Vector Calculus: Differentiation and Integration

Introduction to Vector Calculus

- Welcome to the study of Vector Calculus.
- We will explore two main topics: Vector Differentiation and Vector Integration.
- Understanding vectors is crucial as they denote direction in space.

Understanding Vectors

- As we know the basic concept about vectors that ;
- A vector quantity has both magnitude and direction.
- Components of a vector:
- 'i' represents the x-axis.
- 'j' represents the y-axis.
- 'k' represents the z-axis.
- Scalar quantities lack direction.

Vector Differentiation

- The differentiation operator is denoted by 'del' (∇).
- To differentiate a vector function, we apply the 'del' operator.
- Example: $\nabla f = (\partial f/\partial x)i + (\partial f/\partial y)j + (\partial f/\partial z)k$.

Gradient of a Scalar Function

- The gradient of a scalar function f is given by:
- $\nabla f = (\partial f/\partial x)i + (\partial f/\partial y)j + (\partial f/\partial z)k$.
- This indicates the direction of the steepest ascent of the function.

Divergence of a Vector Field

- The divergence of a vector field F is defined as:
- $\nabla \cdot F = (\partial F_1/\partial x + \partial F_2/\partial y + \partial F_3/\partial z)$.
- It measures the magnitude of a source or sink at a given point.

Curl of a Vector Field

- The curl of a vector field F is given by:
- $\nabla \times F = (\partial F_3/\partial y - \partial F_2/\partial z)i + (\partial F_1/\partial z - \partial F_3/\partial x)j + (\partial F_2/\partial x - \partial F_1/\partial y)k$.
- It represents the rotation of the field around a point.

Applications of Gradient, Divergence, and Curl

- Gradient: Used to find the normal vector (N) of a surface.
- Divergence: Used to find the output of vector differentiation in scalar form.
- Curl: Used to find the output of vector differentiation in vector form.

Finding Angles Between Scalars

- To find the angle between two scalar functions f and g:
- Calculate ∇f and ∇g at a point (a, b, c).
- Use the formula: $\cos(\theta) = (\nabla f \cdot \nabla g) / (|\nabla f|\,|\nabla g|)$.

Unit Normal and Tangent Vectors

- The unit normal vector is calculated as:
- $N = \nabla f / |\nabla f|$.
- The unit tangent vector is derived from the position vector.
- Understanding these vectors is crucial for analyzing curves and surfaces.