Ratcheting and Cyclic Plasticity Considerations for Code Analysis

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Ratcheting Definitions

- "Ratchetting: Unsymmetric cycles of stress between prescribed limits will cause progressive 'creep' or 'ratchetting' in the direction of the mean stress [1]"
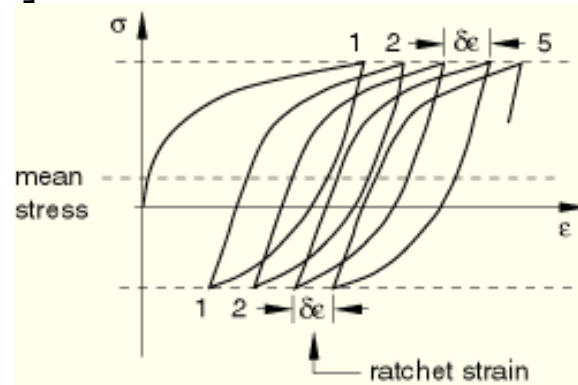

Figure from [1]

- From Paragraph 5.5.1.5, Combinations of steady state and cyclic loadings may result in ratcheting or incremental cyclic growth of **a component**.

- Appears coke drums were first component observed to do this, sodium-cooled fast (nuclear) reactors were then suspected of having the potential to do it

- The failure mechanism is instability from growth and burst from thinning

Bree Diagram

- The Code ratcheting rules (for elastic analysis results) are based on the Bree diagram
  - Bree published his famous paper in 1967 [2] – it re-derived the results of Miller [3] from a decade earlier
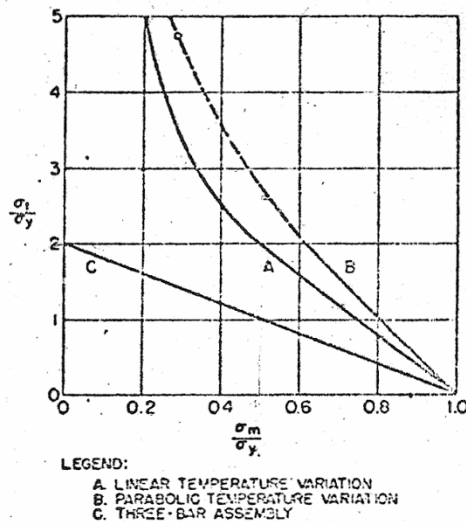 - Original Code criteria document [4] sites the Miller paper as the basis for the Code rules

Figure from [3]

LEGEND:
A. LINEAR TEMPERATURE VARIATION
B. PARABOLIC TEMPERATURE VARIATION
C. THREE-BAR ASSEMBLY

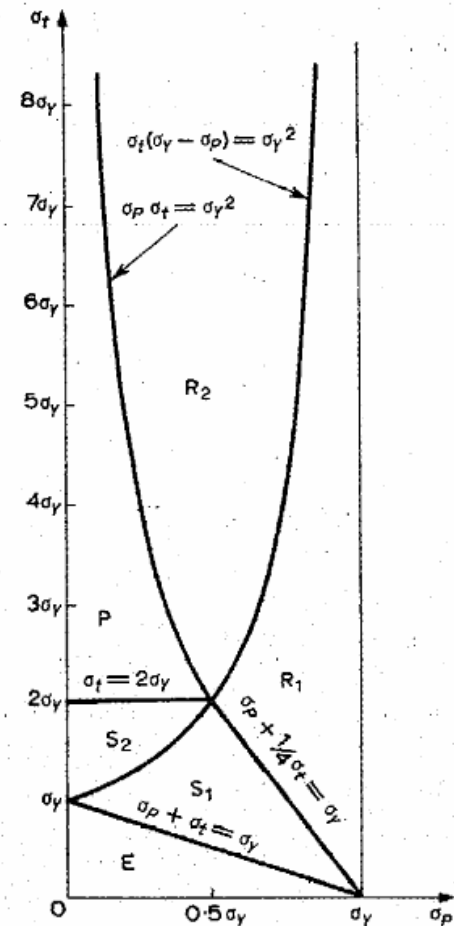Fig. 5 Thermal stress beyond which cyclic growth occurs

Figure from [2]

