Supplemental Materials for 'Synthesizing data from pretest-posttest-control-group designs in mediation meta-analysis'

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S1 Mathematical Derivations

S1.1 Derivations for Equations Used to Construct Correlation Matrices

S1.1.1 Point-biserial Correlations

In this section, we do not consider the justification for degree of freedom for simplicity.

Considering Equation 4 and 5,

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{N_T\left(\hat{\sigma}_T^2 + \left(\hat{\mu}_T - \frac{N_T\hat{\mu}_T + N_C\hat{\mu}_C}{N_T + N_C}\right)^2\right) + N_C\left(\hat{\sigma}_C^2 + \left(\hat{\mu}_C - \frac{N_T\hat{\mu}_T + N_C\hat{\mu}_C}{N_T + N_C}\right)^2\right)}{N_T + N_C}}}\sqrt{\frac{N_T N_C}{(N_T + N_C)^2}},$$

Equation 6 can be obtained with the following algebraic transformations:

(1) Expanding the denominator:

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{N_T\hat{\sigma}_T^2 + N_C\hat{\sigma}_C^2 + N_T\hat{\mu}_T^2 + N_C\hat{\mu}_C^2 - \frac{(N_T\hat{\mu}_T + N_C\hat{\mu}_C)^2}{N_T + N_C}}{N_T + N_C}\frac{(N_T + N_C)^2}{N_T N_C}}};$$

(2) Simplifying the denominator:

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{(N_T + N_C)(N_T\hat{\sigma}_T^2 + N_C\hat{\sigma}_C^2 + N_T\hat{\mu}_T^2 + N_C\hat{\mu}_C^2) - (N_T\hat{\mu}_T + N_C\hat{\mu}_C)^2}{N_T N_C}}};$$

(3) Expanding the simplified equation:

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{N_T^2\hat{\sigma}_T^2 + N_T N_C\hat{\sigma}_T^2 + N_C^2\hat{\sigma}_C^2 + N_T N_C\hat{\sigma}_C^2 + N_T N_C\hat{\mu}_T^2 + N_T N_C\hat{\mu}_C^2 - 2N_T N_C\hat{\mu}_T\hat{\mu}_C}{N_T N_C}}};$$

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{N_T\hat{\sigma}_T^2(N_T + N_C) + N_C\hat{\sigma}_C^2(N_T + N_C)}{N_T N_C} + \hat{\mu}_T^2 + \hat{\mu}_C^2 - 2\hat{\mu}_T\hat{\mu}_C}};$$

(4) Obtaining Equation 6 in the main text

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{(\hat{\mu}_T - \hat{\mu}_C)^2 + (N_T\hat{\sigma}_T^2 + N_C\hat{\sigma}_C^2)\frac{N_T + N_C}{N_T N_C}}}.$$

S1.1.2 Converting from Paired-sample t Values to Bivariate Correlations

Equation 7 in the main text can be obtained by multiplying the *t* value and dividing the SD in both sides of the regular *t*-test equation $t = \frac{\hat{\mu}_{cs}}{\frac{\hat{\sigma}_{cs}}{\sqrt{N}}}$.

S1.1.3 Converting from Confidence Intervals to Bivariate Correlations

The upper or lower bound of the confidence interval of change scores can be computed using $CI_{upper/lower} = \hat{\mu}_{cs} \pm t_{crit}\frac{\hat{\sigma}_{pl,cs}}{\sqrt{N}}$. Therefore, half of the confidence interval can be obtained: $\frac{CI_{upper}-CI_{lower}}{2} = \frac{\hat{\mu}_{cs}+t_{crit}\frac{\hat{\sigma}_{pl,cs}}{\sqrt{N}}-\hat{\mu}_{cs}+t_{crit}\frac{\hat{\sigma}_{pl,cs}}{\sqrt{N}}}{2} = t_{crit}\frac{\hat{\sigma}_{pl,cs}}{\sqrt{N}}$. Then Equation 8 in the main text can be obtained by dividing t_{crit} and multiplying $\sqrt{N}$ in both sides.

S1.1.4 Converting from Regression Coefficients to Bivariate Correlations

The standardized a and c coefficients are in nature bivariate correlations between X and M and between X and Y, respectively. The correlation between M and Y, on the other hand, can be converted from the regression coefficient b using the equation in the main text, which comes from $b_s = \frac{r_{MY}-r_{XM}r_{XY}}{1-r_{XM}^2}$, the regular equation for converting between regression coefficients and Pearson's correlations.

S1.2 Data-generating Mechanisms

S1.2.1 Change-score Group Variances

$$var(M_{cs}) = cov(M_2 - M_1, M_2 - M_1)$$
$$= var(M_1) - 2 \times cov(M_2, M_1) + var(M_2)$$
$$var(Y_{cs}) = cov(Y_2 - Y_1, Y_2 - Y_1)$$
$$= var(Y_1) - 2 \times cov(Y_2, Y_1) + var(Y_2)$$

S1.2.2 Posttest Means of M and Y in the Treatment Group

For each individual study, considering that the pretest both groups and posttest of the control group were fixed at 0, posttest means in the treatment group would be the mean difference of change scores $MD_k = \hat{\mu}_{T,cs} - \hat{\mu}_{C,cs}$, which applies to both M and Y. Considering $d_{cs} = \frac{MD_k}{\sqrt{\frac{\sigma^2_{T,cs}+\sigma^2_{C,cs}}{2}}}$ and $d_{cs} = \frac{2r}{\sqrt{1-r^2}}$, we can obtain $MD_k^M = \frac{2r_{XM,k}}{\sqrt{1-r^2_{XM,k}}} \times \sqrt{\frac{\sigma^2_{M_{cs,T}}+\sigma^2_{M_{cs,C}}}{2}}$, and $MD_k^Y = \frac{2r_{XY,k}}{\sqrt{1-r^2_{XY,k}}} \times \sqrt{\frac{\sigma^2_{M_{cs,T}}+\sigma^2_{M_{cs,C}}}{2}}$.

S1.2.3 Combined Variances of Change Scores of M and Y

Given the fixed pretest and posttest means and variances, $D_T = D_C = \frac{MD_k^{M/Y}}{2}$ in the population when $\pi = 0.5$. Therefore, $\sigma^2_{cb,cs} = \frac{\sigma^2_{\mathrm{C,cs}}+\sigma^2_{\mathrm{T,cs}}}{2} + \frac{(\mathrm{MD_k})^2}{4}$, which applies to both M and Y.

S1.2.4 Generating Pretest Data of Y

We generated pretest data of Y based on change scores of Y using:

$$Y_{1,T/C} = i_{Y_{1,T/C}} + b_{Y_{cs,1,T/C}} Y_{cs,T/C} + e_{Y_{1,T/C}},$$

where $b_{Y_{cs,1,T/C}}$ is the unstandardized regression coefficient when regressing $Y_{cs,T/C}$ on $Y_{1,T/C}$, and the subscript T/C represent the treatment group OR the control group.

Considering the correlation between pretest and posttest in each group is set as ρ_{12}:

$$cor\left(Y_{1,T/C}, Y_{2,T/C}\right) = \rho_{12},$$

the correlation between pretest and change-score in each group can be computed using

$$cor(Y_{1,T/C}, Y_{cs,T/C}) = \frac{cov(Y_{1,T/C}, Y_{2,T/C}) - cov(Y_{1,T/C}, Y_{1,T/C})}{\sigma_{Y_{1,T/C}}\sigma_{Y_{cs,T/C}}} = \frac{\rho_{12}\sigma_{Y_{1,T/C}}\sigma_{Y_{2,T/C}} - \sigma^2_{Y_{1,T/C}}}{\sigma_{Y_{1,T/C}}\sigma_{Y_{cs,T/C}}}.$$

Next, the unstandardized regression coefficient ($b_{Y_{cs,1,T/C}}$) can be obtained:

$$b_{Y_{cs,1,T/C}} = cor(Y_{1,T/C}, Y_{cs,T/C}) \times (\frac{\sigma_{Y_{1,T/C}}}{\sigma_{Y_{cs,T/C}}}) = \frac{\rho_{12}\sigma_{Y_{1,T/C}}\sigma_{Y_{2,T/C}} - \sigma^2_{Y_{1,T/C}}}{\sigma^2_{Y_{cs,T/C}}}.$$

S2 Previous Simulations Designed to Examine the Impact of the c' and β Paths

In addition to the simulations reported in the main text, we also conducted simulations where the magnitude of the direct effect c' and the moderating effect β were manipulated. The data generating process, the estimation methods, true path coefficients for CSMMA and PSMMA, and the performance measures were the same as in the main simulations. All simulations were implemented using R Version 4.2.2 (R code team, 2022), with 5,000 repetitions for each condition.

S2.1 Manipulated Factors

In this simulation, we investigated the impact of several factors on the performance of CSMMA and PSMMA, including standardized change-score (denoted by the subscript cs) path parameters ($a_{s,cs}$, $b_{s,cs}$ and $c'_{s,cs}$), pretest-posttest correlation (ρ_{12}), the between-study heterogeneity (moderating effect, $\beta_{c'_{s,cs}}$) , the number of primary studies (K), and the posttest variance of M and Y in the treatment group ($\sigma^2_{W_{2,T}}$). A 2 $(a_{s,cs}) \times 2$ $(b_{s,cs}) \times 2$ $(c'_{s,cs}) \times 2$ $(\rho_{12}) \times 2$ $(\beta_{c'_{s,cs}}) \times 3$ $(K) \times 2$ $\left(\sigma^2_{W_{2,T}}\right)$ design was used. Specifically, two values were considered for standardized change-score regression coefficients: 0 and 0.3, corresponding to Cohen’s null and moderate correlational benchmarks. For the pretest-posttest correlation ρ_{12}, two values were considered: 0.45 and 0.9, reflecting relatively low and high pretest-posttest correlations based on a meta-analysis in training research (Taylor et al., 2005). The moderating effect was introduced to the c' path to match our illustrative example. Two values $(0, 0.1)$ were considered for $\beta_{c'_{s,cs}}$, which were chosen to avoid negative definite correlation matrices for parameter estimation. We adapted the setting about the number of primary studies in by Jak & Cheung (2020; $K = 10, 30, 50$) by considering smaller values ($K = 5, 10, 30$) to reflect the generally smaller number of primary studies in clinical psychology. Following the setting in Morris (2008), we considered two values for the posttest variance (of M and Y) in the treatment group ($\sigma^2_{W_{2,T}}$): 1 and 1.5. The former was chosen to simulate the scenario of MMA under PPCG designs with equal variances in M and Y over time and between groups, whereas the latter was chosen to simulate MMA with *unequal* variances.

S2.2 Fixed Sample Size and Model Parameters

We used the following settings in data generation: (1) The mean and SD of the K sample sizes (i.e., N_i) were fixed at 80 and 23, respectively, and the minimum sample size was set at $0.4\mu_N$, which were selected to match those in an empirical example (Gu et al., 2016); (2) The SD of the population correlations in random-effects model was set at 0.1; (3) The pretest variances in both groups and posttest variance in the control group were fixed at 1 ($\sigma^2_{M_{1,C}} = \sigma^2_{M_{1,T}} = \sigma^2_{Y_{1,C}} = \sigma^2_{Y_{1,T}} = \sigma^2_{M_{2,C}} = \sigma^2_{Y_{2,C}} = 1$), where the subscripts 1 and 2 represent pretest and posttest, respectively; (4) The pretest means in both groups and the posttest mean in the control group were fixed to be 0 ($\mu_{M_{1,C}} = \mu_{M_{1,T}} = \mu_{Y_{1,C}} = \mu_{Y_{1,T}} = \mu_{M_{2,C}} = \mu_{Y_{2,C}} = 0$); (5) The proportion of the treated (π) was set at 0.5 to mimic random assignment.

