Understanding Vector Formulas in Physics

Components of a Vector

- A vector can be broken down into its components: V_X (X component) and V_Y (Y component).
- The angle θ is crucial for determining these components.
- Formulas:
- $V_X = V * \cos(\theta)$
- $V_Y = V * \sin(\theta)$

Magnitude of a Vector

- The magnitude of a 2D vector is calculated using the Pythagorean theorem.
- Formula: $|V| = \sqrt{(V_x^2 + V_Y^2)}$
- For a 3D vector, include the Z component: $|V| = \sqrt{(V_x^2 + V_Y^2 + V_z^2)}$

Direction of a Vector

- The angle θ can be found using the tangent function.
- Formula: $\theta = \arctan(V_Y / V_x)$
- This angle helps in understanding the vector's orientation in space.

Vector Notation

- Vectors can be represented in component form: $V = V_x i + V_y j$ for 2D.
- For 3D vectors, include the Z component: $V = V_x i + V_y j + V_z k$.
- Unit vectors (i, j, k) indicate direction with a magnitude of 1.

Dot Product of Vectors

- The dot product results in a scalar quantity.
- Formula: $A \cdot B = |A| * |B| * \cos(\theta)$
- It can also be calculated using components: $A \cdot B = A_xB_x + A_YB_Y + A_zB_z$.

Applications of Dot Product

- A practical example of the dot product is calculating work.
- Formula: Work = Force • Displacement = |F| * |D| * cos(θ).
- When vectors are perpendicular, the work done is zero.

Cross Product of Vectors

- The cross product results in a vector quantity.
- Formula: A × B = |A| * |B| * sin(θ).
- This can be visualized using the right-hand rule to determine direction.

Applications of Cross Product

- The cross product is used to calculate torque.
- Formula: Torque = R × F, where R is the lever arm and F is the force.
- Torque is maximized when the angle between R and F is 90°.

Unit Vectors

- A unit vector has a magnitude of 1 and indicates direction.
- Formula: $\hat{R} = R / |R|$, where R is the position vector.
- Unit vectors are essential in expressing vectors in terms of direction.

Position Vectors

- The position vector R defines the location in space.
- Formula: $R = (X^b - X_a)i + (Y^b - Y_a)j + (Z^b - Z_a)k$.
- It represents the difference between two points in space.

Magnitude of Position Vector

- The magnitude of the position vector is calculated as:
- $|R| = \sqrt{(R_x{}^2 + R_Y{}^2 + R_z{}^2)}$.
- This gives the distance from the origin to the point in space.

Graphical Representation of Vectors

- Vectors can be represented graphically with arrows.
- The length of the arrow indicates magnitude, and the direction indicates orientation.
- Graphs help visualize vector addition and subtraction.

Vector Addition

- Vectors can be added using the head-to-tail method.
- The resultant vector is drawn from the tail of the first vector to the head of the last vector.
- Formula: R = A + B.

Vector Subtraction

- Vector subtraction involves reversing the direction of the vector being subtracted.
- Formula: R = A - B = A + (-B).
- This can also be visualized graphically.

Summary of Key Formulas

- Magnitude: $|V| = \sqrt{(V_x^2 + V_y^2 + V_z^2)}$
- Dot Product: $A \cdot B = |A| * |B| * \cos(\theta)$
- Cross Product: $A \times B = |A| * |B| * \sin(\theta)$
- Work: Work = Force $\cdot$ Displacement