Problem 1.

- The gradient and the Hessian is

$$\nabla f(x_1, x_2) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix} = \begin{pmatrix} x_1^3 \\ \frac{4}{9}x_2^3 \end{pmatrix}$$

$$H(f(x_1, x_2)) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{pmatrix} = \begin{pmatrix} 3x_1^2 & 0 \\ 0 & \frac{4}{3}x_2^2 \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \frac{1}{3x_1^2} & 0 \\ 0 & \frac{3}{4x_2^2} \end{pmatrix}$$

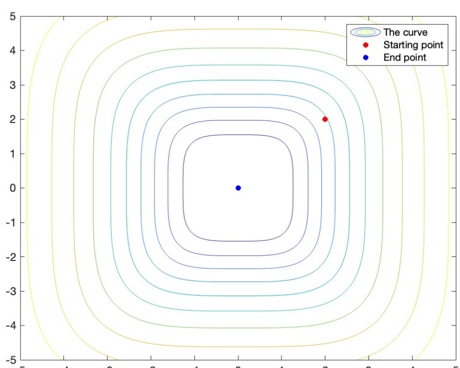
$$H^{-1}\nabla f = \begin{pmatrix} \frac{x_1}{3} \\ \frac{x_2}{3} \end{pmatrix}$$

  Therefore, we have

$$\vec{x}^1 = \vec{x}^0 - \left[\frac{x_1}{3}, \frac{x_2}{3}\right]^T = \left[\frac{4}{3}, \frac{4}{3}\right]^T$$

$$\vec{x}^2 = \vec{x}^1 - \left[\frac{x_1}{3}, \frac{x_2}{3}\right]^T = \left[\frac{8}{9}, \frac{8}{9}\right]^T.$$

- The main function in this problem is the newton_optimizer.m. The test function for this function is **text_newton_f1.m**. The result should be figure 1b and the plotting should be figure 1a. It shows that my calculation is correct.



(a) Problem 1 first plotting

```
>> test_newton_f1
start X = [ 2.00000,  2.00000]
new X = [ 1.33333,  1.33333]
new X = [ 0.88889,  0.88889]
Terminating after 24 iterations because norm(delta) < tol.
Found minimum xstar = [ 0.000119; 0.000119 ]
```

(b) Problem 1 first result

Figure 1: Problem 1 first function

- $$\nabla f(x_1,x_2) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix} = \begin{pmatrix} \frac{x_1}{2} \\ \frac{2}{9}x_2 \end{pmatrix}$$

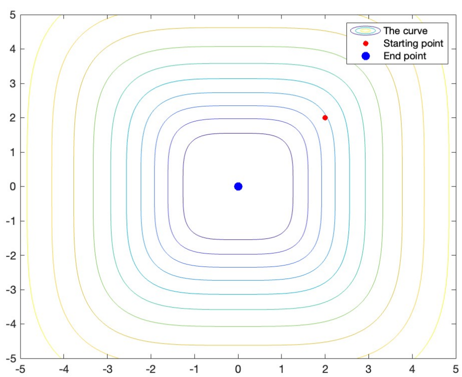
  $$H(f(x_1,x_2)) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{2}{9} \end{pmatrix}$$

  $$H^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & \frac{9}{2} \end{pmatrix}$$

  $$H^{-1}\nabla f = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

  We have
  $$\vec{x}^1 = [0,0]^T.$$
  Which means with one calculation we get the result. However, to truely stop the Newtons' method, we need to make the step size equals to 0. Actually we need another step. Here we have
  $$\vec{x}^2 = [0,0]^T.$$
  The step size $H^{-1}\nabla f$ is 0, which means we get the result, the Newton's method stops.

- The mainfunction is the same as before. The test function is **text_newton_f2.m**. The result should be figure 2b and the plotting should be figure 2a. The result shows that my calculation is correct.



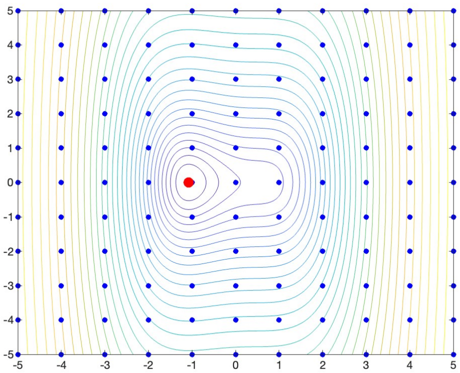
(a) Problem 1 second plotting

```
>> test_newton_f2
start X = [ 2.00000,  2.00000]
new X = [ 0.00000,  0.00000]
new X = [ 0.00000,  0.00000]
Terminating after 2 iterations because norm(delta) < tol.
Found minimum xstar = [ 0.000000; 0.000000 ]
```

(b) Problem 1 second result

Figure 2: Problem 1 second function

Problem 2.