S2.3 Results

S2.3.1 Convergence Rate

Non-convergence was not reported by osmasem. However, a proportion of simulated datasets were reported as having negative definite information matrices[1] (See Supplemental S2.3) and were excluded from the analysis of simulation results. For CSMMA, when $\rho_{12} = 0.45$, the probabilities of reporting negative definite information matrices were around 60%, 30%, and 3% with a K of 5, 10, and 30, respectively; when $\rho_{12} = 0.9$, the chances were around 60%, 20%, and 0% with a K of 5, 10, and 30, respectively. For PSMMA, when $\rho_{12} = 0.45$, the chances of reporting negative definite information matrices were around 70%, 50%, and 20% with a K of 5, 10, and 30, respectively; when $\rho_{12} = 0.9$, the chances were around 80%, 70%, and 55% with a K of 5, 10, and 30, respectively. The posttest variance inflation and the size of $a_{s,cs}$, $b_{s,cs}$, $c'_{s,cs}$, and $\beta_{c'_{s,cs}}$ did not have apparent influence here.

S2.3.2 Results with Equal Posttest Variances

Here, complete results regarding the indirect, direct, moderating effects are shown, including bias, CR, type I error rates, and statistical power. In all figures, "PostVar" represents the posttest variance in the treatment group.

The indirect effect. As shown in Figure S1, bias of CSMMA and PSMMA remained acceptable. The CR of both approaches remained above 0.95 (Figure S2). The type I error rates of CSMMA and PSMMA remained below 0.07 (Figure S3). As shown in Figure S4, while CSMMA had favorable power ($0.9 \sim 1$) in all conditions, the power of PSMMA decreased with a smaller K or a larger ρ_{12}. Specifically, when $\rho_{12} = 0.45$, PSMMA had

[1] This was probably because we allowed heterogeneity in all bivariate correlations by setting the matrix of random effects, i.e., the T matrix, as diagonal, whereas the T matrix of the generated data set was more likely to be close to a zero matrix when $K = 5$, leading to failure to estimate the T matrix.

power around 0.95, 0.55, and 0.3 when K equaled 30, 10, and 5, respectively. Additionally, when $\rho_{12} = 0.9$, the power of PSMMA were 0.088, 0.024, and 0.006 when K equaled to 30, 10, and 5, respectively. The size of $\beta_{c'_{s,cs}}$ and $c'_{s,cs}$ had no apparent impact on the statistical power of the indirect effect (Figure. S4).

Figure S1.

Bias of the Indirect Effect with Equal Posttest Variances

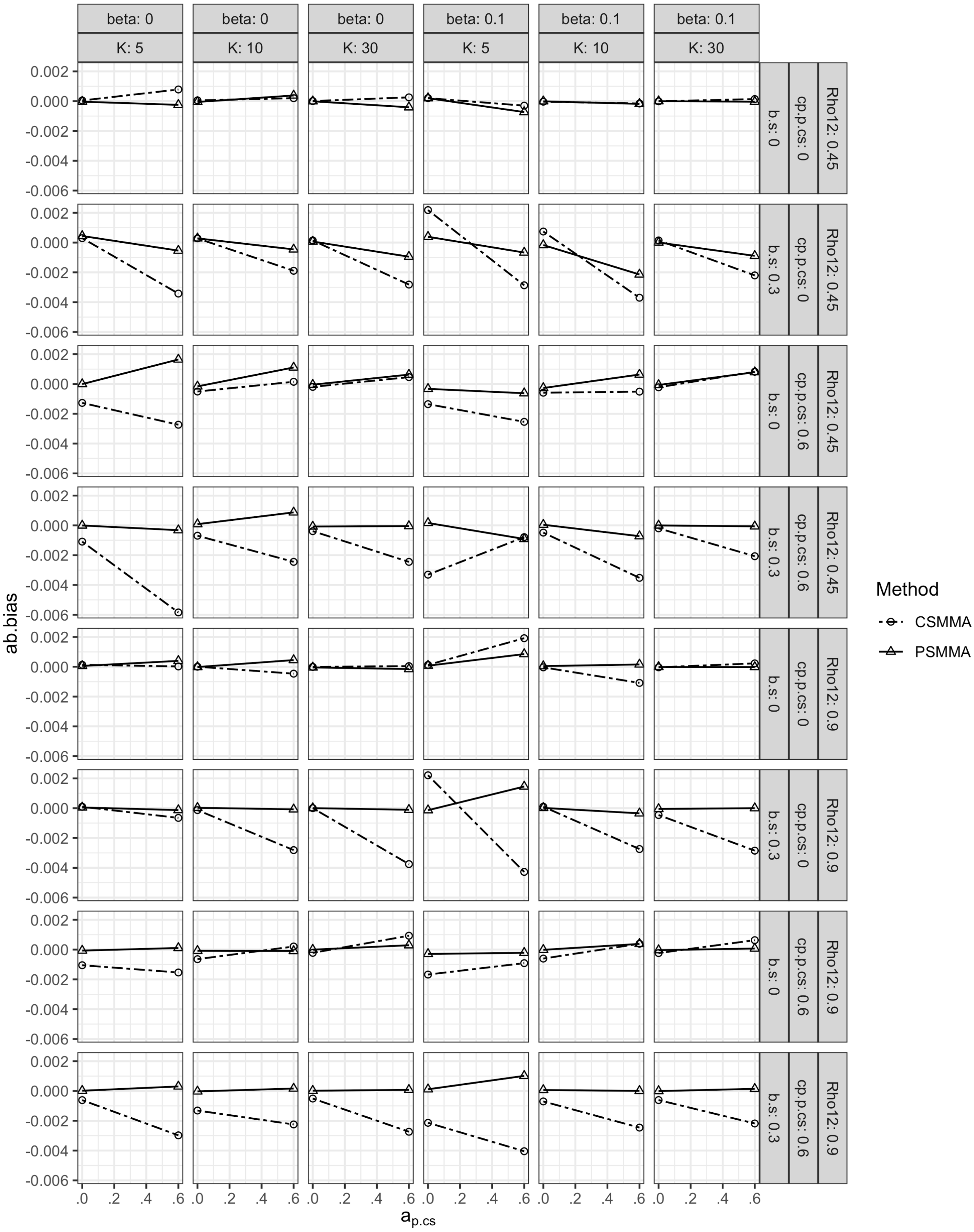


Figure S2.

Coverage Rates of the Indirect Effect with Equal Posttest Variances

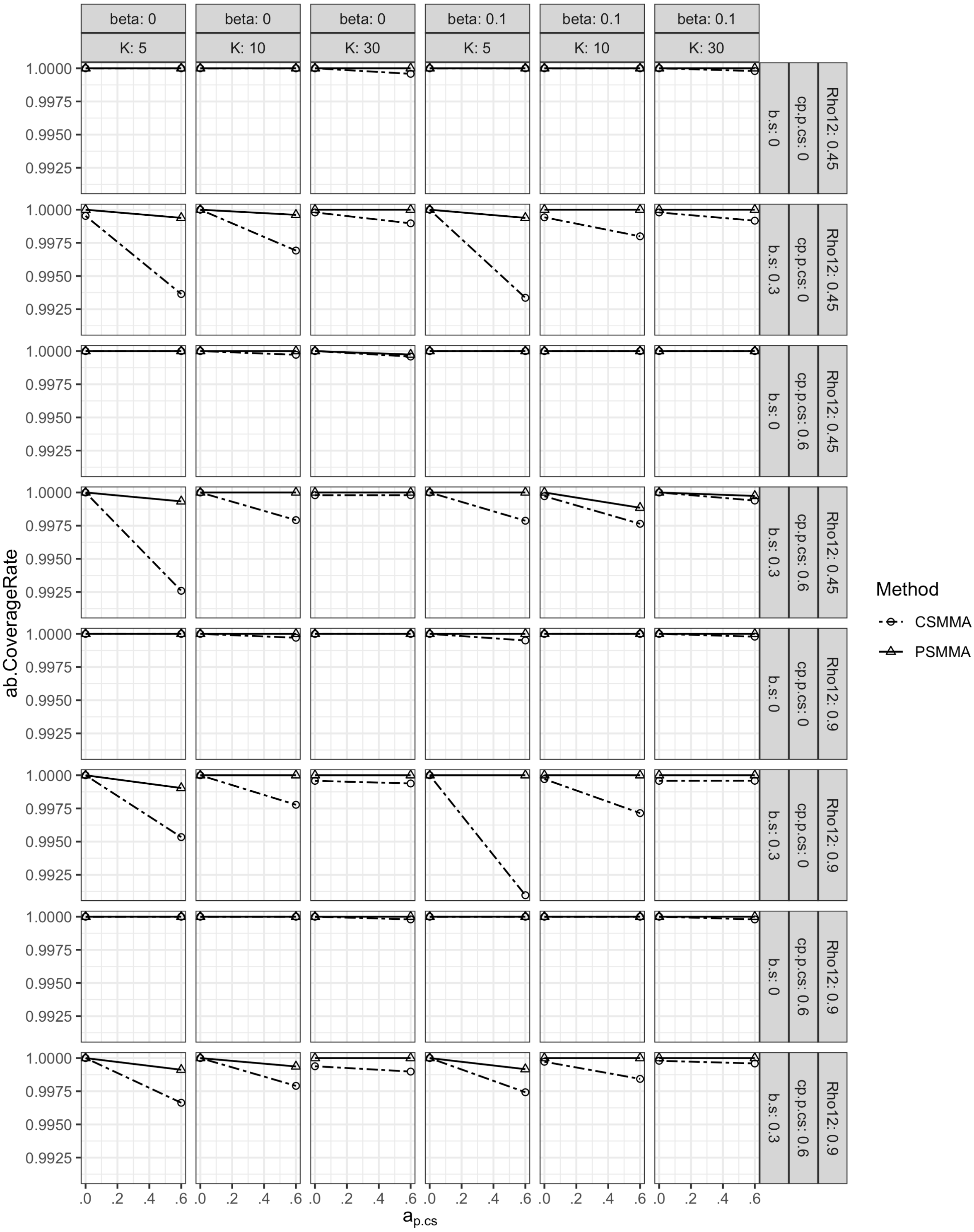


Figure S3.

