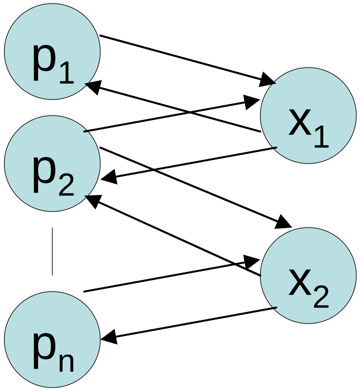
Asynchronous Shared-Memory Systems

Asynchronous Shared-Memory systems

- We've covered basics of non-fault-tolerant asynchronous network algorithms:
  - How to model them.
  - Basic asynchronous network protocols---broadcast, spanning trees, leader election,…
  - Synchronizers (running synchronous algorithms in asynch networks)
  - Logical time
  - Global snapshots
- Now consider asynchronous shared-memory systems:

- Processes, interacting via shared objects, possibly subject to some access constraints.
- Shared objects are typed, e.g.:
  - Read/write (weak)
  - Read-modify-write, compare-and-swap (strong)
  - Queues, stacks, others (in between)



Asynch Shared-Memory systems

- Theory of ASM systems has much in common with theory of asynchronous networks:
  - Similar algorithms and impossibility results.
  - Even with failures.
  - Transformations from ASM model to asynch network model allow ASM algorithms to run in asynchronous networks. • "Distributed Shared Memory".
- Historically, theory for ASM started first.

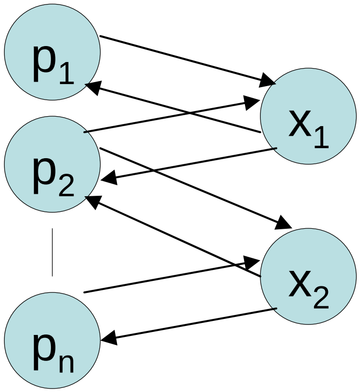
- Arose in study of early operating systems, in which several processes can run on a single processor, sharing memory, with possibly-arbitrary interleavings of steps.
- Currently, ASM models apply to multiprocessor shared-memory systems, in which processes can run on separate processors and share memory.

Topics

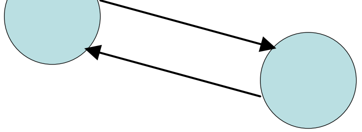
- Define the basic system model, without failures.
- Use it to study basic problems:
  - Mutual exclusion.
  - Other resource-allocation problems.
- Introduce process failures into the model.
- Use model with failures to study basic problems:
  - Distributed consensus
  - Implementing atomic objects:

- Atomic snapshot objects
- Atomic read/write registers
- Wait-free and fault-tolerant computability theory• Modern shared-memory multiprocessors:
  - Practical issues
  - Algorithms
  - Transactional memory

Basic ASM Model, Version 1

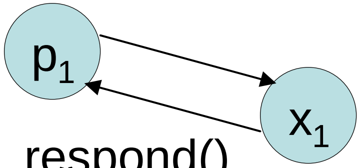


- Processes + objects, modeled as automata.
- Arrows:
  - Represent invocations and responses for operations on the objects.
  - Modeled as input and output actions.
- Fine-granularity model, can describe:
  - Delay between invocation and response. invoke(read)
  - Concurrent (overlapping) operations: p_1



- Object could reorder. x_1
- Could allow them to run concurrently, interfering with respond(v) each other.
- We’ll begin with a simpler, coarser model:

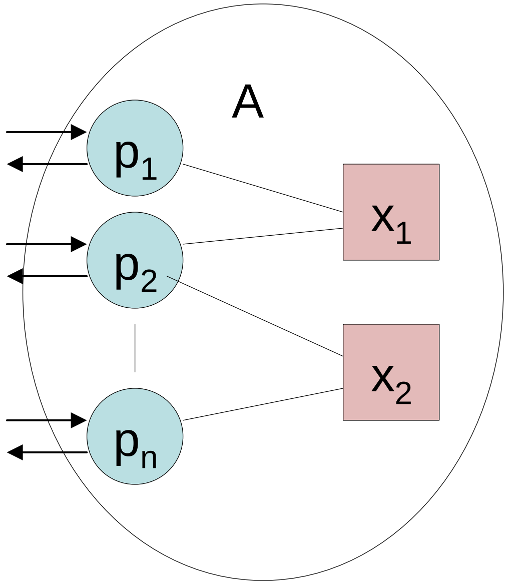
invoke(write,v)



respond()

  - Object runs ops in invocation order, one at a time.– In fact, collapse each operation into a single step.
- Return to the finer model later.

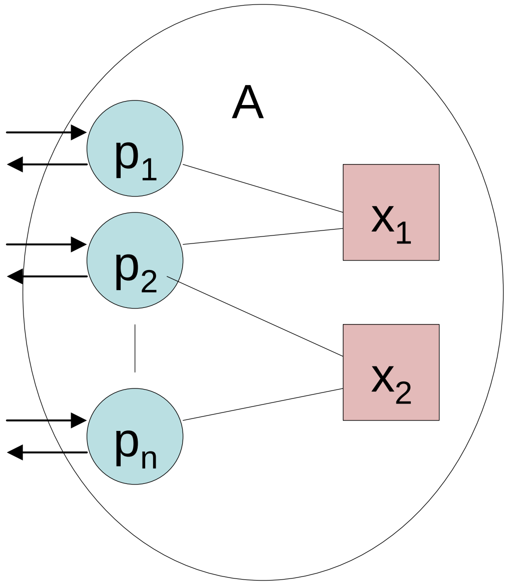
Basic ASM Model, Version 2



- One big shared memory system automaton A.
- External actions at process "ports".
- Each process i has:
  - A set $states_i$ of states.
  - A subset $start_i$, of start states.
- Each variable x has:
  - A set $values_x$ of values it can take on.
  - A subset $initial_x$ of initial values.
- Automaton A:
  - States: State for each process, a value for each variable.
  - Start: Start states, initial values.
  - Actions: Each action associated with one process, and some also with a single shared variable.
  - Input/output actions: At the external boundary.
  - Transitions: Correspond to local process steps and variable accesses.
- Action enabling, which variable is accessed, depend only on process state.
- Changes to variable and process state depend also on variable value.
- Must respect the type of the variable.

