AI61003 Linear Algebra for AI & ML
Assignment 02 - Problem 06

(a) $A \in \mathbb{R}^{MN \times 4}$ (size : $MN \times 4$)

$A_{k1} = 1$, $1 \le k \le MN$

$A_{k2} = x_{\lfloor (k-1)/N \rfloor + 1}$, $1 \le k \le MN$

$A_{k3} = y_{k - N\lfloor (k-1)/N \rfloor}$, $1 \le k \le MN$

$A_{k4} = A_{k2} \cdot A_{k3}$, $1 \le k \le MN$

$$A = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_1 & y_2 & x_1 y_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_1 & y_N & x_1 y_N \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_M & y_1 & x_M y_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_M & y_N & x_M y_N \end{bmatrix}_{MN \times 4}$$

$\theta \in \mathbb{R}^4$ (size : 4×1)

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}_{4 \times 1}$$

(size : MN x 1)

$b \in \mathbb{R}^{MN}$; $b_k = F_{ij}$ $(1 \le k \le MN)$

where $i = \lfloor (k-1)/N \rfloor + 1$

$j = k - N\lfloor (k-1)/N \rfloor$

$F_{ij} = f(P_{ij}) = f(x_i, y_j)$

$(i = 1, 2 \ldots M \; ; \; j = 1, 2 \ldots N)$

$$b = \begin{bmatrix} F_{11} \\ F_{12} \\ \vdots \\ F_{1N} \\ \vdots \\ F_{M1} \\ \vdots \\ F_{MN} \end{bmatrix}_{MN \times 1}$$

Hence the interpolat^{n} conditions can be written in the form $A\theta = b$ where A, θ, b are as stated here.

$$A\theta = b$$

(b) The minimum values of M and N so as to expect a unique sol^{n} to $A\theta = b$ is 2, ∵ only then will all the columns be linearly independent.

If M is 1 then 1st & 2nd columns of A and 3rd & 4th columns of A will be linearly dependent.

Similarly if N is 1 then 1st & 3rd and 2nd & 4th columns of A are linearly dependent.

If M and N are at least two then no two columns can be lin. dep. because the values of x_i's and y_i's are strictly increasing (given). Hence, $A\theta = b$ can have a unique sol^{n}.