Type I Error Rates of the Indirect Effect with Equal Posttest Variances

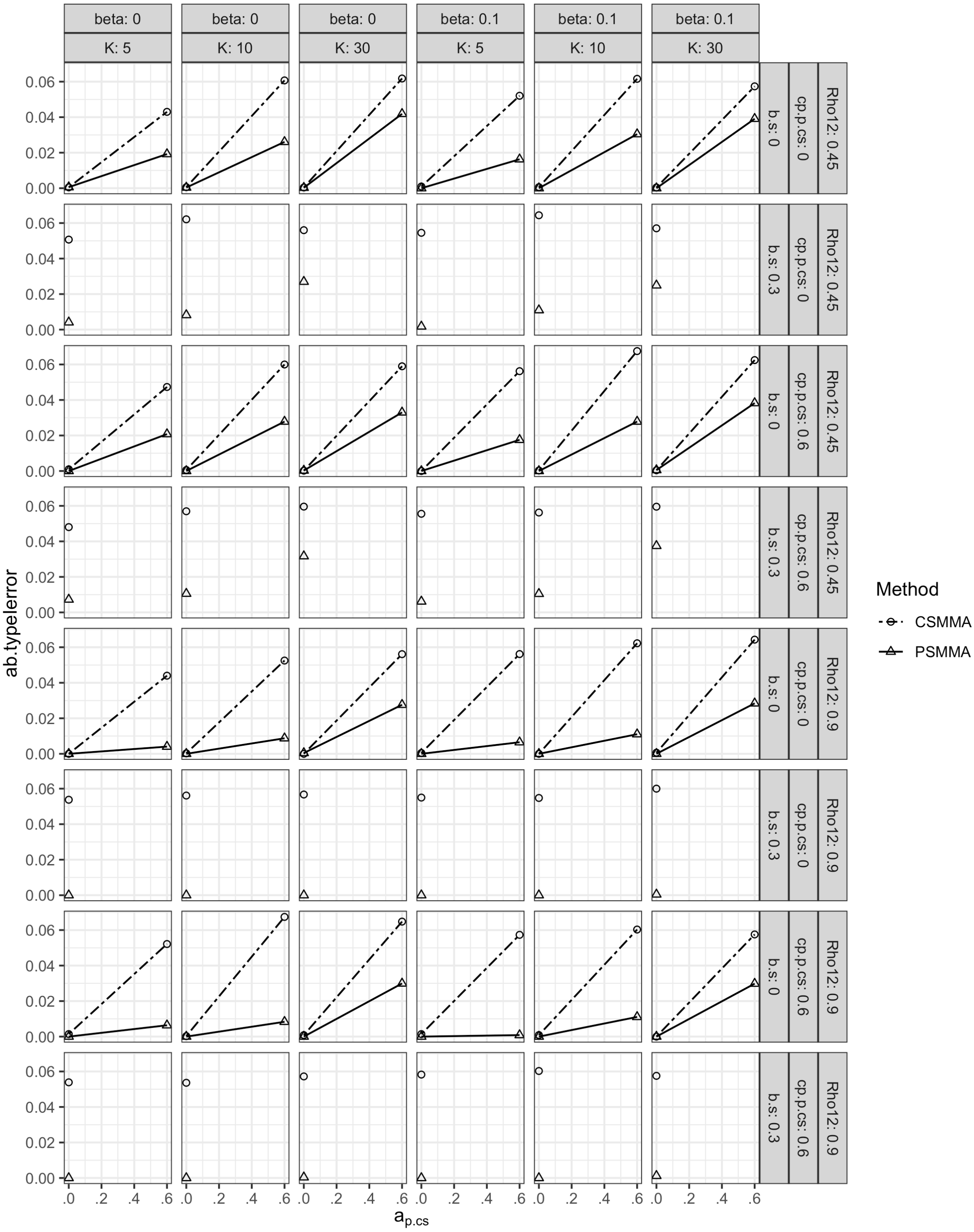
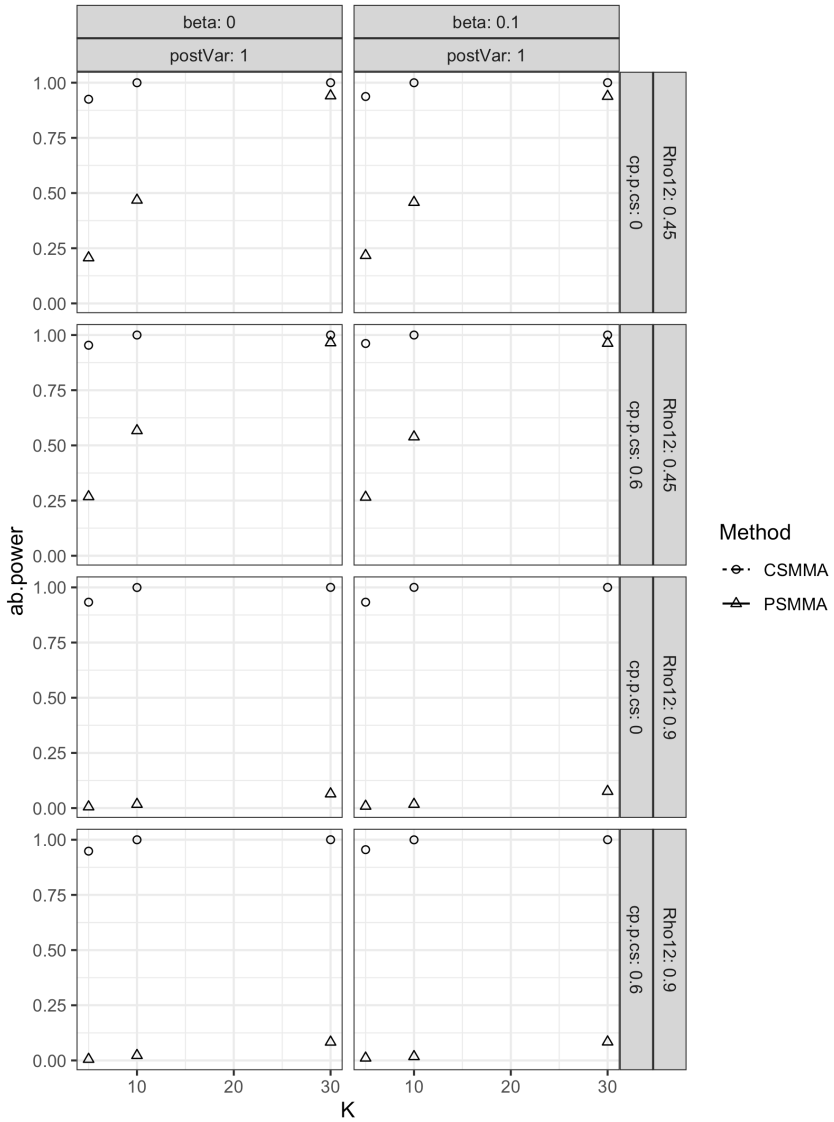


Figure S4.

Statistical Power of the Indirect Effect with Equal Posttest Variances



The Direct Effect. As shown in Figure S5, both approaches had small bias. The CR of both CSMMA and PSMMA remained above 0.85 (Figure S6). The type I error rates of both approaches remained below 0.1 (Figure S7). The pattern of power of CSMMA and PSMMA was demonstrated in the main text. The size of $\beta_{c'_{s,cs}}$ and $c'_{s,cs}$ had no apparent effect on power (Figure S8).

Figure S5.

Bias of the Direct Effect with Equal Posttest Variances

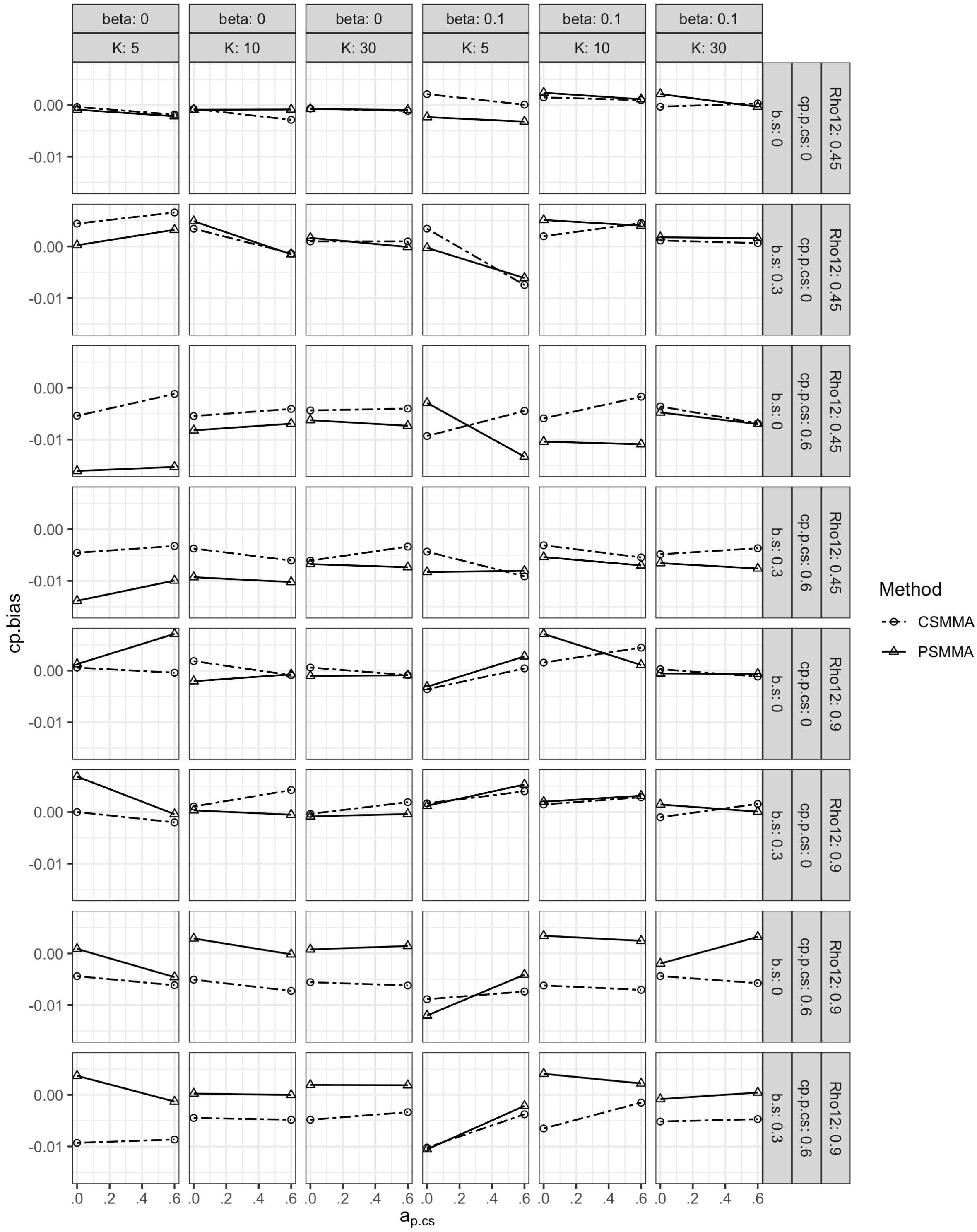


Figure S6.

Coverage Rates of the Direct Effect with Equal Posttest Variances

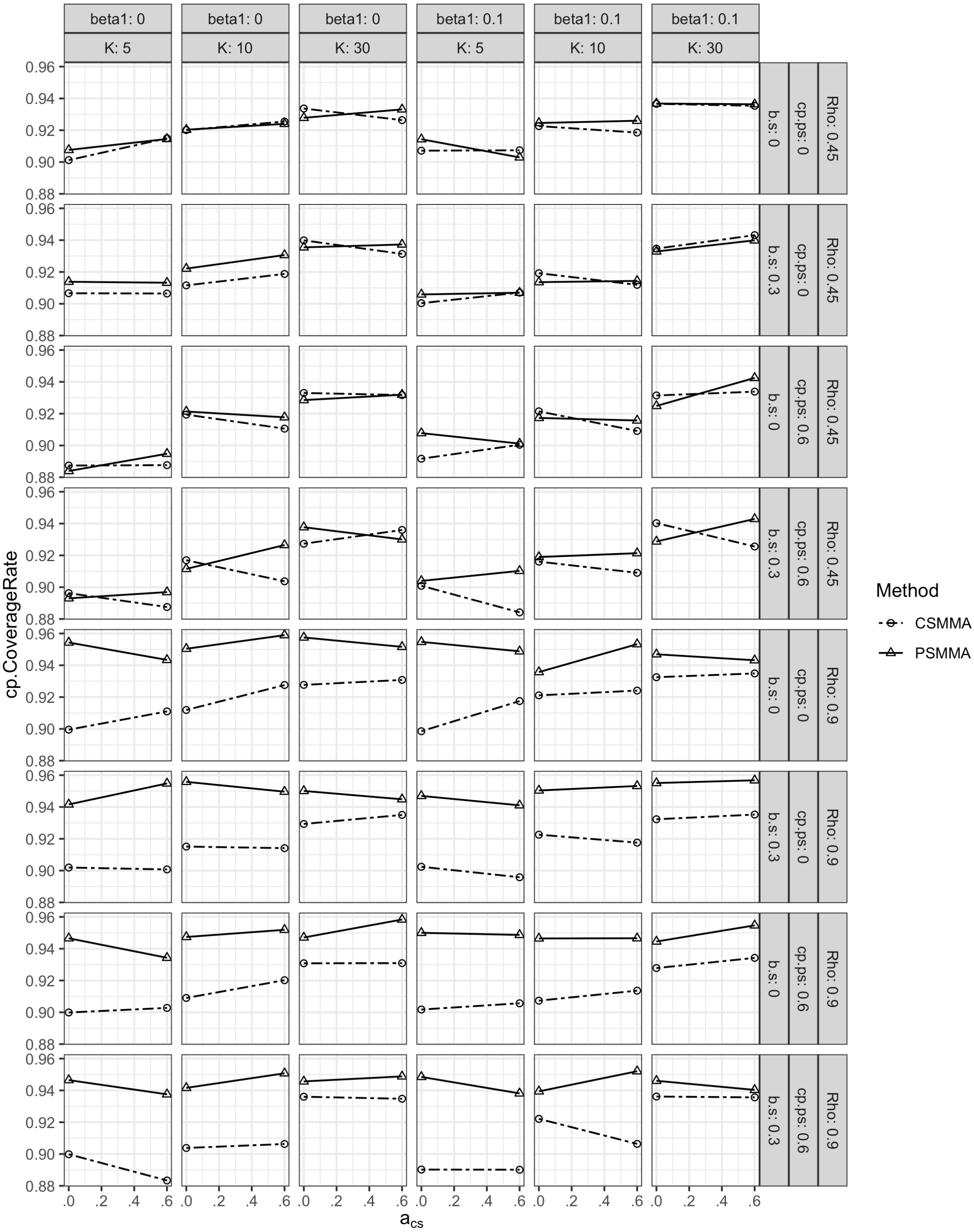


Figure S7.

