Filtering in the spatial domain (Spatial Filtering)

refers to image operators that change the gray value at any pixel (x,y) depending on the pixel values in a square neighborhood centered at (x,y) using a fixed integer matrix of the same size. The integer matrix is called a *filter*, *mask*, *kernel* or a *window*.

The mechanism of spatial filtering, shown below, consists simply of moving the filter mask from pixel to pixel in an image. At each pixel (x,y), the response of the filter at that pixel is calculated using a predefined relationship (linear or nonlinear).
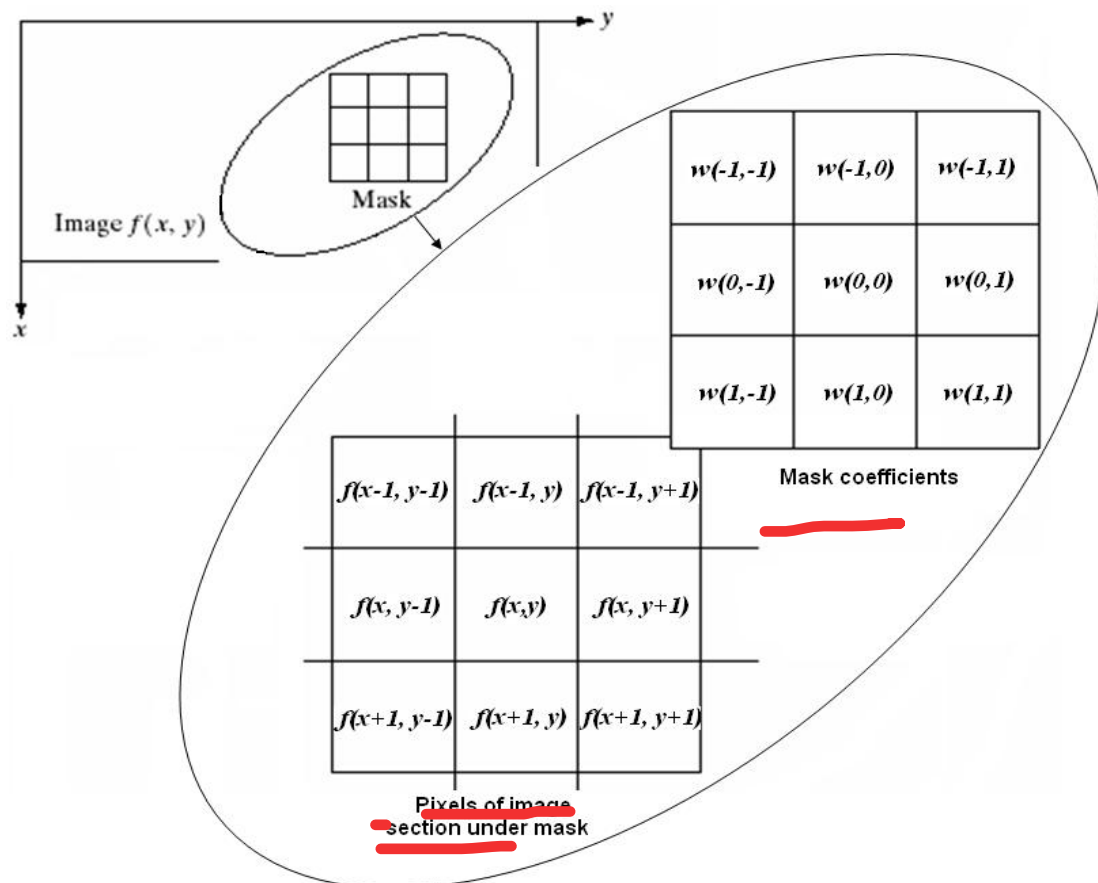
Figure 6.1 Spatial filtering

Note:

The size of mask must be odd (i.e. 3×3, 5×5, etc.) to ensure it has a center. The smallest meaningful size is 3×3.