3

Bree Diagram

- Bree made the following analysis assumptions to obtain a problem that had a close-formed solution:
  - Cylinder wall approximated by 1D slab (hoop direction stress kept)
  - Elastic-perfectly plastic material with constant (temperature-independent) yield stress
  - Constant membrane (pressure) stress, P·R/t
  - Thermal stress equal to E·α·ΔT/(2(1-ν))
  - ΔT assumed to alternate between zero and max. value (i.e. not fully reversed like might happen in a start-up/shut-down cycle)
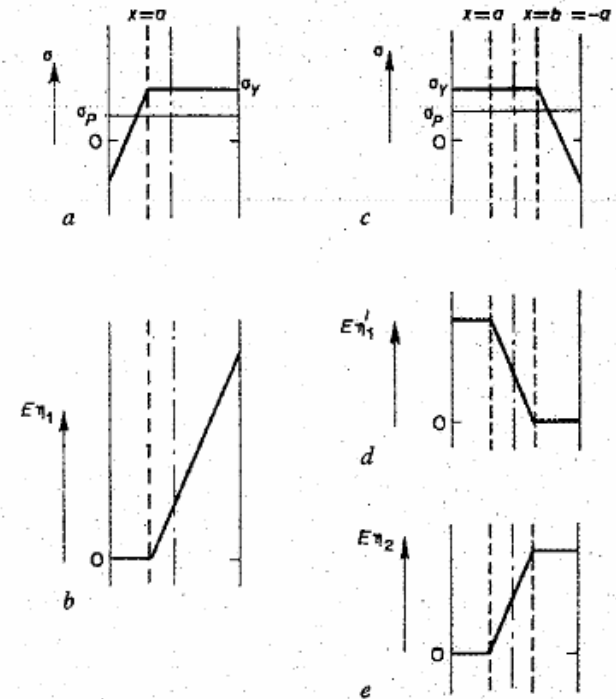
a Stress incurred by first half of each cycle.
b Plastic strain incurred by first half-cycle.
c Stress incurred by second half of each cycle.
d Plastic strain incurred by second half of each cycle.
e Plastic strain incurred by first half of each cycle after the first.

Fig. 4. *Distributions of stress and plastic strain incurred by each half-cycle for the stress régime* R_1 *of Fig. 3*

Figure from [2]

Bree Diagram

- To summarize:
  - Primary stress: membrane only, constant through time
  - Secondary stress: alternates between zero and pure bending (i.e. linear temperature gradient, though parabolic gradient has also been solved and included in Code)
  - Fictitious elastic stresses are always used to enter into the diagram (see Slide 12.8)
- The Bree Diagram is an amazing bit of practical engineering and a great screening tool
  - but it only applies to a pretty specific problem
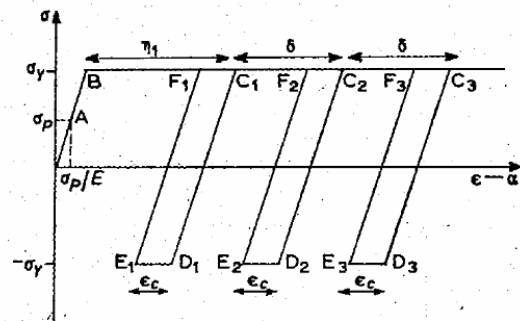 - Different loading conditions can give a very different result!

Fig. 8. Stress–strain path at $x = \frac{1}{2}d$ corresponding to the stress régime R_2 of Fig. 3

Figure from [2]

Extensions of the Bree Diagram

- Moreton verified the Bree/Miller ratchet boundaries experimentally, and showed that the ratchet strains predicted by their model were extremely conservative (which they predicted originally)
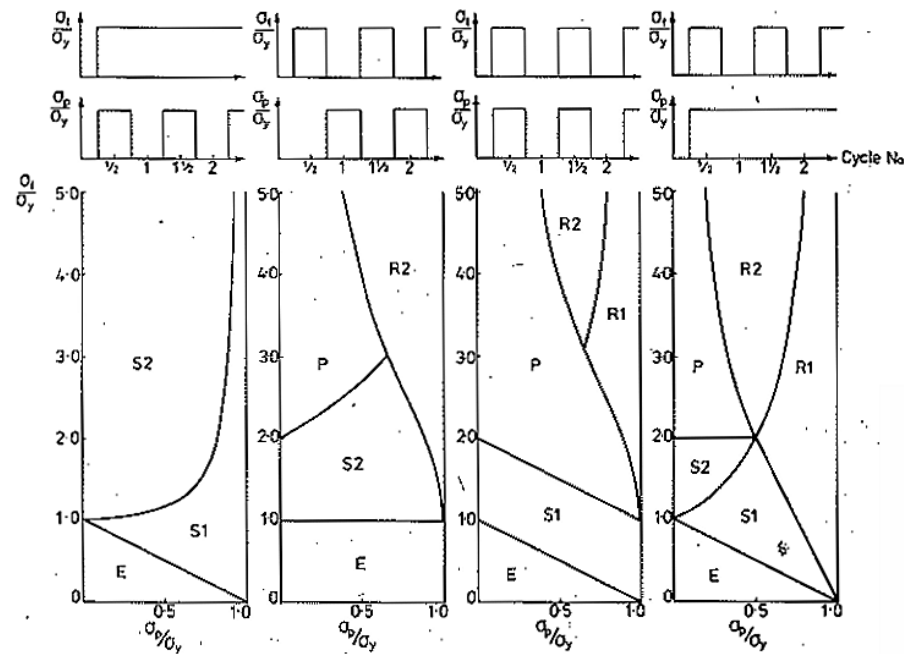- He also numerically generated Bree-type diagrams for alternate loading conditions