Type I Error Rates of the Direct Effect with Equal Posttest Variances

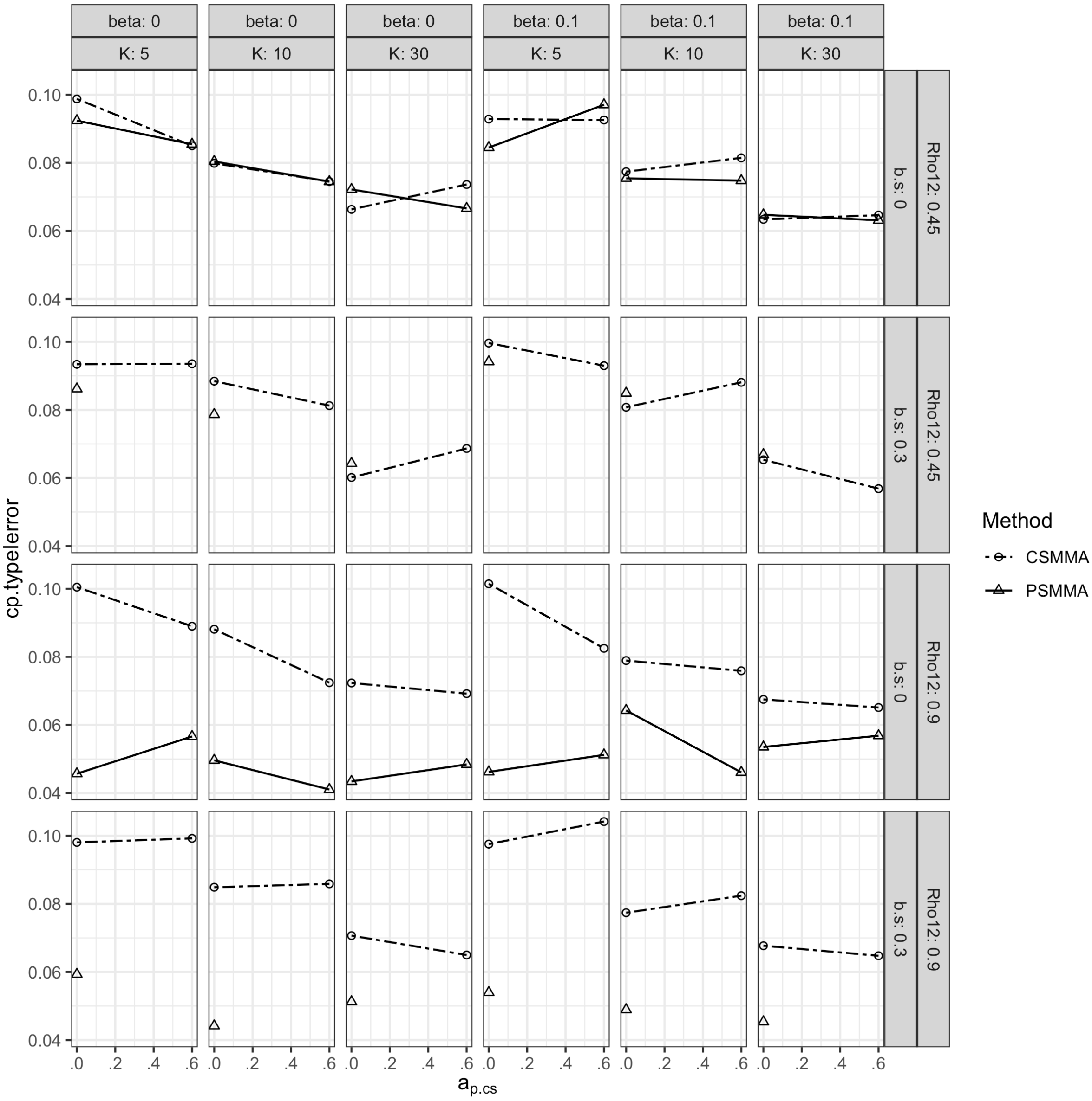
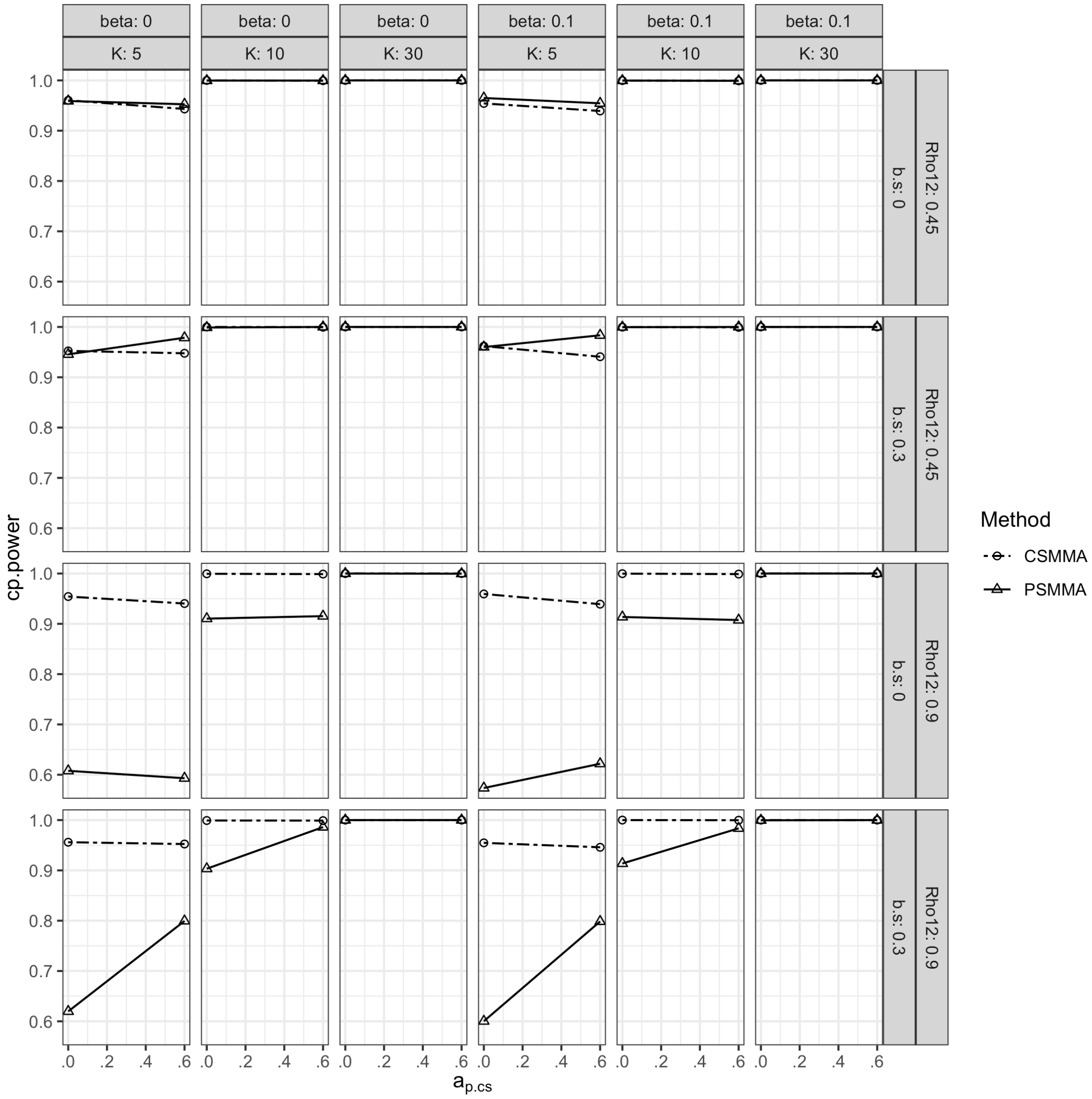


Figure S8.

Statistical Power of the Direct Effect with Equal Posttest Variances



The Moderating Effect. As shown in Figure S9, the bias of both CSMMA and PSMMA were small. The CR of both CSMMA and PSMMA remained above 0.85 (Figure S10). The type I error rates of PSMMA and CSMMA when testing the moderating effect were comparable, fluctuating between $0.05 \sim 0.1$ (Supplemental Figures S11). As shown in figure S12, although CSMMA and PSMMA exhibited comparable power when $\rho_{12} = 0.45$, they differed when $\rho_{12} = 0.9$: While the power of CSMMA was 0.95, 0.6, and 0.4, PSMMA had lower power of 0.7, 0.3, and 0.15 when K was 30, 10, and 5, respectively. The magnitude of $a_{s,cs}$, $b_{s,cs}$, and $c'_{s,cs}$ had no apparent impact on the relative performance on the power of the moderating effect (Figure S12).

Figure S9.

Bias of the Moderating Effect with Equal Posttest Variances

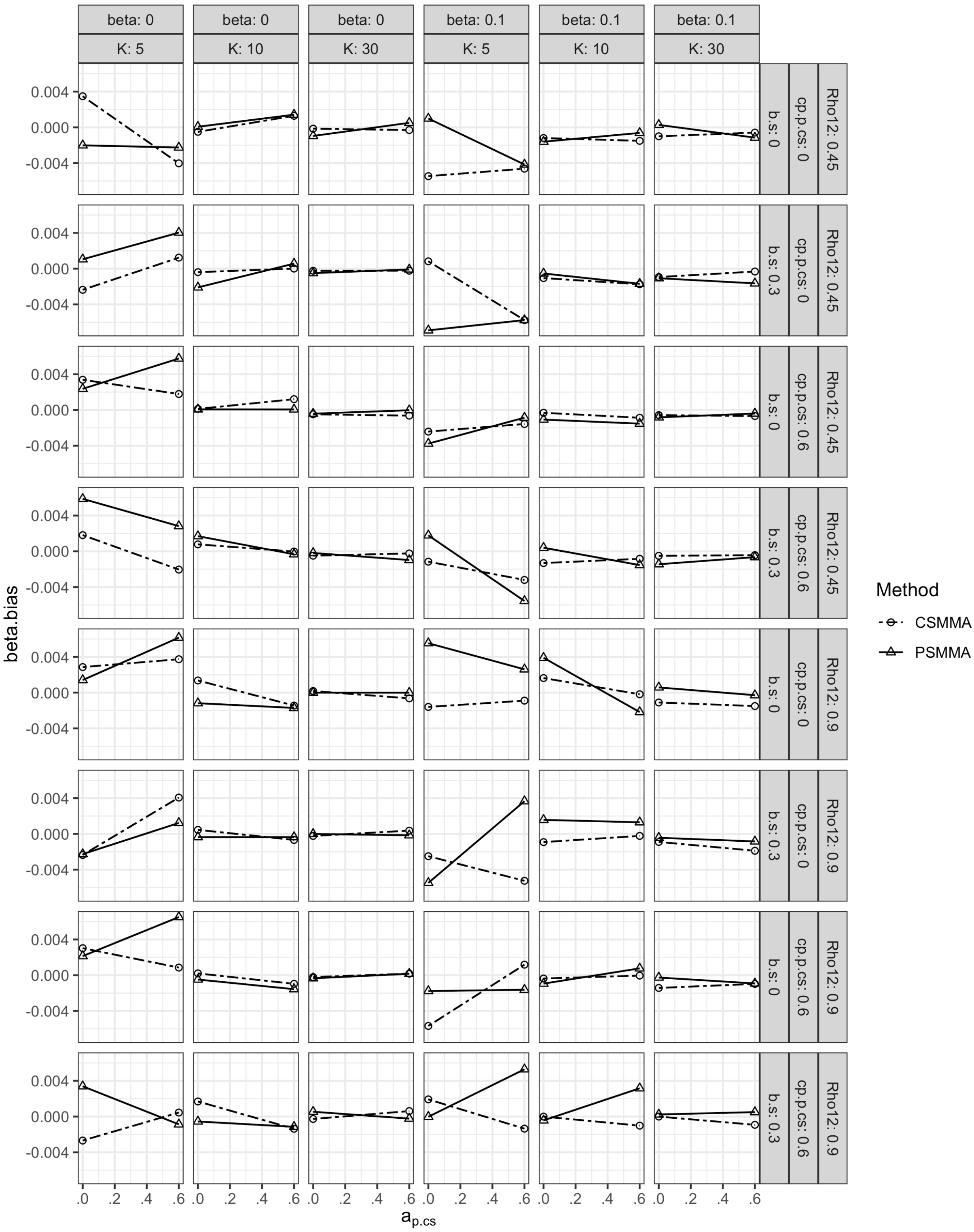


Figure S10.

