Problem 1.

- The expression for the objective function in 2 dimensions is:

$$f(x_1, x_2) = \left[(1+\beta)(x_1 - 1) + (2+\beta)\left(x_2 - \frac{1}{2}\right)\right]^2 + \left[(1+\beta)\left(x_1^2 - 1\right) + (2+\beta)\left(x_2^2 - \frac{1}{4}\right)\right]^2$$

- The problem is:

$$\min_{x_1, x_2} f(x_1, x_2) \text{ subject to } x_2 = x_1^2 - 1.$$

- Let:

$$S_1 = (1+\beta)(x_1 - 1) + (2+\beta)\left(x_2 - \frac{1}{2}\right)$$
$$S_2 = (1+\beta)(x_1^2 - 1) + (2+\beta)\left(x_2^2 - \frac{1}{4}\right)$$

The gradient is:

$$\nabla f = \begin{bmatrix} 2(1+\beta)S_1 + 4(1+\beta)x_1 S_2 \\ 2(2+\beta)S_1 + 4(2+\beta)x_2 S_2 \end{bmatrix}$$

The Hessian matrix is:

$$H = \begin{bmatrix} 2(1+\beta)^2 + 8(1+\beta)^2 x_1^2 + 4(1+\beta)S_2 & 2(1+\beta)(2+\beta) + 8(1+\beta)(2+\beta)x_1 x_2 \\ 2(1+\beta)(2+\beta) + 8(1+\beta)(2+\beta)x_1 x_2 & 2(2+\beta)^2 + 8(2+\beta)^2 x_2^2 + 4(2+\beta)S_2 \end{bmatrix}$$

- The expression for the quadratic model is:

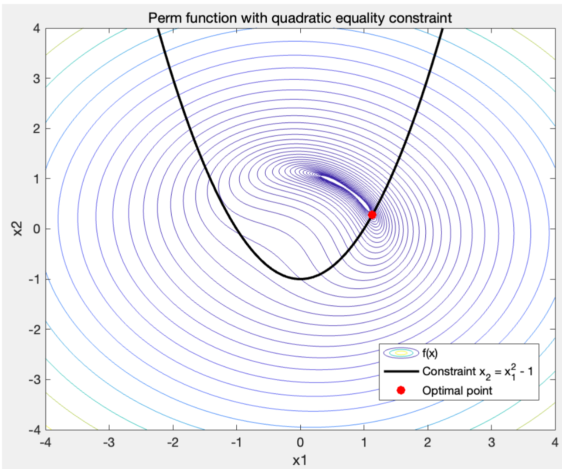
$$m_k(p) = f(\mathbf{x}_k) + \begin{bmatrix} \frac{\partial f}{\partial x_1}(\mathbf{x}_k) & \frac{\partial f}{\partial x_2}(\mathbf{x}_k) \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} p_1 & p_2 \end{bmatrix} \begin{bmatrix} H_{11}(\mathbf{x}_k) & H_{12}(\mathbf{x}_k) \\ H_{21}(\mathbf{x}_k) & H_{22}(\mathbf{x}_k) \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

- The expression for the linearized constraints is:

$$\begin{bmatrix} -2x_{k1}p_1 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = -(x_{k2} - x_{k1}^2 + 1)$$

- The test function in this problem is **test_P1().m**. Here I use $\beta = 10$. The plotting is shown in the 1a. The result is shown in 1b. The red dot is the optimal point.

Problem 2.

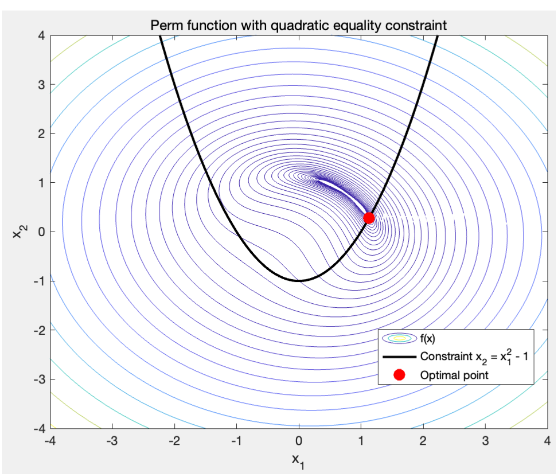


(a) Problem 1 plotting1

```
---------  cnt = 7  -----------
unp1 =
   1.0e+03 *

    0.0011
    0.0003
    3.0224


ustar =

   1.0e+03 *

    0.0011
    0.0003
    3.0224

Optimal solution:
x_1 = 1.132822
x_2 = 0.283285
Minimum function value: f(x) = 2.463097
```