Figure from [5]

Fig. 7   Computed Accumulation of Strain with Cycling for Stainless Steel 316.

Extensions of the Bree Diagram

- The Elastic Code ratcheting rules do not account for any of these cases, but are generally conservative (sometimes extremely so) for them according to the Moreton analysis

- The most extreme illustration of this is the constant thermal, alternating pressure case, which is predicted to **never** show ratcheting, only shakedown

Figure from [5]
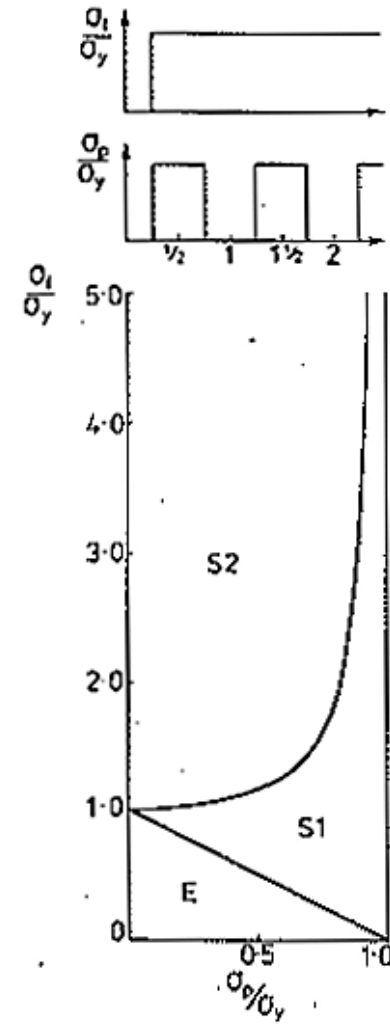
Cyclic Stress-Strain Behavior

- Life is simple if you use an elastic-perfectly plastic model like the Bree diagram is based on
- For this case, the common incremental plasticity models in FEA (isotropic and kinematic hardening) are the same
- For real metals, things are much more complex, and models that try to deal with this reality are correspondingly much more complex
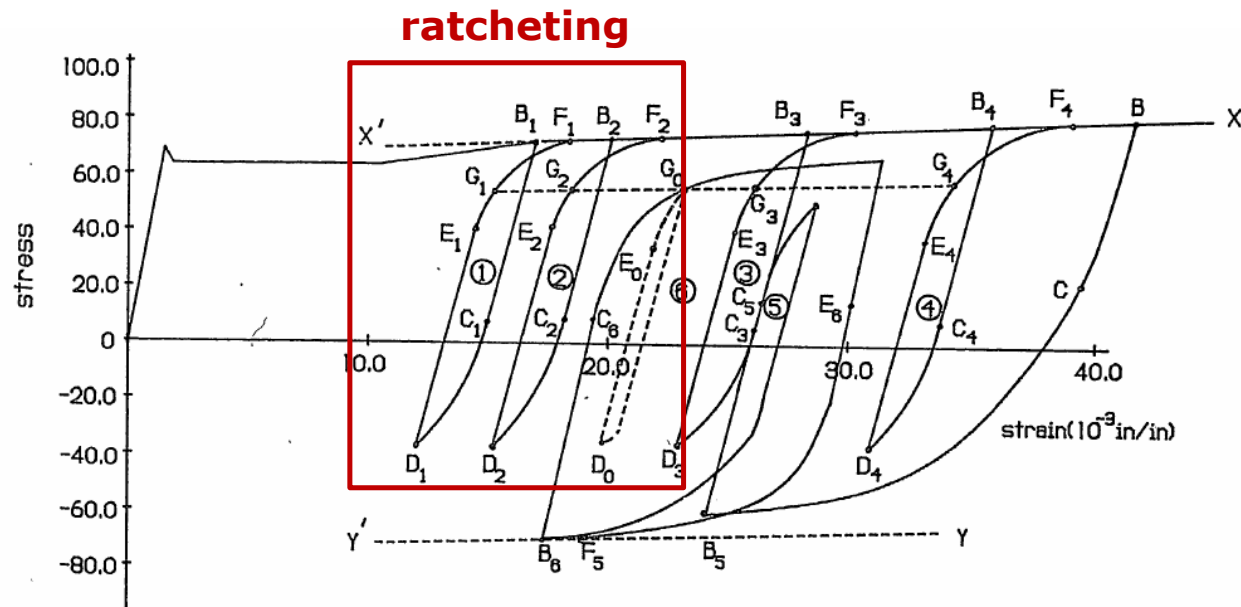
ratcheting