Coverage Rates of the Moderating Effect with Equal Posttest Variances

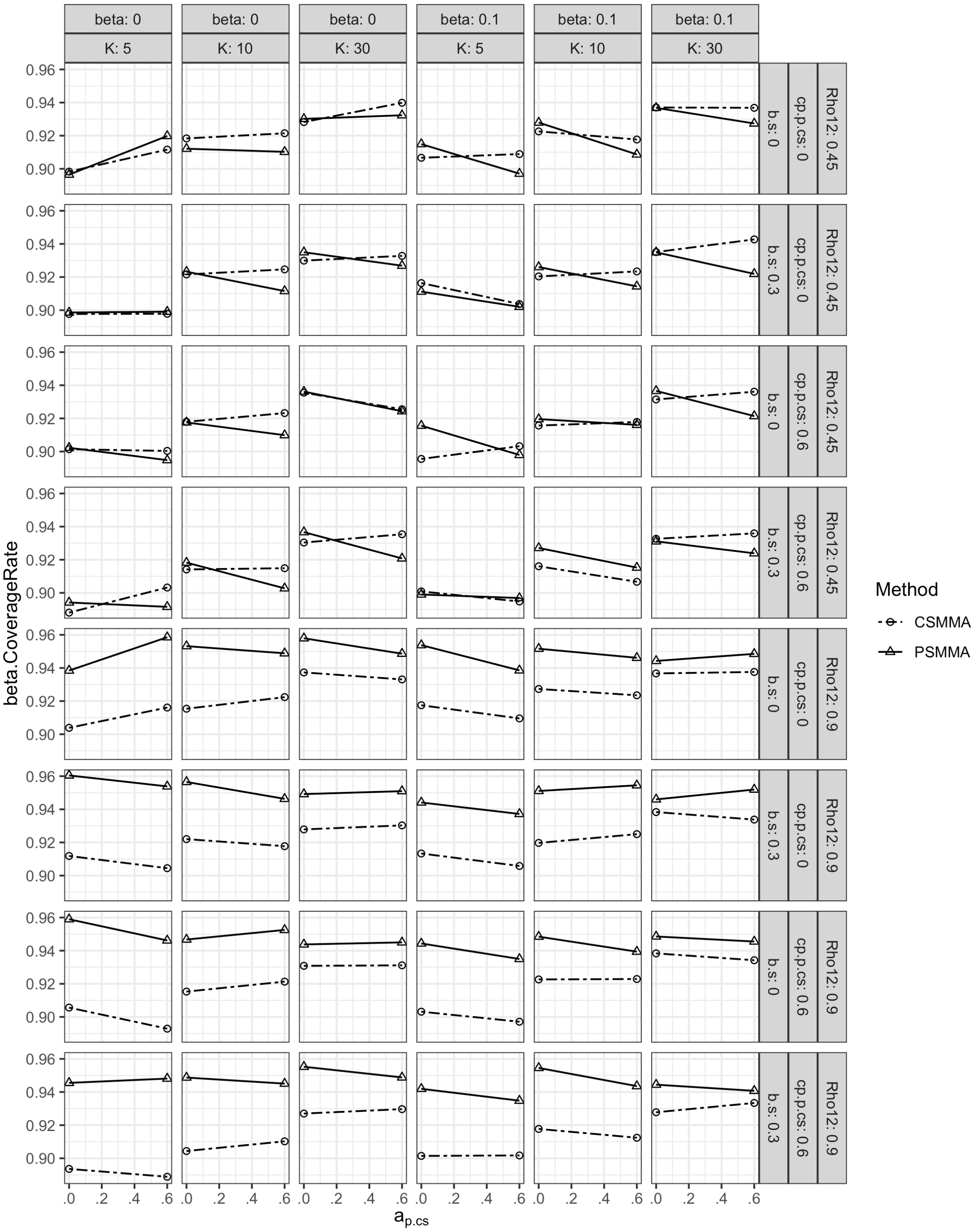


Figure S11.

Type I Error Rates of the Moderating Effect with Equal Posttest Variances

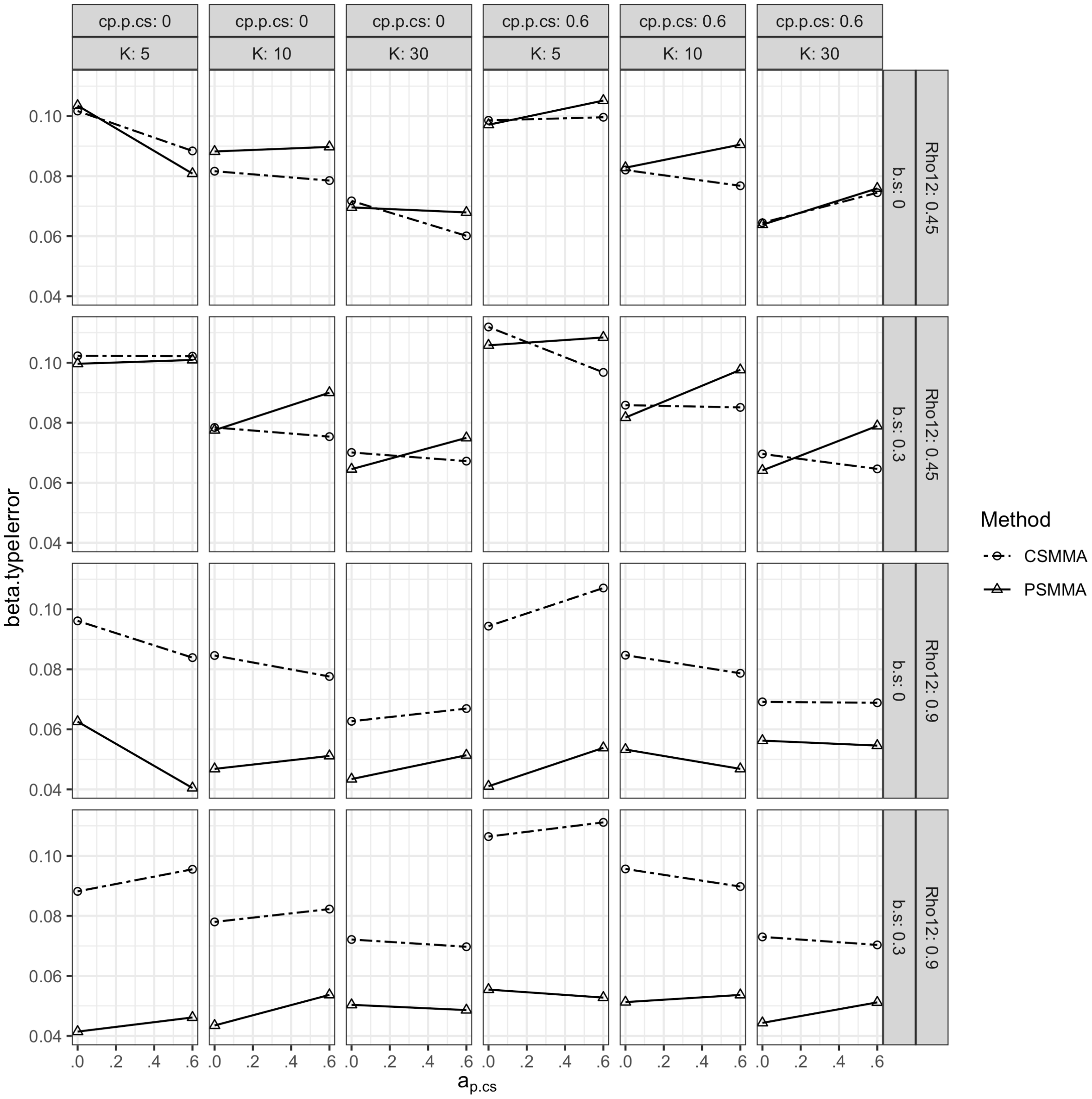
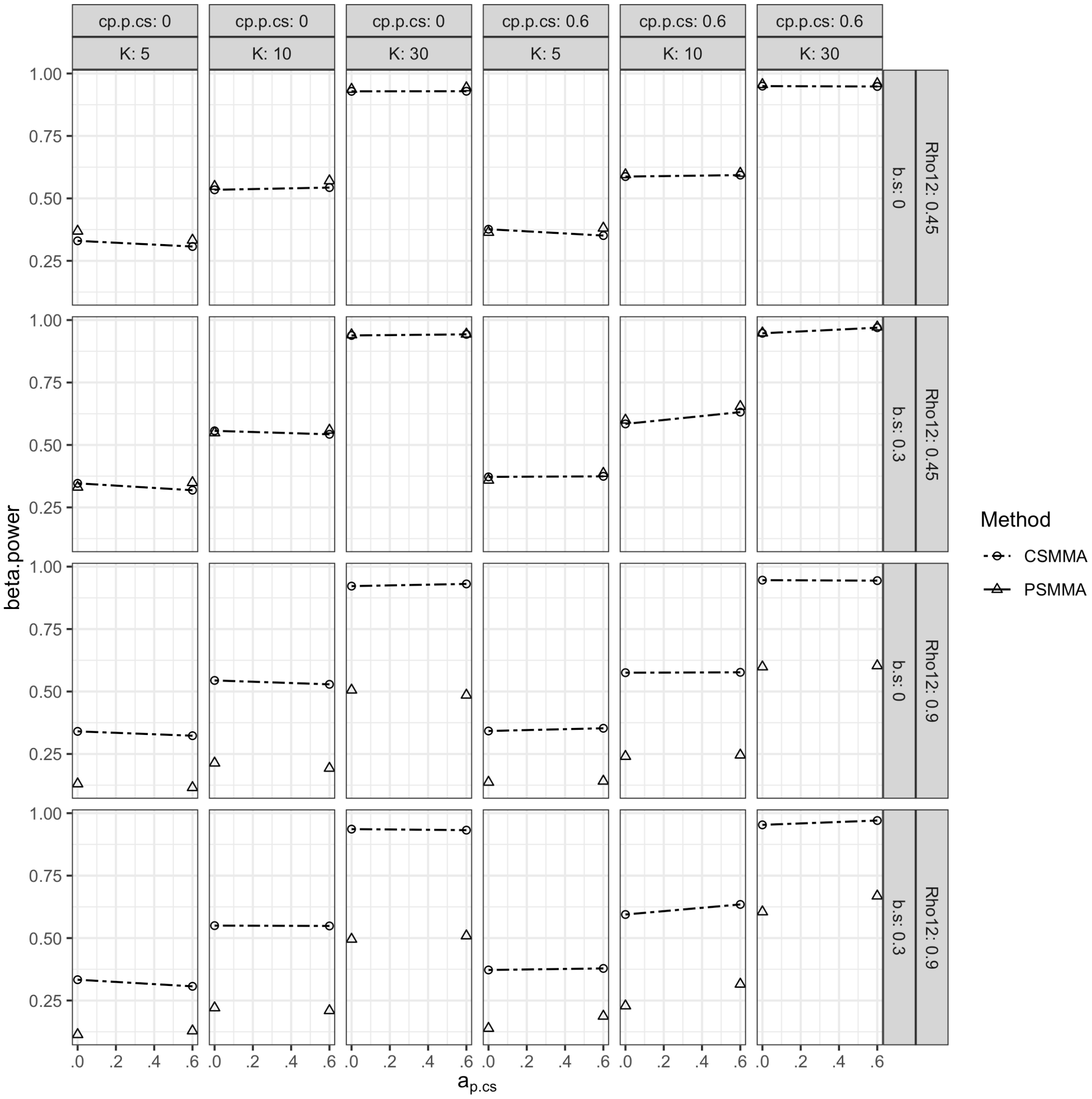


Figure S12.

