Q1. Determine if the system $y(t) = x(t) + \dot{x}$ is time invariant, linear, causal, and/or memory

$\Rightarrow$ • $y(t-t_0) = \dot{x}(t-t_0) + x(t-t_0)$

$x(t-t_0) \rightarrow \text{system} \rightarrow y'(t-t_0) = \dot{x}(t-t_0) + x(t-t_0)$

So, system is time invariant

• $y(t) = \dot{x} + x(t)$ is not memoryless since the derivative of function at a specific point t_0 cannot be determined just from the value ~~at~~ of the function at t_0.

• We know,

$$\frac{dx(t)}{dt} = \lim_{h \to 0^+} \frac{x(t) - x(t-h)}{h}$$

~~with~~ which means $\frac{dx(t)}{dt}$ only depends on the past values of $x(t)$.

So, $y(t) = \dot{x} + x(t)$ is Causal.

• $y_1(t) = \dot{x}_1 + x_1(t)$; $y_2(t) = \dot{x}_2 + x_2(t)$

$\Rightarrow y_1(t) + y_2(t) = \dot{x}_1 + \dot{x}_2 + x_1(t) + x_2(t)$ — (1)

now, $y'[x_1(t) + x_2(t)] = \frac{d}{dt}(x_1(t) + x_2(t)) + x_1(t) + x_2(t)$

$= \dot{x}_1 + \dot{x}_2 + x_1(t) + x_2(t)$ — (2)

~~So, (1) = (2) ... So, System is~~

Now, $Ky(t) = K\dot{x} + x(t)$

$Kx(t) \rightarrow y'(t) = \dot{x} + Kx(t)$

So, the system is Non-Linear.

Q.2) Solve the following differential equation using the Laplace Transform method- $\dot{y} - 2y = 2x$, $x(t) = u(t)$, $y(0) = -1$.

Ans-

$$\dot{y} - 2y = 2x \quad , \quad x(t) = u(t) \quad , \quad y(0) = -1$$

$$\Rightarrow S \cdot Y(S) - y(0) - 2Y(S) = 2X(S) = 2U(S) = 2/S$$

considering $u(t)$ = unit step. so, $L(u(t)) = L(1) = \frac{1}{S}$

$$\Rightarrow Y(S)(S-2) = \frac{2}{S} - 1$$

$$\Rightarrow Y(S) = \frac{2-S}{S(S-2)} = -\frac{1}{S}$$

$$L^{-1}(Y(S)) = y(t) = -1 \cdot -u(t)$$

Q.3> Find initial value of $\frac{df(t)}{dt}$ for $F(S) = \mathcal{L}[f(t)]$

$= \frac{2s+1}{s^2+s+1}$

Using the initial-value theorem,

$$\lim_{t\to 0+} f(t) = f(0+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s(2s+1)}{s^2+s+1} = 2$$

Since the $\mathcal{L}_+$ transform of $df(t)/dt = g(t)$ is given by-

$$\mathcal{L}_+[g(t)] = sF(s) - f(0+)$$
$$= \frac{s(2s+1)}{s^2+s+1} - 2 = \frac{-s-2}{s^2+s+1}$$

the initial value of $df(t)/dt$ is obtained as-

$$\lim_{t\to 0+} \frac{df(t)}{dt} = g(0+) = \lim_{s\to\infty} s[sF(s) - f(0+)]$$
$$= \lim_{s\to\infty} \frac{-s^2-2s}{s^2+s+1} = -1$$

Q.4> Determine whether the system characterized by the differential equation, $\ddot{y}(t) - \dot{y}(t) + 2y(t) = x(t)$ is stable or not? Assume zero initial condition.

Ans-

$$\ddot{y}(t) - \dot{y}(t) + 2y(t) = x(t)$$

Let us define, $x_1 = y$, $\dot{x}_1 = \dot{y} = x_2$

$x_2 = \dot{y}$, $\dot{x}_2 = \ddot{y} = x(t) - 2y(t) + \dot{y}(t)$

$= U - 2x_1 + x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$\dot{X} = A X + B U$$

Now, $|SI - A| = 0$

$\Rightarrow \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} \right| = 0$

$\Rightarrow \begin{vmatrix} s & -1 \\ 2 & s-1 \end{vmatrix} = 0 \Rightarrow s^2 - s + 2 = 0$

$\Rightarrow (s+1)(s-2) = 0$

As, both the eigen values of matrix A did not lie on left-half, System is not stable.

Q. 5) The unit impulse response of an LTI system is the ~~differential~~ unit function $u(t)$. Find response of the system to an excitation $e^{-at}u(t)$.

Ans — As, unit impulse response of LTI system is the unit step function $u(t)$, $h(t) = u(t)$ $[u(t) = 1]$

Excitation, $x(t) = e^{-at}u(t)$

$$y(t) = h(t)x(t)$$

$$\Rightarrow Y(S) = H(S)\,X(S)$$

$$= \frac{1}{S} \times \frac{1}{S+a} = \frac{A_1}{S} + \frac{A_2}{S+a} = \frac{A_1S + A_2a + A_2S}{S(S+a)}$$

$$= \frac{(A_1+A_2)S + A_1a}{S(S+a)}$$

$\therefore (A_1+A_2)S + A_1a = 1$

$\Rightarrow A_1 + A_2 = 0$

$\Rightarrow A_2 = -A_1 = -\frac{1}{a}$

$A_1a = 1$

$\Rightarrow A_1 = \frac{1}{a}$

$\therefore Y(S) = \frac{1}{a}\left(\frac{1}{S} - \frac{1}{S-a}\right)$

$\Rightarrow y(t) = \frac{1}{a}(1 - e^{-at})u(t)$

Q.6) The response of an LTI system to a step input $u(t)$, is $y(t) = (1 - e^{-2t})u(t)$. Find the response of the system to an input, $x(t) = 4u(t) - 4u(t-1)$.