– Tasks: One or more per process (threads).

Basic ASM Model



- Execution of A:
  - By IOA fairness definition, each task gets infinitely many chances to take steps.
  - Model environment as a separate automaton, to express restrictions on environment behavior.

- Commonly-used variable types:
  - Read/write registers: Most basic primitive.
- Allows access using separate read and write operations.
  - Read-modify-write: More powerful primitive:
- Atomically, read variable, do local computation, write to variable.– Compare-and-swap, fetch-and-add, queues, stacks, etc.

- Different computability and complexity results hold for different variable types.

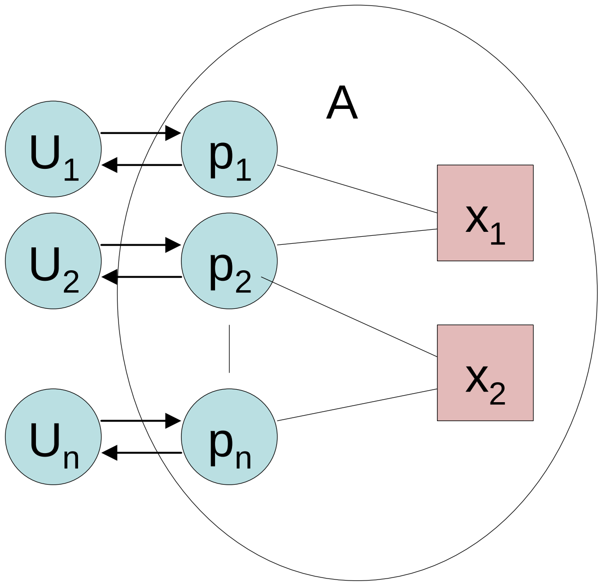
The Mutual Exclusion Problem

- Share one resource among n user processes, U_1, U_2,…,U_n.
  - E.g., printer, portion of a database.
- U_i has four “regions”.
  - Subsets of its states, described by portions of its code.
  - C critical; R remainder; T trying; E exit

Protocols for obtaining and relinquishing the resource

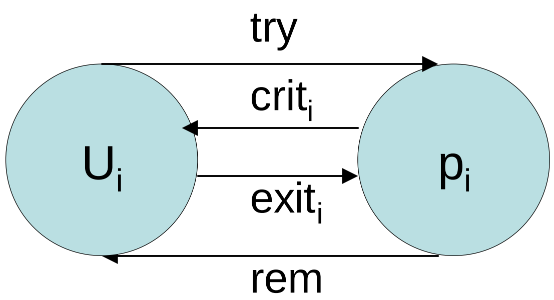
$$\begin{tikzcd} R \arrow[r] & T \arrow[r] & C \arrow[r] & E \arrow[lll, bend left] \end{tikzcd}$$

- Cycle:
- Architecture:
  - U_is and A are IOAs, compose.



The Mutual Exclusion Problem

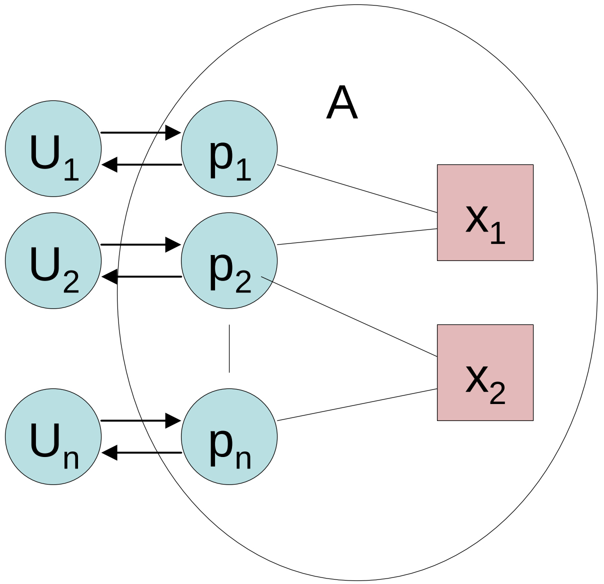
- Actions at user interface:
  - try_i, $crit_i$, $exit_i$, rem_i i
  - U_i interacts with p_i



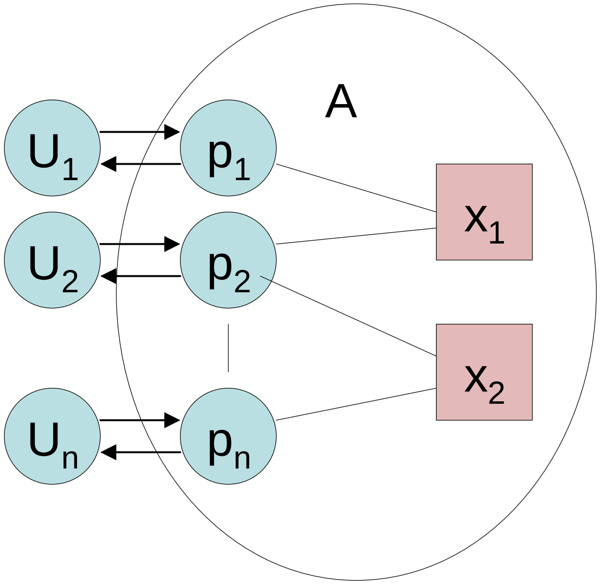
- Correctness conditions:
  - Well-formedness (Safety property):
- System obeys cyclic discipline. i
- E.g., doesn’t grant resource when it wasn’t requested.
  - Mutual exclusion (Safety):

