



AE 244
Low Speed Aerodynamics

Prof: Dhwanil Shukla

Assignment - 2

Team No. 24
Ghoshank Nanhe
22B0073

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Note:

Two Jupyter notebooks are provided:

- **Assignment Main 1** - Functions in section 2.
- **Assignment Main 2** - For verifying rest calculations.

1. Team Contribution

Roll No.	Name	Contribution type	Overall Contribution score(0 to 5)
22B0027	Shivendra Jhala	Algorithm and comments	5
22B4215	Kshitij Pradhan	Functions to plot NACA, slope, Cl and Circulation.	5
22B0073	Ghoshank Nanhe	Functions to plot vector fields, circulation and final function.	5

2. Algorithm

2.1. Algorithm for plotting camber line

The algorithm we have used for plotting the camber line is pretty simple. We first created a function which takes maximum camber and maximum camber position as a fraction of chord length (assumed to be one). Then we created an array of x_values containing 1000 points equally spaced between 0 and 1. Then we used the following formula to compute y coordinate corresponding to each x in our array:

$$\begin{array}{ll} \text{Front } (0 \leq x < p) & \text{Back } (p \leq x \leq 1) \\ \text{Camber } y_c = \frac{M}{p^2} (2Px - x^2) & y_c = \frac{M}{(1-p)^2} (1 - 2P + 2Px - x^2) \end{array}$$

We stored the corresponding values of y in another array and then used python plotting library to plot the camber line. Also, our program gives freedom to the user, if you don't want to use NACA formulae then you can give your own function also as input.

2.2. Algorithm for plotting camber line slope.

We have defined a function such that it takes input_x (the point where you want to find slope), x_values (our previous array of x values) and camber function as input. Now if input_x belongs to x_values (let us say input_x=x_values[i]) then the slope will be

$$\text{Slope} = \frac{y_2 - y_1}{x_values[i+1] - x_values[i]}$$

Where y2 and y1 are corresponding y coordinates. Then we stored the value of slope at every x in another array and then plotted it. Also, if input_x does not belong to x_values then we used formula

$$\text{Slope} = (y_2 - y_1) / (2 * \text{eps})$$

Where y1 and y2 are values of the camber function corresponding to input_x-eps and input_x+eps and eps is a very small positive number.

2.3. Algorithm for computing Cl

(i) Variables:

- ``alpha``: Angle of attack of the airfoil.
- ``s``: Array representing the lengths of the panels on the airfoil.
- ``n``: Number of panels.

(ii) Calculate Panel Angles (theta)

- Iterate over each panel, and calculate the panel angles (theta) using the formula: $\theta[i] = \cos^{-1}(1 - 2 \cdot x[i])$, where $x[i]$ is the location of the panel on the airfoil.

(iii) Calculate Panel Angles Differences (dtheta):

- Compute the differences between consecutive panel angles:

$$d\theta[i] = \theta[i+1] - \theta[i] \text{ for } (i = 0, 1, \dots, n-1).$$

(iv) Calculate (A_0):

- Compute the sum $A_0 = \sum s[i] \cdot d\theta[i]$
- Set $A[0] = \alpha - A_0 / \pi$

(v) $A[i]$ Coefficients:

$$A[i] = (2/\pi) \cdot \sum_j s[j] \cdot d\theta[j] \cdot \cos(i \cdot \theta[j])$$

(vi) Return Results:

- Return the arrays A , θ , and $d\theta$, which represent the coefficients of the Fourier expansion, panel angles, and panel angle differences, respectively.

