CSCI203

Algorithms and Data Structures

Huffman Trees

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Encoding messages

- Encode a message composed of a string of characters
- Codes used by computer systems
  - ASCII
    - uses 8 bits per character
    - can encode 256 characters
  - Unicode
    - 16 bits per character
    - can encode 65536 characters
    - includes all characters encoded by ASCII
- ASCII and Unicode are *fixed-length codes*
  - all characters represented by same number of bits

Problems

- Suppose that we want to encode a message constructed from the symbols **A**, **B**, **C**, **D**, and **E** using a fixed-length code
  - How many bits are required to encode each symbol?
    - at least 3 bits are required
    - 2 bits are not enough (can only encode four symbols)
  - How many bits are required to encode the message **DEAACAAAAABA**?
    - there are twelve symbols, each requires 3 bits
    - 12*3 = 36 bits are required

Drawbacks of fixed-length codes

- Wasted space
  - Unicode uses twice as much space as ASCII
    - inefficient for plain-text messages containing only ASCII characters
- Same number of bits used to represent all characters
  - 'a' and 'e' occur more frequently than 'q' and 'z'

- **Potential solution**: use variable-length codes
  - variable number of bits to represent characters when frequency of occurrence is known
  - short codes for characters that occur frequently

Advantages of variable-length codes

- The advantage of variable-length codes over fixed-
  - length is short codes can be given to characters that occur frequently
  - on average, the length of the encoded message is less than fixed-length encoding
- **Potential problem:** how do we know where one character ends and another begins?
  - not a problem if number of bits is fixed!

A = 00
B = 01
C = 10
D = 11

00101101110011111111111

A C D B A D D D D D

Prefix property

- A code has the **prefix property** if no character code is the prefix (start of the code) for another character
- Example:

Symbol	Code
P	000
Q	11
R	01
S	001
T	10

01001101100010

R S T Q P T

- 000 is not a prefix of 11, 01, 001, or 10
- 11 is not a prefix of 000, 01, 001, or 10 …

Code without prefix property

- The following code does **not** have prefix property

Symbol	Code
P	0
Q	1
R	01
S	10
T	11

- The pattern **1110** can be decoded as **QQQP**, **QTP**, **QQS**, or **TS**

Problem

- Design a variable-length prefix-free code such that the message **DEAACAAAAABA** can be encoded using 22 bits
- Possible solution:
  - **A** occurs eight times while **B**, **C**, **D**, and **E** each occur once
  - represent **A** with a one bit code, say 0
    - remaining codes cannot start with 0
  - represent **B** with the two bit code 10
    - remaining codes cannot start with 0 or 10
  - represent **C** with 110
  - represent **D** with 1110
  - represent **E** with 11110

Encoded message

DEAACAAAAABA

Symbol	Code
A	0
B	10
C	110
D	1110
E	11110

1110111100011000000100

22 bits

Another possible code

DEAACAAAAABA

Symbol	Code
A	0
B	100
C	101
D	1101
E	1111

1101111100101000001000

22 bits

Better code

DEAACAAAAABA

Symbol	Code
A	0
B	100
C	101
D	110
E	111

11011100101000001000

20 bits

What code to use?

- Question:
  - Is there a variable-length code that makes the most efficient use of space?

Answer: Yes!

Huffman coding tree

- Binary tree
  - each leaf contains symbol (character)
  - label edge from a node to its left child with 0
  - label edge from a node to its right child with 1
- Code for any symbol obtained by following path from root to the leaf containing symbol
- Code has prefix property
  - leaf node cannot appear on path to another leaf
  - *note*: fixed-length codes are represented by a complete Huffman tree and clearly have the prefix property

Building a Huffman tree

- Find frequencies of each symbol occurring in message
- Begin with a forest of single node trees
  - each contain symbol and its frequency
- Do recursively
  - select two trees with smallest frequency at the root
  - produce a new binary tree with the selected trees as children and store the sum of their frequencies in the root
- Recursion ends when there is one tree
  - this is the Huffman coding tree

Example

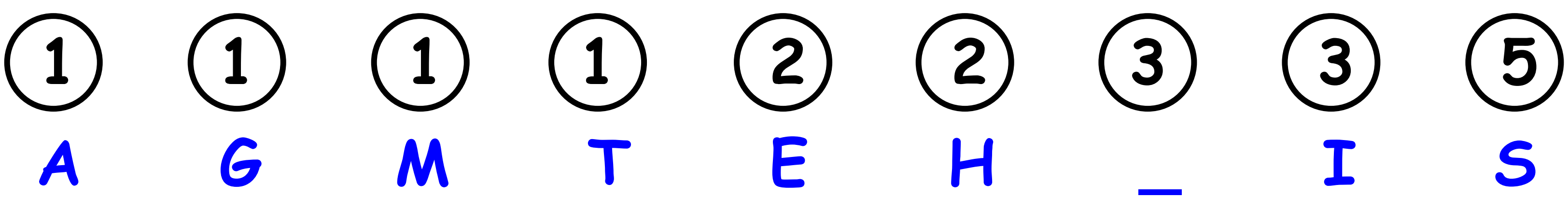
- Build the Huffman coding tree for the message

