```
Success, Matrix is symmetric positive definite

x_1 =

   -1.4340
   -0.6187
    1.8030
   -0.8681
   -1.0180


x_2 =

   -1.4264
   -0.6145
    1.7927
   -0.8613
   -1.0117

Norm between to method is 0.016341
Relative norm between to method is 0.005975
The average of norm dif is 0.118101
The average of relative norm dif is 0.022479
```

Figure 1: P1_result

Problem 1.

- take the gradient of (2), we get

$$\frac{\partial}{\partial x} f(x) = Ax - b$$

  $f(x)$ gets its minimum when $Ax - b = 0$ if A is positive definite.

- If A is not positive definite, $\frac{\partial}{\partial x} f(x) = 0$ doesn't mean it is the minimum value. Some solutions may not be covered, and the argmin can even not be a solution or not exists.

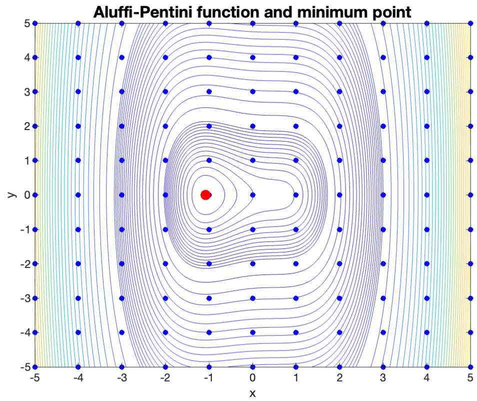
- The main function is gradient_descent.m. The test function is **test_P1.m**. I get the norm difference and the relative difference. It should look like figure 1.

Problem 2.

- The main function is gradient_descent_back.m. The testing program is **test_Back.m**. Here I use $c1 = .00001$. Then it works. The plotting should look like figure 2a and the result looks like figure 2b.

- Also, I tried nesterov momentum, if you just delete **test_Back.m** line 33 and use line 34. This one didn't work quite well. I choose the momentum parameter to be 0.8 and step size to be 0.038 only to get the minimum point after less than 120 iterations. The result is like figure 3b and the plotting is like figure 3a.

Problem 3.

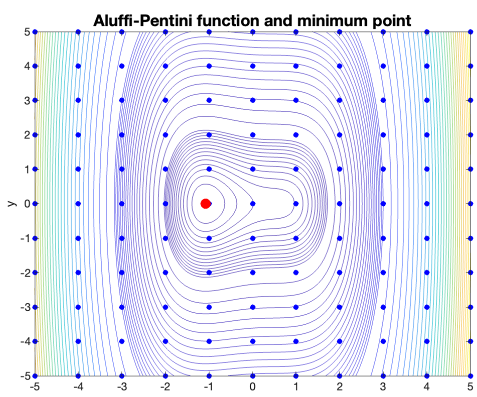


(a) Problem 2 Backtracking Plot

```
Starting point (5.000000, -3.000000), Terminating after 19 iterations.
Found the minimum, the point is (-1.075384, 0.000000)
Starting point (5.000000, -2.000000), Terminating after 19 iterations.
Found the minimum, the point is (-1.075384, 0.000000)
Starting point (5.000000, -1.000000), Terminating after 19 iterations.
Found the minimum, the point is (-1.075384, 0.000000)
Starting point (5.000000, 0.000000), Terminating after 19 iterations.
Found the minimum, the point is (-1.075384, 0.000000)
Starting point (5.000000, 1.000000), Terminating after 19 iterations.
Found the minimum, the point is (-1.075384, 0.000000)
Starting point (5.000000, 2.000000), Terminating after 19 iterations.
Found the minimum, the point is (-1.075384, 0.000000)
Starting point (5.000000, 3.000000), Terminating after 19 iterations.
Found the minimum, the point is (-1.075384, 0.000000)
Starting point (5.000000, 4.000000), Terminating after 19 iterations.
Found the minimum, the point is (-1.075384, 0.000000)
Starting point (5.000000, 5.000000), Terminating after 19 iterations.
Found the minimum, the point is (-1.075384, 0.000000)
Matlab found the minimum, the point is (-1.075382, -0.000001)
```