The Mutual Exclusion Problem

- System never grants to > 1 user simultaneously.
- Trace safety property.
- Or, there's no reachable system state in which >1 user is in C at once. – Progress (Liveness):
- From any point in a fair execution:
  – If some user is in T and no user is in C then at some later point, some user enters C.
  – If some user is in E then at some later point, some user enters R.



The Mutual Exclusion Problem

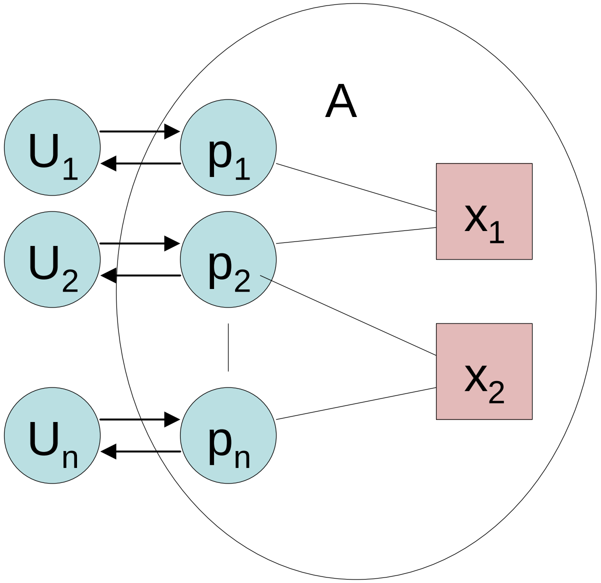


- Well-formedness (Safety):
  - System obeys cyclic discipline.
- Mutual exclusion (Safety):– System never grants to > 1 user.
- Progress (Liveness):
  - From any point in a fair execution:
- If some user is in T and no user is in C then at some later point, some user enters C.
- If some user is in E then at some later point, some user enters R.
- Conditions all constrain the system automaton A, not users.
  - System determines if/when users enter C and R.
  - Users determine if/when users enter T and E.

The Mutual Exclusion Problem

- We don't state any requirements on the users, except that they preserve well-formedness.

- Well-formedness (Safety):
- Mutual exclusion (Safety):
- Progress (Liveness):
  - From any point in a fair execution:
- If some user is in T and no user is in C then at some later point, some user enters C.
- If some user is in E then at some later point, some user enters R.



- Fairness assumption:
  - Progress condition requires fairness assumption (all process tasks continue to get turns to take steps).
  - Needed to guarantee that some process enters C or R.

The Mutual Exclusion Problem

- In general, in the asynchronous model, liveness properties require fairness assumptions.
- Contrast: Well-formedness and mutual exclusion are safety properties, don’t depend on fairness.

One more assumption…

- No permanently active processes.
  - Locally-controlled actions can be enabled only when user is in T or E.
  - No always-awake, dedicated processes.
  - Motivation:
- Multiprocessor settings, where users can run processes at any time, but are otherwise not involved in the protocol.
- Avoid “wasting processors”.

[Dijkstra 65]

- Based on Dekker's 2-process solution.
- Pseudocode, p. 265-266
  – Written in traditional sequential style, must somehow translate into more detailed state/transition description.
- Shared variables: Read/write registers.
  – turn, in {1,2,…,n}, multi-writer multi-reader (mWmR), init anything.
  – for each process i:
    - flag(i), in {0,1,2}, single-writer multi-reader (1WmR), init 0
    - Written by i, read by everyone.
- Process i's Stage 1:
  – Set flag := 1, test to see if turn = i.
  – If not, and turn's current owner is seen to be inactive, then set turn := i.

- Otherwise go back to to testing…
- When you see turn = i, move to Stage 2.

Dijkstra's algorithm

- Stage 2:
  - Set flag(i) := 2.
  - Check (one at a time, any order) that no other process has flag = 2.
  - If check completes successfully, go to C.– If not, go back to beginning of Stage 1.
- Exit protocol:– Set flag(i) := 0.
- Problem with the sequential code style:
  - Unclear what constitutes an atomic step.
- E.g., need three separate steps to test turn, test flag(turn), and set turn.
  - Must rewrite to make this clear:
- E.g., precondition/effect code (p. 268-269)

- E.g., sequential-style code with explicit reads and writes, one per line.

Dijkstra's algorithm, pre/eff code

- One transition definition for each kind of atomic step.
- Explicit program counter, pc.
- E.g.: When pc is:
  - set-flag-1_i: Sets flag to 1 and prepares to test turn.
  - test-$turn_i$: Tests turn, and either moves to Stage 2 or prepares to test the current owner's flag.
  - test-flag$(j)_i$: Tests j's flag, and either goes on to set turn or goes back to test turn again.
  - …
  - set-flag-2_i: Sets flag to 2 and initializes set S, preparing to check all other processes' flags.
  - check$(j)_i$: If flag(j) = 2, go back to beginning.
  - …

- S keeps track of which processes have been successfully checked in Stage 2.