Linear Spatial Filtering (Convolution)

The process consists of moving the filter mask from pixel to pixel in an image. At each pixel (x,y), the response is given by a sum of products of the filter coefficients and the corresponding image pixels in the area spanned by the filter mask.

For the 3×3 mask shown in the previous figure, the result (or response), R, of linear filtering is:

$$R = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \cdots$$
$$+ w(0,0)f(x,y) + \cdots + w(1,0)f(x+1,y) + w(1,1)f(x+1,y+1)$$

In general, linear filtering of an image f of size $M\times N$ with a filter mask of size $m\times n$ is given by the expression:

$$g(x,y) = \sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)f(x+s,y+t)$$

where a = (m - 1)/2 and b = (n - 1)/2. To generate a complete filtered image this equation must be applied for x = 0,1, 2,..., M-1 and y = 0,1, 2,..., N-1.

Nonlinear Spatial Filtering

The operation also consists of moving the filter mask from pixel to pixel in an image. The filtering operation is based conditionally on the values of the pixels in the neighborhood, and they do not explicitly use coefficients in the sum-of-products manner.

For example, noise reduction can be achieved effectively with a nonlinear filter whose basic function is to compute the median gray-level value in the neighborhood in which the filter is located. Computation of the median is a nonlinear operation.

Example:

Use the following 3×3 mask to perform the convolution process on the shaded pixels in the 5×5 image below. Write the filtered image.

0	1/6	0
1/6	1/3	1/6
0	1/6	0

3×3 mask

30	40	50	70	90
40	50	80	60	100
35	255	70	0	120
30	45	80	100	130
40	50	90	125	140

5×5 image

Solution:

$$0 \times 30 + \frac{1}{6} \times 40 + 0 \times 50 + \frac{1}{6} \times 40 + \frac{1}{3} \times 50 + \frac{1}{6} \times 80 + 0 \times 35 + \frac{1}{6} \times 255$$
$$+ 0 \times 70 = 85$$

$$0 \times 40 + \frac{1}{6} \times 50 + 0 \times 70 + \frac{1}{6} \times 50 + \frac{1}{3} \times 80 + \frac{1}{6} \times 60 + 0 \times 255 + \frac{1}{6} \times 70$$
$$+ 0 \times 0 = 65$$

$$0 \times 50 + \frac{1}{6} \times 70 + 0 \times 90 + \frac{1}{6} \times 80 + \frac{1}{3} \times 60 + \frac{1}{6} \times 100 + 0 \times 70 + \frac{1}{6} \times 0$$
$$+ 0 \times 120 =$$

$$0 \times 40 + \frac{1}{6} \times 50 + 0 \times 80 + \frac{1}{6} \times 35 + \frac{1}{3} \times 255 + \frac{1}{6} \times 70 + 0 \times 30 + \frac{1}{6} \times 45$$
$$+ 0 \times 80 = 118$$

and so on …

Filtered image =

30	40	50	70	90
40	**85**	**65**	**61**	100
35	**118**	**92**	**58**	120
30	**84**	**77**	**89**	130
40	50	90	125	140

Spatial Filters

Spatial filters can be classified by effect into:

1. **Smoothing Spatial Filters:** also called lowpass filters. They include:

    1.1   Averaging linear filters

    1.2   Order-statistics nonlinear filters.

2. **Sharpening Spatial Filters:** also called highpass filters. For example, the Laplacian linear filter.

Smoothing Spatial Filters

are used for blurring and for noise reduction. Blurring is used in preprocessing steps to:

- § remove small details from an image prior to (large) object extraction

- § bridge small gaps in lines or curves.

Noise reduction can be accomplished by blurring with a linear filter and also by nonlinear filtering.

Averaging linear filters

The response of averaging filter is simply the average of the pixels contained in the neighborhood of the filter mask.

The output of averaging filters is a smoothed image with reduced "sharp" transitions in gray levels.

Noise and edges consist of sharp transitions in gray levels. Thus smoothing filters are used for noise reduction; however, they have the undesirable side effect that they blur edges.

The figure below shows two 3×3 averaging filters.