Fig. 6.1 Experimental results for uniaxial random cyclic loading on grade 60 steel specimen (Dafalias 1975)

Figure from [7]

Cyclic Stress-Strain Behavior

- "Cyclic" is a bit deceptive – any time the load is not held indefinitely, there is the potential for cyclic behavior because there is unloading
- Cyclic behavior is complex and shows increased (or decreased) stresses at fixed strain range cycling before eventual stabilization
- Stabilized means that the stress amplitude has stopped changing with cycling at a fixed strain range
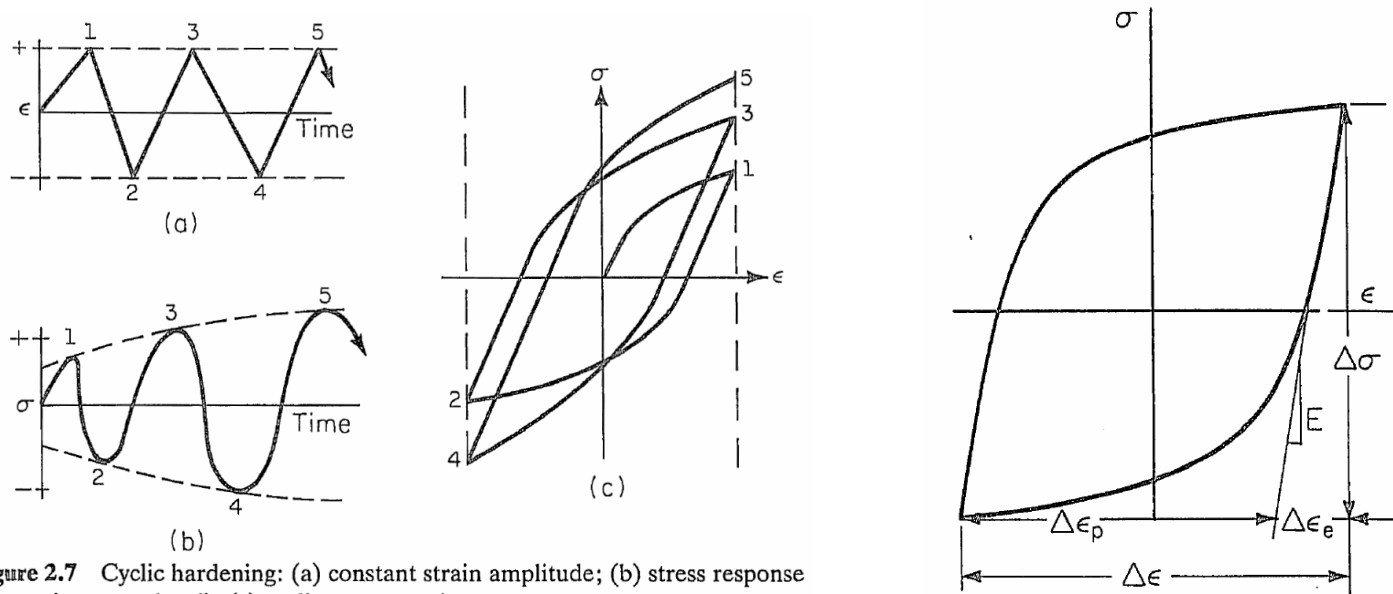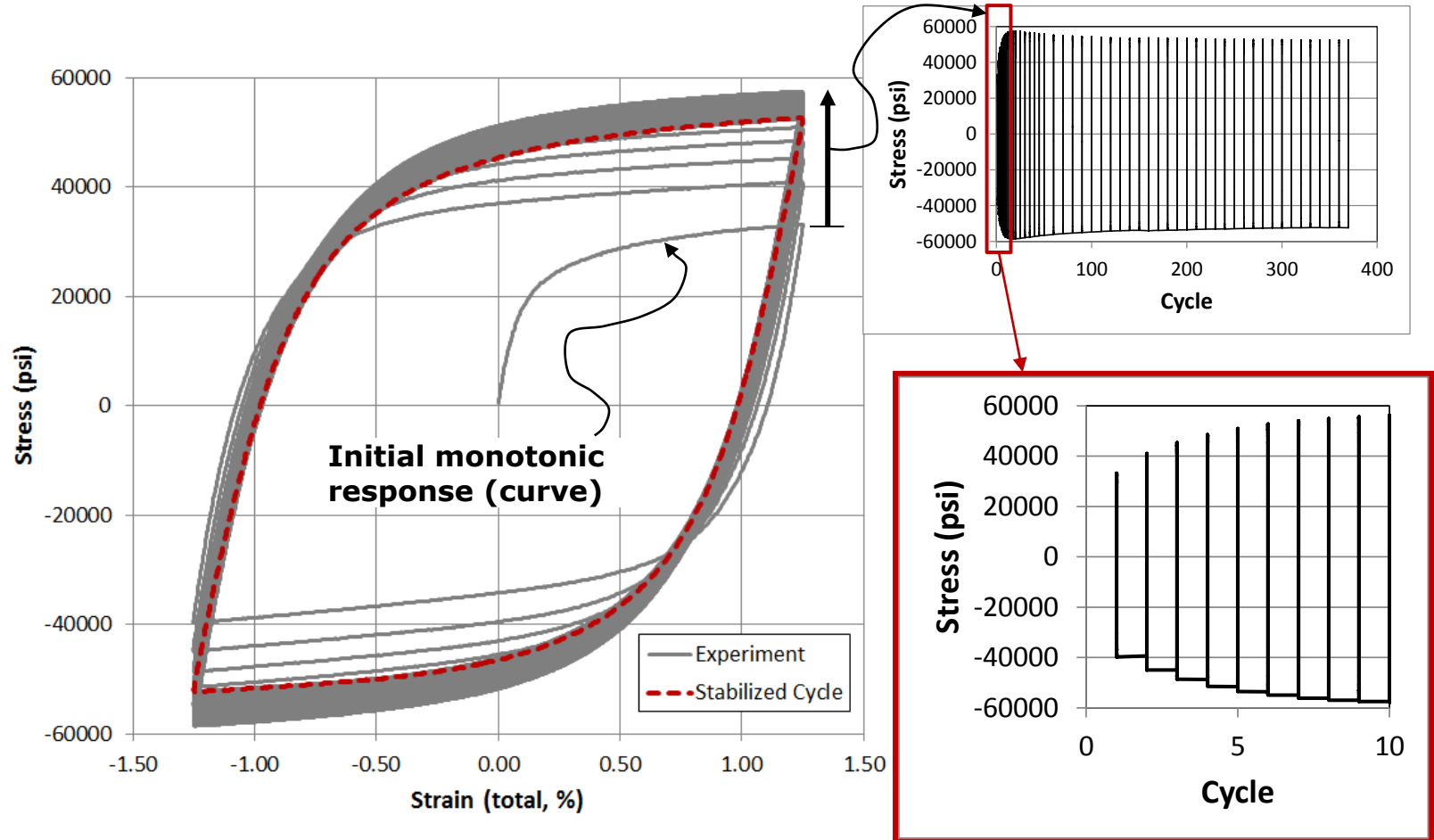

Figure 2.7 Cyclic hardening: (a) constant strain amplitude; (b) stress response (increasing stress level); (c) cyclic stress–strain response.