Precondition/effect code

Shared variables:

turn $\in$ {1,…,n}, initially arbitrary

for every i: flag(i) $\in$ {0,1,2}, initially 0

Actions of process i:

Input: try_i, $exit_i$

Output: $crit_i$, rem_i

Internal: set-flag-1_i, test-$turn_i$, test-flag$(j)_i$, set-$turn_i$, set-flag-2_i, check$(j)_i$, $reset_i$

Precondition/effect code, Dijkstra process i

try_i: Eff: pc := set-flag-1

set-flag-1_i :

Pre: pc = set-flag-1

Eff: flag(i) := 1

pc := test-turn

test-$turn_i$:

Pre: pc = test-turn

Eff: if turn = i then pc := set-flag-2

else pc := test-flag(turn)

test-flag$(j)_i$

Pre: pc = test-flag(j)

Eff: if flag(j) = 0 then pc := set-turn else pc := test-turn

set-$turn_i$:

Pre: pc = set-turn

Eff: turn := i pc := set-flag-2

Pre: pc = set-flag-2

Eff: flag(i) := 2 S := {i} pc := check

set-flag-2$_i$:

More precondition/effect code, Dijkstra process i

check(j)$_i$:

Pre: pc = check

Eff: if flag(j) = 2 then

S := ∅

pc := set-flag-1

exit$_i$

Eff: pc := reset j ∉ S

reset$_i$:

Pre: pc = reset

Eff: flag(i) := 0

S := ∅ else pc

:= leave-exit S := S ∪ {j}

if |S| = n then pc := leave-try

rem_i :

$crit_i$:

Eff: Pre: pcpc :== rem leave-exit

Pre: pc = leave-try

Eff: pc := crit

Note on code style

- Explicit pc makes atomicity clear, but looks somewhat verbose/awkward.
- pc is often needed in invariants.

- Alternatively: Use sequential style, with explicit reads or writes (or other operations), one per line.
- Need line numbers:
  - Play same role as pc.
  - Used in invariants: "If process i is at line 7 then…"

Correctness

Initial state

- Well-formedness: Obvious.
- Mutual exclusion:
  - Based on event order in executions, rather than invariants.
  - By contradiction: Assume U_i, U_j are ever in C at the same time.

U_i, U_j in C

– Both must set-flag-2 before entering C; Initial consider the last time they do this.state

– WLOG, suppose set-flag-2_i comes first. set-flag-2_i – Then flag(i) = 2 from that point onward (until set-flag-2 they are both in C).$_j$

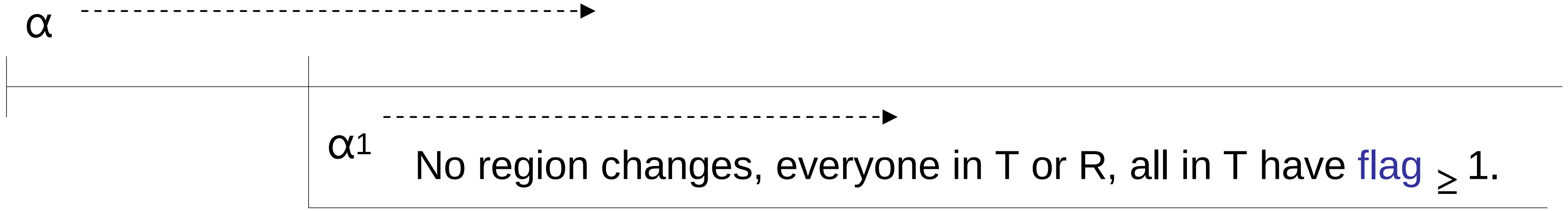
– However, j must see flag(i) ≠ 2, in order to j sees flag(i) ≠ 2 enter C.U_i, U_j

– Impossible. in C

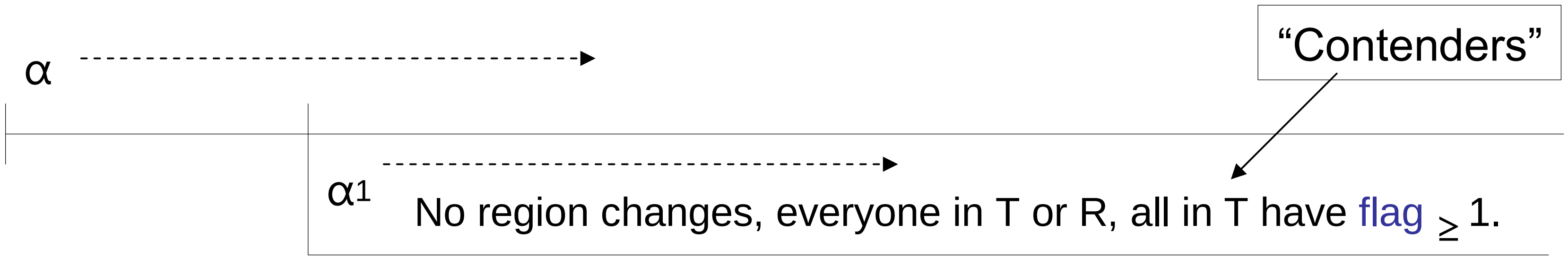
Progress

- Interesting case: Trying region.
- Proof by contradiction:

– Suppose α is a fair execution, reaches a point where some process is in T, no process is in C, and thereafter, no process ever enters C.
– Now start removing complications…
– Eventually, all regions changes stop and all in T keep their flags ≥ 1.
– Then it must be that everyone is in T and R, and all in T have flag ≥ 1.