(b) Problem 1 result

Figure 1: Problem 1



(a) Problem 2 plotting

```
>> test_P2
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
    0           3    4.850000e+02     1.000e+00     1.000e+00     0.000e+00     4.080e+02
    1          21    4.337902e+02     2.157e+00     4.748e-03     1.776e+00     1.663e+03
    2          24    1.658031e+02     5.902e-01     1.000e+00     9.571e-01     3.373e+02
    3          27    2.758469e+00     1.801e-02     1.000e+00     8.710e-01     3.878e+01
    4          30    2.605041e+00     2.324e-04     1.000e+00     2.268e-02     2.095e+01
    5          33    2.463670e+00     4.400e-05     1.000e+00     1.656e-02     8.149e-01
    6          36    2.463097e+00     6.376e-09     1.000e+00     1.585e-04     6.854e-03
    7          39    2.463097e+00     1.416e-12     1.000e+00     2.988e-06     6.658e-05
    8          42    2.463097e+00     1.110e-16     1.000e+00     5.718e-13     7.370e-06

找到满足约束的局部最小值。

优化已完成，因为目标函数沿
可行方向在最优性容差值范围内呈现非递减，
并且在约束容差值范围内满足约束。

<停止条件详细信息>
Optimal solution:
x_1 = 1.132821
x_2 = 0.283285
Minimum function value: f(x) = 2.463097
```

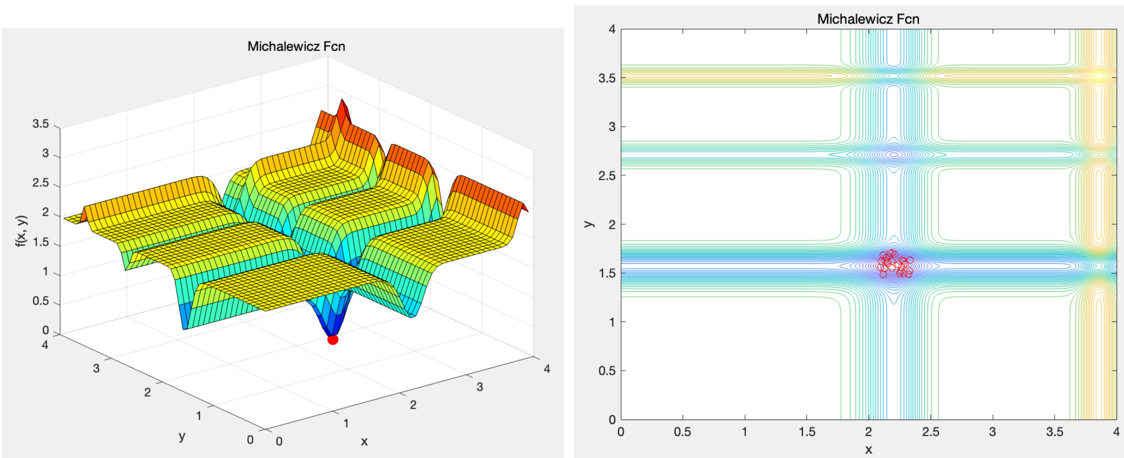
(b) Problem 2 result

Figure 2: Problem 2

The test function in this problem is **test_P2().m**. Here I use $\beta = 10$. The plotting is shown in the 2a. The result is shown in 2b. The result is the same as what in the problem 1.

Problem 3.

Here the test function is **test_P3.m**. The plotting should be like figure 3a and figure 3b. The red dot in figure 3a is the minimum point. The result is like figure 4. Compare to the website result, they are nearly the same. Although it almost always give the true answer, it might sometimes results in a local minimum.



(a) Problem 3 plotting1 (b) Problem 3 plotting2

Figure 3: Problem 3 plotting

```
>> test_P3
Hit any key to continue ...
At end of run, estimated xstar = [2.206250, 1.546285]
Obj fcn value = 2.225630e-01
Original obj fcn value = -1.777437e+00

ans =

    2.2062    1.5463
```

Figure 4: Problem 3 result