Problem 1.

- The constraint is:

$$\max_{\mathbf{x}} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{subject to} \quad \begin{bmatrix} 1 & 3 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

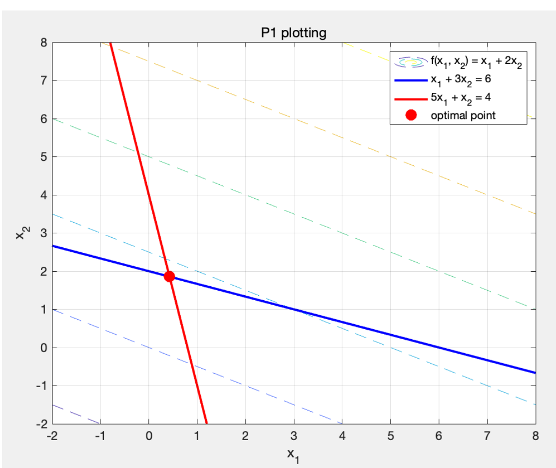
- The contour plot of the primal objective and lines corresponding to the constraints is shown in figure 1a.
- The optimal point is the intersection of the two constraints which is $\left(\frac{3}{7}, \frac{13}{7}\right)$.
- The dual problem is:

$$\max_{\mathbf{x}} \begin{bmatrix} 6 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad \text{subject to} \quad \begin{bmatrix} 1 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

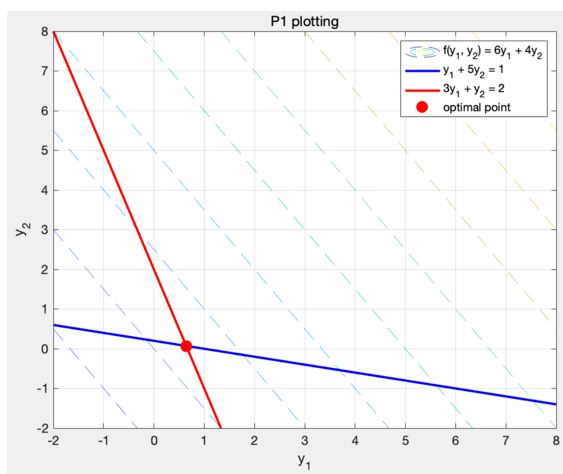
  We also have:

$$\Delta L = \begin{bmatrix} 1 & 3 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} - \begin{bmatrix} 6 \\ 4 \end{bmatrix} = 0$$

- The plotting is shown in figure 1b.
- The optimal point is the intersection of the two constraints which is $\left(\frac{1}{14}, \frac{9}{14}\right)$.
- Let $\Delta L = 0$, we have $\lambda = \left(\frac{3}{7}, \frac{13}{7}\right)$, so results are the same.



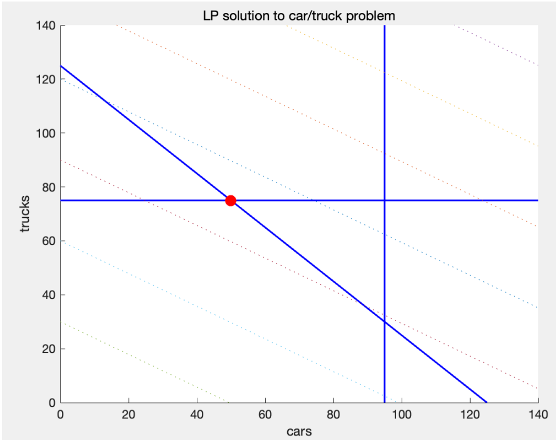
(a) Problem 1 plotting1



(b) Problem 1 plotting2

Figure 1: Problem 1

Problem 2.



(a) Problem 2 plotting

```
>> test_P2
Initial tableau =
     -2000     -3300         0         0         0         1         0
         1         0         1         0         0         0        95
         0         1         0         1         0         0        75
         1         1         0         0         1         0       125

pivot_col = 2, pivot_row = 3, pivot = 1.000000
pivot_col = 1, pivot_row = 4, pivot = 1.000000
Final tableau =
         0         0         0      1300      2000         1    347500
         0         0         1         1        -1         0        45
         0         1         0         1         0         0        75
         1         0         0        -1         1         0        50

The optimal is [50, 75]
The result is fval = 347500
Total test is 45, pass test is 45, fail 0 times
```

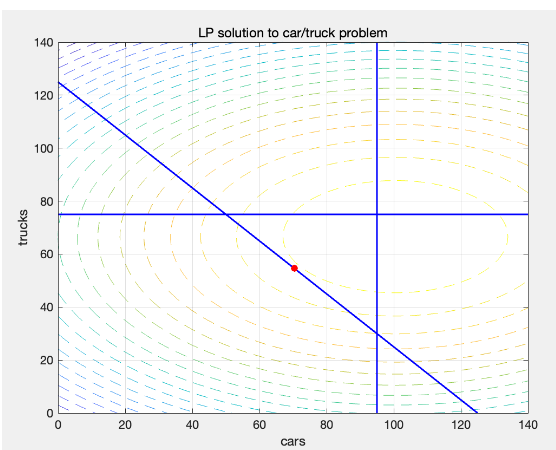
(b) Problem 2 result

Figure 2: Problem 2

The plotting is shown in figure 2a. The red dot is the optimal point. The method I use in this function is the code on Canvas. The test function is **test_P2().m**. Here I use all the possible (x, y) pair to show that it is the optimal point. The optimal choice is 50 cars and 75 trucks, the profit is 347500. It is shown in the result figure 2b.

Problem 3.

The plotting is shown in figure 3a. The red dot is the optimal point. The method I use in this function is the "quadprog" in Matlab. The test function is **test_P3().m**. Here I use all the possible (x, y) pair to show that it is the optimal point. The optimal choice is 70.42 cars and 54.57 trucks, the profit is 199231.42. It is shown in the result figure 3b.



(a) Problem 3 plotting

```
>> test_P3
Solved 0 variables, 0 equality, and 2 inequality constraints during the presolve.

 Iter            Fval  Primal Infeas    Dual Infeas  Complementarity
    0   -2.111112e+05   7.191251e+01   1.309886e+02     3.923097e+01
    1   -2.111571e+05   3.218723e+01   5.862903e+01     2.986214e+00
    2   -1.985864e+05   1.609361e-02   2.931452e-02     2.877371e+01
    3   -1.992220e+05   8.046807e-06   1.465726e-05     8.968977e+00
    4   -1.992314e+05   4.023406e-09   7.328698e-09     4.979802e-03
    5   -1.992314e+05   2.017941e-12   3.606881e-12     2.489901e-06
    6   -1.992314e+05   1.421085e-14   1.476637e-13     1.244951e-09

找到满足约束的最小值。

优化已完成，因为目标函数沿
可行方向在最优性容差值的范围内呈现非递减，
并且在约束容差值范围内满足约束。

<停止条件详细信息>
Total test is 95, pass test is 46, fail 0 times
The optimal is [70.428571, 54.571429]
The maximal profit is fval = 199231.428571
```

(b) Problem 3 result

Figure 3: Problem 3