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

   Standard average filter   Weighted average filter

Note:

Weighted average filter has different coefficients to give more importance (weight) to some pixels at the expense of others. The idea behind that is to reduce blurring in the smoothing process.

Averaging linear filtering of an image *f* of size *M×N* with a filter mask of size *m×n* is given by the expression:

$$g(x, y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

To generate a complete filtered image this equation must be applied for x = 0,1, 2,..., *M*-1 and y = 0,1, 2,..., *N*-1.

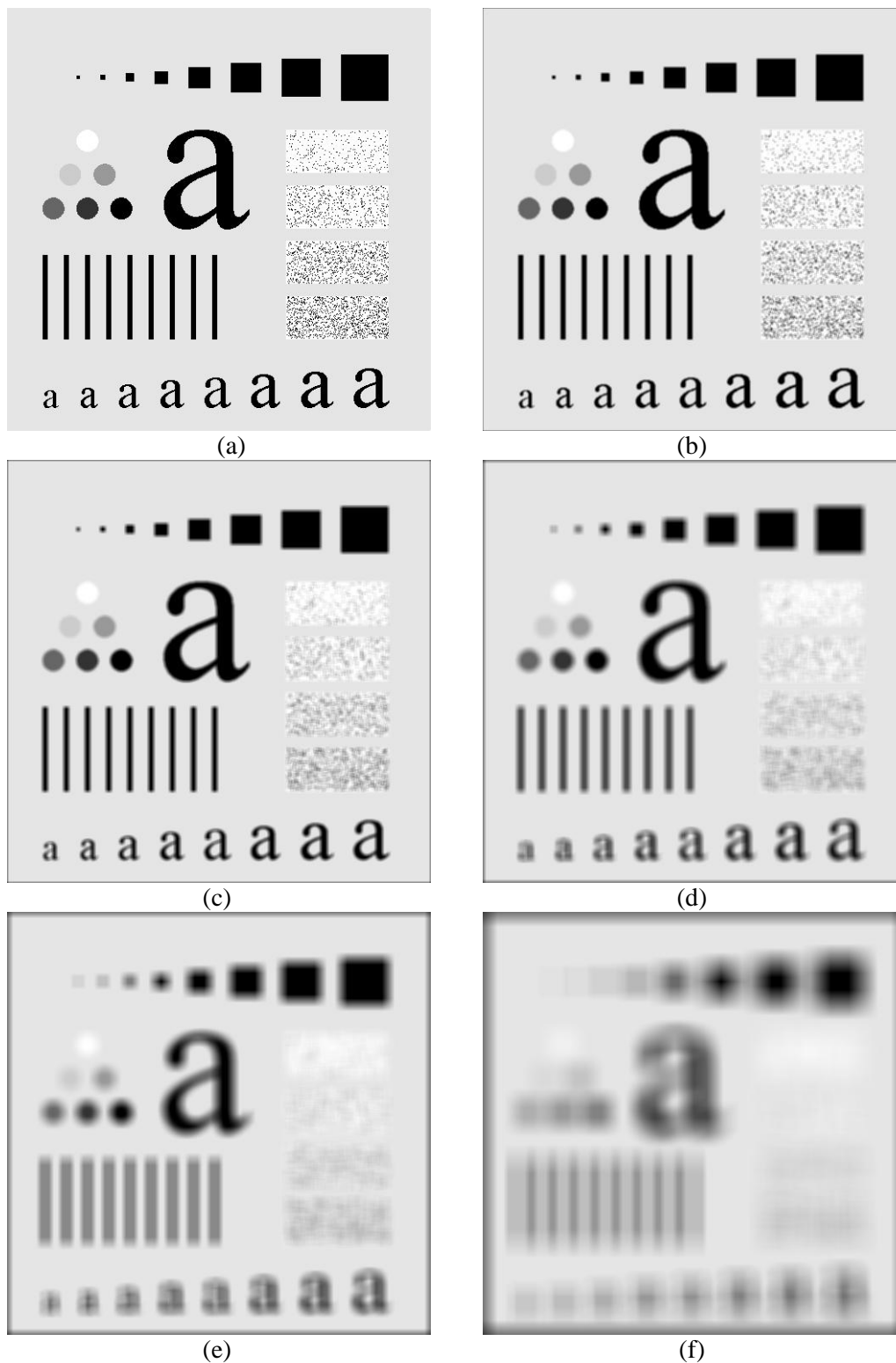Figure below shows an example of applying the standard averaging filter.