Progress, cont'd

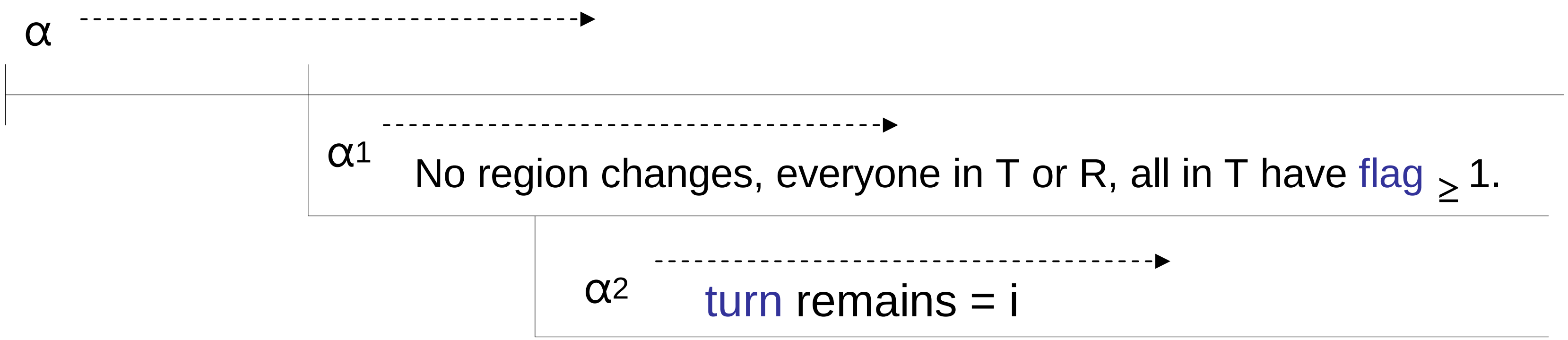


- Then whenever turn is reset in α_1, it must be set to a contender's index.
- Claim: In α_1, turn eventually acquires a contender's index.
- Proof:
  - Suppose not---stays non-contender forever.
  - Consider any contender i.
  - If it ever reaches test-turn, then it will set turn := i, since it sees an inactive process.
  - Why must process i reach test-turn?
- It's either that, or it succeeds in reaching C.

- But we have assumed no one reaches C.
  - – Contradiction.

Progress, cont'd

- In α_1, once turn = contender's index, it is thereafter always = some contender's index. – Because contenders are the only processes that can change turn.
- May change several times.
- Eventually, turn stops changing (because tests come out negative), stabilizes to some value, say i.



- Thereafter, all contenders ≠ i wind up looping in Stage 1.
  - If j reaches Stage 2, it returns to Stage 1, since it doesn't go to C.
  - But then j's tests always fail, so j stays in Stage 1.
- But then nothing stops process i from entering C.

Mutual exclusion, Proof 2

- Use invariants.
- Must show they hold after any number of steps.
- Main goal invariant: $|\{i : pc_i = crit\}| \leq 1$. • To prove by

  induction, need more:

  1. If pc_i = crit (or leave-try or reset) then $|S_i| = n$.
  2. There do not exist i, j, $i \neq j$, with i in S_j and j in S_i.

- 1 and 2 easily imply mutual exclusion.

- Proof of 1: Easy induction
- Proof of 2:
  - Needs some easy auxiliary invariants saying what S-values go with what flag values and what pc values.
  - Key step: When j gets added to S_i, by $\text{check}(j)_i$ event.
- Then must have $\text{flag}(j) \neq 2$.
- But then $S_j = \emptyset$ (by auxiliary invariant), so $i \notin S_j$, can't break invariant.

Running Time

- Upper bound on time from when some process is in T until some process is in C.

- Assume upper bound of l on successive turns for each process task (here, all steps of each process are in one task).
- Time upper bound for [Dijkstra]: O(l n).
- Proof: LTTR

Adding fairness guarantees [Peterson]

- Dijkstra algorithm does not guarantee fairness in granting the resource to different users.
- Might not be important in practice, if contention is rare.
- Other algorithms add fairness guarantees.

- E.g., [Peterson]: a collection of algorithms guaranteeing lockout-freedom.
- Lockout-freedom: In any (low-level) fair execution:
  - If all users always return the resource then any user that enters T eventually enters C.
  - Any user that enters E eventually enters R.

Peterson 2-process algorithm

- Shared variables:
  - turn, in {0,1}, 2W2R read/write register, initially arbitrary.
  - for each process i = 0,1:
    - flag(i), in {0,1}, 1W1R register, initially 0
    - Written by i, read by 1-i.

- Process i's trying protocol:
  - Sets flag(i) := 1, sets turn := i.

- Waits for either $\underbrace{\text{flag(1-i)} = 0}_{\text{Other process not active.}}$ or $\underbrace{\text{turn} \neq i}_{\text{Other process has the turn variable.}}$.
- Toggles between the two tests.

