PAGE NO. :

DATE : / /

AI61003 Linear Algebra for AI & ML
Assignment 01 - Problem 04

Consider $Ax = b$ where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.

Let $A = \begin{bmatrix} | & | & & | & & | \\ u_1 & u_2 & \dots & u_i & \dots & u_n \\ | & | & & | & & | \end{bmatrix}$

where $u_i \in \mathbb{R}^m$ for $i = 1 \dots n$.

col(A) is the columnspace of A defined as the span of all its column vectors.

$col(A) = span\{u_1, u_2 \dots, u_i, \dots u_n\}$

★ At least one solution of $Ax = b$ exists if and only if $b \in col(A)$.

★ A unique solution of $Ax = b$ exists if and only if

1. $b \in col(A)$
2. $\{u_1, u_2 \dots, u_i \dots, u_n\}$ is a set of linearly independent vectors, i.e. matrix A is of full column rank (and equivalently A has a left inverse).

These conditions can be equivalently written as following for a square matrix $A \in \mathbb{R}^{m \times m}$.

A unique solⁿ of $Ax = b$ exists if and only if A is invertible.

PAGE NO. :
DATE : / /

This is because in a square matrix ($\in R^{m\times m}$) if the column vectors are linearly independent then

$$col(A) = \mathbb{R}^m$$

and $\therefore$ $b \in col(A)$. (conditⁿ ① ✓)

And for a square matrix, it is invertible <u>iff</u> it has full rank.

(conditⁿ ② ✓)