Offering a new formula in the vertex color number in some graphs

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Abstract

In graph theory, **graph coloring** is a special case of graph labeling; it is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints. In its simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices share the same color; this is called a **vertex coloring**.

This research is presented in formula for some graphics that limits the number of vertices with vertex chromatic number of the graph gives us and if you want to head higher number of vertex color they must also obtain a formula to extend.

در این تحقیق فرمولی ارائه شده است که برای بعضی از گراف ها با محدودیت تعداد راس ها عدد رنگی راسی آن گراف را به ما می دهد و اگر بخواهیم برای تعداد راس های بالاتر عدد رنگی راسی آنها را نیز به دست آوریم باید فرمول را تعمیم دهیم.

Keywords: Graph, vertex color number, a new formula

کلید واژه: گراف، عدد رنگی راسی، فرمول جدید

۱.Introduction

The first results about graph coloring deal almost exclusively with planar graphs in the form of the coloring of *maps*. While trying to color a map of the counties of England, Francis Guthrie postulated the four color conjecture, noting that four colors were sufficient to color the map so that no regions sharing a common border received the same color. Guthrie's brother passed on the question to his mathematics teacher Augustus de Morgan at University College, who mentioned it in a letter to William Hamilton in ۱۸۵۲. Arthur Cayley raised the problem at a meeting of the London Mathematical Society in ۱۸۷۹. The same year, Alfred Kempe published a paper that claimed to establish the result, and for a decade the four color problem was considered solved. For his accomplishment Kempe was elected a Fellow of the Royal Society and later President of the London Mathematical Society.

In ۱۸۹۰, Heawood pointed out that Kempe's argument was wrong. However, in that paper he proved the five color theorem, saying that every planar map can be colored with no more than *five* colors, using ideas of Kempe. In the following century, a vast amount of work and theories were developed to reduce the number of colors to four, until the four color theorem was finally proved in ۱۹۷۶ by Kenneth Appel and Wolfgang Haken. The proof went back to the ideas of Heawood and Kempe and largely disregarded the intervening developments. The proof of the four color theorem is also noteworthy for being the first major computer-aided proof.

In ۱۹۱۲, George David Birkhoff introduced the chromatic polynomial to study the coloring problems, which was generalised to the Tutte polynomial by Tutte, important structures in algebraic graph theory. Kempe had already drawn attention to the general, non-planar case in ۱۸۷۹, and many results on generalisations of planar graph coloring to surfaces of higher order followed in the early ۲۰th century.

In ۱۹٦۰, Claude Berge formulated another conjecture about graph coloring, the *strong perfect graph conjecture*, originally motivated by an information-theoretic concept called the zero-error capacity of a graph introduced by Shannon. The conjecture remained unresolved for ٤۰ years, until it was established as the celebrated strong perfect graph theorem by Chudnovsky, Robertson, Seymour, and Thomas in ۲۰۰۲.

Graph coloring has been studied as an algorithmic problem since the early ۱۹۷۰s: the chromatic number problem is one of Karp's ۲۱ NP-complete problems from ۱۹۷۲, and at approximately the same time various exponential-time algorithms were developed based on backtracking and on the deletion-contraction recurrence of Zykov (۱۹٤۹). One of the major applications of graph coloring, register allocation in compilers, was introduced in ۱۹۸۱.

When used without any qualification, a **coloring** of a graph is almost always a *proper vertex coloring*, namely a labeling of the graph's vertices with colors such that no two vertices sharing the same edge have the same color. Since a vertex with a loop (i.e. a connection directly back to itself) could never be properly colored, it is understood that graphs in this context are loopless.

The terminology of using *colors* for vertex labels goes back to map coloring. Labels like *red* and *blue* are only used when the number of colors is small, and normally it is understood that the labels are drawn from the integers {۱, ۲, ۳, ...}.

A coloring using at most *k* colors is called a (proper) ***k*-coloring**. The smallest number of colors needed to color a graph *G* is called its **chromatic number**, and is often denoted $\chi(G)$. Sometimes $\gamma(G)$ is used, since $\chi(G)$ is also used to denote the Euler characteristic of a graph. A graph that can be assigned a (proper) *k*-coloring is ***k*-colorable**, and it is ***k*-chromatic** if its chromatic number is exactly *k*. A subset of vertices assigned to the same color is called a *color class*, every such class forms an independent set. Thus, a *k*-coloring is the same as a partition of the vertex set into *k* independent sets, and the terms *k-partite* and *k-colorable* have the same meaning.