- Exit protocol:
  - Sets flag(i) := 0

Precondition/effect code

Shared variables:

turn $\in$ {0,1}, initially arbitrary

for every i $\in$ {0,1}: flag(i) $\in$ {0,1}, initially 0

Actions of process i:

Input: try_i, $exit_i$

Output: $crit_i$,rem_i

Internal: $set\text{-}flag_i$,$set\text{-}turn_i$, $check\text{-}flag_i$,$check\text{-}turn_i$, $reset_i$

Precondition/effect code, Peterson 2P, process i

try_i:

Eff: pc := set-flag

$set\text{-}flag_i$:

Pre: pc = set-flag

Eff: flag(i) := 1

pc := set-turn

$set\text{-}turn_i$:

Pre: pc = set-turn

Eff: turn := i pc := check-flag

$check\text{-}flag_i$

Pre: pc = check-flag

Eff: if flag(1-i) = 0 then pc := leave-try else pc := check-turn

More precondition/effect code, Peterson 2P, process i

$check\text{-}turn_i$:

Pre: pc = check-turn

Eff: if turn ≠ i
then pc := leave-try else
pc := check-flag

$crit_i$:

Pre: pc = leave-try Eff: pc := crit

$exit_i$ Eff: pc := reset

$reset_i$:

Pre: pc = reset

Eff: flag(i) := 0 pc := leave-exit

rem_i:

Pre: pc = leave-exit

Eff: pc := rem

Correctness: Mutual exclusion

- Key invariant:
  - If $pc_i \in$ {leave-try, crit, reset} (essentially in C), and
  - $pc_{1-i} \in$ {check-flag, check-turn, leave-try, crit, reset} (engaged in the competition or in C), – then $turn \neq i$.
- That is:
  - If i has won and 1-i is currently competing then turn is set favorably for i---which means it is set to 1-i.

- Implies mutual exclusion: If both are in C then turn must be set both ways, contradiction.

- Proof of invariant: All cases of inductive step are easy.
  - E.g.: a successful check-$turn_i$, causing i to advance to leave-try.

– This explicitly checks that turn ≠ i, as needed.

Correctness: Progress

- By contradiction:
  - Suppose someone is in T, and no one is ever thereafter in C.
  - Then the execution eventually stabilizes so no new region changes occur.
  - After stabilization:
- If exactly one process is in T, then it sees the other's flag = 0 and enters C.

- If both processes are in T, then turn is set favorably to one of them, and it enters C.

Correctness: Lockout-freedom

- Argue that neither process can enter C three times while the other stays in T, after setting its flag := 1.
- Bounded bypass.
- Proof: By contradiction.
  - Suppose process i is in T and has set flag := 1, and subsequently process (1-i) enters C three times.
  - In each of the second and third times through T, process (1-i) sets turn := 1-i but later sees turn = i.
  - That means process i must set turn := i at least twice during that time.

- But process i sets turn := i only once during its one execution of T.
  - Contradiction.
- Bounded bypass + progress imply lockout-freedom.

Time complexity

- Time from when any particular process i enters T until it enters C: c + O(l), where:
  - c is an upper bound on the time any user remains in the critical section, and
  - l is an upper bound on local process step time.
- Detailed proof: See book.
- Rough idea:

- Either process i can enter immediately, or else it has to wait for (1-i).
- But in that case, it only has to wait for one critical-section time, since if (1-i) reenters, it will set turn favorably for i.

Peterson n-process algorithms

- Extend 2-process algorithm for lockout-free mutual exclusion to n-process algorithm, in two ways:
  - Using linear sequence of competitions, or – Using binary tree of competitions.

Sequence of competitions

- Competitions 1,2,…,n-1.
- Competition k has one loser, up to n-k winners.
- Thus, only one can win in competition n-1, implying mutual exclusion.

- Shared vars:
  - For each competition k in {1,2,…,n-1}:
    - turn(k) in {1,2,…n}, mWmR register, written and read by all, initially arbitrary.
  - For i in {1,2,…n}:
    - flag(i) in {0,1,2,…,n-1}, 1WmR register, written by i and read by all, initially 0.

- Process i trying protocol:
  - For each level k:
    - Set flag(i) := k, indicating i is competing at level k.
    - Set turn(k) := i.
    - Wait for either turn(k) ≠ i, or everyone else's flag < k (check flags one at a time).

- Exit protocol:

– Set flag(i) := 0

Correctness: Mutual exclusion

- Definition: Process i is a winner at level k if either:
  - $level_i > k$, or
  - $level_i = k$ and $pc_i \in$ {leave-try, crit, reset}.
- Definition: Process i is a competitor at level k if either:
  - Process i is a winner at level k, or
  - $level_i = k$ and $pc_i \in$ {check-flag, check-turn}.

- Invariant 1: If process i is a winner at level k, and process $j \neq i$ is a competitor at level k, then $turn(k) \neq i$.
- Proof: By induction, similar to 2-process case.
  - Complication: More steps to consider.

- Now have many flags, checked in many steps.
- Need auxiliary invariants saying something about what is true inthe middle of checking a set of flags.

Correctness: Mutual exclusion

- Invariant 2: For any k, $1 \leq k \leq n-1$, there are at most n-k winners at level k.
- Proof: By induction, on level number, for a particular reachable state (not induction on number of steps).

– Basis: k = 1:

- Suppose false, for contradiction.
- Then all n processes are winners at level 1.
- Then Invariant 1 implies that turn(1) is unequal to all indices,contradiction.

– Inductive step: …

Correctness: Mutual exclusion

- Invariant 2: For any k, $1 \le k \le n-1$, there are at most n - k winners at level k.
- Inductive step: Assume for k, $1 \le k \le n-2$, show for k+1.
  - Suppose false, for contradiction.
  - Then more than $n - (k + 1)$ processes, that is, at least $n - k$ processes, are winners at level k + 1: $|Win_{k+1}| \ge n - k$.
  - Every level k+1 winner is also a level k winner: $Win_{k+1} \subseteq Win_k$.
  - By inductive hypothesis, $|Win_k| \le n-k$.
  - So $Win_{k+1} = Win_k$, and $|Win_{k+1}| = |Win_k| = n - k$.
  - Q: What is the value of turn(k+1) ?
- Can't be the index of any process in Win_{k+1}, by Invariant 1.
- Must be the index of some competitor at level k+1 (Invariant, LTTR).

