1.1) Let M be the mass of carbon monoxide.

$$M' = R_{in} \cdot C_{in} - R_{out} \cdot C_{out} \qquad C = \frac{M}{V} \quad C' = \frac{M'}{V}$$

$$M' = R_{in} \cdot 4\% - R_{out} \cdot \frac{M}{V} \qquad M' = C'V$$

$$C' = \frac{R_{in} \cdot 4\% - R_{out} \cdot C}{V} \qquad R_{in} = 0.006 m^3/min. \quad R_{out} = 10 R_{in}$$

$$C'V + R_{out} \cdot C = R_{in} \cdot 4\% \qquad \text{product } e^{\frac{R_{out}}{V}t}$$

$$(Ce^{\frac{R_{out}}{V}t})' = (R_{in} \cdot 4\% \cdot e^{\frac{R_{out}}{V} \cdot t}) \cdot \frac{1}{V}$$

$$\Rightarrow \quad Ce^{\frac{R_{out}}{V}t} = R_{in} \cdot 4\% \cdot \frac{1}{R_{out}} \cdot e^{\frac{R_{out}}{V} \cdot t} + K$$

$$\Rightarrow \quad C = \frac{R_{in}}{R_{out}} \cdot 4\% + \frac{K}{e^{\frac{R_{out}}{V}t}} \qquad V = 1200 \ m^3$$

$$C(0) = 0 \Rightarrow \quad K = -0.004 \qquad C = 0.004 - \frac{0.004}{e^{0.00005t}}$$

2) $C = 0.012\% \Rightarrow$

$$0.012\% = 0.004 - \frac{0.004}{e^{0.00005t}}$$

$$t = -20000 \log \frac{97}{100}$$

$$\approx 609 \text{ min.} \approx 10 \text{ hours}$$

2. (a). $M'(t) = 0.01 r M(t)$ $\quad M' - 0.01 r M = 0$

$\Rightarrow \quad (Me^{-0.01rt})' = 0$

$\Rightarrow \quad M = Ke^{0.01rt}$ $\quad K$ is a constant

$M(t) = 2M(0)$

$\Rightarrow \quad K \cdot e^{0.01rt} = 2 \cdot K \Rightarrow t = \frac{100}{r} \ln 2$

$\frac{100 \ln 2}{r} \approx \frac{69.314}{r} < \frac{72}{r}$

So the rules always overestimates

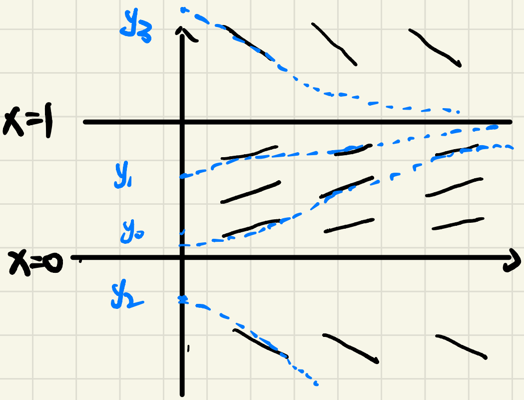
(b) The rate range from 7% to 10%.

