Question 2:

Give a θ characterization, in terms of *n*, of the running time of the following fou functions:

```
def example1(lst):
 """Return the sum of the prefix sums of sequence S."""
        n = len(lst)  O(1)
        total = 0     O(1)
(n) ←for j in range(n):                    =O(n^2)
  O(n^2) ← for k in range(1+j):
        O(n^2) ← total += lst[k]
O(1)    return total
```

```
def example2(lst):
"""Return the sum of the prefix sums of sequence S."""
O(1)    n = len(lst)
O(1)    prefix = 0
O(1)    total = 0                          O(n)
O(n)    for j in range(n):
          O(n) prefix += lst[j]
          O(n) total += prefix
O(1)    return total
```

```
def example3(n):
 O(1) i = 1
 O(1) sum = 0
O(log(n)) while (i < n*n):               O(log(n))
            i *= 2
            sum += i
        return sum
```

Explanation:

K_1	K_2	K_3	....
2i	4i	8i	....
2^1	2^2	2^3	....

$\log_2 2^K < \log_2 n^2$

$K < 2\log_2 n$

```
def example4(n):
O(1)    i = n
O(1)    sum = 0                            O(n log(n))
n/2^k ←[while (i > 1): O(log n)
      O(i) for j in range(i):
          O(2n) sum += i*j
      O(1) i //= 2
   O(1) return sum
```

$n(\underbrace{1+\frac{1}{2}+\frac{1}{4}+\dots+\frac{1}{2^k}}_{2})=2n$

Q1

a) $5n^3 + 2n^2 + 3n = O(n^3)$

$5n^3 + 2n^2 + 3n \leq (5+2+3)n^3 \leq Cn^2$

where C is a constant $= 10$ when $n \geq n_0 = 1$

$5n^3 + 2n^2 + 3n \leq 10n^3$ for $n \geq 1$

Thus, $5n^3 + 2n^2 + 3n$ is $O(n^3)$, $C = 10$, $n_0 = 1$

b) $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

$c_1(g(n)) \leq \sqrt{7n^2 + 2n - 8} \leq c_2(g(n))$

- Bound Below / $\Omega(n)$

$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7n^2}$

$\sqrt{7n^2 + 2n - 8} \geq n\sqrt{7}$

$c_1 = \sqrt{7}$ $n_0 = 0$

- Bound Above / $O(n)$

$\sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n^2 - 8n^2}$, $n_0 \geq 1$

$\sqrt{7n^2 + 2n - 8} \leq \sqrt{n^2}$

$\sqrt{7n^2 + 2n - 8} \leq n$ $c_2 = 1$ $n_0 = 1$

Thus $\sqrt{7n^2 + 2n - 8}$ is $\Theta(n)$, $c_1 = \sqrt{7}$, $c_2 = 1$, $n_0 = 1$

d) Show that if *d(n)=O(f(n))* and *e(n)=O(g(n))*, then the product *d(n)e(n)* is O(*f (n)g(n)*).

$d(n) \leq Cf(n)$, $n \geq n_0 = 1$.

$e(n) \leq Cg(n)$, $n \geq n_0 = 1$.

] C is a different constant

$d(n) \cdot e(n) \equiv C \cdot f(n) \cdot C \cdot g(n)$ # By Substitution

$\equiv C \cdot f(n) g(n)$ # ReWrite

Therefore, $d(n)e(n) = O(f(n)g(n))$