- But every competitor at level k+1 is a winner at level k, so is in Win_k.
- Contradiction, since $Win_{k+1} = Win_k$.

Progress, Lockout-freedom

- Lockout-freedom proof idea:
  - Let k be the highest level at which some process i gets stuck.
  - Then turn(k) must remain = i.
  - That means no one else ever reenters the competition at level k.
  - Eventually, winners from level k will finish, since k is the highest level at which anyone gets stuck.
  - Then all other flags will be < k, so i advances.

- Alternatively, prove lockout-freedom by showing a time bound for each process, from →T until →C. (See book)
  - Define T(0) = maximum time from when a process →T until →C.
  - Define T(k), $1 \leq k \leq n-1$ = max time from when a process wins at level k until →C.

- $T(n-1) \leq l$.
- $T(k) \leq 2\,T(k+1) + c + (3n+2)\,l$, by detailed analysis.
- Solve recurrences, get exponential bound, good enough for showing lockout-freedom.

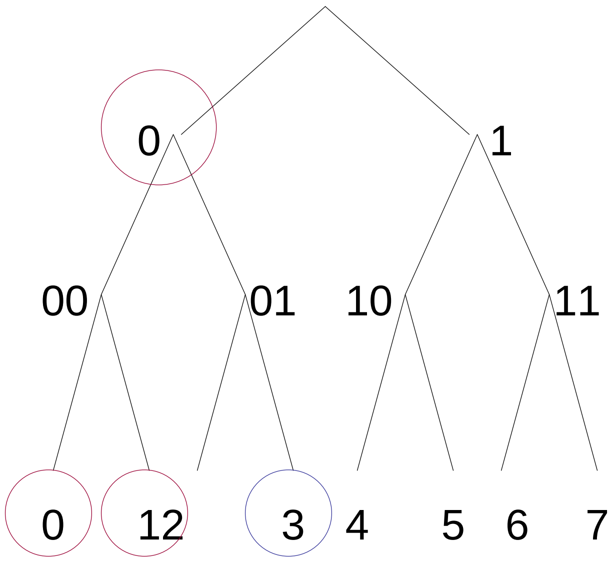
Peterson Tournament Algorithm

- Assume $n = 2^h$.
- Processes = leaves of binary tree of height h.
- Competitions = internal nodes, labeled by binary strings.
- Each process engages in log n competitions, following path up to root.
- Each process i has: λ

– A unique competition x at each level k. – A unique role in x (0 = left, 1 = right).

– A set of potential opponents in x.

Peterson Tournament Algorithm



- Shared variables:
  - For each process i, flag(i) in {0,…,h}, indicating level, initially 0

– For each competition x, turn(x), a Boolean, initially arbitrary. •

Process i’s trying protocol: For each level k:

- Set flag(i) := k.
- Set turn(x) := b, where:

• x is i’s level k competition, λ

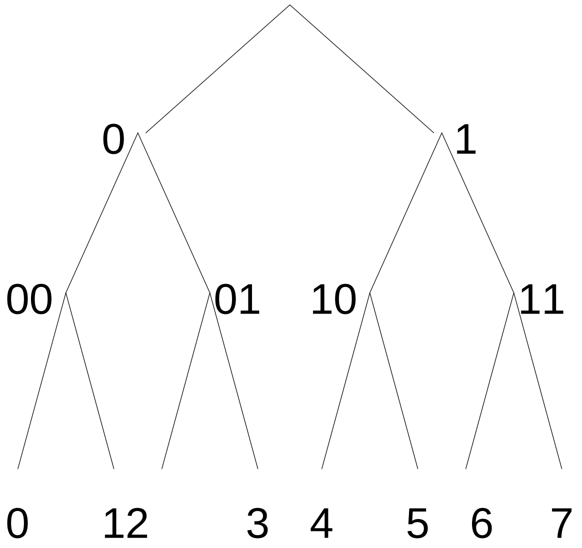
• b is i’s “role”, 0 or 1– Wait for either:

• turn(x) = opposite role, or

• all flags of potential opponents in x are < k.

• Exit protocol:

– Set flag(i) := 0.



Correctness

- Mutual exclusion:
  - Similar to before.
  - Key invariant: At most one process from any particular subtree rooted at level k is currently a winner at level k.

- Time bound (from →T until →C): $(n-1)\,c + O(n^2\,l)$
  - Implies progress, lockout-freedom.
  - Define: $T(0)$ = max time from →T until →C.
  - $T(k)$, $1 \le k \le \log n$ = max time from winning at level k until →C.
  - $T(\log n) \le l$.
  - $T(k) \le 2\,T(k+1) + c + (2^{k+1} + 2^k + 7)\,l$ (see book).
- Roughly: Might need to wait for a competitor to reach C, then finish C, then for yourself to reach C.
  - Solve recurrences.

Bounded Bypass?

- Peterson's Tournament algorithm has a low time bound from →T until →C:

$$(n-1)\,c + O(n^2\,l)$$

- Implies lockout-freedom, progress.

- Q: Does it satisfy bounded bypass?
- No! There's no upper bound on the number of times one process could bypass another in the trying region. E.g.:
  - Process 0 enters, starts competing at level 1, then pauses.
  - Process 7 enters, quickly works its way to the top, enters C, leaves C.
  - Process 7 enters again…repeats any number of times.
  - All while process 0 is paused.

- No contradiction between small time bound and unbounded bypass.
  - Because of the way we're modeling timing of asynchronous executions, using upper bound assumptions.
  - When processes go at very different speeds, we say that the slow processes are going at normal speed, faster processes are going very fast.