Ans — For input $x_1(t)$; $y_1(t) = 4(1 - e^{-2t})u(t)$

For input $-x_2(t)$, $y_2(t) = -4(1 - e^{-2(t-1)})u(t-1)$

$\therefore$ Here, required response —

$$y(t) = y_1(t) + y_2(t)$$

$$= 4\left[(1 - e^{-2t})u(t) - (1 - e^{-2(t-1)})u(t-1)\right]$$

Q.7) Find the state equation for the following system –
$\ddot{y}(t) + 2\dot{y}(t) + 4y(t) = 2u(t)$.

$$\ddot{y}(t) + 2\dot{y}(t) + 4y(t) = 2u(t)$$

Let us define vectors as –

$$u \rightarrow \boxed{\text{control}} \rightarrow y$$

$$x_1 = y \Rightarrow \dot{x}_1 = \dot{y} = x_2$$

$$x_2 = \dot{y} \Rightarrow \dot{x}_2 = \ddot{y} = 2U - 4x_1 - 2x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} U$$

$$\dot{X} = A \quad X + B \; U$$

$$\& \quad Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$ System Equation

$$Y = C \quad X$$

Q.10) Find the unit impulse response of the system characterized by the following differential equation $\dot{y} + ay = x$. Assume zero initial condition.

$$\dot{y} + ay = x$$

Taking Laplace transform,

$$sY(s) + aY(s) = X(s)$$

Taking zero initial condition –

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{s+a}$$

For the response to be unit impulse, Input function should be Unit step –

So, $X(s) = 1$

$$\Rightarrow Y(s) = \frac{1}{s+a}$$

$$L^{-1}(Y(s)) = L^{-1}\left[\frac{1}{s+a}\right]$$

$$\Rightarrow \boxed{y(t) = e^{-at}}$$

Q. 9) Find the equations for the following system

$\ddot{y}(t) - 4y(t) = u(t)$

$\ddot{y}(t) - 4y(t) = u(t)$

Assume, $x_1 = y \Rightarrow \dot{x}_1 = x_2$

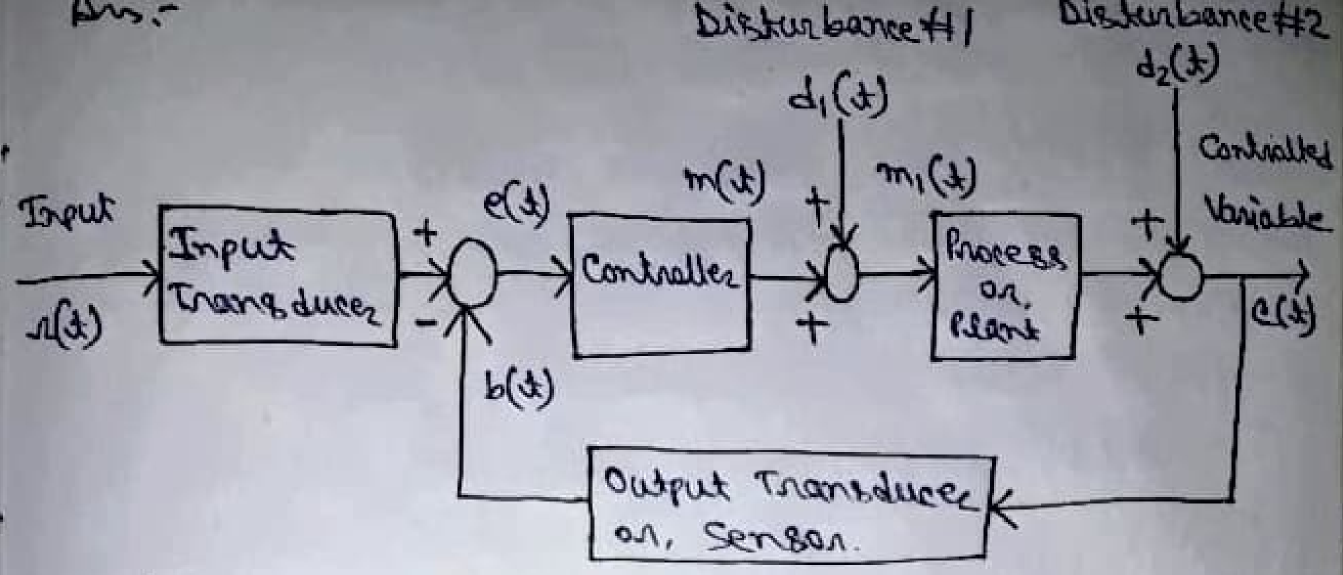
$x_2 = \dot{y} \Rightarrow \dot{x}_2 = U + 4x_1$

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$\& \ Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

ii) Schematic diagram and definitions of dif. var. of CLCS.

Ans:-



$r(t)$ = reference / input signal

$e(t)$ = actuating / error signal

$b(t)$ = feedback signal

$c(t)$ = controlled variable / output

$m(t)$ = manipulated variable

$d_1(t)$, $d_2(t)$ = disturbance signals.

<u>$r(t)$</u> :- Reference / Input signal is an external signal applied to the control system to produce a specific action. It represents the desired value of the controlled variable and is also called as "Set Point" for the CLCS.

<u>$c(t)$</u> :- Controlled Variable / output is the quantity or the condition of the plant which is to be controlled or maintained at the desired value set by the reference point.

<u>$b(t)$</u> :- Feedback signal is a function of the output signal as identified by the feedback elements.

e(t) :- Actuating / Error Signal represents the control action of the control loop as determined by the difference between the set point and feedback signal.

m(t) :- Manipulated variable is that variable of the process which is acted upon by the controller to maintain the output at the desired value.

$d_1(t)$, $d_2(t)$:- Disturbace signals are the undesirable input signals that upsets the value of the controlled variable.

If a disturbace is generated within the system, it is called internal disturbance, while an external disturbance is generated outside the system and is treated as an input.

12) PID is called "Gain - Reset - Preact".

Ans: The PID (Proportional Integral Derivative) controller has 3 modes of controls – Proportional Control, Integral control and Derivative Control. ~~The parallel form of the PID cont~~ The control equation is,

$$p(t) = \bar{p} + K_c e(t) + \frac{K_c}{\tau_I}\int_0^t e(t)\,dt + K_c \tau_D \frac{de(t)}{dt}$$

The term $K_c e(t)$ or proportional control or P-gain determines how much change the output (O/P) will make due to a change in error. "Gain" implies that a larger number will have more effect.

The term $\frac{K_c}{\tau_I}\int_0^t e(t)\,dt$ or integral control is a reset action where τ_I is the integral or reset time. It aims to "reset" the control to eliminate the error.

The term $K_c \tau_D \frac{de(t)}{dt}$ or derivative control or rate action or "preact". Its purpose is to anticipate where the process is heading by looking at the time rate of change of error, its derivative.

Since all 3 fn, gain, reset, preact are acting together, PID controller is called "Gain, Reset, Preact" controller.

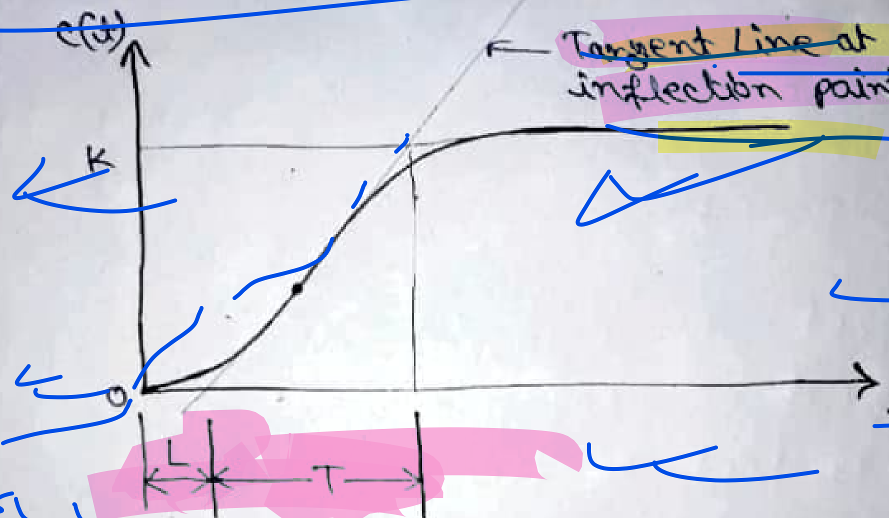
13) Ziegler Nicholas method of PID. Unit Step Test.

