PAGE NO. :
DATE : / /

AI61003 Linear Algebra for AI & ML
Assignment 01 - Problem 06

Consider $A \in \mathbb{R}^{m \times n}$. Left inverse of A is matrix $X \in \mathbb{R}^{n \times m}$ such that $XA = I_n$, where $I_n \in \mathbb{R}^{n \times n}$ is an identity matrix.

(a) $A^T = [1\ 0\ 0\ 1\ 0]$

Let A be left invertible with left inverse $X \in \mathbb{R}^{1 \times 5}$.

Let $X = [a\ b\ c\ d\ e]$

By definition.

$XA = [1]$

$\Rightarrow 1.a + 0.b + 0.c + 1.d + 0.e = 1$

$\Rightarrow a + d = 1$ (possible)

$\therefore$ A is left invertible (choose a, b, c, d, e with constraints)

All left inverses of A can be represented as $[a, b, c, 1-a, e]$ where $a, b, c, e \in \mathbb{R}$.

$\therefore$ Left Inv $(A) = \{ X \mid X \in \mathbb{R}^{1 \times 5}$ and $X.e_1 = 1 - X.e_4 \}$

(characterizat^n of all left inverses of A)

(b) $A = \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ 3 & 3 \end{bmatrix}$

Let A be left invertible with left inverse $X \in \mathbb{R}^{2 \times 3}$

Let $X = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

PAGE NO. :
DATE : / /

By definit^{n}

$$XA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a+3c & 3c-2b \\ 2d+3f & 3f-2e \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $2a + 3c = 3f - 2e = 1$
$3c - 2b = 2d + 3f = 0$

$\Rightarrow$ $b = 3c/2$, $a = (1-3c)/2$ (possible)
$d = -3f/2$, $e = (3f-1)/2$ (possible)

$\therefore$ A is invertible. (choose a, b, c, d, e, f with constraints)

All left inverses of A can be characterized as $\begin{bmatrix} (1-3c)/2 & 3c/2 & c \\ -3f/2 & (3f-1)/2 & f \end{bmatrix}$

$$\therefore \text{Left Inv}(A) = \left\{ \begin{bmatrix} -(3\alpha-1)/2 & 3\alpha/2 & \alpha \\ -3\beta/2 & (3\beta-1)/2 & \beta \end{bmatrix} \;\middle|\; \alpha, \beta \in \mathbb{R} \right\}$$

(characterizat^{n} of all left inverses of A)