$M = K \cdot e^{0.01rt} = 7000\, e^{0.01 \times 8 \times 65} \approx 1268906.$

if the rate is 8% or higher, it is true. however. if the rate is 7%

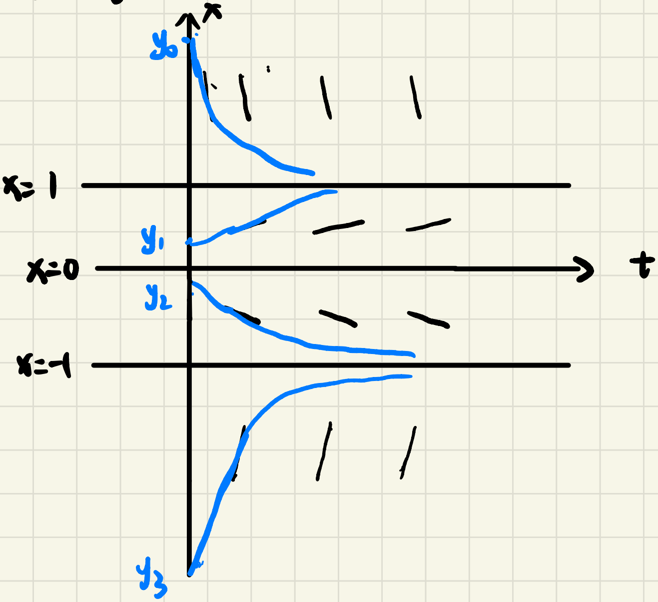
then $7000 e^{0.01 \times 7 \times 65} \approx 0.66$ million it is not true, but still close

3 (a). $[a.1] = \dot{x} = x(1-x) \Rightarrow x=0$ or $x=1$



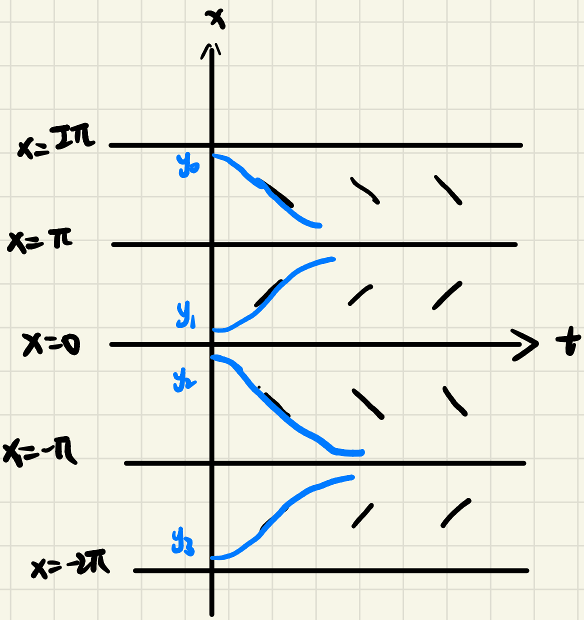
x	$\dot{x} = x(1-x)$
$-\frac{1}{2}$	$-\frac{3}{4}$
$\frac{1}{4}$	$\frac{3}{16}$
$\frac{1}{2}$	$\frac{1}{4}$
$\frac{3}{4}$	$\frac{3}{16}$
$\frac{3}{2}$	$-\frac{3}{4}$

[a.2] $x(1-x^2) = x(1-x)(1+x)$ $\quad x=0,\ x=-1,\ x=1$



x	$\dot{x} = x - x^3$
-2	6
$-\frac{1}{2}$	$-\frac{3}{8}$
$\frac{1}{2}$	$\frac{3}{8}$
2	-6

[a.3] $\dot{x} = \sin(x)$ $\quad \dot{x} = 0 \Rightarrow x = k\pi$



x	$\dot{x} = \sin x$
$-\frac{3\pi}{2}$	1
$-\frac{\pi}{2}$	-1
$\frac{\pi}{2}$	1
$\frac{3}{2}\pi$	-1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$-\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$-\frac{3}{4}\pi$	$-\frac{\sqrt{2}}{2}$
$-\frac{3}{4}\pi$	$-\frac{\sqrt{2}}{2}$

CC 1. $\begin{cases} \dot{Q} = \frac{V_0}{R} - \frac{Q}{RC} \\ Q(0) = 0 \end{cases}$ $\dot{Q} + \frac{Q}{RC} = \frac{V_0}{R}$

$$\Rightarrow \dot{Q} e^{\frac{t}{RC}} + \frac{Q}{RC} \cdot e^{\frac{t}{RC}} = \frac{V_0}{R} e^{\frac{t}{RC}}$$

$$(Q e^{\frac{t}{RC}})' = \frac{V_0}{R} e^{\frac{t}{RC}}$$

$$Q e^{\frac{t}{RC}} = RC \frac{V_0}{R} e^{\frac{t}{RC}} + K$$

$$Q e^{\frac{t}{RC}} = CV_0 e^{\frac{t}{RC}} + K$$

$$Q = CV_0 + \frac{K}{e^{\frac{t}{RC}}} \qquad Q(0) = 0 \Rightarrow K = -CV_0$$

$$\Rightarrow Q = CV_0 - \frac{CV_0}{e^{\frac{t}{RC}}}$$

4. $f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(x) + \frac{h^4}{24} f^{(4)}(x) + \frac{h^5}{120} f^{(5)}(x) \cdots$

