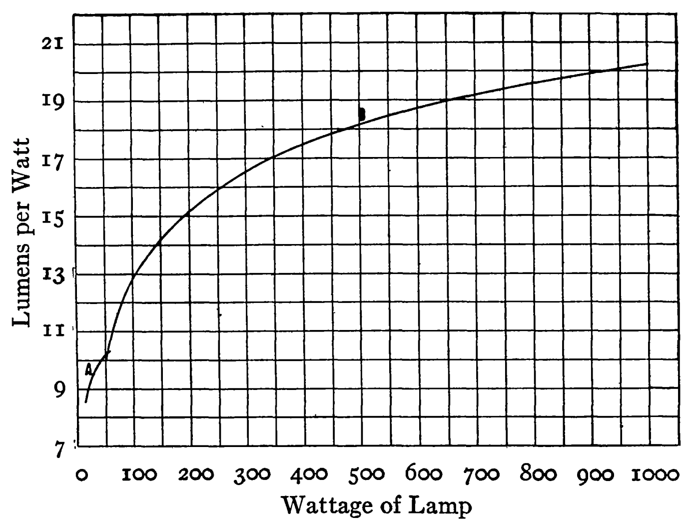

FIG. 14.—Efficiency of 115-volt tungsten lamps as a function of the wattage. Curve *A*, vacuum lamps; Curve *B*, gas-filled lamps.

Data on some special lamps are given in the second and third parts of the table. The 10- and 30-kw lamps were designed and constructed by the Lamp Development Laboratory of the National Lamp Works. They have four coils in the same plane covering an area 4×5 cm in the 10-kw lamp and 7×11 cm in the 30-kw lamp. The 10-kw lamp has a luminous intensity of about 40,000 candles in a direction perpendicular to the coils, while that for the 30-kw lamp is about 100,000 candles. This means that the 30-kw lamp gives the same illumination at a distance of $3\frac{1}{2}$ feet from the plane of the coils as does the sun in midsummer at noon.

Table XIII gives data on the brightness of the bulbs and filaments of some of the tungsten lamps with some other sources included for purposes of comparison.

TABLE XIII

BRIGHTNESS OF FILAMENTS AND BULBS OF SOME TUNGSTEN LAMPS AND OF SOME OTHER SOURCES FOR COMPARISON

Lamp	Brightness Measured at	Brightness Candles/cm²
Kerosene flame	Flat wick	1.2
4-watt per candle carbon lamp	Filament	55.0
40-watt vacuum tungsten lamp	Filament	206.
40-watt vacuum tungsten lamp	Bulb-frosted	2.5
40-watt golden Mazda	Bulb	2.0
50-watt white Mazda	Filament	408.
50-watt white Mazda	Bulb	1.3
75-watt white Mazda sprayed	Filament	563.
75-watt white Mazda sprayed	Bulb	2.1
2000-watt gas-filled Mazda	Filament	1350.
2000-watt gas-filled Mazda	Between coil	3000.
2000-watt gas-filled Mazda	Bulb-frosted	130.
Sun as observed at earth's surface	....	165,000.
Clear sky, average	....	0.4

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