۲. problem statement

First, the assumption that the type of graphs that can be used in the formula must also be adhered to as follows:

- The graph should not be multicast.
- Vertices should be linked together that it must be drawn around and in and out around, vertices does not exist.
- This formula only covers graphs up to seven vertex and for the number of vertices above formula should be applied.

ابتدا فرضیاتی که در نوع گراف هایی که از این فرمول می تواند استفاده شود باید رعایت نیز شود به شرح زیر می باشد:

- گراف نباید چندبخشی باشد.
- راس ها باید طوری با هم ارتباط داشته باشند که دور آن حتما رسم شده باشد و در داخل و خارج دور، راسی وجود نداشته باشد.
- این فرمول فقط گراف های تا هفت راس را پوشش می دهد و برای تعداد راس های بالاتر باید فرمول را تعمیم داد.

Now, vertex Formula chromatic number of these graphs is calculated as follows:

$$K = (n - (m2 - m1) * (٠.٠٧٨٥ * n^{٢} - ١.٠٦٤٠ * n + ٣.٩٥٦٥))$$
(١)

In formula (١) n (n <= ٧) the number of vertex of the graph, m٢ maximum number of edges in the graph, m١ the number of edges in the graph and K Vertex of the graph is the color number. However, be had a K value in the formula (١) to a constant:

$$K1 = fix(n - (m2 - m1) * (٠.٠٧٨٥ * n^{٢} - ١.٠٦٤٠ * n + ٣.٩٥٦٥))$$
(٢)

Using the equations (١) and (٢) reduce the amount of K of K١ If the value was greater than or equal to ٠,٥, we add a number to K١ and put its value in K Otherwise, put the value of K١ at K.

حال فرمول عدد رنگی راسی این نوع گراف ها به صورت زیر محاسبه می شود:

$$K = (n - (m2 - m1) * (٠.٠٧٨٥ * n^{٢} - ١.٠٦٤٠ * n + ٣.٩٥٦٥))$$
(١)

در فرمول (١) n(n<=٧) تعداد راس گراف، m٢ حداکثر تعداد یال های گراف ، m١ تعداد یال های گراف و K عدد رنگی راسی گراف می باشد.

حال مقدار K را در فرمول (١) به عدد ثابت تبدیل می کنیم:

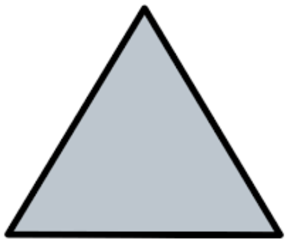
$$K1 = fix(n - (m2 - m1) * (٠.٠٧٨٥ * n^{٢} - ١.٠٦٤٠ * n + ٣.٩٥٦٥))$$
(٢)

با استفاده از روابط (١) و (٢) مقدار K را از K١ کم کرده اگر مقدار آن بزرگتر مساوی ٠,٥ شد به K١ یک عدد اضافه می کنیم و مقدار آن را در K قرار می دهیم در غیر اینصورت مقدار K١ را در K قرار می دهیم.

حال چند مثال با استفاده از روابط بالا حل می کنیم:

n=۳ m۲=۳ m۱=۳

K=۳ K۱=۳ K-K۱=۰<۰٫۵ ==> K=۳

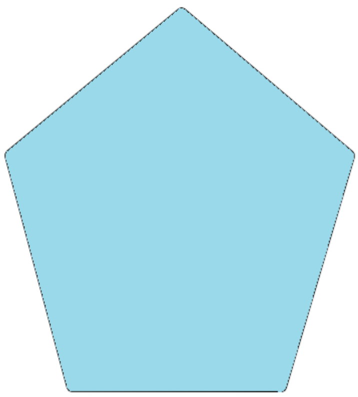


بنابراین عدد رنگی راسی مثلث عدد سه می باشد.

شکل (۱) : شکل یک مثلث که عدد رنگی راسی آن عدد سه می باشد

n=۵ m۲=۱۰ m۱=۵

K=۲٫۰۰۵۰ K۱=۲ K-K۱=۰٫۰۰۵۰ < ۰٫۵ ==> K=۲



بنابراین عدد رنگی راسی پنج ضلعی عدد دو می باشد.

شکل (۲): شکل یک پنج ضلعی که عدد رنگی راسی آن عدد دو می باشد.

۳. References

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