This is his message

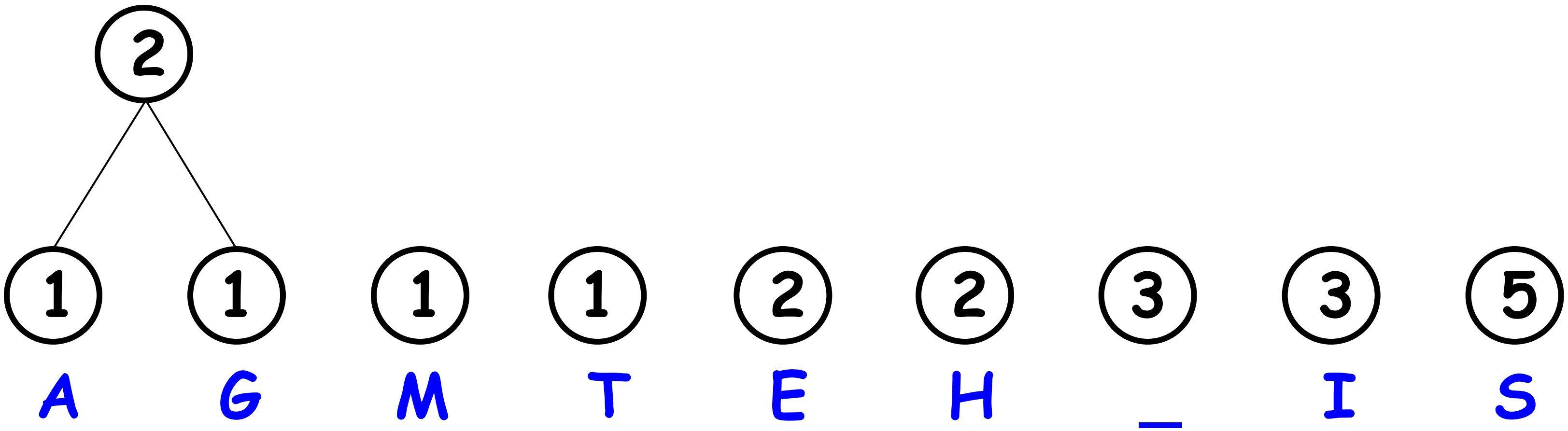
- Character frequencies

A	G	M	T	E	H	_	I	S
1	1	1	1	2	2	3	3	5

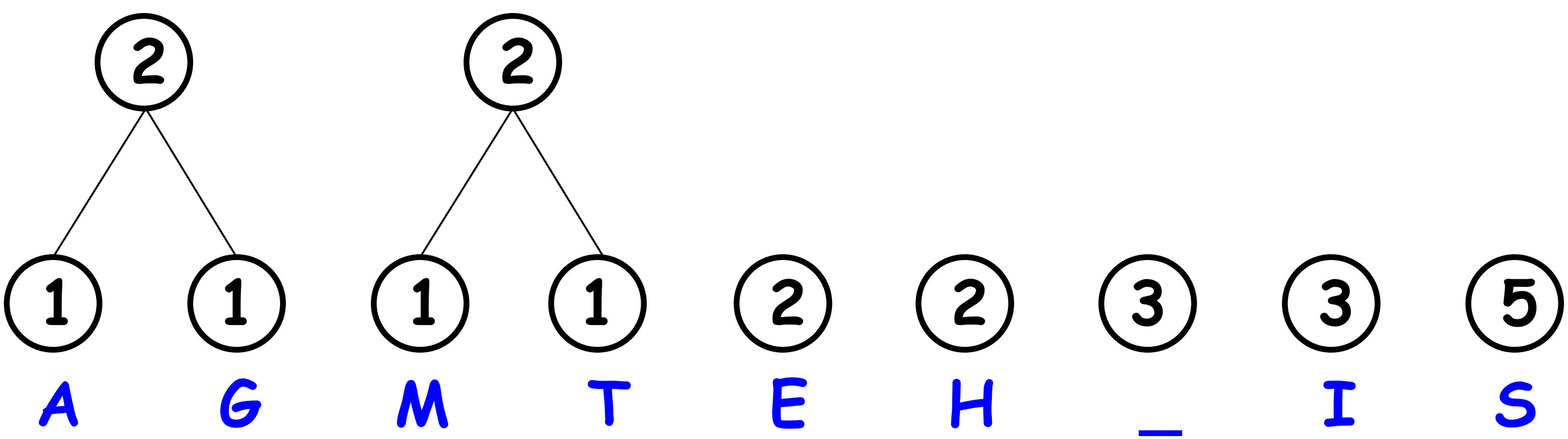
- Begin with forest of single trees



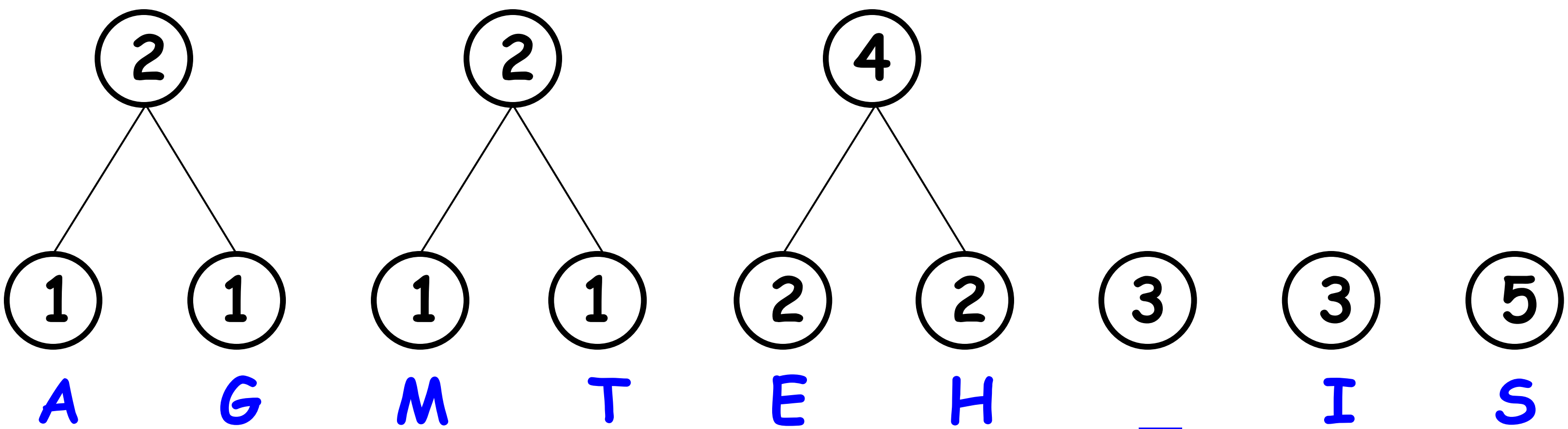
Step 1



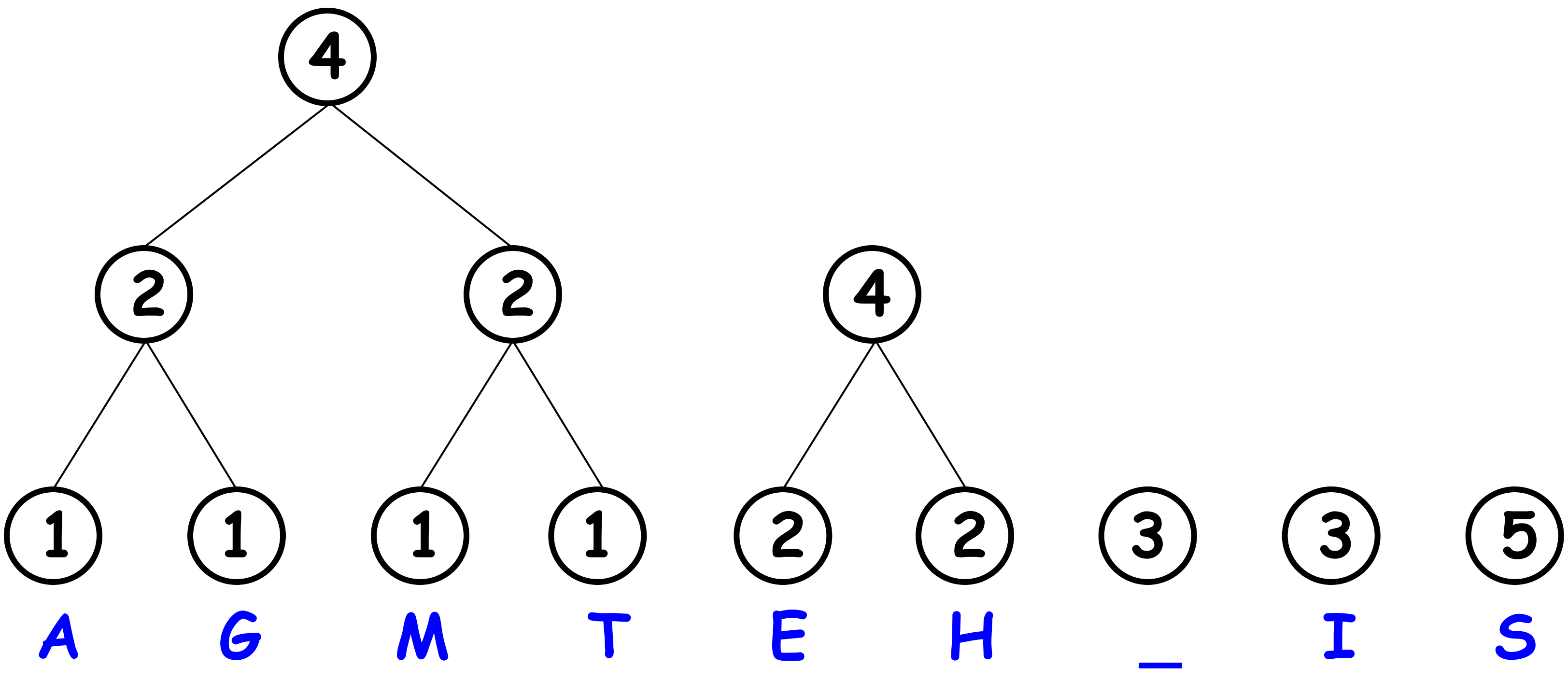
Step 2