Figure 6.2 Effect of averaging filter. (a) Original image. (b)-(f) Results of smoothing with square averaging filter masks of sizes n = 3,5,9,15, and 35, respectively.

As shown in the figure, the effects of averaging linear filter are:

1. Blurring which is increased whenever the mask size increases.
2. Blending (removing) small objects with the background. The size of the mask establishes the relative size of the blended objects.
3. Black border because of padding the borders of the original image.
4. Reduced image quality.

Order-statistics filters

are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the neighborhood, and then replacing the value of the center pixel with the value determined by the ranking result. Examples include Max, Min, and Median filters.

Median filter

It replaces the value at the center by the median pixel value in the neighborhood, (i.e. the middle element after they are sorted). Median filters are particularly useful in removing impulse noise (also known as salt-and-pepper noise). Salt = 255, pepper = 0 gray levels.

In a 3×3 neighborhood the median is the 5th largest value, in a 5×5 neighborhood the 13th largest value, and so on.

For example, suppose that a 3×3 neighborhood has gray levels (10, 20, 0, 20, 255, 20, 20, 25, 15). These values are sorted as (0,10,15,20,20,20,20,25,255), which results in a median of 20 that replaces the original pixel value 255 (salt noise).

Example:

Consider the following 5×5 image:

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Apply a 3×3 median filter on the shaded pixels, and write the filtered image.

Solution

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Sort:
20, 25, 30, 30, **30**, 70, 80, 80, 255

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Sort
0, 20, 30, 70, **80**, 80, 100, 100, 255

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Sort
0, 70, 80, 80, **100**, 100, 110, 120, 130

Filtered Image =

20	30	50	80	100
30	20	80	100	110
25	**30**	**80**	**100**	120
30	30	80	100	130
40	50	90	125	140

Figure below shows an example of applying the median filter on an image corrupted with salt-and-pepper noise.