The resulting A array can be used to calculate the lift coefficient C_l distribution on the airfoil using the formula:

$$C_l = \pi(2A[0] + A[1])$$

$$A_0 = \alpha - \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} d\theta \quad A_n = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos n\theta d\theta$$

$$C_l = 2\pi \left(A_0 + \frac{A_1}{2} \right) \quad C_m = -\frac{\pi}{2} \left(A_0 + A_1 - \frac{A_2}{2} \right) = -\frac{C_l}{4} - \frac{\pi}{4} (A_1 - A_2)$$

2.4. Algorithm for plotting vector field.

We defined a function called `streamlines` to visualise velocity vectors around an airfoil using the vortex panel method. It takes two parameters: `vel` (velocity magnitude) and `A` (an array containing circulation coefficients). The function calculates the streamlines by iteratively summing the contribution of each panel's circulation to the velocity field. It then sets up a grid to plot velocity vectors and adjusts them based on the airfoil's circulation distribution. The resulting velocity vectors are plotted with the NACA camber line. The code includes adjustments for the freestream velocity and takes care of handling different quadrants of the airfoil. The final visualisation represents the vector field around the airfoil, showing the direction and intensity of the airflow. In the end our function returns circulation distribution 'gamma' in the form of an array.

2.5. Algorithm for calculating circulation through line integral.

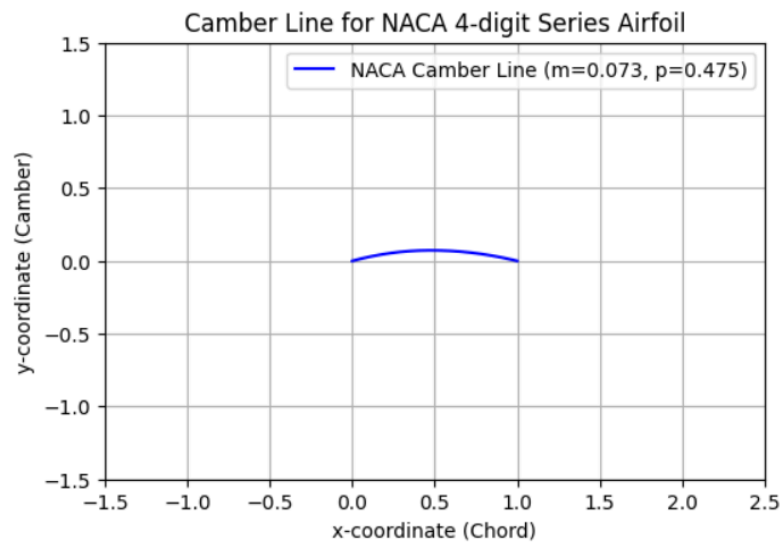
To do this we just used the formula:

$$\text{Circulation} = \int_0^1 \gamma(x) dx = \sum \text{gamma}[i] * \sin(\theta) * d\theta/2$$

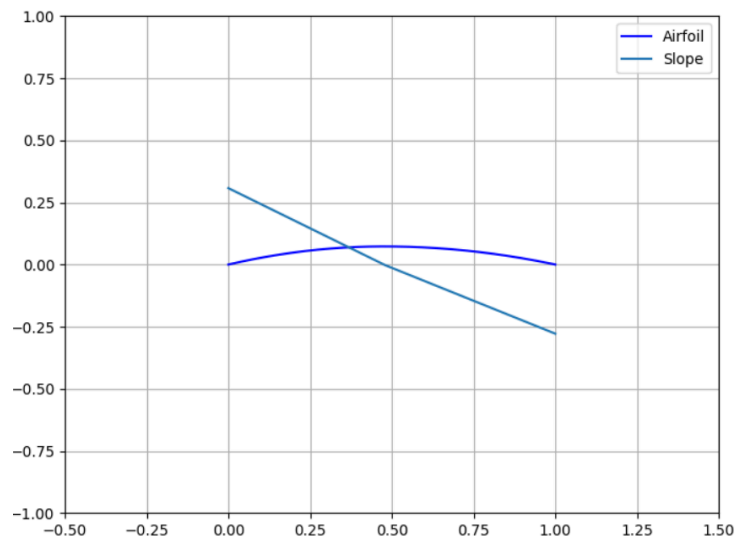
3. Airfoil Simulation and Results.