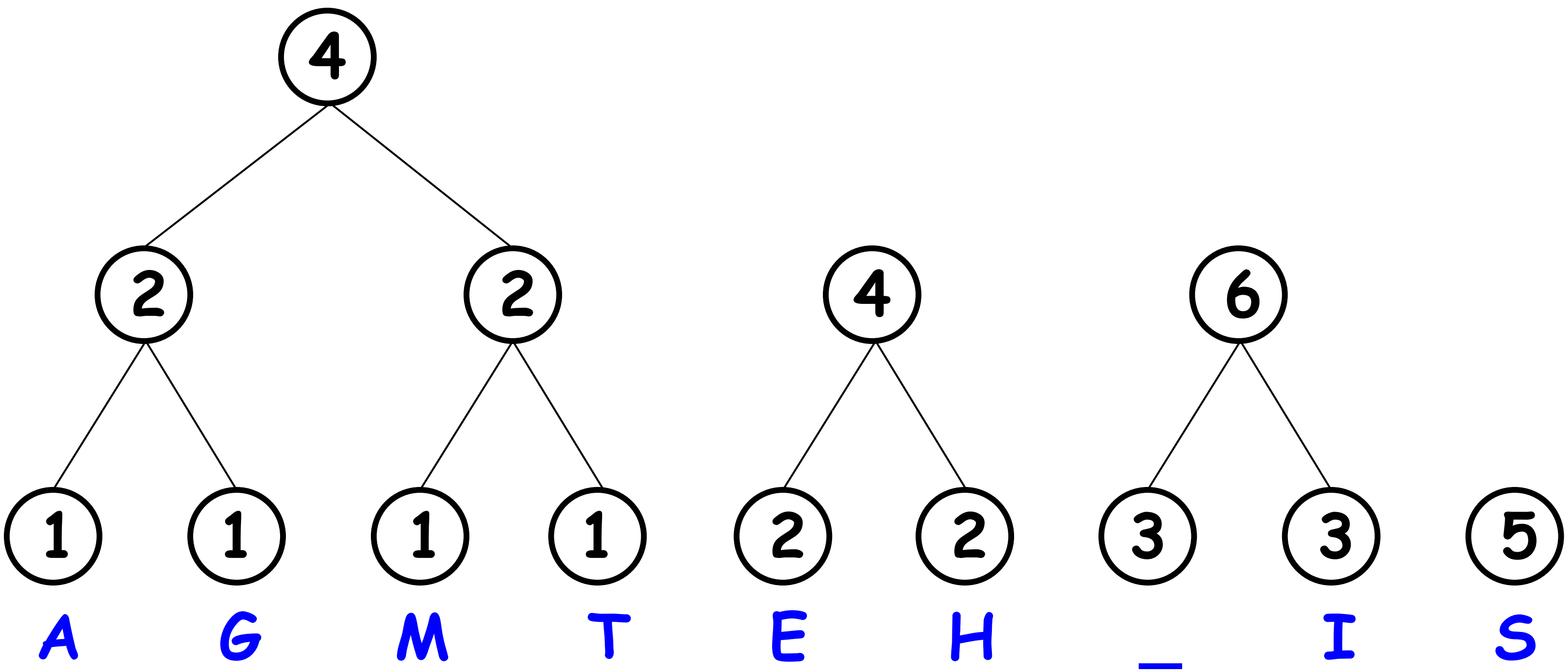
Step 3



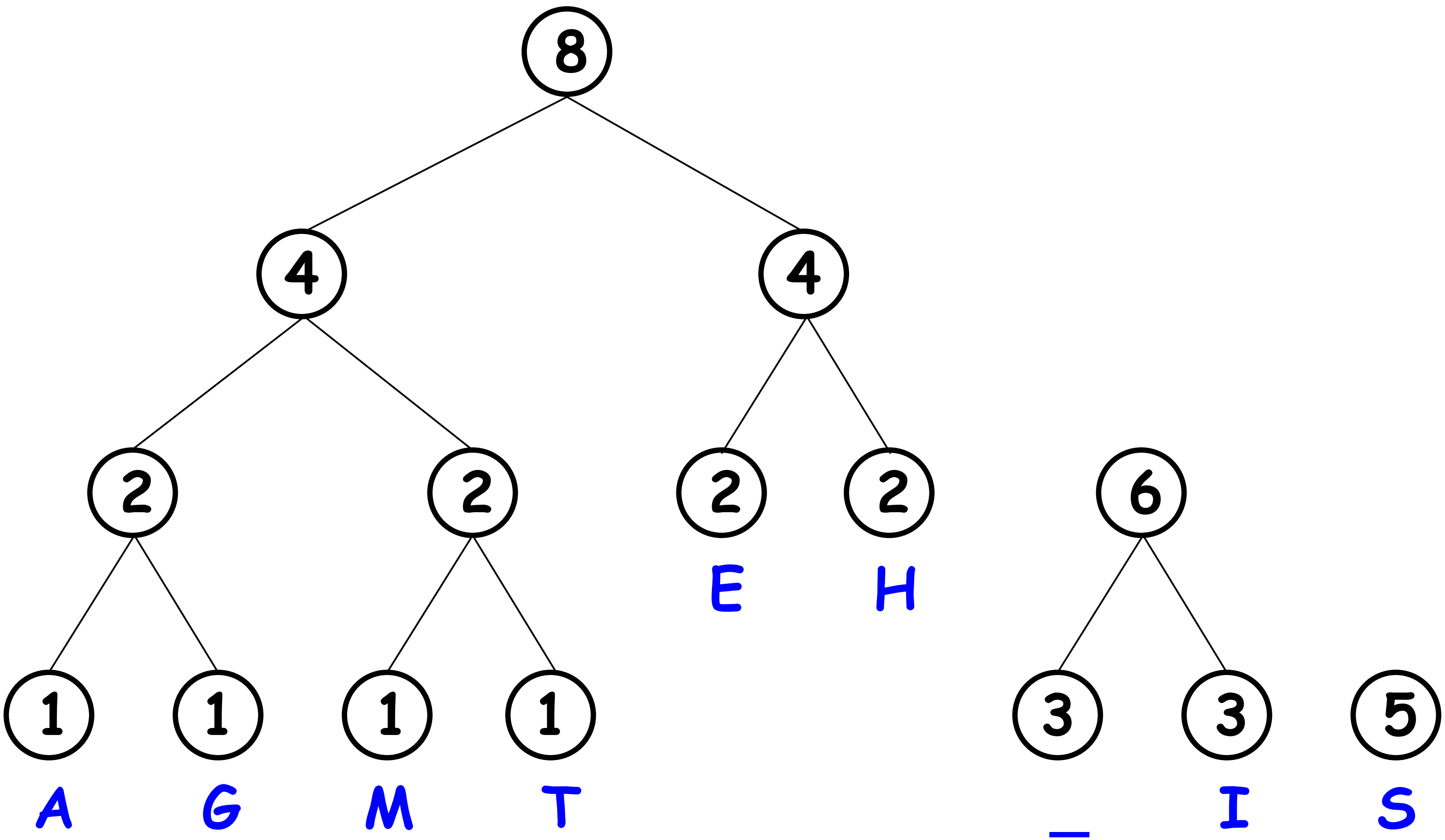
Step 4



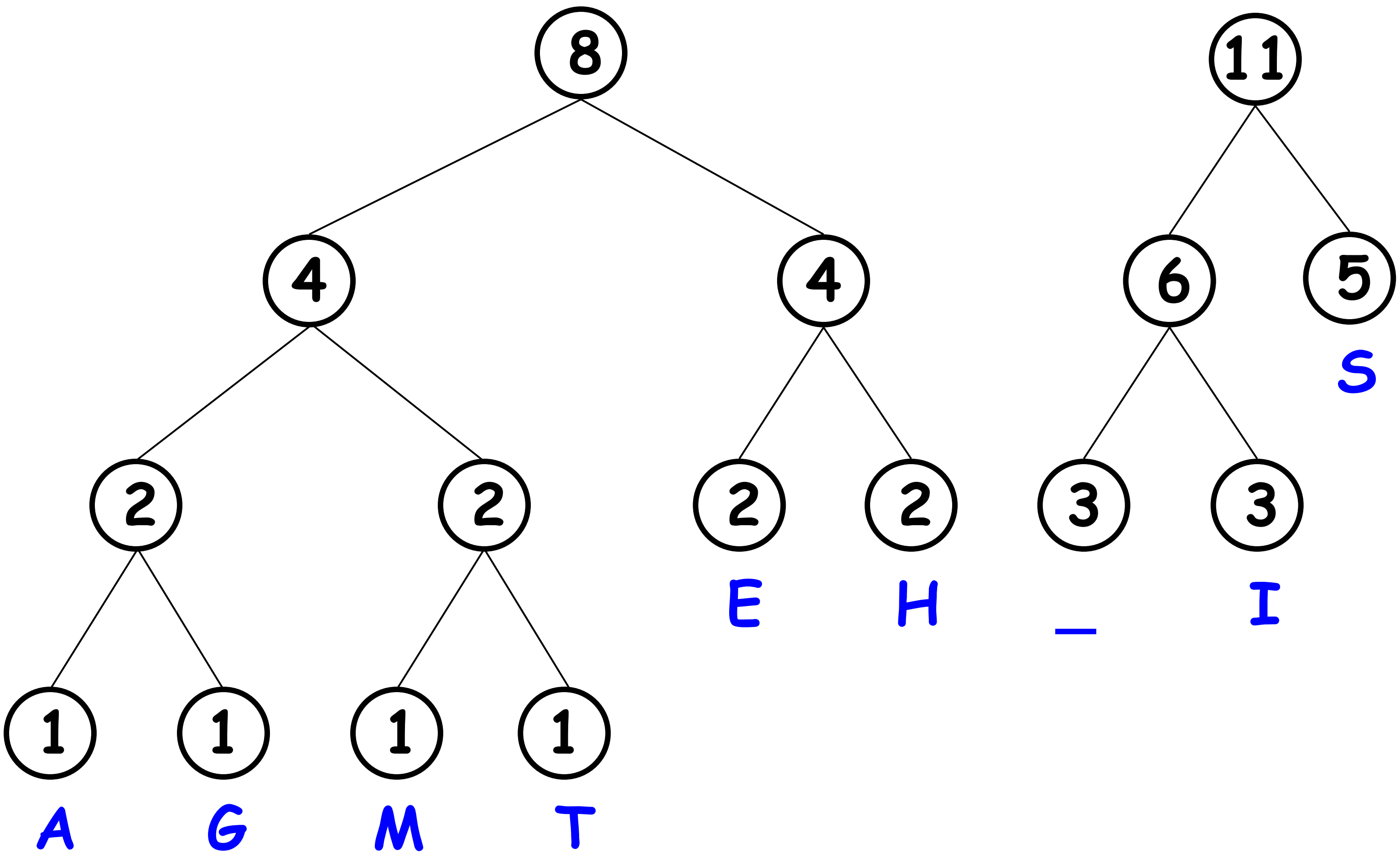
Step 5



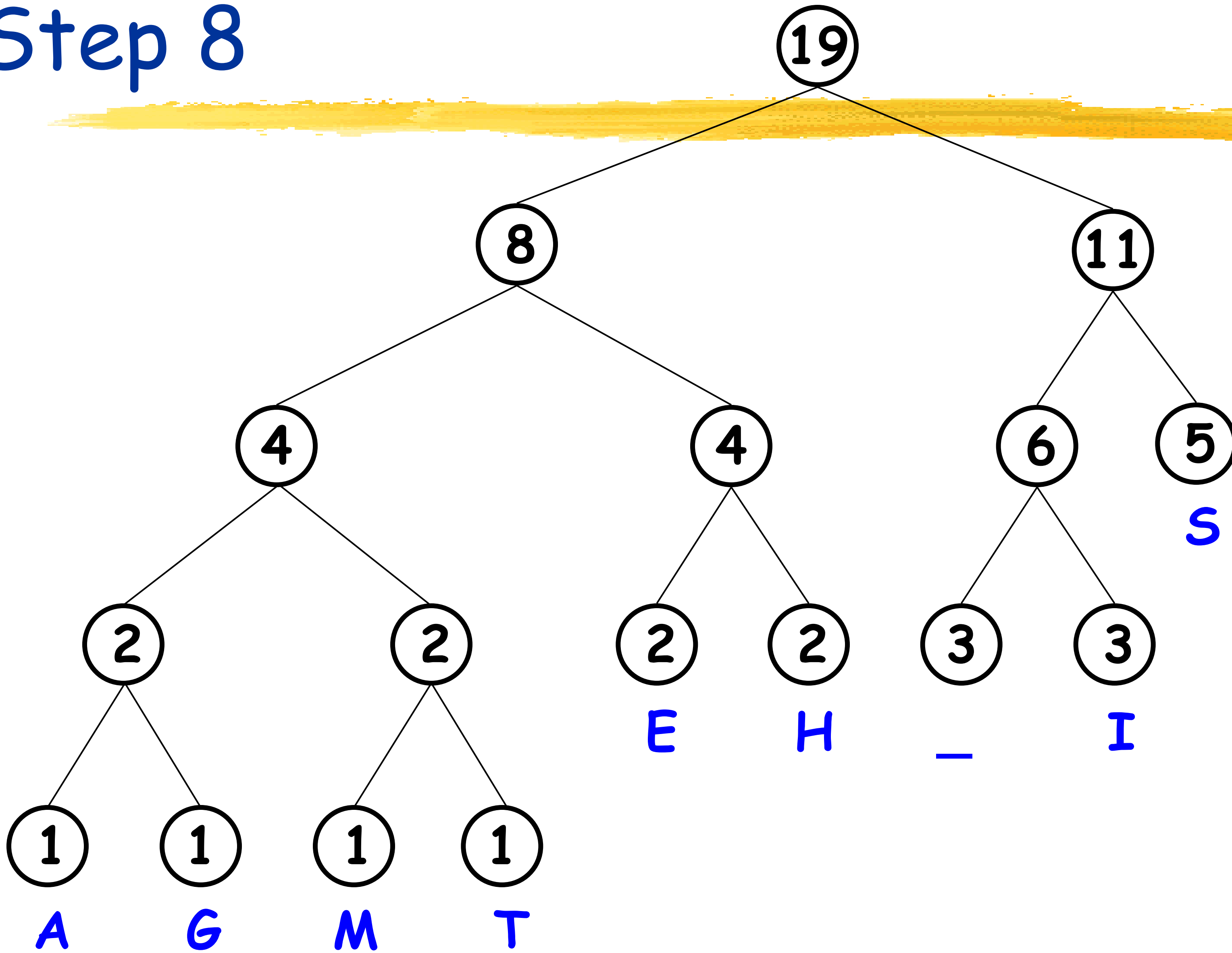
Step 6



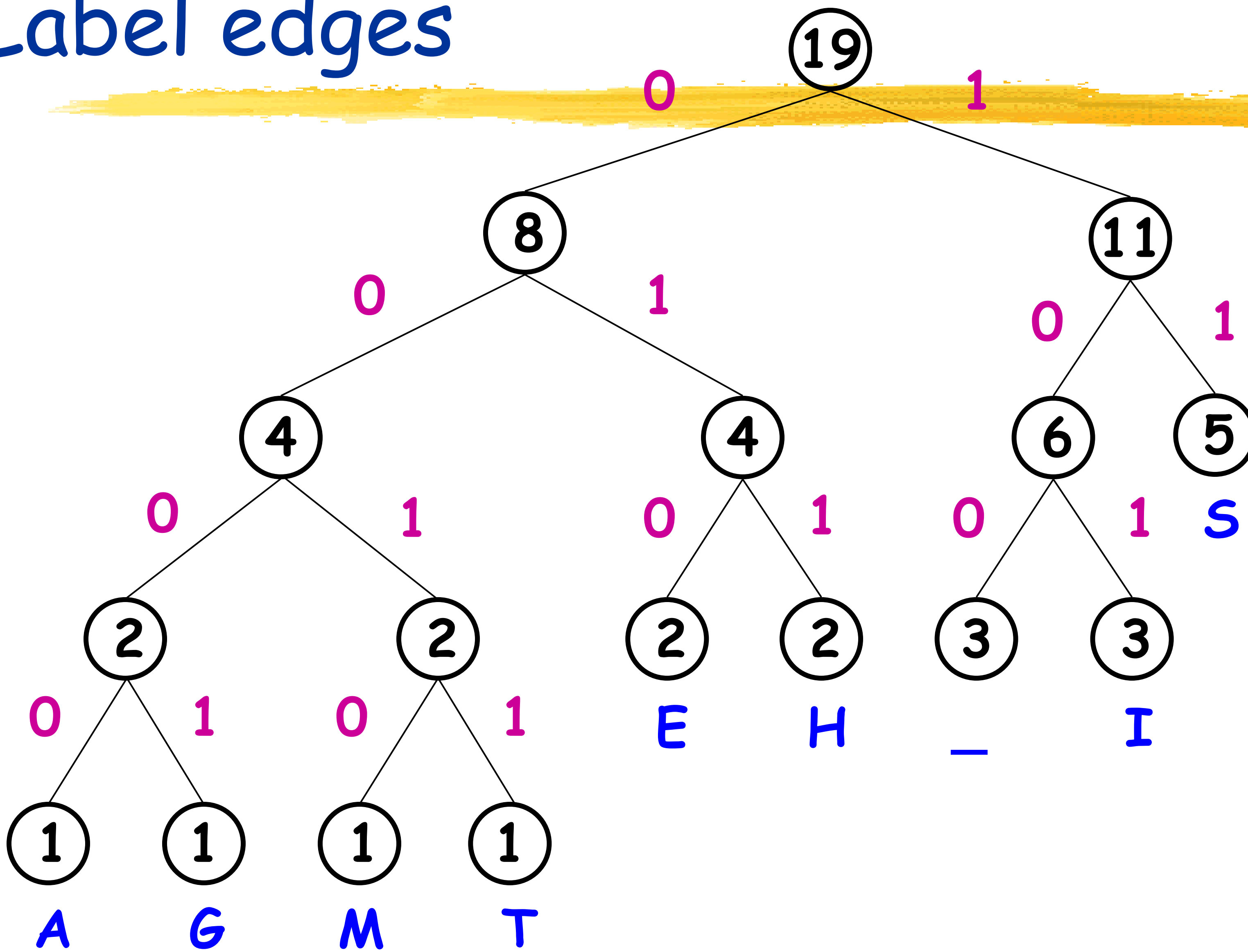
Step 7



Step 8



Label edges



Huffman code & encoded message

This is his message