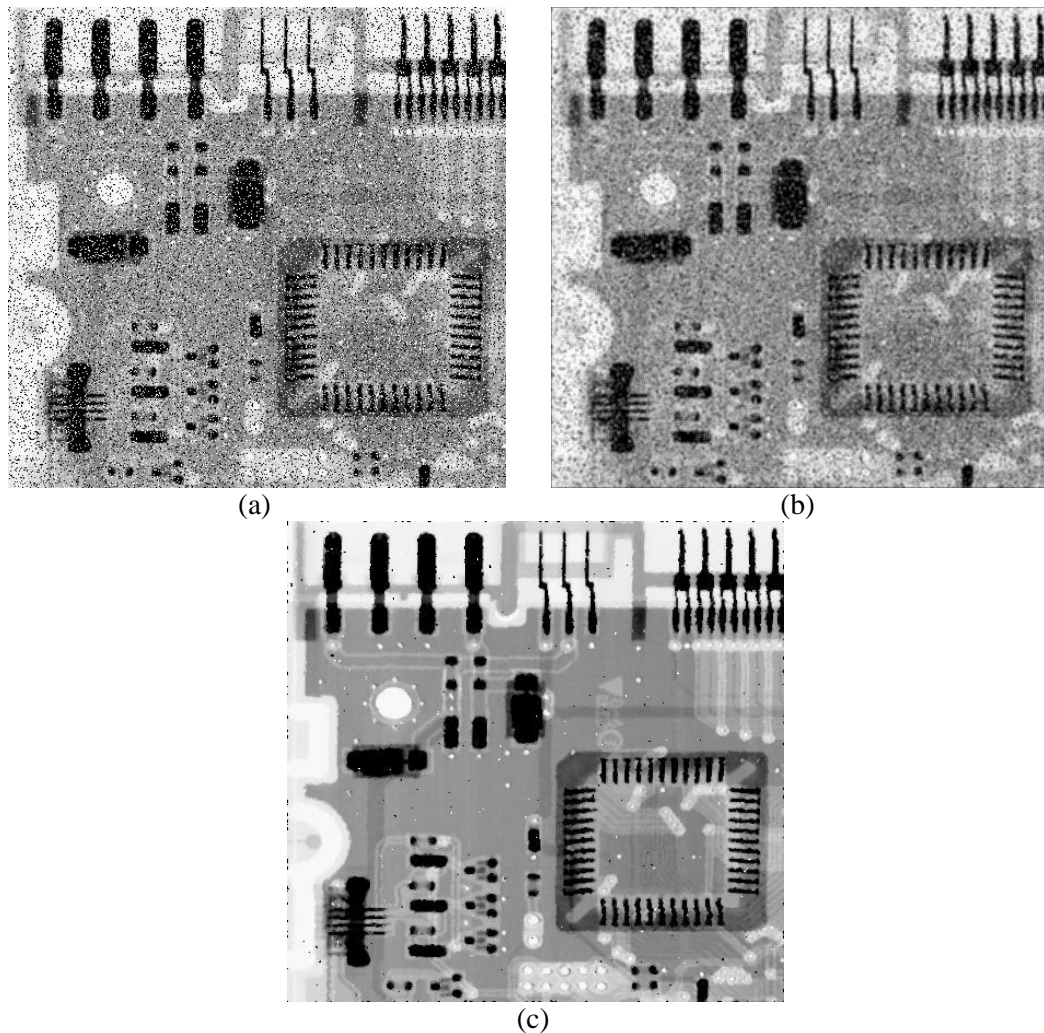(a)   (b)

(c)

Figure 6.3 Effect of median filter. (a) Image corrupted by salt & pepper noise. (b) Result of applying 3×3 standard averaging filter on (a). (c) Result of applying 3×3 median filter on (a).

As shown in the figure, the effects of median filter are:

1. Noise reduction
2. Less blurring than averaging linear filter

Sharpening Spatial Filters

Sharpening aims to highlight fine details (e.g. edges) in an image, or enhance detail that has been blurred through errors or imperfect capturing devices.

Image blurring can be achieved using averaging filters, and hence sharpening can be achieved by operators that invert averaging operators. In mathematics, averaging is equivalent to the concept of integration, and differentiation inverts integration. Thus, sharpening spatial filters can be represented by partial derivatives.

Partial derivatives of digital functions

The first order partial derivatives of the digital image $f(x,y)$ are:

$$\frac{\partial f}{\partial x} = f(x + 1, y) - f(x, y) \quad \text{and} \quad \frac{\partial f}{\partial y} = f(x, y + 1) - f(x, y)$$

The first derivative must be:

1) zero along flat segments (i.e. constant gray values).
2) non-zero at the outset of gray level step or ramp (edges or noise)
3) non-zero along segments of continuing changes (i.e. ramps).

The second order partial derivatives of the digital image $f(x,y)$ are:

$$\frac{\partial^2 f}{\partial x^2} = f(x + 1, y) + f(x - 1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y + 1) + f(x, y - 1) - 2f(x, y)$$

The second derivative must be:

1) zero along flat segments.
2) nonzero at the outset and end of a gray-level step or ramp;
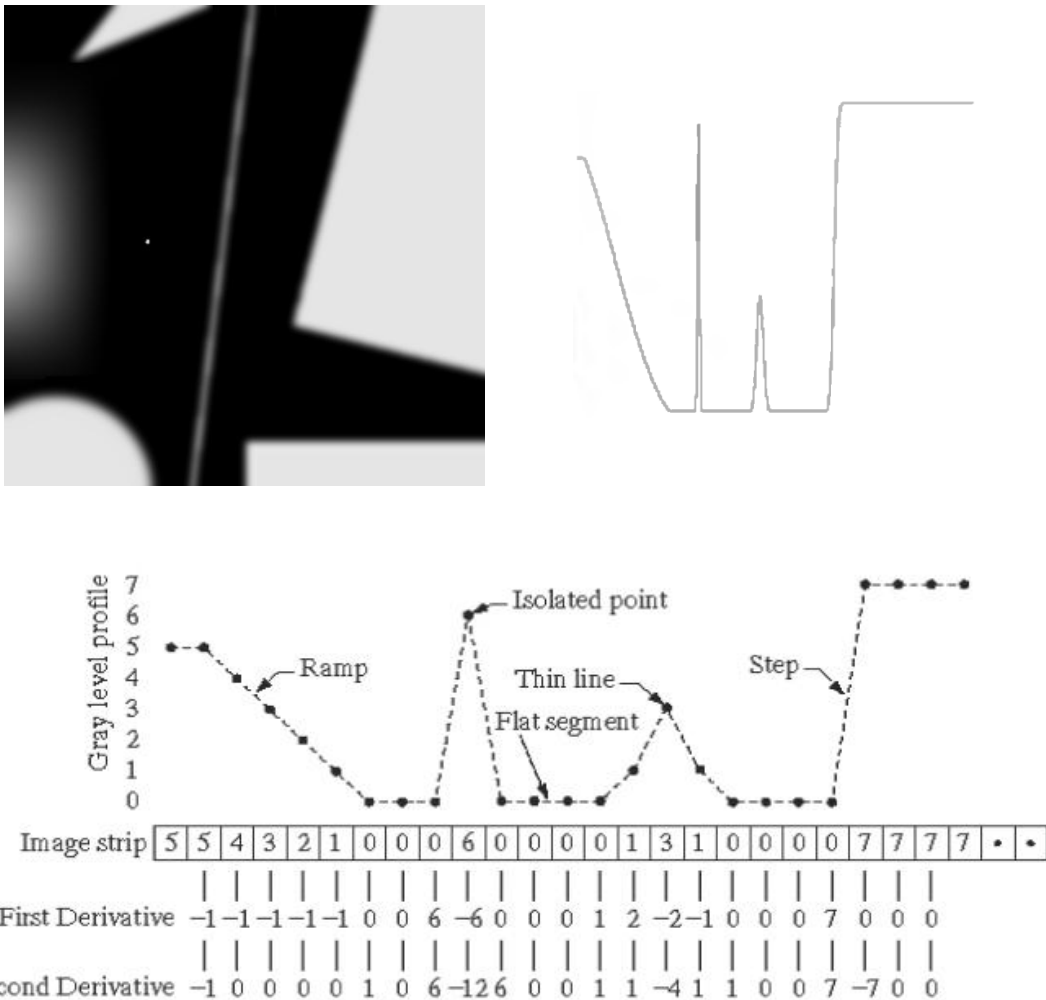
3) zero along ramps

Consider the example below:

Figure 6.4 Example of partial derivatives

We conclude that:

- 1st derivative detects thick edges while 2nd derivative detects thin edges.
- 2nd derivative has much stronger response at gray-level step than 1st derivative.

Thus, we can expect a second-order derivative to enhance fine detail (thin lines, edges, including noise) much more than a first-order derivative.

The Laplacian Filter

The Laplacian operator of an image *f(x,y)* is:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

This equation can be implemented using the 3×3 mask:

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Since the Laplacian filter is a linear spatial filter, we can apply it using the same mechanism of the convolution process. This will produce a laplacian image that has grayish edge lines and other discontinuities, all superimposed on a dark, featureless background.

Background features can be "recovered" while still preserving the sharpening effect of the Laplacian operation simply by adding the original and Laplacian images.

$$g(x, y) = f(x, y) + \nabla^2 f(x, y)$$

The figure below shows an example of using Laplacian filter to sharpen an image.

