

④ Transitive Relation :-

A Relation R on a set A is called transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for all $a,b,c \in A$.

$$aRb, bRc$$

$$\swarrow \searrow$$

$$\underline{aRc}$$

$$\underline{aRb \text{ and } bRc} \rightarrow \underline{aRc}$$

Consider the following relations on $\{1, 2, 3, 4\}$:

$$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4), (4,4)\},$$

Not Transitive $\because (3,1) \notin R_1$

$$R_2 = \{(1,1), (1,2), (2,1)\},$$

Not Transitive $\because (2,1) (1,2) \in R_2$ but $(2,2) \notin R_2$

$$R_3 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\},$$

$$R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\},$$

Transitive

$$R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\},$$

Transitive

$$R_6 = \{(3,4)\} \rightarrow \text{Transitive}$$

R_3

$$(1,1) (1,2) \rightarrow (1,2) \checkmark$$

$$(1,1) (1,4) \rightarrow (1,4) \checkmark$$

$$(1,2) (2,1) \rightarrow (1,1) \checkmark$$

$$(1,2) (2,2) \rightarrow (1,2)$$

$$(1,4) (4,1) \rightarrow (1,1)$$

$$(1,4) (4,4) \rightarrow (1,4) \checkmark$$

$$(2,1) (1,1) \rightarrow (2,1) \checkmark$$

$$(2,1) (1,2) \rightarrow (2,2) \checkmark$$

$$(2,1) (1,4) \rightarrow (2,4)$$

R_3 is not transitive

$$R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\},$$

Transitive Relation

$$(2,1) \mid (3,2) (2,1) \rightarrow (2,1) \checkmark$$

$$(3,1) \mid (4,1)$$

$$(4,2) (2,1) \rightarrow (4,1)$$

$$(4,3) (3,1) \rightarrow (4,1)$$

$$(4,3) (3,2) \rightarrow (4,2)$$

#

Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

$$(a,b) \in R \text{ then } (b,a) \in R$$

a) a is taller than b .

→ b) a and b were born on the same day.

→ c) a has the same first name as b .

d) a and b have a common grandparent.

Reflexive, Symmetric, not antisymmetric, Transitive
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Reflexive, Symmetric, not antisymmetric, Transitive

$$\textcircled{a} R = \{ (a,b) \mid a \text{ is taller than } b \}$$

i) not Reflexive $\because a$ is not taller than a
 $(a,a) \notin R$.

ii) not Symmetric $\because (a,b) \in R$ then $(b,a) \notin R$

iii) Transitive a is taller than b , b is taller than c

→ a is taller than c

$$(a,c) \in R$$

antisymmetric

How many reflexive relations are there on a set with 3 elements?