Ans: Assumption,

The transfer function, $\frac{C(s)}{U(s)}$ may be assumed as,

$$\boxed{\frac{C(s)}{U(s)} = \frac{K e^{-Ls}}{Ts+1}}$$

The method of PID controller tuning is applied if the response a step input exhibits an S-shaped curve.



Type of controller	K_p	T_I	T_D
P	T/L	∞	0
PI	$0.9\, T/L$	$L/0.3$	0
PID	$1.2\, T/L$	$2L$	$0.5L$

This is the rule suggested by Ziegler Nichols for first method (unit step test) of PID controller turning.

$$G_c(s) = K_p\left(1 + \frac{1}{T_I s} + T_d s\right)$$

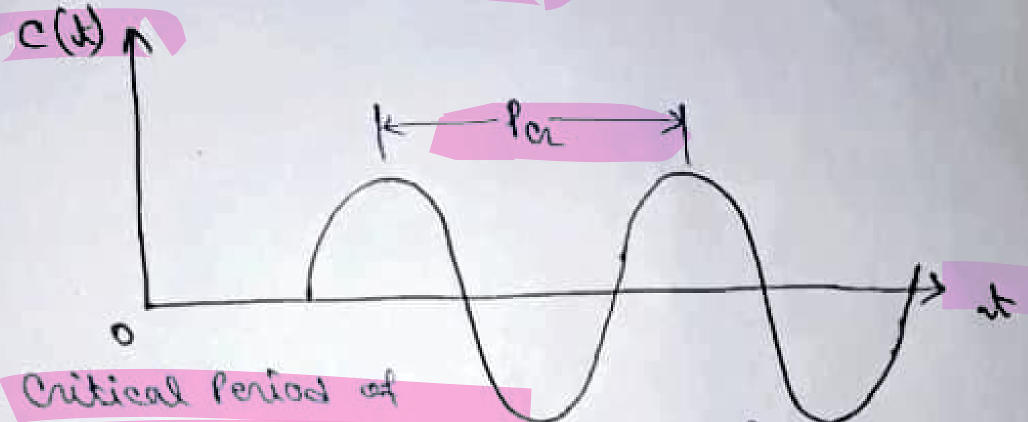
$$= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right)$$

$$= 0.6T\frac{1}{s}\left(s + \frac{1}{L}\right)^2$$

14) Ziegler Nichols method of PID. Critical Gain Test.

Ans: In this method T_I is set to α and $T_D = 0$. Using proportional control action, K_p is increased from 0 to critical value K_{cr} at which the output first exhibits sustained oscillations.

If the output does not exhibit sustained oscillations for any value of K_p, then this method does not apply.



P_{cr} = Critical Period of oscillations for corres. value of K_{cr}

Types of controller	K_p	T_I	T_D
P	$0.5\,K_{cr}$	α	0
PI	$0.45\,K_{cr}$	$\frac{1}{1.2} P_{cr}$	0
PID	$0.6\,K_{cr}$	$0.5\,P_{cr}$	$0.125\,P_{cr}$

Ziegler Nichols rule for critical gain test.

$$G_c(s) = K_p\left(1 + \frac{1}{T_I s} + T_d s\right)$$

$$= 0.6\,K_{cr}\left(1 + \frac{1}{0.5\,P_{cr}\,s} + 0.125\,P_{cr}\,s\right)$$

$$= 0.075\,K_{cr}\,P_{cr}\,\frac{1}{s}\cdot\left(s + \frac{4}{P_{cr}}\right)^2$$

16) "Direct Acting" and "Reverse Acting". Example

Ans:- The output of a process controller is directly proportional to the error. Therefore, if the error increases, the output is expected to increase.

If the control output is increased by a positive error, the controller is said to be "Direct Acting".

In the opposite case, the controller is said to be "Reverse Acting" when the +ve error decreases the output.

<u>Example</u>:- It depends ~~of~~ on the placement of the control valve in the case of a tank level control. If the valve controls the flow out of the tank, we would like to see a +ve error in order to increase the control output, open the valve and leave more fluid out of the tank.

If the valve controls the flow into the tank, reverse acting controller would be used to respond a high level by closing the valve and reducing the flow into the vessel.

18) "Modelling is a compromise between complexity and accuracy". Justify.

Ans: During modeling, one must make a compromise between simplicity/complexity and accuracy of the results of the analysis. In solving a reasonably simplified mathematical model, it becomes necessary to ignore certain inherent physical properties.

In general, it is desirable to first build a simplified model so one gets general idea of the solution.

A more complete and complex mathematical model can be built after that and used for accurate analysis.

17) Advantages of Closed-loop Control Systems over Open-loop Control.

Ans: i) Disturbance rejection.

ii) Reduced sensitivity to parameter variation.

iii) Guaranteed performance even with model uncertainties.

iv) Improved reference tracking performance.

v) Control over system dynamics.

$\boxed{\text{Limitations:-}}$

i) CLS are more complex and expensive.

ii) CLS, at times are dificult to stabilize.

Step Response of Second Order System

Consider the unit step signal as an input to the second order system.

Laplace transform of the unit step signal is,

$$R(s) = \frac{1}{s}$$

We know the transfer function of the second order closed loop control system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Case 1: δ = 0

Substitute, $\delta = 0$ in the transfer function.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$\Rightarrow C(s) = \left(\frac{\omega_n^2}{s^2 + \omega_n^2}\right) R(s)$$

Substitute, $R(s) = \frac{1}{s}$ in the above equation.

$$C(s) = \left(\frac{\omega_n^2}{s^2 + \omega_n^2}\right)\left(\frac{1}{s}\right) = \frac{\omega_n^2}{s(s^2 + \omega_n^2)}$$

Apply inverse Laplace transform on both the sides.

$$c(t) = (1 - \cos(\omega_n t))\, u(t)$$

So, the unit step response of the second order system when $/delta = 0$ will be a continuous time signal with constant amplitude and frequency.

