

1) Find all upper bounds of $\{a, b, c\}$. = k, l, m

2) Find all lower bounds of $\{f, g, h\}$.

= NO lower bound

Least upper bound of $\{a, b, c\}$
= k

$x R f$
 $x R g$
 $x R h$

Least upper bound and greatest lower bound \rightarrow
(l.u.b) (g.l.b)

x is the least upper bound of A if $a R x$ whenever $a \in A$ and $x R z$ whenever z is an upper bound of A .

y is the greatest lower bound of A if $y R a$ whenever $a \in A$ and $z R y$ whenever z is a lower bound of A .

• Answer these questions for the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$.

a) Find the maximal elements.

= $\{72, 60, 48, 27\}$ ✓

b) Find the minimal elements.

= $\{2, 9\}$

c) Is there a greatest element?

= No.

d) Is there a least element?

= No.

e) Find all upper bounds of $\{2, 9\}$.

= $\{18, 36, 72\}$

f) Find the least upper bound of $\{2, 9\}$, if it exists.

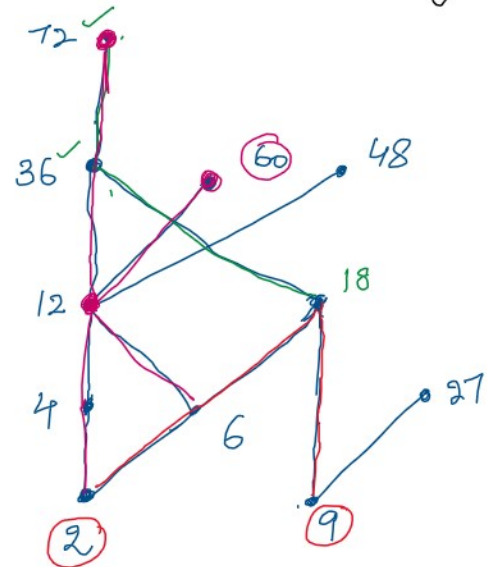
= 18

g) Find all lower bounds of $\{60, 72\}$.

= $\{12, 6, 4, 2\}$

h) Find the greatest lower bound of $\{60, 72\}$, if it exists.

= 12



Find the greatest lower bound and the least upper bound of the sets $\{3, 9, 12\}$ and $\{1, 2, 4, 5, 10\}$, if they exist, in the poset $(\mathbb{Z}^+, |)$. ✓

$$\mathbb{Z}^+ = \{1, 2, 3, 4, 5, \dots\}$$

$$A = \{3, 9, 12\}$$

$$\text{Lower bound of } A = \{1, 3\}$$

$$\text{g.l.b of } A = 3.$$

$$\text{upper bound of } A = 36, 72, \dots$$

$$\text{least upper bound} = \underline{\underline{36}}$$

$$B = \{1, 2, 4, 5, 10\}$$

$$\text{Lower bound of } B = 1.$$

$$\text{g.l.b} = 1$$

$$\text{upper bound of } B = 20, 40, \dots$$

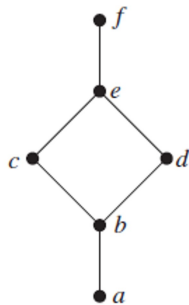
$$\text{least upper bound} = \underline{\underline{20}}$$

Lattice

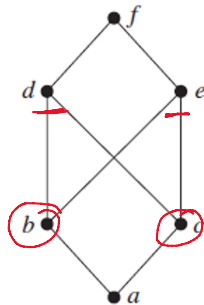
A partial ordered set in which every pair of elements has both a least upper bound and a greatest lower bound is called a Lattice

upper bounds of $\{b, c\}$
= d, e, f .

l.u.b = No



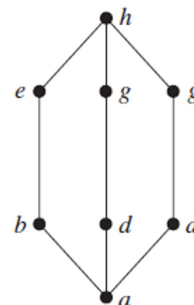
(a)



(b)

not a

Lattice



(c)