3.1 Camber line ('y' vs 'x') for the NACA airfoil from assignment 1

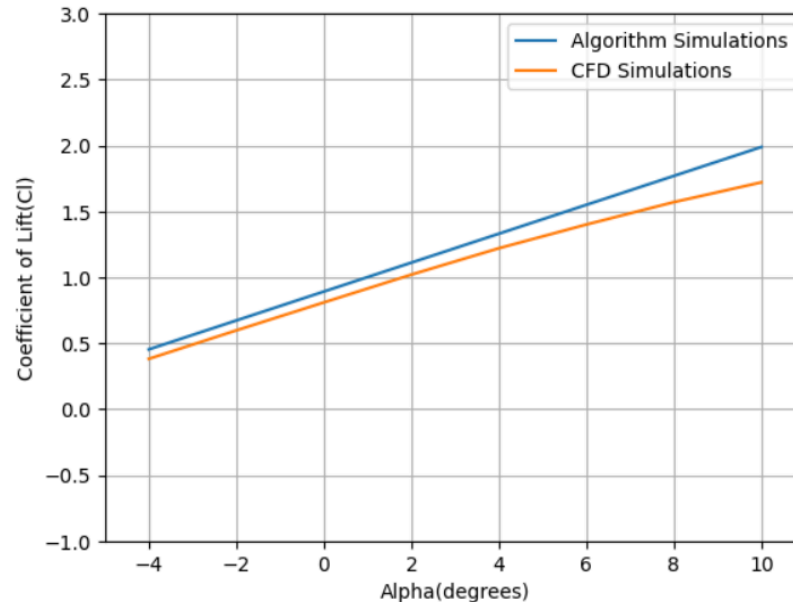
Airfoil	NACA 7411
Max Camber	7.3% of chord
Camber Position	47.5% of chord



3.2. Slope of the camber line along x (dy/dx vs x)



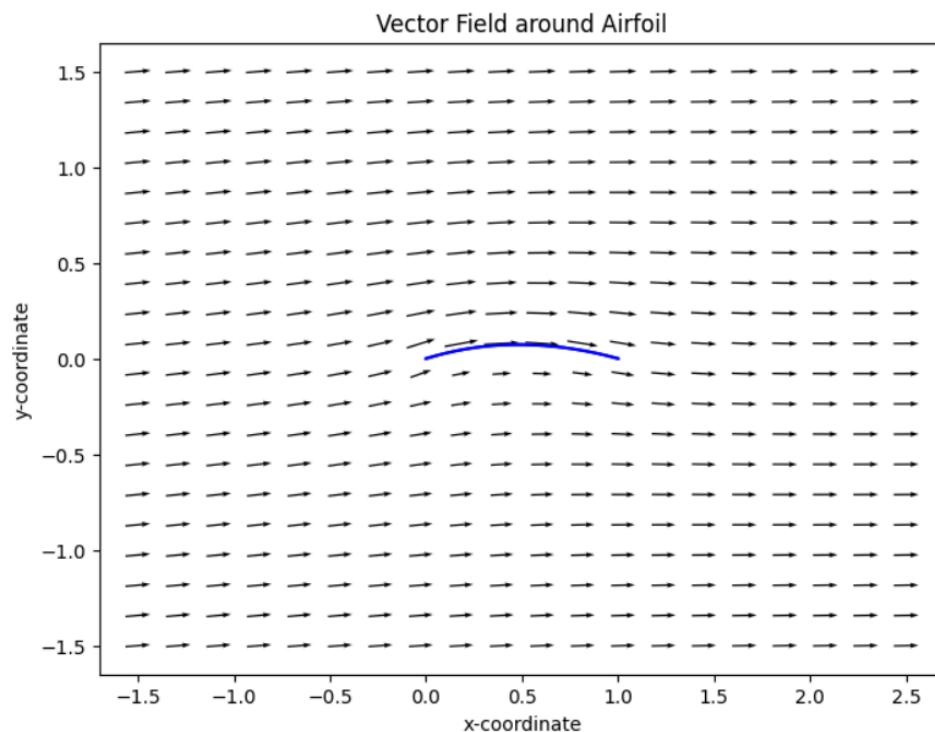
3.3. C_l vs α for the NACA airfoil from the program and CFD simulations.