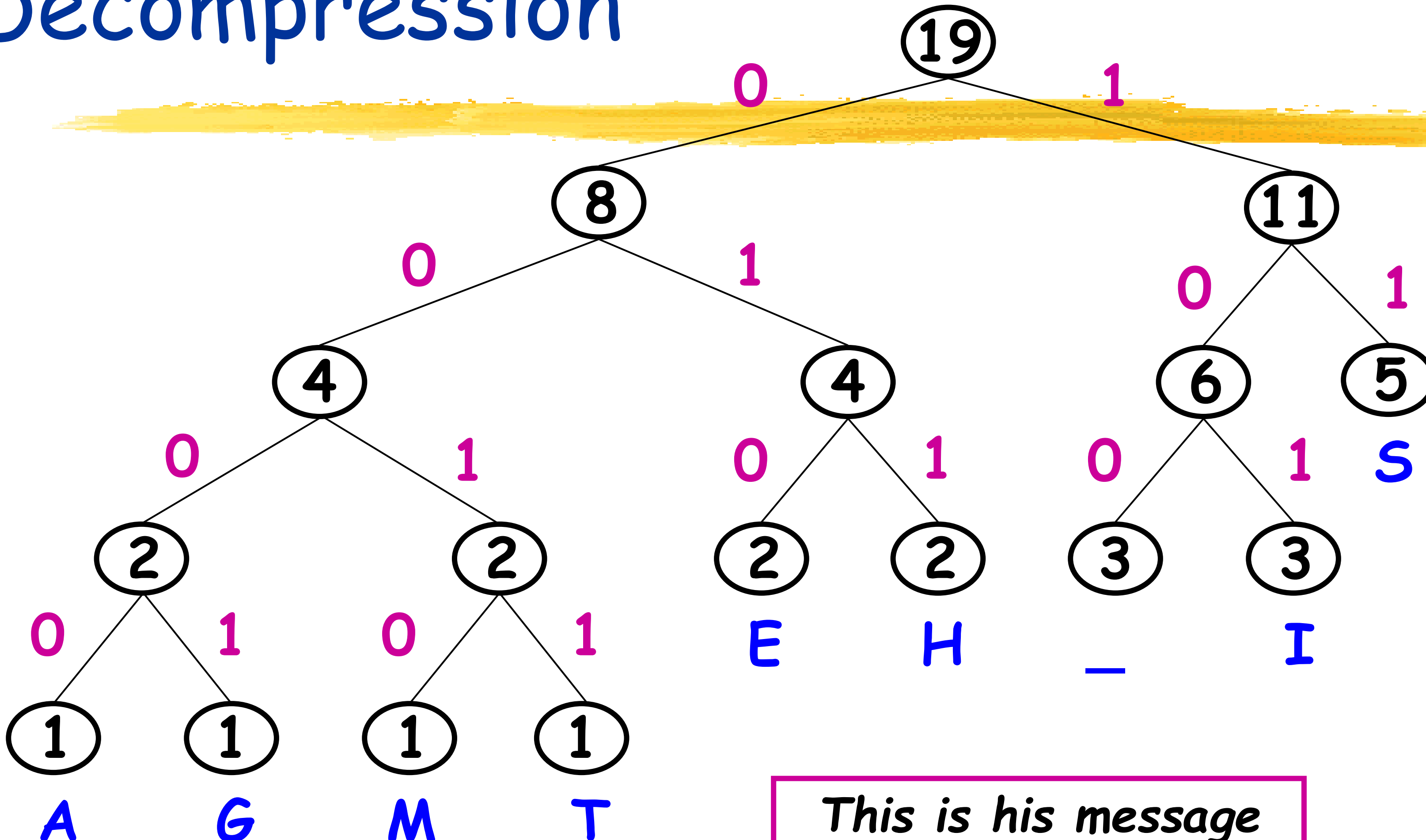
S	11
E	010
H	011
_	100
I	101
A	0000
G	0001
M	0010
T	0011

00110111011110010111100011101111000010010111100000001010

Huffman code

```
Procedure Huffman(C):
  // C is the set of n characters and their frequencies
  n = C.size
  Q = priority_queue()
  for i = 1 to n
      n = node(C[i])