Statistical Power of the Moderating Effect with Equal Posttest Variances



Coefficients of the a Path and the b Path. As shown in Figure S13-15, the bias, CR and type I error rates of a path estimates of CSMMA and PSMMA both remained favorable under all conditions. The statistical power when estimating the a path (Figure S16), on the other hand, decreased with a larger ρ_{12}, due to small true posttest-score coefficient of the a path.

Figure S13.

Bias of the a Path with Equal Posttest Variances

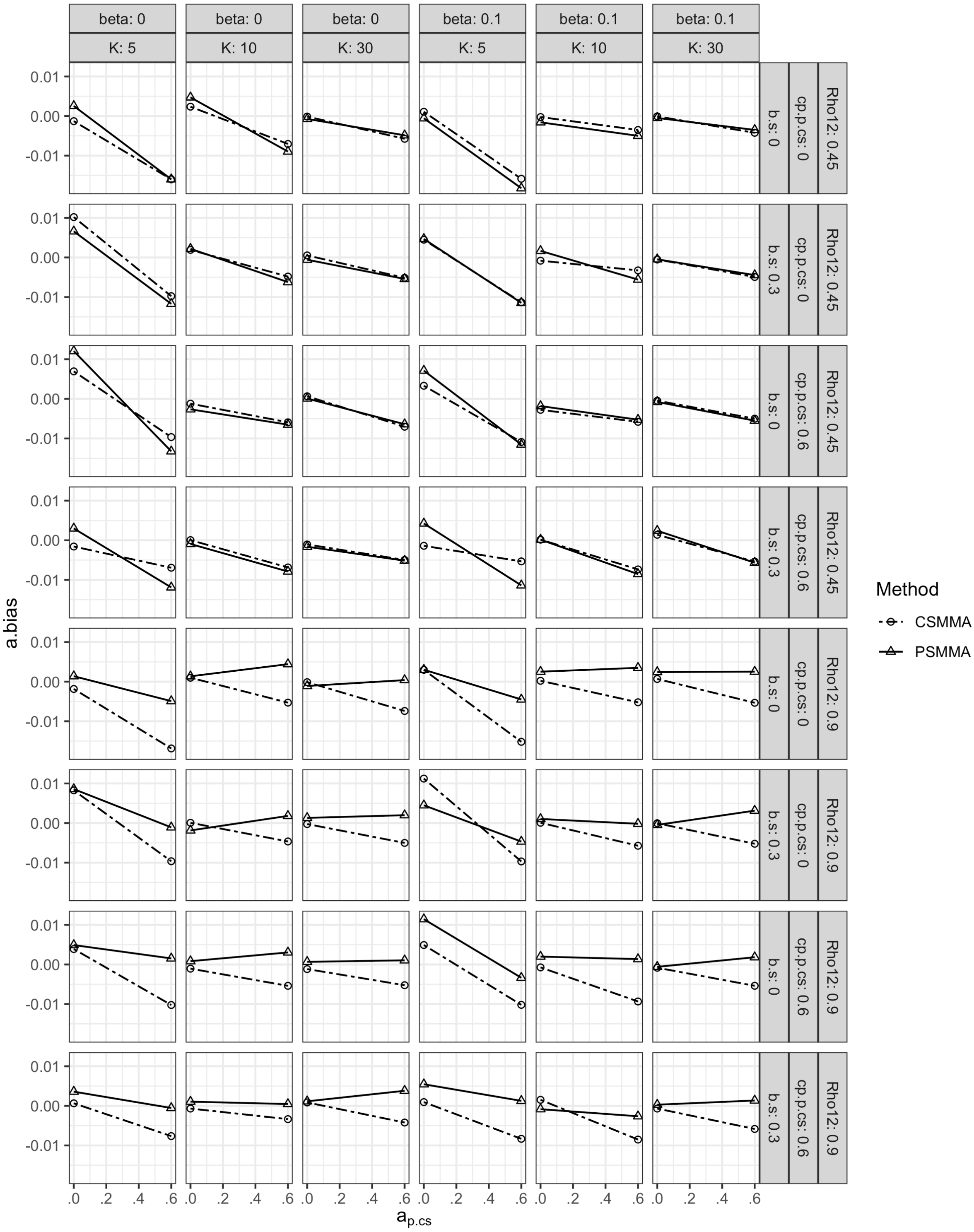


Figure S14.

Coverage Rates of the a Path with Equal Posttest Variances

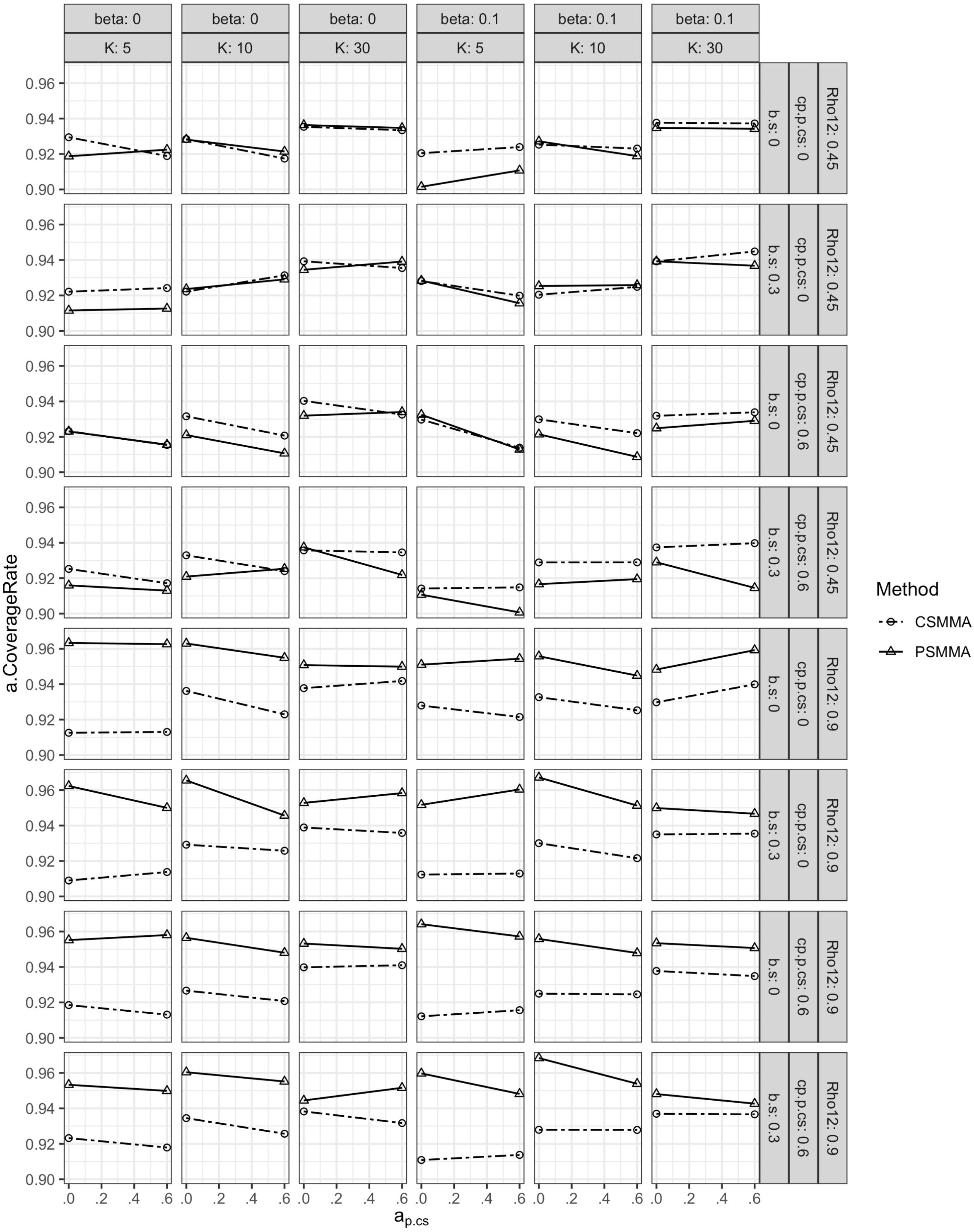


Figure S15.