Case 2: δ = 1

Substitute, $/delta = 1$ in the transfer function.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

$$\Rightarrow C(s) = \left(\frac{\omega_n^2}{(s+\omega_n)^2}\right) R(s)$$

Substitute, $R(s) = \frac{1}{s}$ in the above equation.

$$C(s) = \left(\frac{\omega_n^2}{(s+\omega_n)^2}\right)\left(\frac{1}{s}\right) = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

Do partial fractions of $C(s)$.

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{s+\omega_n} + \frac{C}{(s+\omega_n)^2}$$

After simplifying, you will get the values of A, B and C as $1, -1\, and\, -\omega_n$ respectively.

Substitute these values in the above partial fraction expansion of $C(s)$.

$$C(s) = \frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2}$$

Apply inverse Laplace transform on both the sides.

$$c(t) = (1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t})u(t)$$

So, the unit step response of the second order system will try to reach the step input in steady state.

Case 3: $0 < \delta < 1$

We can modify the denominator term of the transfer function as follows –

$$s^2 + 2\delta\omega_n s + \omega_n^2 = \left\{s^2 + 2(s)(\delta\omega_n) + (\delta\omega_n)^2\right\} + \omega_n^2 - (\delta\omega_n)^2$$

$$= (s + \delta\omega_n)^2 + \omega_n^2(1 - \delta^2)$$

The transfer function becomes,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \delta\omega_n)^2 + \omega_n^2(1 - \delta^2)}$$

$$\Rightarrow C(s) = \left(\frac{\omega_n^2}{(s + \delta\omega_n)^2 + \omega_n^2(1 - \delta^2)}\right) R(s)$$

Substitute, $R(s) = \frac{1}{s}$ in the above equation.

$$C(s) = \left(\frac{\omega_n^2}{(s + \delta\omega_n)^2 + \omega_n^2(1 - \delta^2)}\right)\left(\frac{1}{s}\right) = \frac{\omega_n^2}{s\left((s + \delta\omega_n)^2 + \omega_n^2(1 - \delta^2)\right)}$$

Do partial fractions of $C(s)$.

$$C(s) = \frac{\omega_n^2}{s\left((s + \delta\omega_n)^2 + \omega_n^2(1 - \delta^2)\right)} = \frac{A}{s} + \frac{Bs + C}{(s + \delta\omega_n)^2 + \omega_n^2(1 - \delta^2)}$$

After simplifying, you will get the values of A, B and C as $1, -1\,and\, -2\delta\omega_n$ respectively.

Substitute these values in the above partial fraction expansion of C(s).

$$C(s) = \frac{1}{s} - \frac{s + 2\delta\omega_n}{(s + \delta\omega_n)^2 + \omega_n^2(1 - \delta^2)}$$

$$C(s) = \frac{1}{s} - \frac{s + 2\delta\omega_n}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)}$$

$$C(s) = \frac{1}{s} - \frac{s + \delta\omega_n}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)} - \frac{\delta\omega_n}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)}$$

$$C(s) = \frac{1}{s} - \frac{(s+\delta\omega_n)}{(s+\delta\omega_n)^2 + (\omega_n\sqrt{1-\delta^2})^2} - \frac{\delta}{\sqrt{1-\delta^2}}\left(\frac{\omega_n\sqrt{1-\delta^2}}{(s+\delta\omega_n)^2 + (\omega_n\sqrt{1-\delta^2})^2}\right)$$

Substitute, $\omega_n\sqrt{1-\delta^2}$ as ω_d in the above equation.

$$C(s) = \frac{1}{s} - \frac{(s+\delta\omega_n)}{(s+\delta\omega_n)^2 + \omega_d^2} - \frac{\delta}{\sqrt{1-\delta^2}}\left(\frac{\omega_d}{(s+\delta\omega_n)^2 + \omega_d^2}\right)$$

Apply inverse Laplace transform on both the sides.

$$c(t) = \left(1 - e^{-\delta\omega_n t}\cos(\omega_d t) - \frac{\delta}{\sqrt{1-\delta^2}} e^{-\delta\omega_n t}\sin(\omega_d t)\right) u(t)$$

$$c(t) = \left(1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\left((\sqrt{1-\delta^2})\cos(\omega_d t) + \delta\sin(\omega_d t)\right)\right) u(t)$$

If $\sqrt{1-\delta^2} = \sin(\theta)$, then 'δ' will be cos(θ). Substitute these values in the above equation.

$$c(t) = \left(1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}(\sin(\theta)\cos(\omega_d t) + \cos(\theta)\sin(\omega_d t))\right) u(t)$$

$$\Rightarrow c(t) = \left(1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)\right) u(t)$$

So, the unit step response of the second order system is having damped oscillations (decreasing amplitude) when 'δ' lies between zero and one.

Case 4: δ > 1

We can modify the denominator term of the transfer function as follows −

$$s^2 + 2\delta\omega_n s + \omega_n^2 = \left\{s^2 + 2(s)(\delta\omega_n) + (\delta\omega_n)^2\right\} + \omega_n^2 - (\delta\omega_n)^2$$

$$= (s + \delta\omega_n)^2 - \omega_n^2\left(\delta^2 - 1\right)$$

The transfer function becomes,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\delta\omega_n)^2 - \omega_n^2(\delta^2-1)}$$

$$\Rightarrow C(s) = \left(\frac{\omega_n^2}{(s+\delta\omega_n)^2 - \omega_n^2(\delta^2-1)}\right) R(s)$$

Substitute, $R(s) = \frac{1}{s}$ in the above equation.

$$C(s) = \left(\frac{\omega_n^2}{(s+\delta\omega_n)^2 - (\omega_n\sqrt{\delta^2-1})^2}\right)\left(\frac{1}{s}\right) = \frac{\omega_n^2}{s(s+\delta\omega_n+\omega_n\sqrt{\delta^2-1})(s+\delta\omega_n-\omega_n\sqrt{\delta^2-1})}$$

Do partial fractions of $C(s)$.

$$C(s) = \frac{\omega_n^2}{s(s+\delta\omega_n+\omega_n\sqrt{\delta^2-1})(s+\delta\omega_n-\omega_n\sqrt{\delta^2-1})}$$

$$= \frac{A}{s} + \frac{B}{s+\delta\omega_n+\omega_n\sqrt{\delta^2-1}} + \frac{C}{s+\delta\omega_n-\omega_n\sqrt{\delta^2-1}}$$

After simplifying, you will get the values of A, B and C as 1, $\frac{1}{2(\delta+\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}$ and $\frac{-1}{2(\delta-\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}$ respectively. Substitute these values in above partial fraction expansion of

$$C(s) = \left(\frac{\omega_n^2}{(s+\delta\omega_n)^2-(\omega_n\sqrt{\delta^2-1})^2}\right)\left(\frac{1}{s}\right) = \frac{\omega_n^2}{s(s+\delta\omega_n+\omega_n\sqrt{\delta^2-1})(s+\delta\omega_n-\omega_n\sqrt{\delta^2-1})}$$

