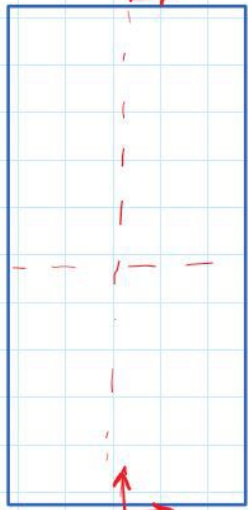


left $\{1\} (0, 0, D)$



Right $\{2\} (0, 0, -D)$

$$D = \frac{\text{track-width}}{2}$$

$$T_{bR} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & D \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{bL} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -D \\ 0 & 0 & 1 \end{bmatrix}$$

$$Ad_{bR} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A_{bL} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow Ad_{Rb} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A_{Lb} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Body Twist } V_b = \begin{bmatrix} \dot{\theta} \\ v_m \\ v_y \end{bmatrix}$$

Forward Kinematics

$$\begin{bmatrix} v_{m_i} \\ v_{y_i} \end{bmatrix} = \begin{bmatrix} r \dot{\phi}_i \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{\theta} \\ r \dot{\phi}_L \\ 0 \end{bmatrix} = A_{Lb} V_b = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_m \\ v_y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{\phi}_L \\ 0 \end{bmatrix} \begin{bmatrix} D/r & 1/r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_m \\ v_y \end{bmatrix}$$

Similarly:

$$\begin{bmatrix} \dot{\theta} \\ r \dot{\phi}_R \\ 0 \end{bmatrix} = A_{Rb} V_b = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_m \\ v_y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{\phi}_R \\ 0 \end{bmatrix} = \begin{bmatrix} -D/r & 1/r & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} u_L \\ u_R \end{bmatrix} = H V_b \Rightarrow V_b = \begin{bmatrix} u_L \\ u_R \end{bmatrix} H^T$$

$$H = \begin{bmatrix} -D/r & 1/r & 0 \\ D/r & 1/r & 0 \end{bmatrix} \Rightarrow H^T = \frac{r}{2} \begin{bmatrix} -1/D & 1/D \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} = \frac{r}{2} \begin{bmatrix} -1/D & 1/D \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\phi_L \\ \Delta\phi_R \end{bmatrix}$$

$$\Rightarrow \dot{\theta} = \left(-\frac{\Delta\phi_L}{D} + \frac{\Delta\phi_R}{D} \right) \frac{r}{2}$$

$$\Rightarrow v_x = (\Delta\phi_L + \Delta\phi_R) \frac{r}{2}$$

$$\Rightarrow v_y = 0$$

$$T_{bb'} = T(\Delta\phi_L, \Delta\phi_R, \Delta y_b) \rightarrow \text{integrate twist (VL)} \quad \text{--- (i)}$$

$$q' = q + \Delta q$$

$$\Delta q = A(\theta, 0, 0) \Delta q_b$$

$$q_b = \begin{bmatrix} \Delta\theta_b \\ \Delta\phi_b \\ \Delta y_b \end{bmatrix} \text{ from eqn (i)}$$

Inverse Kinematics

$$u = H v_b$$

$$= \begin{bmatrix} -D/r & 1/r & 0 \\ D/r & 1/r & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \\ y \end{bmatrix}$$

$$\Delta\phi_L = -\frac{D}{r} \theta + \frac{\phi}{r}$$

$$\Delta\phi_R = \frac{D}{r} \theta + \frac{\phi}{r}$$