Type I Error Rates of the a Path with Equal Posttest Variances

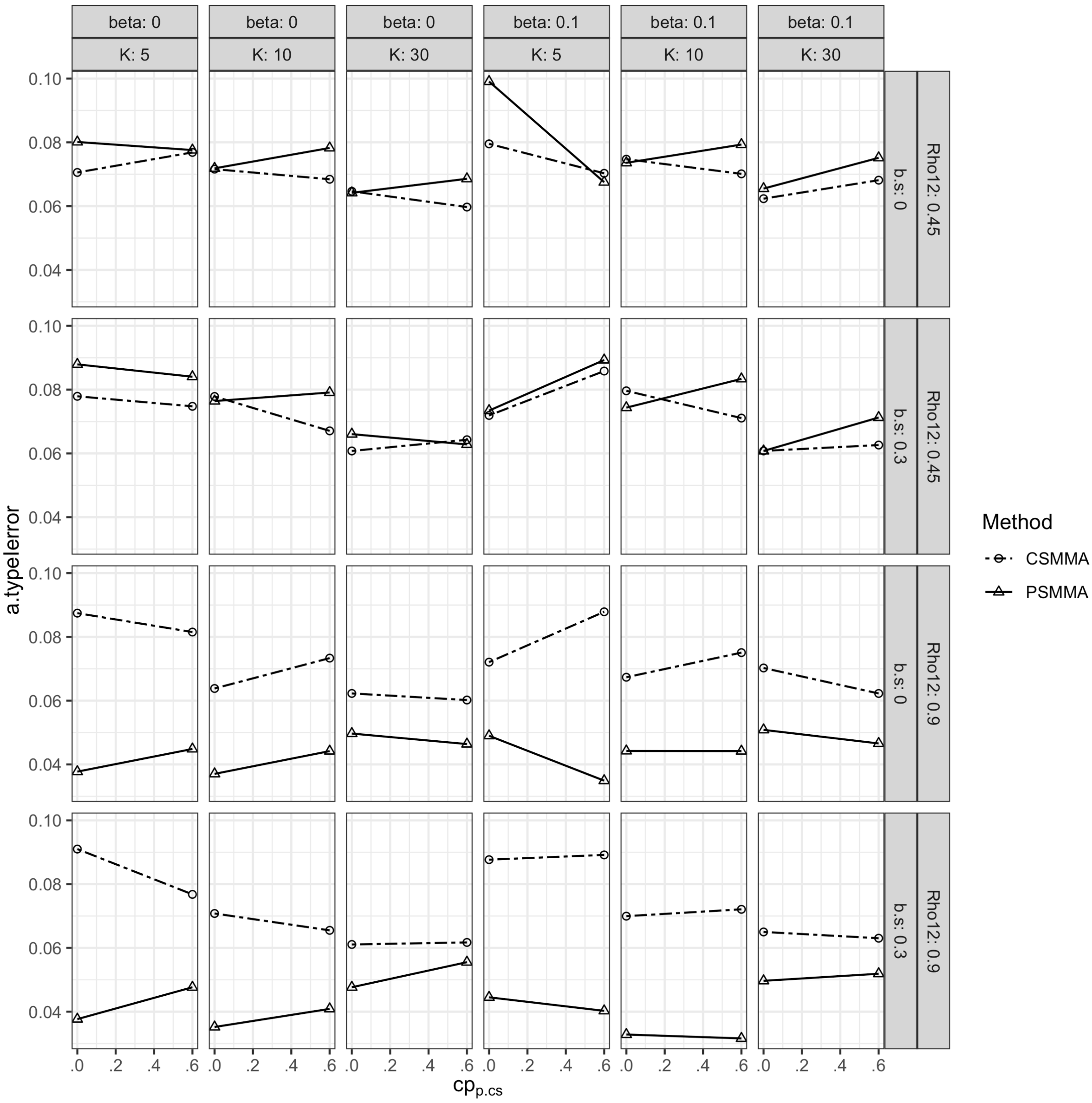
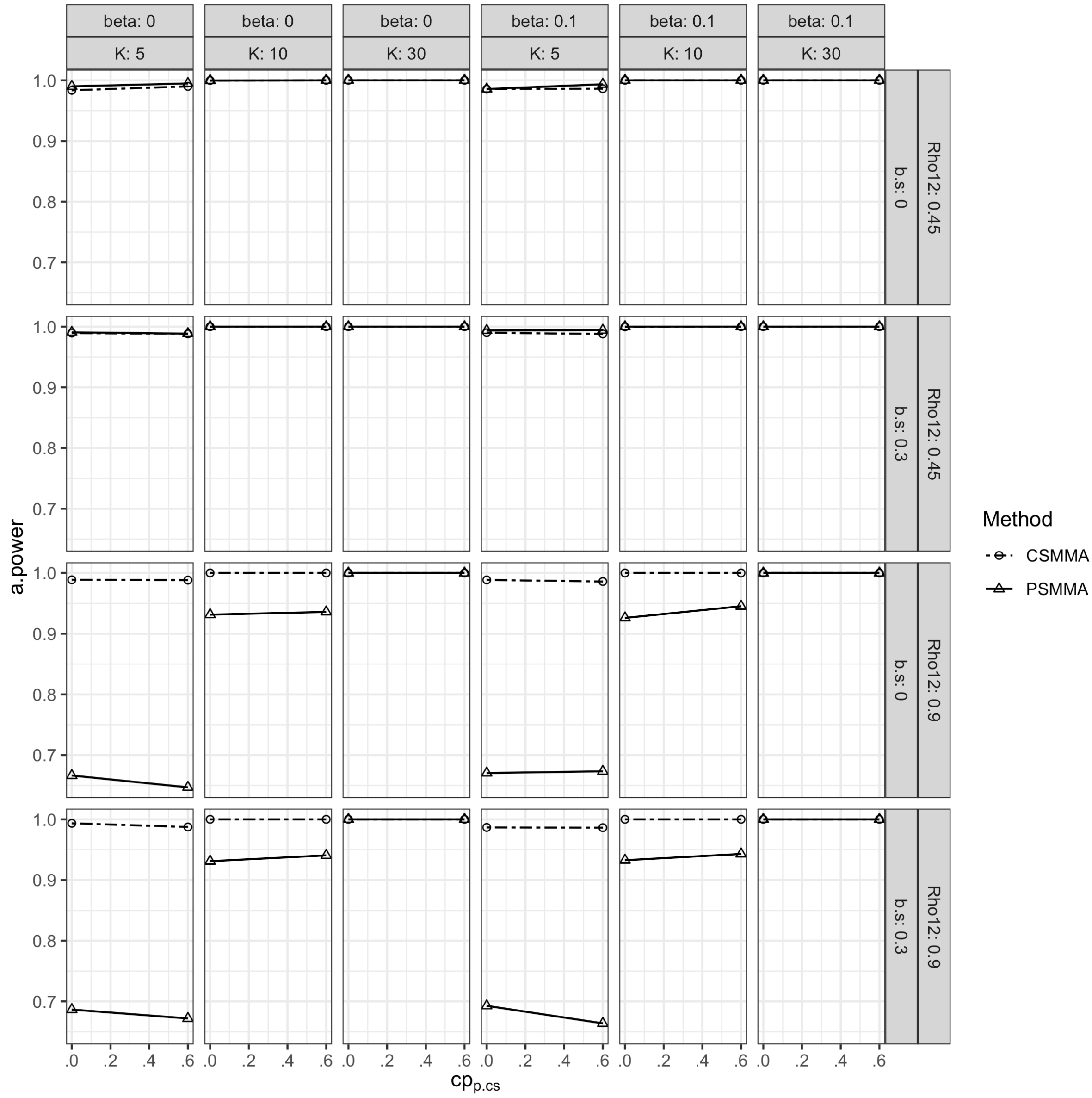


Figure S16.

Statistical Power of the a Path with Equal Posttest Variances



When estimating the b path, both CSMMA and PSMMA had small bias (Figure S17). The CR of both CSMMA and PSMMA remained above 0.9 (Figure S18). The type I error rates of CSMMA and PSMMA fluctuated from 0.02 to 0.1 (Figure S19). While the power of CSMMA remained above 0.9, that of PSMMA dropped with a smaller K and a larger ρ_{12} (Figure S20).

Figure S17.

Bias of the b Path Coefficient with Equal Posttest Variances

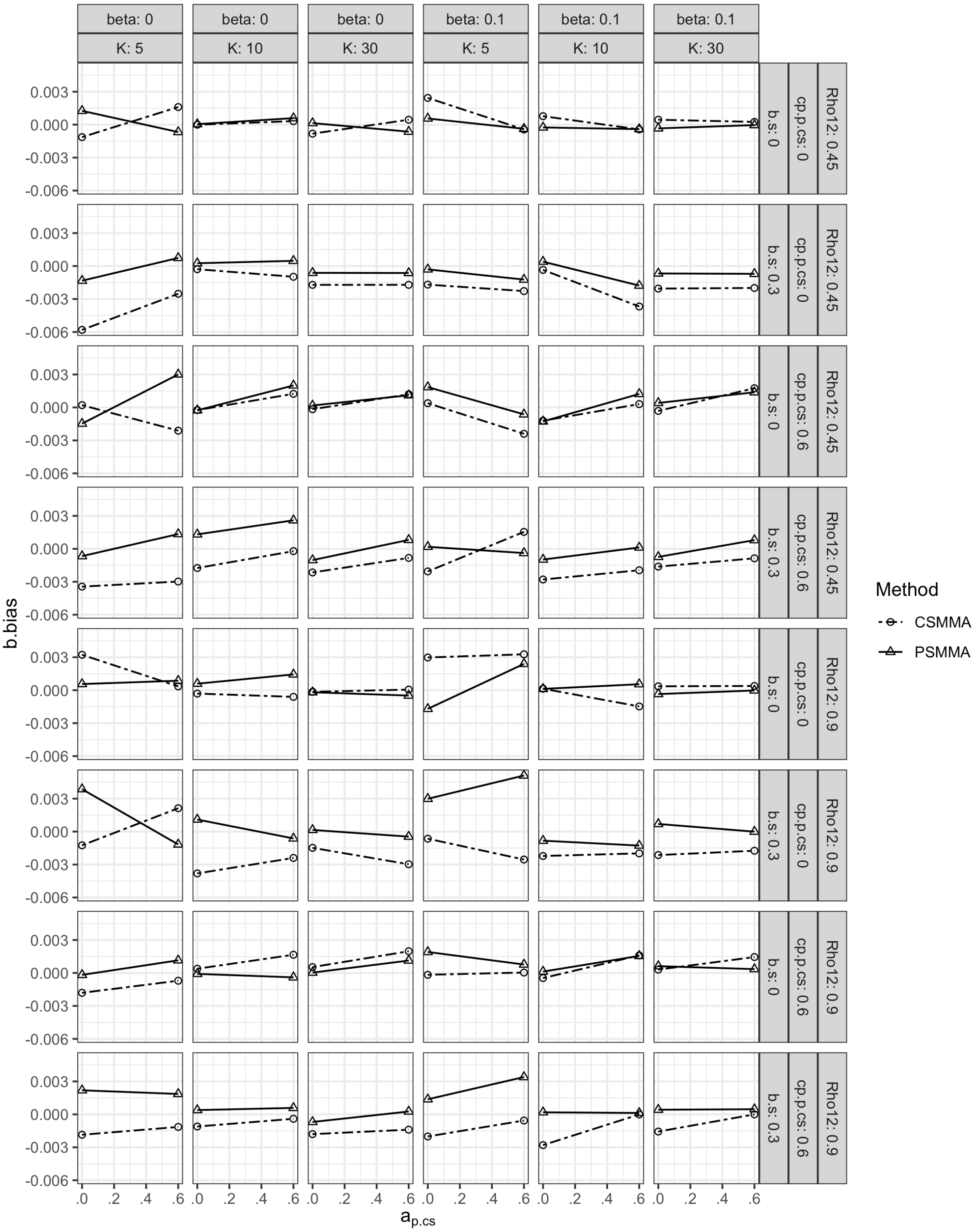


Figure S18.

Coverage Rates of the b Path Coefficient with Equal Posttest Variances

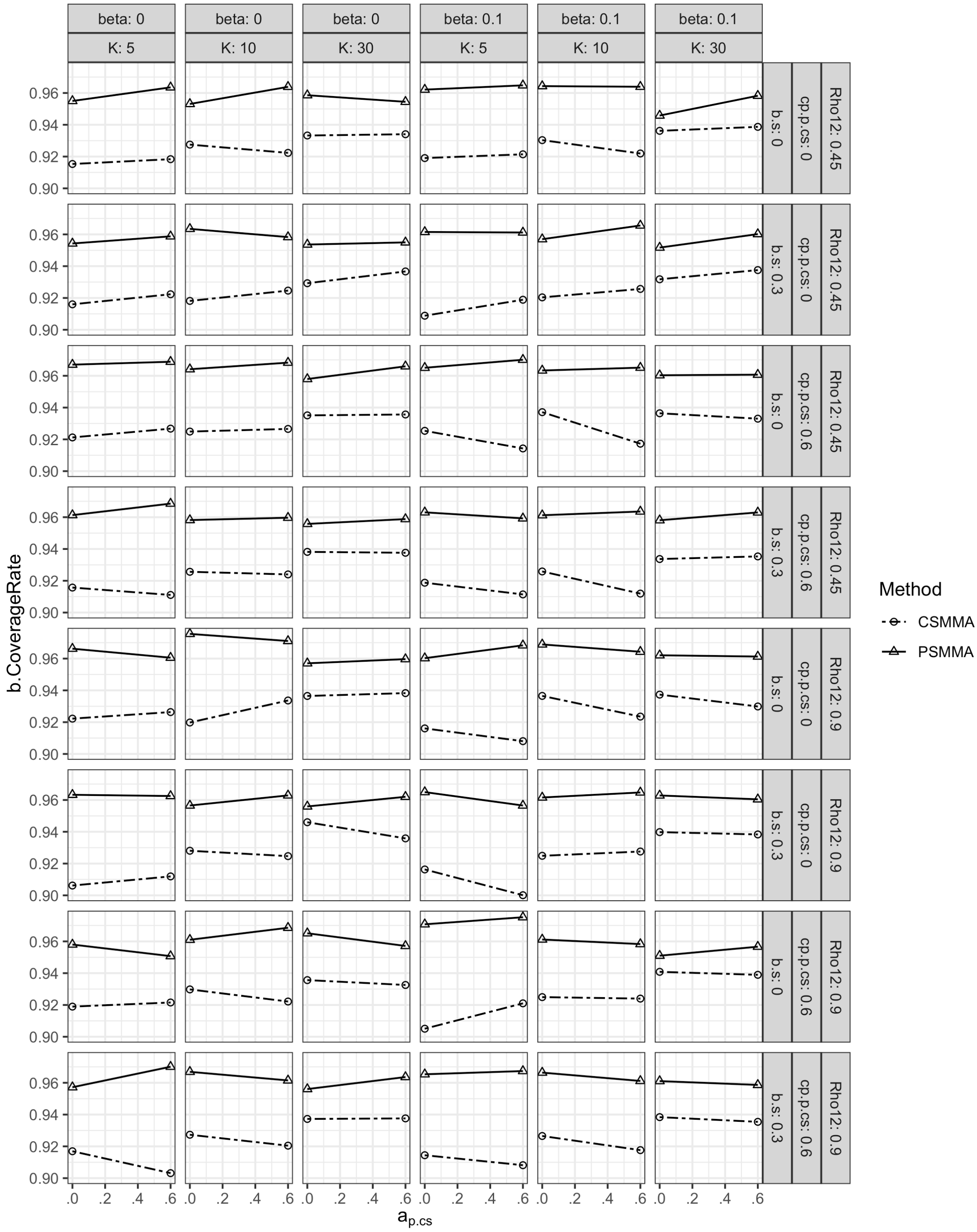


Figure S19.