3.4 Comparing plots obtained from both the simulations.

- The plot obtained from the written algorithm is shifted upward as compared to plot by CFD simulations.
- C_l obtained from algorithms have higher values.
- Algorithm plot is linear as it is obtained from thin airfoil theory assumption.
- As angle of attack increases, deviation increases between plots obtained from both the simulation.
- This is mostly because Our algorithm does not include flow separation and so if we increase α the C_l values keep on increasing.
- Here is deviation from ansys simulation because it is not thin airfoil.
- As we can see C_l vs α curve after a certain angle the slope of it decreases because of flow separation.

3.5 Vector Field Plot around the airfoil.



3.6 Interpretation from Vector Field Plot.

- Arrows are representing the direction of flow of air at a point present on their tail.
- The air is flowing tangential to the airfoil because it experiences viscous effects due to the airfoil's surface friction. The boundary layer tends to adhere to the airfoil's surface and follows its contours, resulting in tangential airflow.
- The velocity generated here is due to the velocity induced by vortices and the free stream flow.
- Camber is positive means it is convex, due to which circulation is negative means anti-clockwise.
- It induces velocity in such a way that velocity above camber is more as compared to velocity below.
- Increasing camber increases circulation due to the fact that this algorithm does not include flow separation.

3.7.Circulation around the entire airfoil at $\alpha = 3^\circ$.

- Circulation is calculated around the airfoil using line integral approach along a rectangular region of $4c \times 3c$ in dimension.

$$\text{Circulation} = 18.418$$

3.8. Bound circulation by integrating circulation distribution along camber at $\alpha = 3^\circ$

- Circulation calculated around the airfoil by integrating circulation distribution along the airfoil.