(from <u>Fundamentals of Metal Fatigue Analysis</u> by Bannantine et. al., Prentice Hall 1989)

Cyclic Material Data

Initial monotonic response (curve)

Experiment

Stabilized Cycle

Cyclic Plasticity

- The cyclic stress-strain curve (and its equations) simply represents the results of a fully reversed strain controlled 1D test – nothing about these equations will reproduce ratcheting
- Ratcheting is simulated by taking the cyclic stress-strain results and using them in an appropriate kinematic hardening incremental plasticity model

$$\varepsilon_{tr} = \frac{\sigma_a}{E_y} + 2\left[\frac{\sigma_r}{2K_{css}}\right]^{\frac{1}{n_{css}}}$$

$$\varepsilon_{ta} = \frac{\sigma_a}{E_y} + \left[\frac{\sigma_a}{K_{css}}\right]^{\frac{1}{n_{css}}}$$
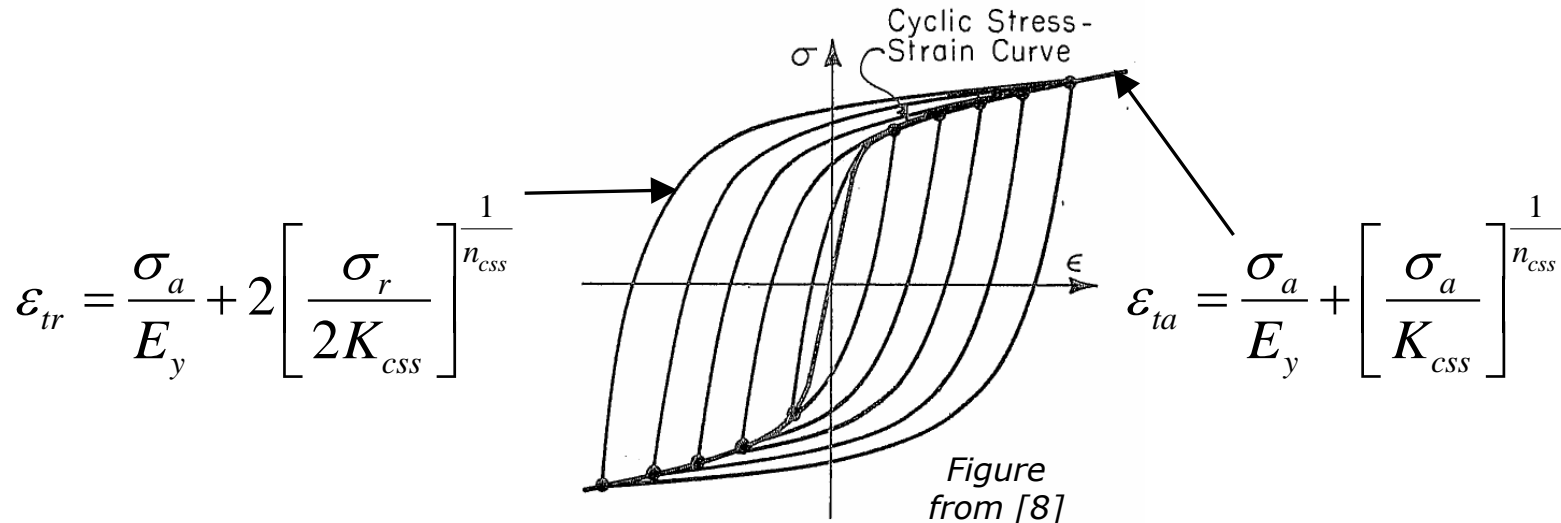
Figure from [8]

Figure 2.11 Cyclic stress–strain curve obtained by connecting tips of stabilized hysteresis loops.

Isotropic vs. Kinematic Hardening

- Essential difference between the two models is treatment of elastic range for unloading/reloading
- In the isotropic model, elastic range increases with plastic straining (total elastic range is twice the value of the largest equivalent stress obtained)
- In the kinematic model, the elastic strain range is fixed at twice the initial yield stress value and never increases:
  - Any stress increase beyond yield (termed the backstress or drag stress) will be removed from the yield strength upon reverse loading
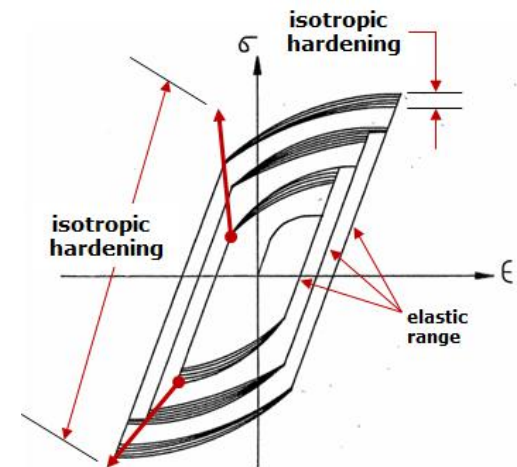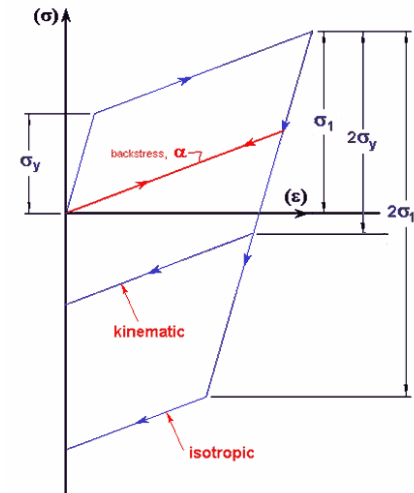 - Termed the Bauschinger Effect