$f(x+2h) = f(x) + 2h f'(x) + \frac{4h^2}{2} f''(x) + \frac{8h^3}{6} f'''(x) + \frac{16h^4}{24} f^{(4)}(x) + \frac{32h^5}{120} f^{(5)}(x)$

$f(x-h) = f(x) - h f'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(x) + \frac{h^4}{24} f^{(4)}(x) - \frac{h^5}{120} f^{(5)}(x) \cdots$

$f(x-2h) = f(x) - 2h f'(x) + \frac{4h^2}{2} f''(x) - \frac{8h^3}{6} f'''(x) + \frac{16h^4}{24} f^{(4)}(x) - \frac{32h^5}{120} f^{(5)}(x) \cdots$

(★)

assume that:

$$k_0 f(x) + k_1 h f'(x) + k_2 h^2 f''(x) + k_3 h^3 f'''(x)$$

$$= a_1 f(x+h) + a_2 f(x-h) + a_3 f(x+2h) + a_4 f(x-2h)$$

we get: $k_0 = 0 \Rightarrow a_1 + a_2 + a_3 + a_4 = 0$

$k_1 = 0 \Rightarrow a_1 - a_2 + 2a_3 - 2a_4 = 0$

$k_2 = 0 \Rightarrow a_1 + a_2 + 4a_3 + 4a_4 = 0$

$k_3 = 1 \Rightarrow \frac{a_1}{6} - \frac{a_2}{6} + \frac{4}{3}a_3 - \frac{4}{3}a_4 = 1$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & -2 \\ 1 & 1 & 4 & 4 \\ 1 & -1 & 8 & -8 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 6 \end{pmatrix} \Rightarrow \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

$$\Rightarrow h^3 f^{(3)}(x) \approx -f(x+h) + f(x-h) + \frac{1}{2} f(x+2h) - \frac{1}{2} f(x-2h)$$

$$\Rightarrow \quad f^{(3)}(x) \approx \frac{f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)}{2h^3}$$

$$e = \left| \frac{f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)}{2h^3} - f^{(3)}(x) \right|$$

$$= \left| \frac{k_0 f(x) + k_1 h f'(x) + k_2 h^2 f''(x) + k_3 h^3 f^{(3)}(x) + k_4 h^4 f^{(4)}(x) + k_5 h^5 f^{(5)}(x) + O(h^6)}{2h^3} \right.$$

$$\left. - f^{(3)}(x) \right|$$

$k_0 = k_1 = k_2 = 0 \quad \frac{k_3}{2} = 1$ as calutated before

$$\frac{k_4}{2} = \frac{1}{24}a_1 + \frac{1}{24}a_2 + \frac{16}{24}a_3 + \frac{16}{24}a_4 = 0$$

$$\frac{k_5}{2} = \frac{1}{120}a_1 - \frac{1}{120}a_2 + \frac{32}{120}a_3 - \frac{32}{120}a_4 = \frac{1}{4}$$

Also $\frac{k_6}{2} = \frac{1}{720}a_1 + \frac{1}{720}a_2 + \frac{64}{720}a_3 + \frac{64}{720}a_4 = 0$

so $$e = f^{(3)}(x) + \frac{\frac{1}{4}h^5 f^{(5)}(x)}{h^3} + \frac{O(h^7)}{2h^3} - f^{(3)}(x)$$

$$= \frac{1}{4}h^2 f^{(5)}(x) + O(h^4)$$

$$= \frac{h^2}{4} f^{(5)}(x) + O(h^4)$$