Do partial fractions of $C(s)$.

$$C(s) = \frac{\omega_n^2}{s(s+\delta\omega_n+\omega_n\sqrt{\delta^2-1})(s+\delta\omega_n-\omega_n\sqrt{\delta^2-1})}$$

$$= \frac{A}{s} + \frac{B}{s+\delta\omega_n+\omega_n\sqrt{\delta^2-1}} + \frac{C}{s+\delta\omega_n-\omega_n\sqrt{\delta^2-1}}$$

After simplifying, you will get the values of A, B and C as 1, $\frac{1}{2(\delta+\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}$ and $\frac{-1}{2(\delta-\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}$ respectively. Substitute these values in above partial fraction expansion of $C(s)$.

$$C(s) = \frac{1}{s} + \frac{1}{2(\delta+\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}\left(\frac{1}{s+\delta\omega_n+\omega_n\sqrt{\delta^2-1}}\right) - \left(\frac{1}{2(\delta-\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}\right)\left(\frac{1}{s+\delta\omega_n-\omega_n\sqrt{\delta^2-1}}\right)$$

Apply inverse Laplace transform on both the sides.

$$c(t) = \left(1+\left(\frac{1}{2(\delta+\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}\right)e^{-(\delta\omega_n+\omega_n\sqrt{\delta^2-1})t} - \left(\frac{1}{2(\delta-\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}\right)e^{-(\delta\omega_n-\omega_n\sqrt{\delta^2-1})t}\right)u(t)$$

Since it is over damped, the unit step response of the second order system when δ > 1 will never reach step input in the steady state.

Q-4 $G(s) = \frac{s^2+9s+15}{s^3+7s^2+16s+4}$ ← no. of zeroes = 2
← no. of poles = 3

or, $\frac{W(s)}{R(s)} = \frac{s^2+9s+15}{s^3+7s^2+16s+4} = \frac{W(s)}{V(s)} \times \frac{V(s)}{R(s)}$

Assume, $\frac{V(s)}{R(s)} = \frac{1}{s^3+7s^2+16s+4} \Rightarrow \dddot{v}+7\ddot{v}+16\dot{v}+4v = r$

Assume state vectors are x_1, x_2, x_3.

$x_1 = v \Rightarrow \dot{x}_1 = x_2$

$x_2 = \dot{v} \Rightarrow \dot{x}_2 = x_3$

$x_3 = \ddot{v} \Rightarrow \dot{x}_3 = r - 4x_1 - 16x_2 - 7x_3$

So,
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -16 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [r]$$

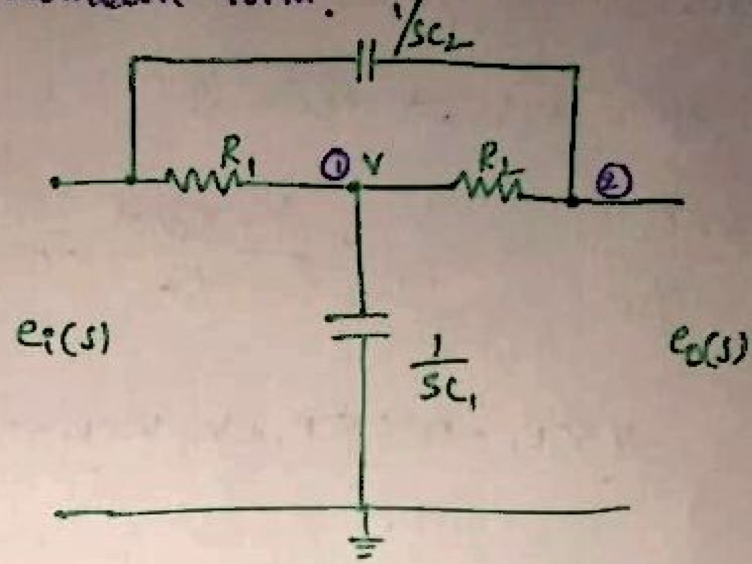
$$\dot{X} = A \quad X + B \quad U$$

And, $\frac{W(s)}{V(s)} = s^2+9s+15 \Rightarrow w = \underset{\downarrow\, x_3}{\ddot{v}} + 9\underset{\downarrow\, x_2}{\dot{v}} + 15\underset{\downarrow\, x_1}{v}$

$$y = \begin{bmatrix} 15 & 9 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Leftarrow \Rightarrow y = w = 15x_1 + 9x_2 + x_3$$

$$Y = C \quad X$$

a) Draw the fig (a) in Laplace domain form.



Apply KCL at node ①

$$\Rightarrow \frac{V - e_i(s)}{R_1} + \frac{V}{1/sC_1} = 0$$

$$\Rightarrow \frac{V - e_i(s) + sC_1R_1V}{R_1} = 0$$

$$\Rightarrow e_i(s) = V(1 + sC_1R_1) \quad — ①$$

Apply KCL at node ②

$$\Rightarrow \frac{e_o(s) - V}{R_2} + \frac{e_o(s) - e_i(s)}{1/sC_2} = 0$$

$$\Rightarrow \frac{e_o(s) - V + sC_2R_2\, e_o(s) - sC_2R_2\, e_i(s)}{R_2} = 0$$

$$\Rightarrow V = e_o(s)[1 + sC_2R_2] - e_i(s)\, sC_2R_2 \quad — ②$$

Substitute equation ② in ①

$$e_i(s) = (1 + sC_1R_1)\left[e_o(s)(1 + sC_2R_2) - e_i(s)(sC_2R_2)\right]$$

$$e_i(s)\left[1 + (1 + sC_1R_1)(sC_2R_2)\right] = e_o(s)(1 + sC_2R_2)(1 + sC_1R_1)$$

$$\therefore \frac{E_o(s)}{e_i(s)} = \frac{1 + (1 + sC_1R_1)\, sC_2R_2}{(1 + sC_2R_2)(1 + sC_1R_1)}$$

Q.6 Draw the asymptotic Bode magnitude plot for the system having a transfer function.

$$G(s) = \frac{20s}{(s+1)(s+3)^2(s+10)}$$

⇒ put, $s = j\omega$

$$G(j\omega) = \frac{20j\omega}{(j\omega+1)\,9\left(\frac{j\omega}{3}+1\right)^2 10\left(\frac{j\omega}{10}+1\right)}$$