Fig. 6.3   Controlled strain testing on 316 L steel at 20°C, (Chaboche 1979)

Kinematic Hardening

- There are MANY kinematic hardening plasticity models
- The first and simplest is still widely used:
  - Linear kinematic hardening
  - Single plastic stress-strain slope
- The linear model is very useful, BUT:
  - It cannot predict several cyclic plasticity phenomena
  - Most importantly, it cannot predict ratcheting
- There are many nonlinear kinematic hardening (NLKH) models as well:
  - Several of these also fundamentally cannot predict ratcheting
  - The most widely known nonlinear model of this class is the Mroz or multilinear model
  - The most widely known NLKH model that CAN predict ratcheting is the Frederick-Armstrong model
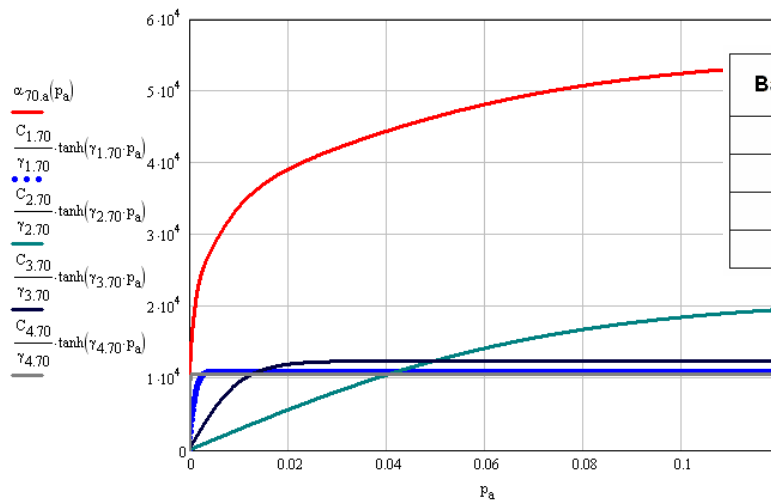
Nonlinear Kinematic Hardening

- Basic Frederick-Armstrong model predicts the back stress or drag stress (stress past yield, denoted here as σ_o):

$$d\boldsymbol{\alpha} = \frac{C}{\sigma_o}(\boldsymbol{\sigma} - \boldsymbol{\alpha})dp - \gamma \cdot \boldsymbol{\alpha} \cdot dp$$

- Chaboche generalized the model as a sum of multiple backstress so that realistic curve shapes could be predicted

$$d\boldsymbol{\alpha} = \sum_{i=1}^{n} \frac{C_i}{\sigma_o}(\boldsymbol{\sigma} - \boldsymbol{\alpha})dp - \gamma_i \cdot \boldsymbol{\alpha}_i \cdot dp$$

Backstress	C (psi)	γ
1	$4.586 \cdot 10^5$	24.34
2	$1.292 \cdot 10^6$	135.2
3	$5.027 \cdot 10^6$	593.4
4	$2.921 \cdot 10^7$	3156

Example (2.25Cr-1Mo) showing total curve (red) and 4 separate backstress curves that sum to it.

Chaboche NLKH

- This model is widely used and has been shown to represent the behaviors of many engineering alloys quite well
  - It can be integrated to give the monotonic stress-strain response (curve) as a simple function
  - If the integration is continued for reversed loading(s), an analytical function can be obtained for the cyclic stress-strain curve
  - Regression can be used to determine the model constants from curve fitting to the monotonic curve or the cyclic curve (or ideally, both)
- **Monotonic curve** form resulting from integration:

$$\sigma(p) = \sigma_o + \sum_i^n \frac{C_i}{\gamma_i}\left(1 - e^{-\gamma_i p}\right)$$