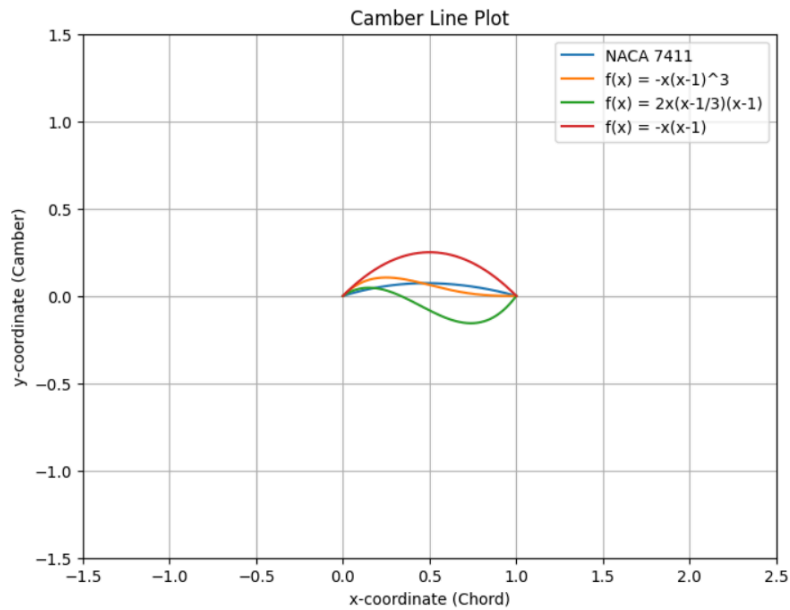
$$\text{Circulation} = 18.423$$

3.9 Comparing values of circulation obtained from both the circulation.

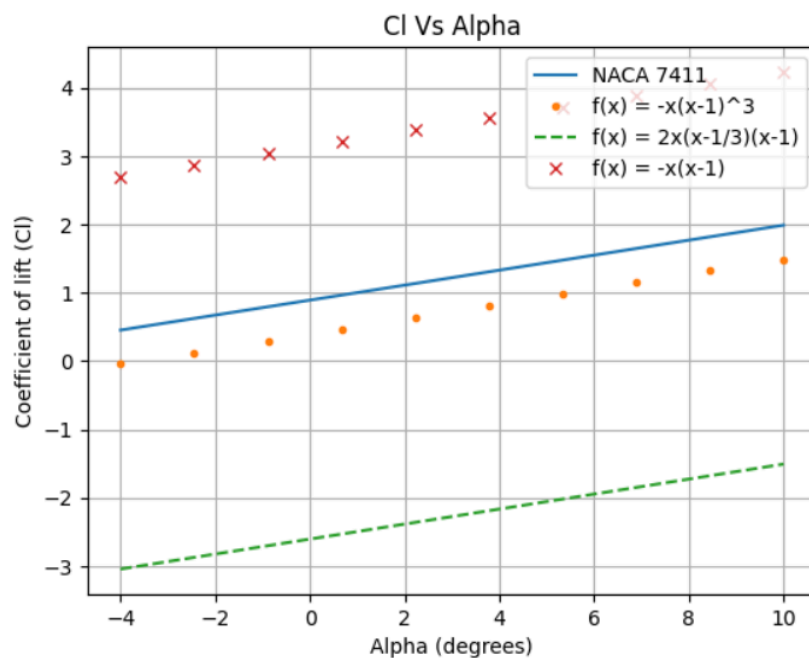
- Value of circulation obtained from the circulation distribution along the airfoil is more as compared to circulation value obtained by using integral approach along the rectangular region.
- In reality both the circulation should be the same because circulation is caused by the airfoil and it should have the same value if we integrate about a region containing it to maintain the integrity of conservation of mass and momentum.
- As we increase the number of divisions of ds both values converge and give better results.
- As we increase the size of the region about which we are applying the line integral approach this value reaches value obtained from circulation distribution.