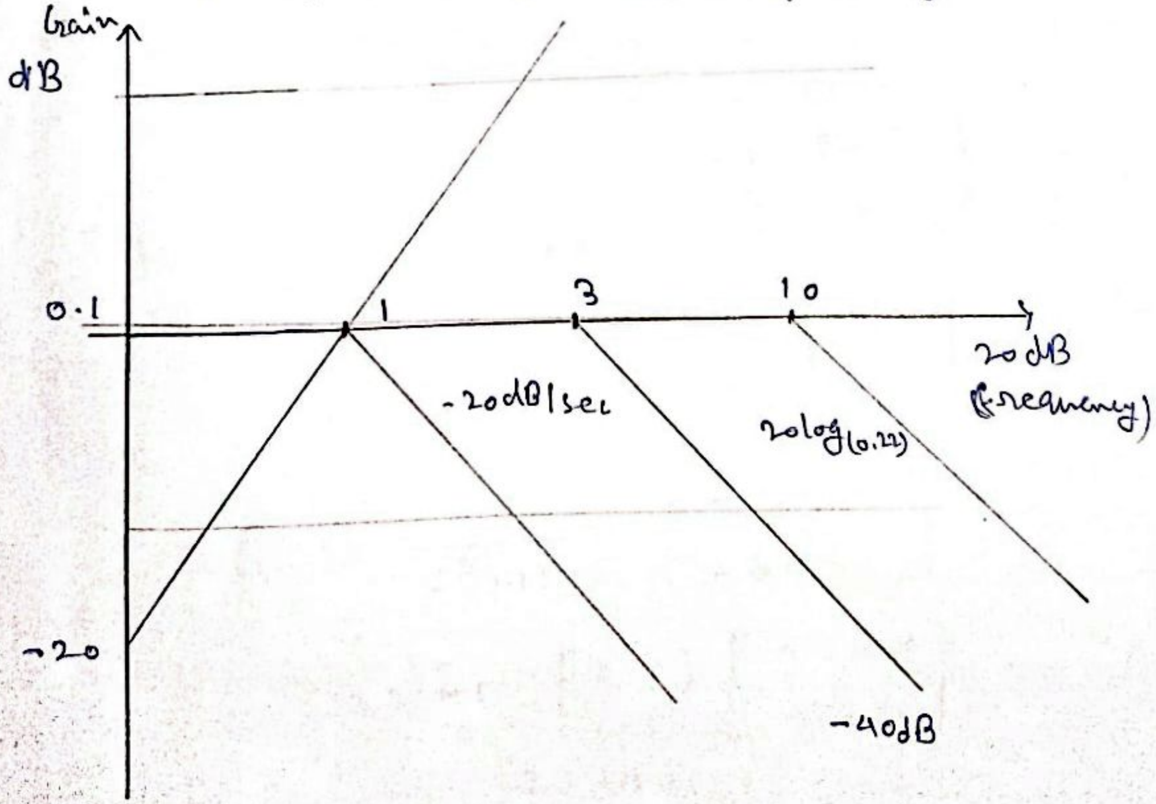
$$= \frac{0.22\,j\omega}{(j\omega+1)\left(\frac{j\omega}{3}+1\right)^2\left(\frac{j\omega}{10}+1\right)}$$

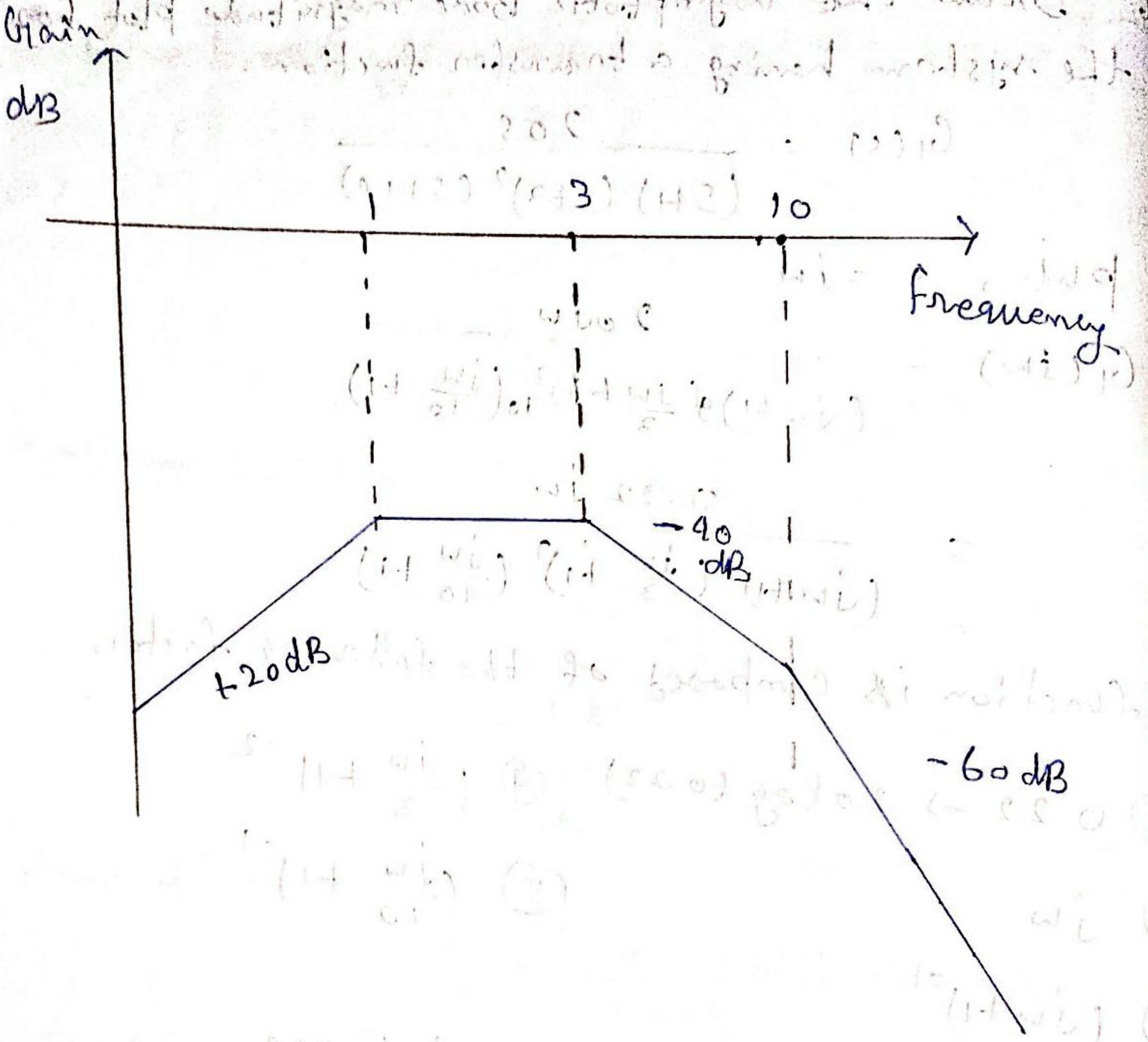
The function is composed of the following factors

1. $0.22 \rightarrow 20\log(0.22)$
2. $j\omega$
3. $(j\omega+1)^{-1}$
4. $\left(\frac{j\omega}{3}+1\right)^{-2}$
5. $\left(\frac{j\omega}{10}+1\right)^{-1}$

The corner frequency of 3, 4 & 5 are,

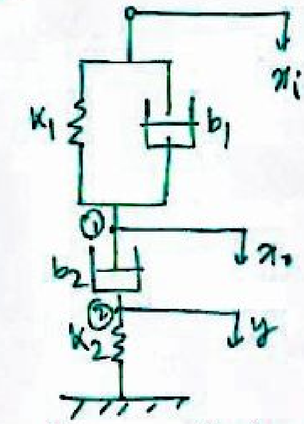
$\omega = 1$; $\omega = 3$ & $\omega = 10$ respectively.





