Problem 1.

- $e(\beta) = \sum_{n=1}^{N} [\beta_1(1-\cos(\beta_2 x_n)) - y_n]^2$
- $r_n(\beta) = \beta_1(1-\cos(\beta_2 x_n)) - y_n$
- The Gradient is:

$$\nabla e(\beta) = \begin{pmatrix} \sum_{n=1}^{N} 2(\beta_1(1-\cos(\beta_2 x_n)) - y_n)(1-\cos(\beta_2 x_n)) \\ \sum_{n=1}^{N} 2x_n\beta_1 \sin(\beta_2 x_n)(\beta_1(1-\cos(\beta_2 x_n)) - y_n) \end{pmatrix}$$

- The Hessian ignoring the higher term is

$$H(\beta) = \begin{pmatrix} \sum_{n=1}^{N} 2(1-\cos(\beta_2 x_n))^2 & \begin{matrix} \sum_{n=1}^{N} 4x_n\beta_1 \sin(\beta_2 x_n)(1-\cos(\beta_2 x_n)) \\ -2y_n x_n \sin(\beta_2 x_n) \end{matrix} \\ \sum_{n=1}^{N} 2x_n \sin(\beta_2 x_n)(2\beta_1(1-\cos(\beta_2 x_n)) - y_n) & 2\sum_{n=1}^{N} \beta_1^2 x_n^2 \sin^2(\beta_2 x_n) \end{pmatrix}$$

-

$$J = \begin{pmatrix} 1-\cos(\beta_2 x_1) & \beta_1 x_1 \sin(\beta_2 x_1) \\ 1-\cos(\beta_2 x_2) & \beta_1 x_2 \sin(\beta_2 x_2) \\ \vdots & \vdots \\ 1-\cos(\beta_2 x_N) & \beta_1 x_N \sin(\beta_2 x_N) \end{pmatrix}$$

-

$$B = \begin{pmatrix} 2\sum_{n=1}^{N}(1-\cos(\beta_2 x_n))^2 & 2\sum_{n=1}^{N}(1-\cos(\beta_2 x_n))\beta_1 x_n \sin(\beta_2 x_n) \\ 2\sum_{n=1}^{N}(1-\cos(\beta_2 x_n))\beta_1 x_n \sin(\beta_2 x_n) & 2\sum_{n=1}^{N} \beta_1 x_n^2 \sin^2(\beta_2 x_n) \end{pmatrix}$$

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$$H - B = \begin{pmatrix} 0 & 2\sum_{n=1}^{N} x_n \sin(\beta_2 x_n)(\beta_1(1-\cos(\beta_2 x_n)) - y_n) \\ 2\sum_{n=1}^{N} x_n \sin(\beta_2 x_n)(\beta_1(1-\cos(\beta_2 x_n)) - y_n) & 0 \end{pmatrix}$$

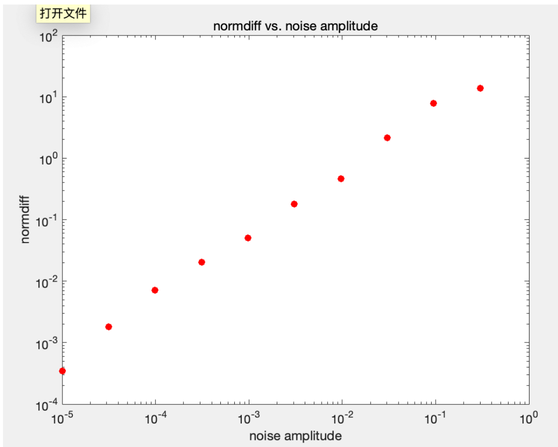
  Here we can see that when $(\beta_1(1-\cos(\beta_2 x_n)) - y_n)$ close to 0, H is nearly B. Which means that when the result we estimate is close to the truth $B \approx H$ valid.

Problem 2.

Here we have the **test_p2** is the test function. The result should be like figure 1b abd the plotting should be like figure 1a.

Problem 3.

Here we have the main function is the levenberg_marquardt.m. The test function is the **test_data.m**. The plot should be like figure 2a, and the result should be like figure 2b. Compare to the result in the website which is $[2.0196866396E-01, -6.1953516256E-06, 1.2044556708E+03, -1.8134269537E+02]$, my result is nearly the same.



(a) Problem 2 plotting

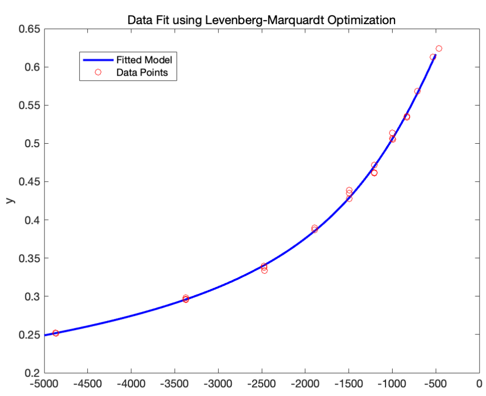
```
>> test_p2

normdiff =

    0.0003    0.0018    0.0072    0.0204    0.0501    0.1793    0.4657    2.1651    7.7381   13.6769
```

(b) Problem 2 result

Figure 1: Problem 2



(a) Problem 3 plotting

```
Terminating after 11 iterations because norm(delta) < tol.
Beta is 0.201969, -0.000006, 1204.455674, -181.342697>>
```

(b) Problem 3 result

Figure 2: Problem 3