(a) Problem 2 plotting

```
start X = [ 5.00000, -3.00000]
Terminating after 8 iterations because norm(delta) < tol.
Startpoint (5, -3). Found the minimum, the point is (-1.075380, 0.000000)
start X = [ 5.00000, -2.00000]
Terminating after 8 iterations because norm(delta) < tol.
Startpoint (5, -2). Found the minimum, the point is (-1.075380, 0.000000)
start X = [ 5.00000, -1.00000]
Terminating after 7 iterations because norm(delta) < tol.
Startpoint (5, -1). Found the minimum, the point is (-1.075380, 0.000000)
start X = [ 5.00000,  0.00000]
Terminating after 3 iterations because norm(delta) < tol.
Startpoint (5, 0). Found the minimum, the point is (-1.075380, 0.000000)
start X = [ 5.00000,  1.00000]
Terminating after 5 iterations because norm(delta) < tol.
Startpoint (5, 1). Found the minimum, the point is (-1.075380, 0.000000)
start X = [ 5.00000,  2.00000]
Terminating after 7 iterations because norm(delta) < tol.
Startpoint (5, 2). Found the minimum, the point is (-1.075380, 0.000000)
start X = [ 5.00000,  3.00000]
Terminating after 9 iterations because norm(delta) < tol.
Startpoint (5, 3). Found the minimum, the point is (-1.075380, 0.000000)
start X = [ 5.00000,  4.00000]
Terminating after 8 iterations because norm(delta) < tol.
Startpoint (5, 4). Found the minimum, the point is (-1.075380, 0.000000)
start X = [ 5.00000,  5.00000]
Terminating after 9 iterations because norm(delta) < tol.
Startpoint (5, 5). Found the minimum, the point is (-1.075380, 0.000000)
f_fail =0, f_success = 121
Matlab found the minimum, the point is (-1.075379, -0.000004)
```

(b) Problem 2 result

Figure 3: Problem 2

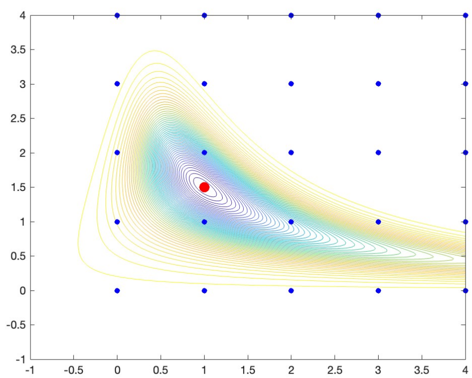
In this problem, I use the line search technique to ensure that every starting point can get to the minimum. The main function is the newton_optimizer.m. The test function is the **test_newton_f.m**. The plotting should be figure 3a. The result should be like figure 3b. In the plotting, the blue dots are the starting point and the red dot are the end point. We can see that they all end at the minimum.

Problem 3.

In this problem, I should use the newton's method with line search. The main function is the newton_optimizer.m. The test function is the **test_newton_f.m**. The plotting should be figure 4a. The result should be figure 4b. The red dot is the successful end point. We can see from the result that only 8 starting point successfully get to the result.

For the reason why it is not correct, check the eigenvalues in 5. We can see that most of the eigenvalues of the start points are negative definite and extremely small. The Hessian are also close to singular. These things make it hard to converge.

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -2f(x_1x_2 - a)x_2 \\ -2f\left[x_1(x_1x_2 - a) + (x_2 - a)\right] \end{bmatrix}$$



(a) Problem 3 plotting

```
ans =

   1.0e-89 *

   -0.2647
   -0.0001

Terminating after 1 iterations because norm(delta) < tol.
Startpoint (4, 4). Fail to find the minimum, the point is (4.080181, 0.431940)
f_fail =17, f_success = 8
Startpoint (0, 1). Found the minimum, the point is (1.000000, 1.500000)
Matlab found the minimum, the point is (0.999999, 1.499998)
Startpoint (0, 2). Found the minimum, the point is (1.000000, 1.500000)
Matlab found the minimum, the point is (0.999999, 1.499998)
Startpoint (0, 3). Found the minimum, the point is (1.000000, 1.500000)
Matlab found the minimum, the point is (0.999999, 1.499998)
Startpoint (1, 3). Found the minimum, the point is (1.000000, 1.500000)
Matlab found the minimum, the point is (0.999999, 1.499998)
Startpoint (1, 4). Found the minimum, the point is (1.000000, 1.500000)
Matlab found the minimum, the point is (0.999999, 1.499998)
Startpoint (2, 1). Found the minimum, the point is (1.000000, 1.500000)
Matlab found the minimum, the point is (0.999999, 1.499998)
Startpoint (2, 2). Found the minimum, the point is (1.000000, 1.500000)
Matlab found the minimum, the point is (0.999999, 1.499998)
Startpoint (3, 1). Found the minimum, the point is (1.000000, 1.500000)
Matlab found the minimum, the point is (0.999999, 1.499998)
```

(b) Problem 3 result

Figure 4: Problem 3

```
ans =

   1.0e-46 *

   -0.2950
   -0.0002

Terminating after 1 iterations because norm(delta) < tol.
Startpoint (3, 4). Fail to find the minimum, the point is (3.102354, 0.579665)
start X = [ 4.00000,  0.00000]

ans =

   -2.1218
         0

Hessian matrix is close to singular, exiting after 1 iterations.
Startpoint (4, 0). Fail to find the minimum, the point is (4.000000, 0.000000)
```

Figure 5: The eigenvalues