Gain
dB
1
3
10
frequency
+20dB
-40 dB
-60 dB

Q-7



Displacement x_i, x_0 and y are measured from their respective steady state positions.

Equations of motion for this mechanical system will be –

$$b_1(\dot{x}_i - \dot{x}_0) + K_1(x_i - x_0) = b_2(\dot{x}_0 - \dot{y})$$

and, $b_2(\dot{x}_0 - \dot{y}) = K_2 y$

By taking Laplace transform –

$$b_1\left(sX_i(s) - sX_0(s)\right] + K_1\left(X_i(s) - X_0(s)\right) = b_2\left[sX_0(s) - sY(s)\right] \longrightarrow (i)$$

$$\text{and, } b_2\left(sX_0(s) - sY(s)\right) = K_2 Y(s) \longrightarrow (ii)$$

Taking all initial conditions to be zero.

Eliminate $Y(s)$ from (i) and (ii), we obtain –

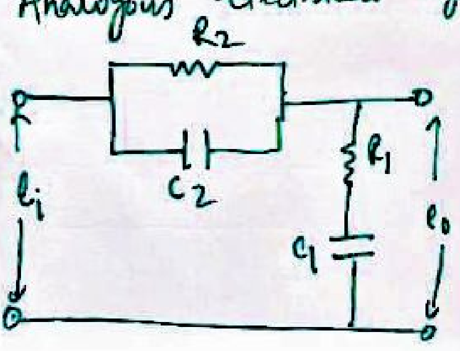
$$b_1\left(sX_i(s) - sX_0(s)\right) + K_1\left(X_i(s) - X_0(s)\right) = b_2 s X_0(s) - b_2 s \times \frac{b_2 s X_0(s)}{b_2 s + K_2}$$

$$\text{or, } (b_1 s + K_1)X_i(s) = \left(b_1 s + K_1 + b_2 s - b_2 s \times \frac{b_2 s}{b_2 s + K_2}\right)X_0(s)$$

Hence, the transfer function $X_0(s)/X_i(s)$ can be obtained as —

$$\frac{X_0(s)}{X_i(s)} = \frac{\left(\frac{b_1}{K_1}s + 1\right)\left(\frac{b_2}{S_2}s + 1\right)}{\left(\frac{b_1}{K_1}s + 1\right)\left(\frac{b_2}{K_2} + 1\right) + \frac{b_2}{K_1}s}$$

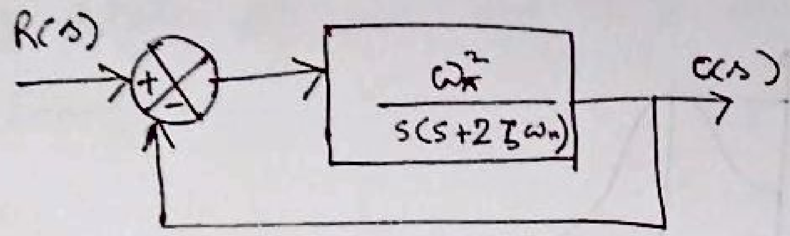
□ Analogous electrical system is –



$K \longrightarrow 1/C$

$b \longrightarrow R$

Mechanical Electrical



Raise Time —

$$t_r = \frac{\pi - \beta}{\omega_n}, \quad \beta = \tan^{-1}\frac{\omega_d}{\xi\omega_n}$$

Peak Time —

$$t_p = \frac{\pi}{\omega_d}$$

Maximum overshoot —

$$M_p = e^{-(\xi\omega_n/\omega_d)\pi}$$

Setting Time —

$$t_s = 4T = \frac{4}{\xi\omega_n} \text{ (For 2\% criteria)}$$

$$t_s = 3T = \frac{3}{\xi\omega_n} \text{ (For 3\% criteria)}$$

$\xi = 0.5$, $\omega_n = 8$ rad/sec, $\omega_d = \omega_n\sqrt{1-\xi^2}$

$= 8\sqrt{1-(0.5)^2} = 6.92$ rad/s

Rise time, $t_r = \frac{\pi - \beta}{\omega_d}$; $\beta = \tan^{-1}\left(\frac{6.92}{0.5\times 8}\right) = 1.04$ rad

$= \frac{3.14 - 1.04}{6.92}$

$= 0.30$ sec

Peak time, $t_p = \frac{\pi}{\omega_d} = \frac{3.14}{6.92} = 0.45$ sec

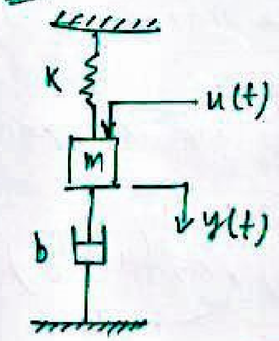
Maximum overshoot, $M_p = e^{-\left(\frac{0.5\times 8}{6.92}\right)\pi} = 0.16$

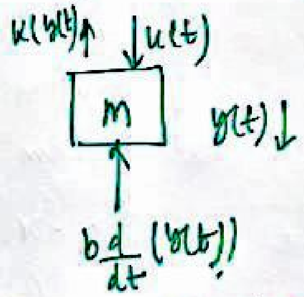
Setting time, $t_s = \frac{4}{0.5\times 8} = 1$ sec (For 2% criteria)

$t_s = 3/0.5\times 8 = 0.75$ sec (For 3% criteria)

Q-10

The free body diagram of mass m.





considering the system is in equilibrium,

$$u(t) - Ky(t) - b\frac{dy(t)}{dt} = m\frac{d^2y(t)}{dt^2}$$

or, $$m\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + Ky(t) = u(t)$$

Let us define, state variables as –

$x_1(t)$, $x_2(t)$ such that

$$x_1(t) = y(t) \Rightarrow \dot{x}_1(t) = x_2(t)$$

$$x_2(t) = \frac{d}{dt}y(t) \Rightarrow \dot{x}_2(t) = \frac{u(t)}{m} - \frac{K}{m}x_1(t) - \frac{b}{m}x_2(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} [u(t)]$$

As, $y = x_1(t)$

So, $$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

The state equations are –

$$\dot{X} = AX + BU \quad \text{and} \quad Y = CX$$

Where, $A = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -b/m \end{bmatrix}$, $B = \begin{bmatrix} b \\ 1/m \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

Q.11. Consider the system.

$$\dot{x} = Ax + Bu \; ; \; y = cx.$$

where, $A = \begin{bmatrix} 2 & 1 \\ -3 & -4 \end{bmatrix}$; $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$; $c = [1 \;\; 1]$.