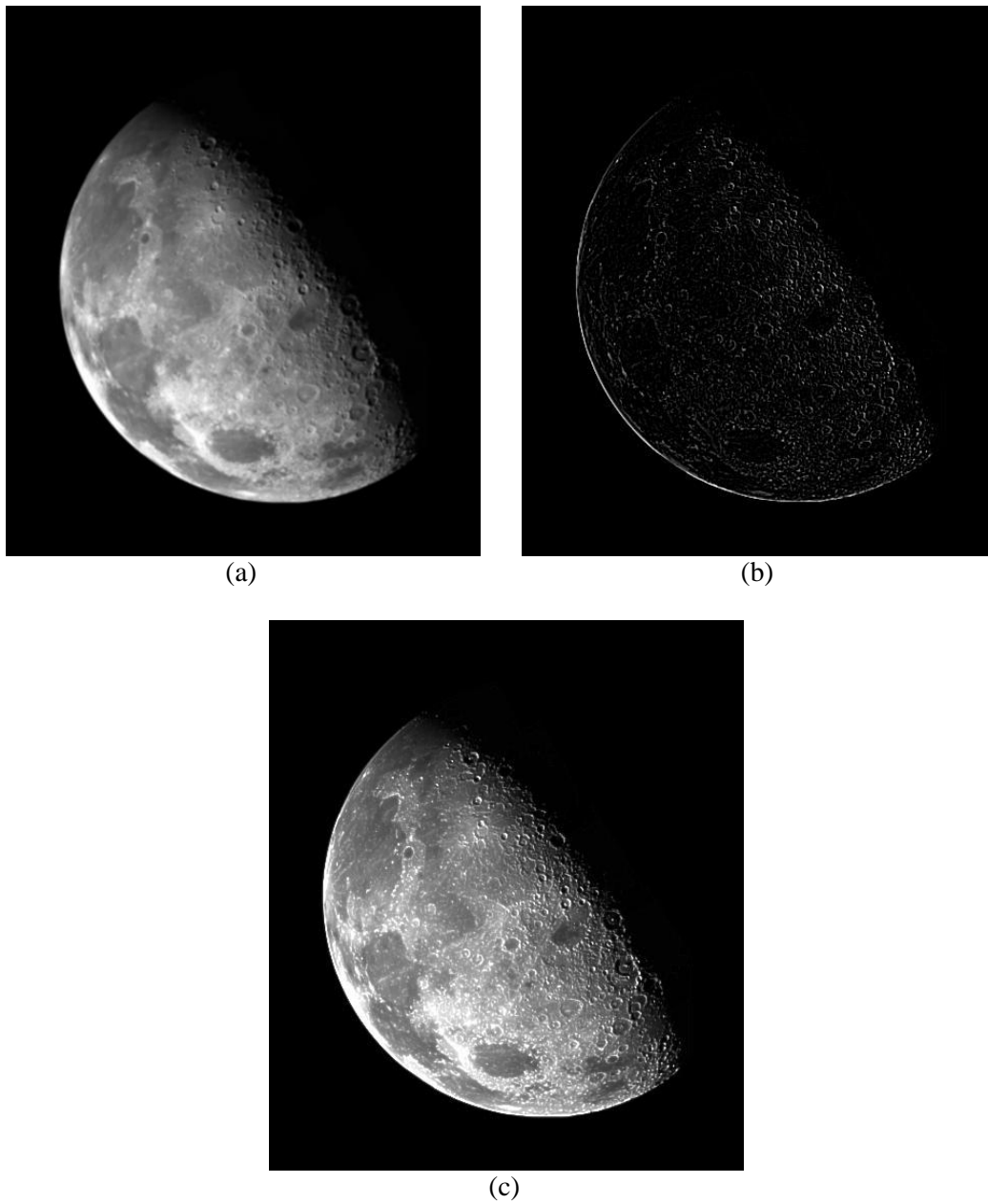(a)   (b)

(c)

Figure 6.5 Example of applying Laplacian filter. (a) Original image. (b) Laplacian image.

(c) Sharpened image.