4. Novel Airfoil Properties.

4.1 Camber line ('y' vs 'x') for your 3 airfoils and the NACA airfoil on the same slope.



4.2 C_l vs α of the three airfoils along with that of the NACA airfoil on the same chart.

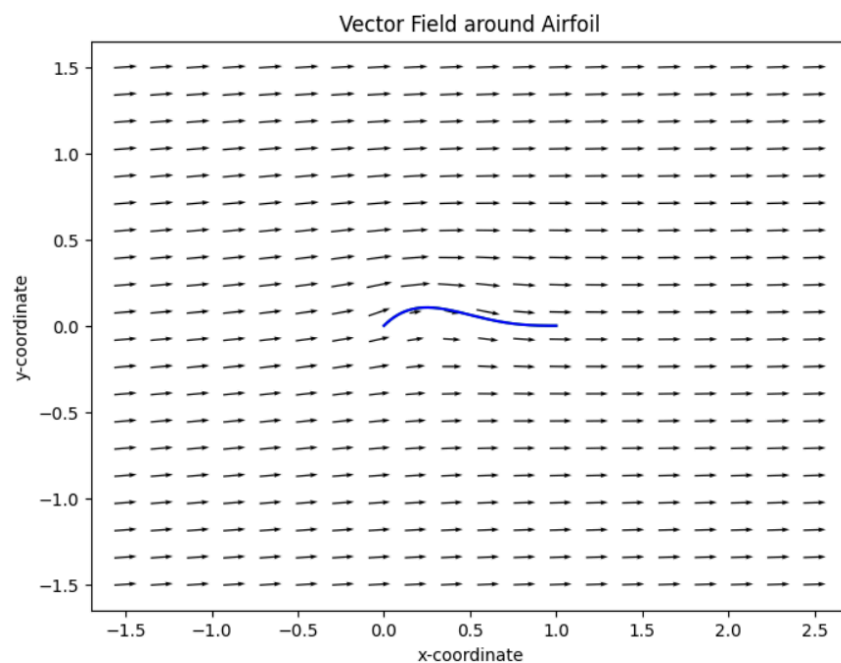


4.3 Interpretation from the Cl vs Alpha curve.

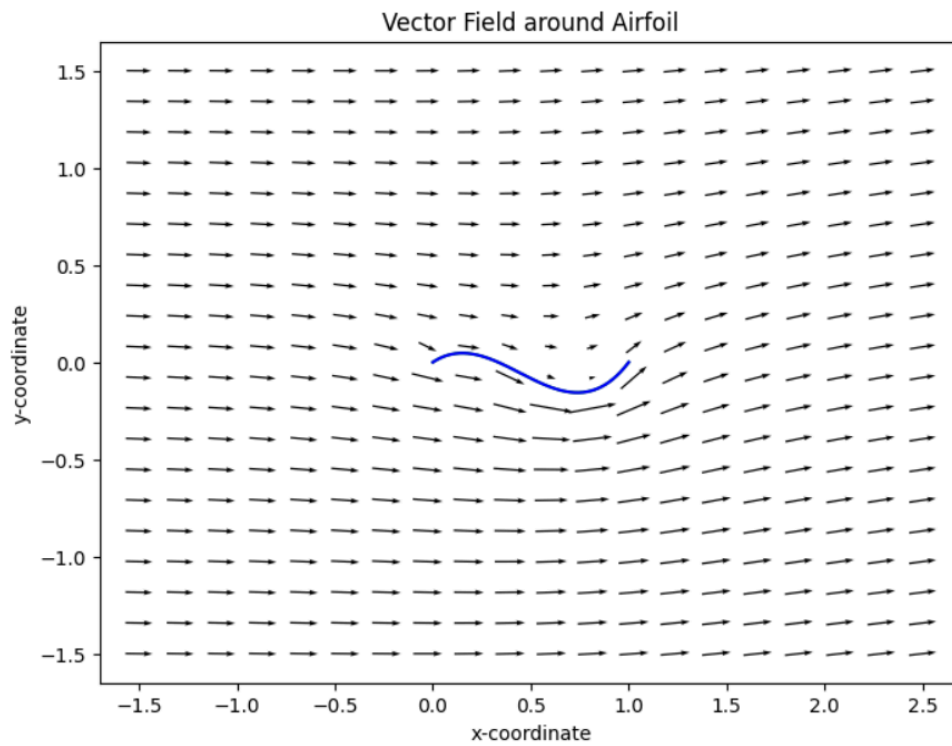
- In the plot for naca 7411 and function 1 is near because both are of same nature and are thin airfoils.
- Both have approximately the same shape; just the back part of function 1 is concave.
- According to Thin Airfoil Theory we have calculated the Cl and we can see that curves have the same slope and they are just shifted.
- Cl vs Alpha curve for function 1 ,the Cl have higher values as compared to others, because it has more camber.
- This is because a more cambered airfoil can create a greater pressure difference between the top and bottom surfaces, resulting in higher lift.
- For function 3 we can observe that the Cl vs Alpha curve is below and it is giving -ve Cl at 0 degree angle of attack.
- This is mostly because the airfoil of function 3 is like an airfoil having inverted camber which results in a -ve Cl and as we increase Alpha slope increases.

4.4.Vector field plot around the three airfoils.

