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AI61003 Linear Algebra for AI & ML
Assignment 02 - Problem 05

Consider $Ax = b$, where $A \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$
The advantages in this case when A is orthogonal is/are as follows.

(1) colspace $(A) = \mathbb{R}^n$. $\therefore$ $b \in$ colspace (A).
Besides columns of A are linearly independent.
$\therefore$ if A is orthogonal, a unique sol^n of $Ax = b$ always exists $\forall\ b \in \mathbb{R}^n$.

(2) The unique sol^n of $Ax = b$ will be $x = A^{-1}b$ that for an orthogonal matrix can be written as $x = A^T b$.
Computat^n complexity of A^T is less than of A^{-1}. (easy availability of A^{-1}).

(3) $k_2(A) = 1$ for orthogonal matrix A.
$\Rightarrow \|\delta x\|_2 / \|x\|_2 \leq \|\delta b\|_2 / \|b\|_2$.
$\therefore$ The relative change in the 2-norm of the solution x is upper bounded by the relative change in the 2-norm of the response vector b.
Besides $k_2(A) = 1$ is the least possible value. So for orthogonal matrices the solutions are the least sensitive to perturbations in b.