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AI61003 Linear Algebra for AI & ML
Assignment 01 – Problem 02

(a) To prove: $\text{avg}(\alpha x + \beta 1_n) = \alpha\, \text{avg}(x) + \beta$
where $x, 1_n \in \mathbb{R}^n$; $\alpha, \beta \in \mathbb{R}$

Let $y = \alpha x + \beta 1_n$ $(y \in \mathbb{R}^n)$

$$\text{avg}(y) = \frac{1_n^T \cdot y}{n}$$

$$= \frac{1_n^T}{n} \cdot (\alpha x + \beta 1_n)$$

($\because$ scalar multiplicat^n is distributive)

$$= \frac{1}{n}(\alpha 1_n^T x + \beta 1_n^T 1_n)$$

($\because 1_n^T 1_n = n$)

$$= \frac{1}{n}(\alpha 1_n^T x + \beta n)$$

$$= \alpha \left(\frac{1_n^T x}{n}\right) + \beta$$

(by definit^n of avg(x))

$$= \alpha\, \text{avg}(x) + \beta$$

$\Rightarrow \text{avg}(\alpha x + \beta 1_n) = \alpha\, \text{avg}(x) + \beta$
Hence proved.

(b) To prove: $\text{std}(\alpha x + \beta 1_n) = |\alpha|\, \text{std}(x)$
where $x, 1_n \in \mathbb{R}^n$; $\alpha, \beta \in \mathbb{R}$

Let $y = \alpha x + \beta 1_n$ $(y \in \mathbb{R}^n)$
& $\hat{y} = y - \left(\frac{1_n^T y}{n}\right) 1_n$

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By definition,

$$\text{std}(y) = \text{rms}(\hat{y}) = \|\hat{y}\|_2 / \sqrt{n}$$

$$\|\hat{y}\|_2^2 = \hat{y}^T \hat{y}$$

Now $\hat{y} = \alpha x + \beta 1_n - \frac{1}{n}\left(1_n^T(\alpha x + \beta 1_n)\right) 1_n$

($\because$ scalar multiplicat^n is distributive)

$$= \alpha x + \beta 1_n - \frac{1}{n}(\alpha 1_n^T x + \beta 1_n^T 1_n) 1_n$$

($\because 1_n^T 1_n = n$)

$$= \alpha x + \cancel{\beta 1_n} - \frac{1}{n}(\alpha 1_n^T x) 1_n - \cancel{\beta 1_n}$$

(by definition of $\hat{x}$)

$$= \alpha\left(x - \frac{1}{n}(1_n^T x) 1_n\right) = \alpha \hat{x}$$

$$\Rightarrow \|\hat{y}\|_2^2 = (\alpha\hat{x})^T(\alpha\hat{x}) = \alpha^2 \hat{x}^T \hat{x}$$

$$\|\hat{y}\|_2 = \sqrt{\alpha^2 \hat{x}^T \hat{x}} = |\alpha| \, \|\hat{x}\|_2$$

$$\Rightarrow \text{std}(y) = |\alpha| \, \|\hat{x}\|_2 / \sqrt{n}$$

(by definit^n of std(x))

$$= |\alpha| \, \text{std}(x)$$

$$\Rightarrow \text{std}(\alpha x + \beta 1_n) = |\alpha| \, \text{std}(x)$$

Hence proved.