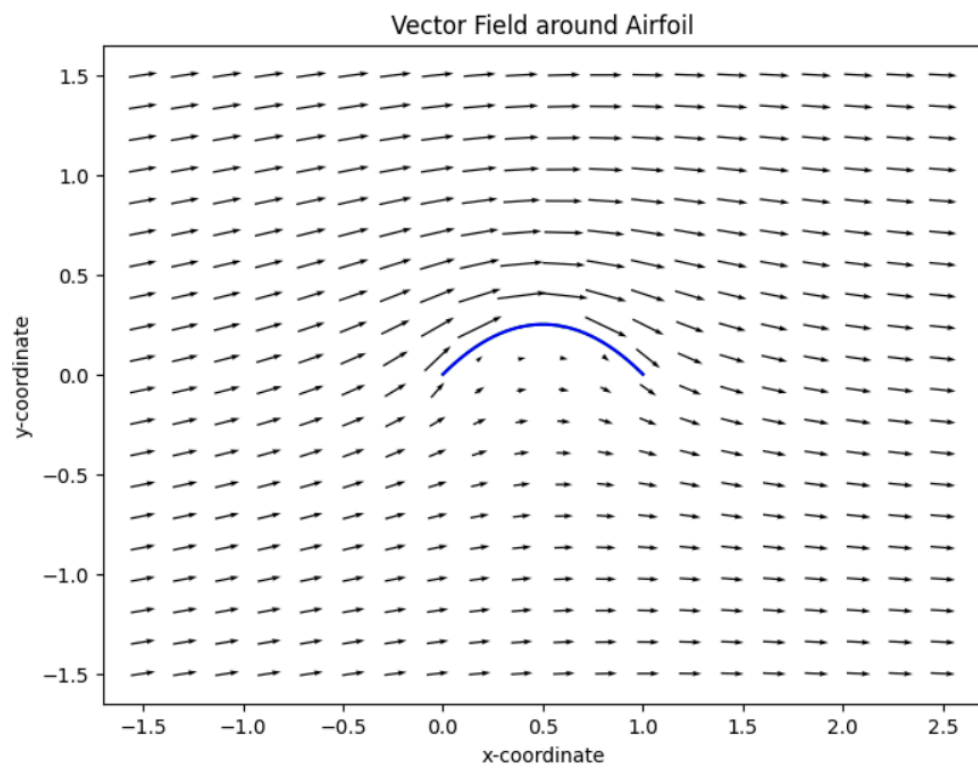
$$f_1(x) = -x(x-1)^3$$



$$f_2(x) = 2x\left(x - \frac{1}{3}\right)(x - 1)$$



$$f_3(x) = -x(x - 1)$$



4.5 Interpretations from above vector plots.

- For function 1, the airfoil is thin and the thin airfoil theory will be more accurate. camber of this is positive means it is convex , and it gives clockwise circulation, velocity of points above camber is more as compared below points.
- For function 2, camber is negative means it is concave , and it gives anti-clockwise circulation due to which velocity of points above camber is less as compared to points which are below camber.
- For function 3, it may not give proper result because it is not thin, the camber of this point is positive means it is convex, and it gives clockwise circulation, velocity of points above camber is more as compared below points.
- We can observe here as it does not show flow separation so the circulation values increase as camber increases as seen in function 3, which has more circulation which leads to more velocity above camber.

5. Conclusion

5.1 Overall take on your code's performance as compared to Ansys simulation, and possible reasons for deviations, if any

- Here we are using Thin Airfoil Theory in our code , which means it will give less error in accuracy if we use thin airfoil. For airfoils with large camber results will deviate from its actual values and we will get accuracy errors.
- The Ansys or other CFD software may give better results because they are for general airfoils and can also be applicable to airfoil with large camber.
- This softwares uses different computational techniques like meshing , finite volume and finite element methods.
- Our algorithm does not contain any turbulence model and so we can not see the flow separation and similar things.
- Assumption and simplification made in our algorithm is different from other cfd softwares.

5.2 Overall take on the performance of your airfoil as compared to the NACA airfoil


- The naca airfoil is 7411 and my airfoils are user defined.
- We can clearly compare them by using C_l vs Alpha curve for NACA and user defined.
- Function 1 is similar to NACA airfoil and Function 2 it is giving negative C_l at zero alpha because its camber is inverted and for function 3 camber is more which gives more lift.
- Function 3 is more cambered and the circulation increases which is due to the fact that our algorithm does not include flow separation and deviates from actual results.

6. Acknowledgement

- Nikhil Jha (22B0002)
- Devesh Mithal (22B0070)
- Shrivardhan Kondekar (22B0054)
- Shreyas N.B (210010061)

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7. References

- [What is an Algorithm? \(programiz.com\)](https://programiz.com)
- [Plotting Vector Fields in Python · Ajit Kumar \(krajit.github.io\)](https://krajit.github.io)
-  Thin airfoil theory [Aerodynamics #12]
- https://web.stanford.edu/~cantwell/AA200_Course_Material/The%20NACA%20airfoil%20series.pdf
- <https://www.desmos.com/calculator/mcyhgmq8va>