Lamport's Bakery Algorithm

- Like taking tickets in a bakery.
- Nice features:

- Uses only single-writer, multi-reader registers.
- Extends to even weaker registers, in which operations have durations, and a read that overlaps a write receives an arbitrary response.
- Guarantees lockout-freedom, in fact, almost-FIFO behavior.

- But:
  - Registers are unbounded size.
  - Algorithm can be simulated using bounded registers, but not easily (uses bounded concurrent timestamps). • Shared variables:
  - For each process i:

- choosing(i), a Boolean, written by i, read by all, initially 0
- number(i), a natural number, written by i, read by all, initially 0

Bakery Algorithm

- First part, up to choosing(i) := 0 (the “Doorway”, D):

- Process i chooses a number number greater than all the numbers it reads for the other processes; writes this in number(i).
- While doing this, keeps choosing(i) = 1.
- Two processes could choose the same number (unlike real bakery).
- Break ties with process ids.

- Second part:
  - Wait to see that no others are choosing, and no one else has a smaller number.
  - That is, wait to see that your ticket is the smallest.
  - Never go back to the beginning of this part---just proceed step by step, waiting when necessary.

Code

Shared variables: for every $i \in \{1,\ldots,n\}$: choosing(i) $\in \{0,1\}$, initially 0, writable by i, readable by all $j \neq i$ number(i), a natural number, initially 0, writable by i, readable by $j \neq i$.

try_i

choosing(i) := 1 number(i) := 1 +

$\max_{j \neq i}$ number(j) choosing(i) := 0

for $j \neq i$ do waitfor

choosing(j) = 0

waitfor number(j) = 0 or (number(i), i) < (number(j), j)

$crit_i$

$exit_i$

number(i) := 0

rem_i

Correctness: Mutual exclusion

- Key invariant: If process i is in C, and process j $\neq$ i is in $\underbrace{(T - D) \cup C}_{\text{Trying region after doorway, or critical region}}$,

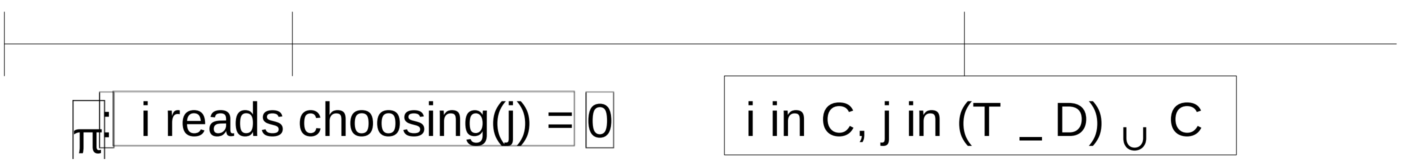
  then (number(i),i) < (number(j),j).

- Proof:
  - Could prove by induction.
  - Instead, give argument based on events in executions.

– This argument extends to weaker registers, with concurrent accesses.

Correctness: Mutual exclusion

- Invariant: If i is in C, and $j \neq i$ is in $(T - D) \cup C$, then (number(i),i) < (number(j),j).
- Proof:
  – Consider a point where i is in C and $j \neq i$ is in $(T - D) \cup C$.
  – Then before i entered C, it must have read choosing(j) = 0, event π.



  – Case 1: j sets choosing(j) := 1 (starts choosing) after π.
- Then number(i) is set before j starts choosing.

- So j sees the "correct" number(i) and chooses something bigger.
- That suffices.

– Case 2: j sets choosing(j) := 0 (finishes choosing) before π.
  - Then when i reads number(j) in its second waitfor loop, it gets the "correct" number(j).
  - Since i decides to enter C, it must see (number(i),i) < (number(j),j).

Correctness: Mutual exclusion

- Invariant: If i is in C, and j ≠ i is in (T − D) ∪ C, then (number(i),i) < (number(j),j).
- Proof of mutual exclusion:
  – Apply invariant both ways.
  – Contradictory requirements.

Liveness Conditions

- Progress:
  - By contradiction.
  - If not, eventually region changes stop, leaving everyone in T or R, and at least one process in T.
  - Everyone in T eventually finishes choosing.
  - Then nothing blocks the smallest (number, index) process from entering C.

- Lockout-freedom:
  - Consider any i that enters T
  - Eventually it finishes the doorway.
  - Thereafter, any newly-entering process picks a bigger number.
  - Progress implies that processes continue to enter C, as long as i is still in T.

- In fact, this must happen infinitely many times!
- But those with bigger numbers can't get past i, contradiction.

FIFO Condition

- Not really FIFO (→T vs. →C), but almost:
  - FIFO after the doorway: if j leaves D before i →T, then j →C before i →C.
- But the "doorway" is an artifact of this algorithm, so this isn't a meaningful way to evaluate the algorithm!
- Maybe say "there exists a doorway such that"…
- But then we could take D to be the entire trying region, making the property trivial.
- To make the property nontrivial:

– Require D to be “wait-free”: a process is guaranteed to complete D it if it keeps taking steps, regardless of what any other processes do.
– D in the Bakery Algorithm is wait-free.

- The algorithm is FIFO after a wait-free doorway.

Impact of Bakery Algorithm

- Originated important ideas:
  – Wait-freedom
    - Fundamental notion for theory of fault-tolerant asynchronous distributed algorithms.
  – Weakly coherent memories

- Beginning of formal study: definitions, and some algorithmic strategies for coping with them.

Next time…

- More mutual exclusion algorithms:
  - Lamport's Bakery Algorithm, cont'd
  - Burns' algorithm
- Number of registers needed for mutual exclusion.
- Reading: Sections 10.6-10.8