(1) obtain Transfer function.

(2) Transform the system equations into the controllable cannonical form.

=> (1) Transfer function :-

$$\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$$

$$(sI-A) = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} s-2 & -1 \\ 3 & s+4 \end{bmatrix}$$

$$(sI-A)^{-1} = \begin{bmatrix} s-2 & -1 \\ 3 & s+4 \end{bmatrix}^{-1}$$

$$= \frac{1}{(s-2)(s+4)+3}\begin{bmatrix} s+4 & 1 \\ -3 & s-2 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = [1 \;\; 1]\frac{1}{s^2+2s-5}\begin{bmatrix} s+4 & 1 \\ -3 & s-2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= [1 \;\; 1]\frac{1}{s^2+2s-5}\begin{bmatrix} 2s+8+1 \\ -6+s-2 \end{bmatrix}.$$

$$= \frac{1}{s^2+2s+5}[1 \;\; 1]\begin{bmatrix} 2s+9 \\ s-8 \end{bmatrix}$$

$$= \frac{1}{s^2+2s-5}[2s+9+s-8]$$

$$= \frac{3s+1}{s^2+2s-5}$$

Now for CCF,

$$\frac{Y(s)}{U(s)} = \frac{(3s+1)\,X(s)}{(s^2+2s-5)\,X(s)}$$

$$Y(s) = 3sX(s) + X(s)$$

$$= 3x_2 + x_1$$

$$U(s) = s^2X(s) + 2sX(s) - 5X(s)$$

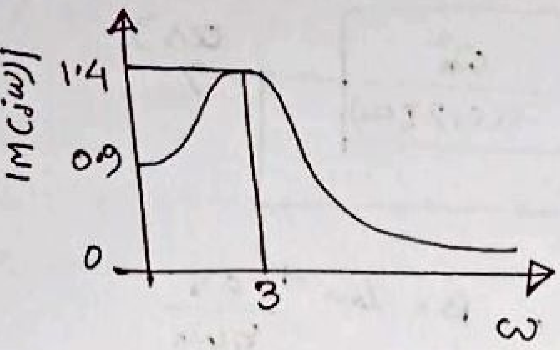
$$= \dot{x}_2 + 2x_2 - 5x_1$$

$$\dot{x}_1 = x_2 \quad [\,sx_1 = \dot{x}_1\,]$$

so,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -5 & 6\end{bmatrix}\begin{bmatrix}x_1\\ \dot{x}_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$Y(s) = \begin{bmatrix}1 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$



$$M_p = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.4$$

$$\Rightarrow \frac{1}{\xi^2(1-\xi^2)} = 2.8^2$$

$$\Rightarrow \xi^2 - \xi^4 = 0.12$$

$$\Rightarrow \xi^4 - \xi^2 + 0.12 = 0$$

let $t = \xi^2$

$$\Rightarrow t^2 - t + 0.12 = 0$$

$$\Rightarrow t_1 = 0.8605, \; t_2 = 0.14$$

$$\xi_1 = 0.927 \;\Big|\; \xi_2 = 0.374$$

% maximum peak overshoot —

$$\%M_p = e^{-\frac{\pi\xi_2}{\sqrt{1-\xi_2^2}}} \times 100$$

$$\boxed{\%M_{p_2} = 28.17\,\%}$$

Peak Time —

$$\omega_r = \omega_n\sqrt{1-2\xi_2^2}$$

$$3 = \omega_n\sqrt{1-2\times 0.374^2}$$

$$\Rightarrow \omega_n = 3.53 \text{ rad/sec}$$

$$\omega_d = \omega_n\sqrt{1-\xi_2^2}$$

$$= 3.27 \text{ rad/s}$$

∴ Peak Time,

$$t_p = \frac{\pi}{\omega_d}$$

$$= \frac{\pi}{3.27}$$

$$\Rightarrow t_p = 0.9607 \text{ sec}$$

Steady state error due to a unit step input —

$$e_{ss} = \frac{1}{1+K_P}$$

$$K_P = \lim_{s \to 0} G(s)$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s}$$

$$K_P = \lim_{s \to 0} \frac{\omega_n^2}{s^2 + 2\xi\omega_n s} = \infty$$

$$\boxed{e_{ss} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0}$$

31) Consider a PID controller used to control the plant,

$$G(s) = \frac{1}{s(s+2)(s+4)}$$

Determine parameters, Ziegler Nichols.

Ans: Given, $G(s) = \frac{1}{s(s+2)(s+4)}$.

Type of controller	K_p	T_I	T_D
P	$0.5K_{cr}$	α	0
PI	$0.45\,K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6\,K_{cr}$	$0.5\,P_{cr}$	$0.125\,P_{cr}$

∴ By setting $T_i = \alpha$ and $T_d = 0$

$$\frac{G(s)}{R(s)} = \frac{K_p}{s(s+2)(s+4)}$$

The value of the K_p that makes the system marginally stable so that sustained oscillation occurs, can be obtained by use of Routh's stability criteria.

Characteristic eq. of closed loop system,

$$s(s+2)(s+4) + K_p = 0$$

$$\text{or, } s(s^2+6s+8) + K_p = 0$$

$$\text{or, } s^3 + 6s^2 + 8s + K_p = 0 \text{ —— } i)$$

s^3	1	8
s^2	6	K_p
s^1	$\frac{48-K_p}{6}$	0
s^0	K_p	

$\therefore \frac{48-K_p}{6} = 0$

or, $K_p = 48$

Now, Critical Gain, $K_{cr} = K_p = 48$

Put, $s = j\omega$,

$\therefore (j\omega)^3 + 6(j\omega)^2 + 8j\omega + 48 = 0$

or, $-j\omega^3 - 6\omega^2 + 8j\omega + 48 = 0$

or, $j(8\omega - \omega^3) + (48 - 6\omega^2) = 0$

Frequency of sustained oscillation,

$8\omega - \omega^3 = 0$

or, $\omega^2 = 8$ | or, $\omega = 2.828$ rad/s

Hence, Period of sustained oscillation,

$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{2.828} = 2.221$$

From the above table,

$K_p = 0.6\ K_{cr} = 0.6 \times 48 = 28.8$

$T_i = 0.5\ P_{cr} = 0.5 \times 2.221 = 1.11$

$T_d = 0.125\ P_{cr} = 0.125 \times 2.221 = 0.277$