2. $\begin{cases} \dot{x} = x(2-x-y) \\ \dot{y} = y(3-2x-2y) \end{cases}$ fixed points: $\begin{cases} x=0 \\ y=0 \end{cases}$ $\begin{cases} x=0 \\ 3-2x-2y=0 \end{cases}$ $\begin{cases} 2-x-y=0 \\ y=0 \end{cases}$

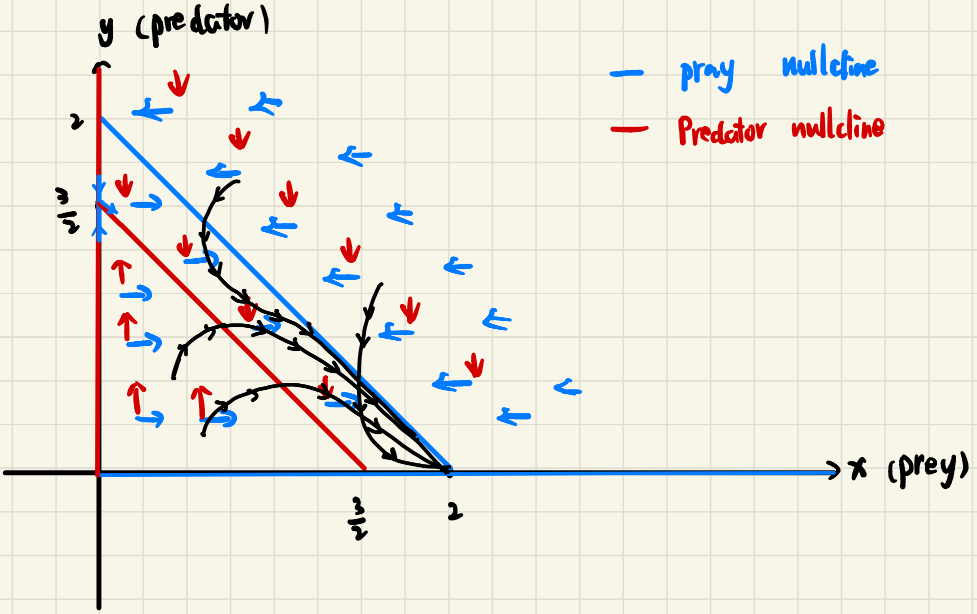
$\begin{cases} 2-x-y=0 \\ 3-2x-2y=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases}$ $\begin{cases} x=0 \\ y=\frac{3}{2} \end{cases}$ $\begin{cases} x=2 \\ y=0 \end{cases}$

The Jacobi matrix is : $\begin{bmatrix} -2x+2-y & -x \\ -2y & -2x+3-4y \end{bmatrix}$

$[0,0]$: $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ $T=5$ $\Delta=6$ $\lambda_1=2$ $\lambda_2=3$ It's a source. eigenvector: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$[0,\frac{3}{2}]$: $\begin{pmatrix} \frac{1}{2} & 0 \\ -3 & -3 \end{pmatrix}$ $T=-\frac{5}{2}$ $\Delta=-\frac{3}{2}$ $\lambda_1=\frac{1}{2}$ $\lambda_2=-3$ It's a saddle point eigenvector: $\begin{bmatrix} 7 \\ -6 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$[2,0]$: $\begin{pmatrix} -2 & -2 \\ 0 & -1 \end{pmatrix}$ $T=-3$ $\Delta=2$ $\lambda_1=-1$ $\lambda_2=-2$ It's a stable node eigenvector: $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$



3\. (a) $\dot{h}=0 \Rightarrow h=0$ or $s=0$

$\dot{s}=0 \Rightarrow kh-l=0$ or $s=0$ $\Rightarrow$ fixed point $(H,0)$ (H is what ever a number s.t $H\geq 0$)

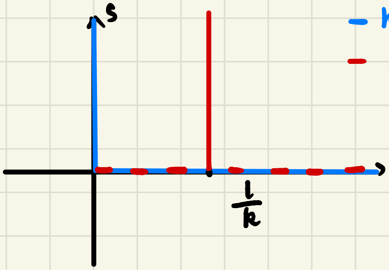
Jacobi matrix is $\begin{pmatrix} -ks & -kh \\ ks & kh-l \end{pmatrix}$ Take $\begin{cases} h=H \\ s=0 \end{cases}$

It is $\begin{pmatrix} 0 & -kH \\ 0 & kH-l \end{pmatrix}$ $T=kH-l$ $\Delta=0$ $\lambda_1=0$, $\lambda_2=kH-l$ eigen vecter: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} kH \\ l-kH \end{bmatrix}$

if $H>\frac{l}{k}$, then its an unstable node

if $H<\frac{l}{k}$, then its a stable node.

(b)



— h nullclines

— s nullclines

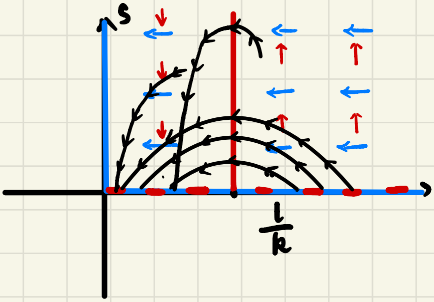
(x axis is h nullclines as well as s nullclines)

(c) $\frac{ds}{dh}=\frac{khs-ls}{-khs}=-1+\frac{l}{kh} \Rightarrow ds=-dh+\frac{l}{k}\cdot\frac{dh}{h}$

$\int ds=\int -dh+\frac{l}{k}\cdot\frac{dh}{h} \Rightarrow s=-h+\frac{l}{k}\ln h+C$

$C=s+h-\frac{l}{k}\ln h$ is the conserved quantity.

(d)



(e) On the phase portrait we know that if $h_0>\frac{l}{k}$, the epidemic occur.