      Q.push(n)
  end for
  while Q.size() is not equal to 1
      Z = new node()
      Z.left = x = Q.pop
      Z.right = y = Q.pop
      Z.frequency = x.frequency + y.frequency
      Q.push(Z)
  end while
Return Q
```

Decompression



This is his message

0011011101111001011110001110111100001001011110000001010

Decompression

```
Procedure HuffmanDecompression(root, S):
  // root represents the root of Huffman Tree
  // S refers to bit-stream to be decompressed
  n := S.length
  for i := 1 to n
    current = root
    while current.left != NULL and current.right != NULL
        if S[i] is equal to '0'
            current := current.left
        else
            current := current.right
        endif
        i := i+1
    endwhile
    print current.symbol
  endfor
```

Average Bits Per Symbol

Symbol	Frequency	Probability (p_i)	Code	Code $(l_i\ bits)$
A	1	1/19=0.0526	0000	4
G	1	0.0526	0001	4
M	1	0.0526	0010	4
T	1	0.0526	0011	4
E	2	0.105	010	3
H	2	0.105	011	3
_	3	0.158	100	3
I	3	0.158	101	3
S	5	0.263	11	2
	19	1.0		

Average bits per symbol $= \sum_1^9 p_i l_i$

Related References

- Introduction to the Design and Analysis of Algorithms, A. Levitin, 3rd Ed., Pearson 2011.
  - Chapters 9.4

- Some slides are based on the ones prepared by Dr Steve Goddard@cse.unl.edu