(b) Problem 2 Backtracking result

Figure 2: P2 Backtracking



(a) Problem 2 Nesterov Plot

```
Startpoint (4.000000, 5.000000). Successfully terminating after 90 iterations.
Startpoint (4, 5). Found the minimum, the point is (-1.075381, 0.000008)
Startpoint (5.000000, -5.000000). Successfully terminating after 109 iterations.
Startpoint (5, -5). Found the minimum, the point is (-1.075376, 0.000001)
Startpoint (5.000000, -4.000000). Successfully terminating after 109 iterations.
Startpoint (5, -4). Found the minimum, the point is (-1.075376, 0.000001)
Startpoint (5.000000, -3.000000). Successfully terminating after 109 iterations.
Startpoint (5, -3). Found the minimum, the point is (-1.075376, 0.000001)
Startpoint (5.000000, -2.000000). Successfully terminating after 109 iterations.
Startpoint (5, -2). Found the minimum, the point is (-1.075376, 0.000000)
Startpoint (5.000000, -1.000000). Successfully terminating after 109 iterations.
Startpoint (5, -1). Found the minimum, the point is (-1.075376, 0.000000)
Startpoint (5.000000, 0.000000). Successfully terminating after 109 iterations.
Startpoint (5, 0). Found the minimum, the point is (-1.075376, 0.000000)
Startpoint (5.000000, 1.000000). Successfully terminating after 109 iterations.
Startpoint (5, 1). Found the minimum, the point is (-1.075376, -0.000000)
Startpoint (5.000000, 2.000000). Successfully terminating after 109 iterations.
Startpoint (5, 2). Found the minimum, the point is (-1.075376, -0.000000)
Startpoint (5.000000, 3.000000). Successfully terminating after 109 iterations.
Startpoint (5, 3). Found the minimum, the point is (-1.075376, -0.000001)
Startpoint (5.000000, 4.000000). Successfully terminating after 109 iterations.
Startpoint (5, 4). Found the minimum, the point is (-1.075376, -0.000001)
Startpoint (5.000000, 5.000000). Successfully terminating after 109 iterations.
Startpoint (5, 5). Found the minimum, the point is (-1.075376, -0.000001)
Matlab found the minimum, the point is (-1.075382, -0.000001)
```

(b) Problem 2 Nesterov result

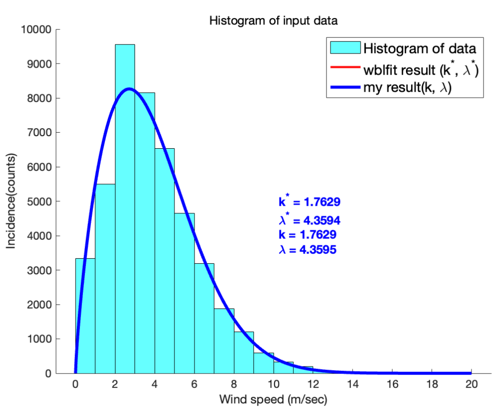
Figure 3: P2 Nesterov

- $$\begin{aligned}
L(k, \lambda; x_i) &= \log \prod_{i=1}^{N} f(x_i) \\
&= \sum_{i=1}^{N} \log \left(\frac{k}{\lambda}\right) \left(\frac{x}{\lambda}\right)^{k-1} - \left(\frac{x}{\lambda}\right)^{k} \\
&= \sum_{i=1}^{N} \log k + (k-1)\log x - k \log \lambda - \left(\frac{x}{\lambda}\right)^{k}
\end{aligned}$$

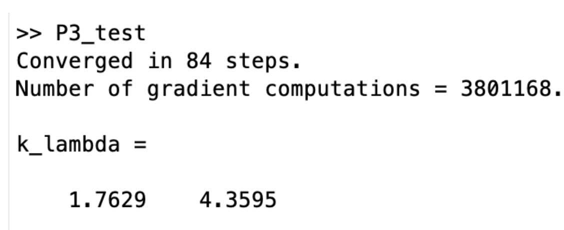
  Then we have:

  $$\begin{aligned}
\frac{\partial L}{\partial \lambda} &= \sum_{i=1}^{N} -\frac{k}{\lambda} + \frac{kx^k}{\lambda^{k+1}} = \sum_{i=1}^{N} \frac{k}{\lambda}\left(\left(\frac{x}{\lambda}\right)^{k} - 1\right) \\
\frac{\partial L}{\partial k} &= \sum_{i=1}^{N} \frac{1}{k} + \log x - \log \lambda - \left(\frac{x}{\lambda}\right)^{k} \log \frac{x}{\lambda} \\
&= \sum_{i=1}^{N} \frac{1}{k} + \ln \frac{x}{\lambda}\left(1 - \left(\frac{x}{\lambda}\right)^{k}\right)
\end{aligned}$$

- Here the main function is mygd.m. The test function is **P3_test.m**. Here I use the general gradient descent. I found that the if I set the $tol = 1e-5$ and the $\eta = 0.00002$, the result will be the same as the wblfit function in Matlab. The plotting is overlapping each other like 4a. The result should be like 4b.



(a) Problem 3 Plotting



```
>> P3_test
Converged in 84 steps.
Number of gradient computations = 3801168.

k_lambda =

    1.7629    4.3595
```

(b) Problem 3 result

Figure 4: Problem 3