- **Cyclic curve** form resulting from continued integration:

$$\frac{\Delta\sigma}{2}(\Delta p) = \sigma_o + \sum_i^n \frac{C_i}{\gamma_i}\tanh\left(\gamma_i \frac{\Delta p}{2}\right)$$

$$\sigma_{\max} = \sigma_o + \sum_{i=1}^n \frac{C_i}{\gamma_i}$$

Verification

- How do you know for sure if your chosen hardening model can predict ratcheting?
- Test it in a simple 1 element test that enforces an applied STRESS range that is NOT fully-reversed
- In general, it's essential to check your model in this fashion; for example, does application of fully reversed STRAIN ranges give back the intended cyclic stress strain curve that the C's and γ's were calibrated to in the first place?

$$\Delta\boldsymbol{\sigma} = \Delta\boldsymbol{\alpha}_{total} = \sum_{j=1}^{N} \frac{C_j}{\sigma_o} \left(\boldsymbol{\sigma} - \boldsymbol{\alpha}_{total} \right) \Delta p - \gamma_j \cdot \boldsymbol{\alpha}_j \cdot \Delta p$$

(From FEA single element test results)

```
*Plastic, hardening=combined, data type=parameters, number backstresses=4
  42351.4, 4.586E+05,24.341, 1.292E+06,135.2, 5.027E+06,593.4, 2.921E+07,
        3156
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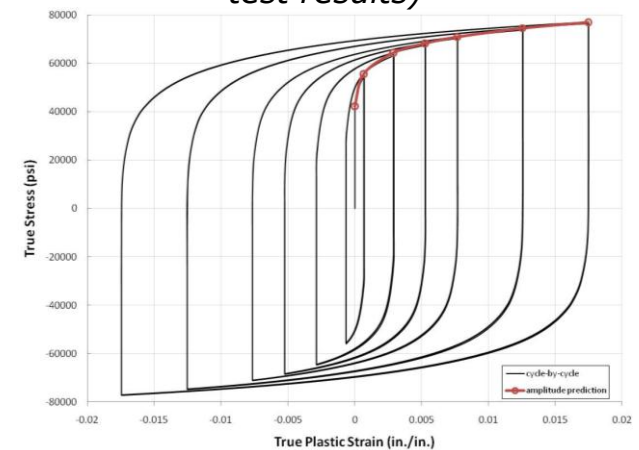
Code Implementation

- The only generally available form of cyclic stress-strain data is the cyclic stress-strain curve
  - These curves come from tests on "typical" (not minimum) strength material
  - They further include (isotropic) hardening for many materials (like carbon and stainless steel) and so are even further strengthened relative to a Code minimum yield strength
- Use of any cyclic data for a ratcheting assessment REQUIRES a nonlinear kinematic model as a minimum
  - Even then, not all nonlinear models can fundamentally predict ratcheting
  - Calibration of these models is not trivial either
    + For example, in the F-A or Chaboche models, the γ's should not vary with temperature
    + Additionally, the C's need to vary smoothly with temperature, or when the FEA interpolates for temperatures other than those specified, nonsense will likely be returned
    + Not a trivial task

Code Implementation

- And even after all of that work:
  - Only very advanced models with very specialized (and overwhelmingly non-existent) material test data can accurately predict actual ratcheting strain magnitudes per cycle
  - Or deal with known complexities like non-proportional loading
- As a Code working group, we also need to ensure that we're not biased towards a particular commercial software package when trying to make sense (and rules) with respect to all of the preceding
- If guidance is developed, it should probably focus on required verification and demonstration activities, what strength level is intended, etc.

Code Implementation

- Perhaps the simplest rule we could make as a start would recommend perfect-plasticity (as required for any FEA ratcheting analysis in the current Section VIII Division 2)
  - Code minimum strength is used unambiguously
  - Margin is consistent with Bree simplified rules
  - You literally cannot screw up the hardening model implementation – both isotropic and kinematic give the same answer in this case, and ratcheting IS predicted
  - Substantial benefit is often gained relative to the simplified Bree rules:
    - FEA even in this simple case will capture the Bree diagram extensions discussed earlier (pressure/thermal loading pattern)
    - Biggest benefit is that if any elastic core exists, shakedown will always be predicted

References

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5. Moreton, D.N. and Ng, H.W., "The Extension and Verification of the Bree Diagram," Trans. of the 6th International Structural Mechanics in Reactor Technology (SMiRT) Conference, Paris, 1981.
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8. Bannantine, J., Comer, J. and Handrock, J., 1989, Fundamentals of Metal Fatigue Analysis, Prentice Hall, New York.
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