Type I Error Rates of the b Path Coefficient with Equal Posttest Variances

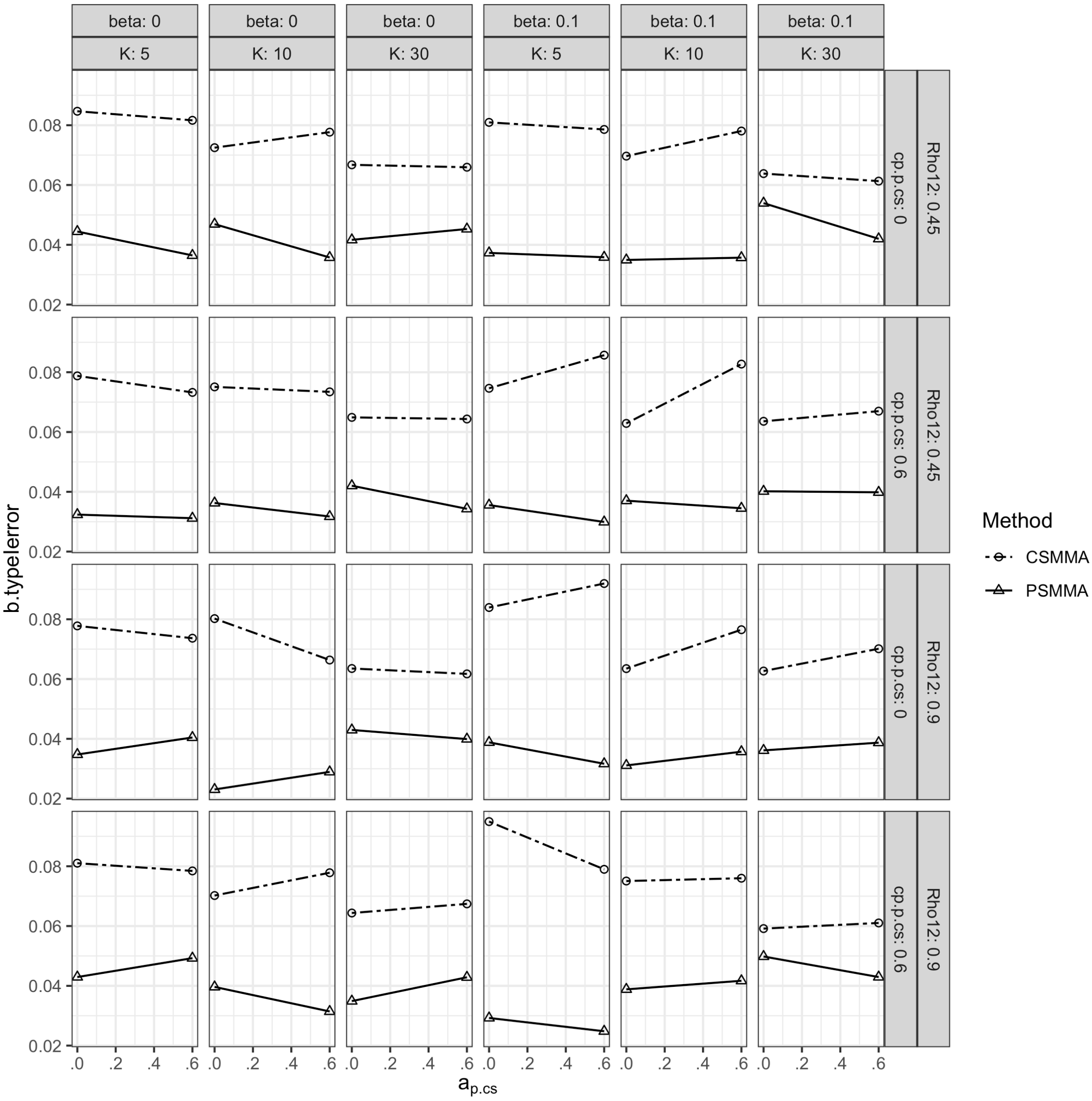
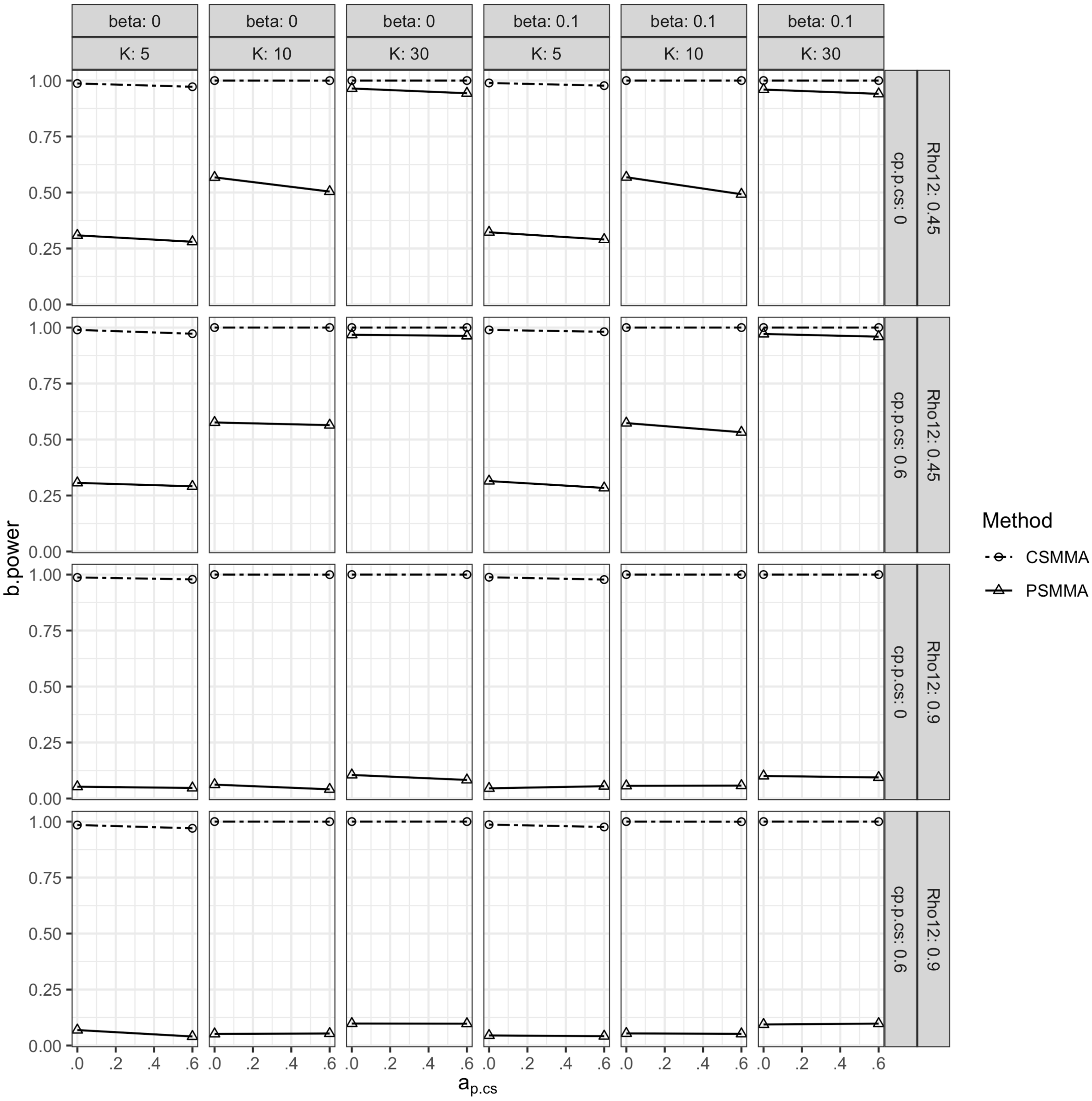


Figure S20.

Statistical Power of the b Path Coefficient with Equal Posttest Variances



S2.3.2 Results with Unequal Posttest Variances

The Indirect Effect. As shown in Figure S21-23, the patterns of bias, CR, type I error rates and power of CSMMA and PSMMA in Study 2 were similar to those with equal posttest variances. However, the inflation of posttest variances increased the power of PSMMA, and $\beta_{c'_{s,cs}}$ and $c'_{s,cs}$ had no apparent effect on this pattern (Figure S24).

Figure S21.

Bias of the Indirect Effect with Unequal Posttest Variances

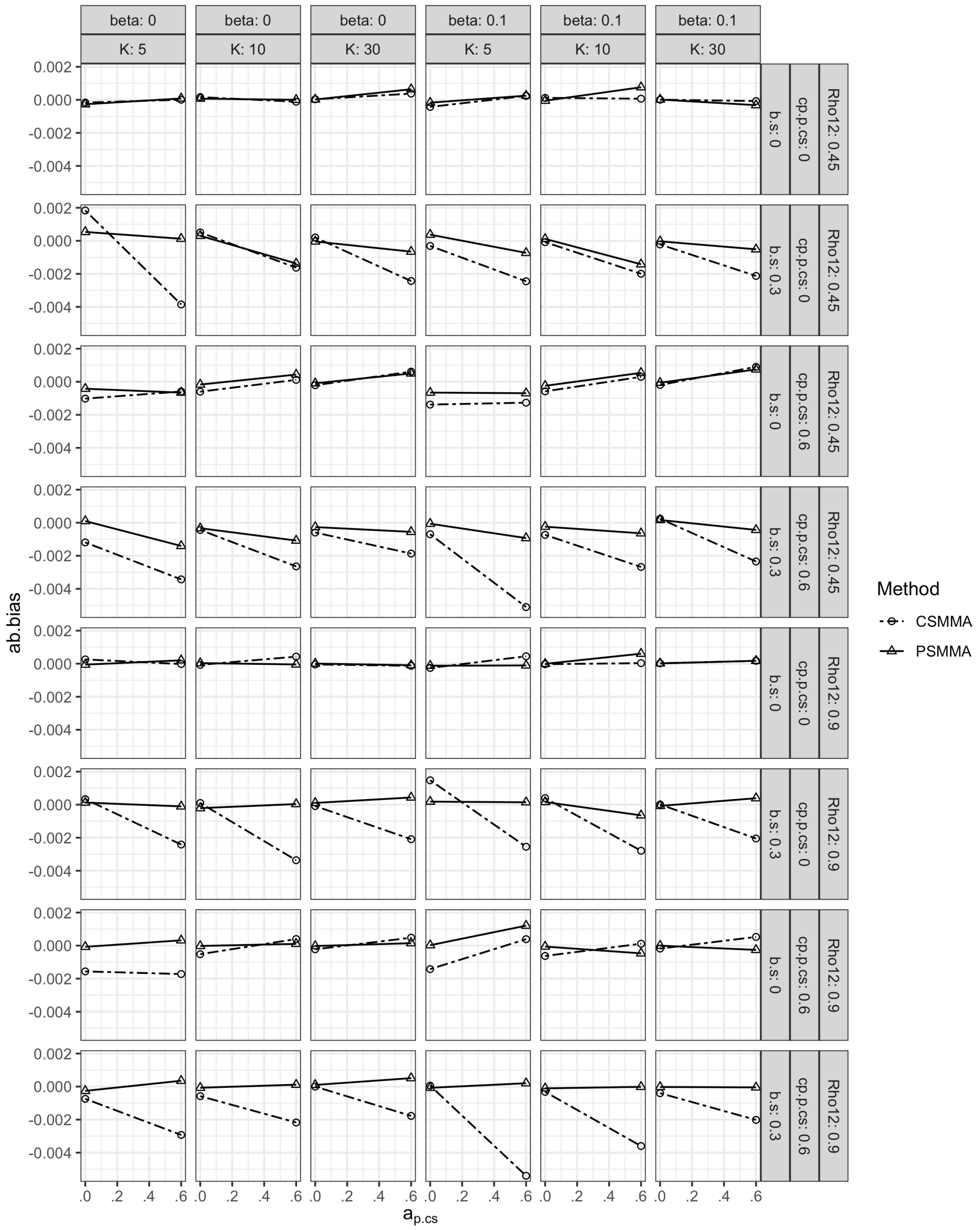


Figure S22.

Coverage Rates of the Indirect Effect with Unequal Posttest Variances

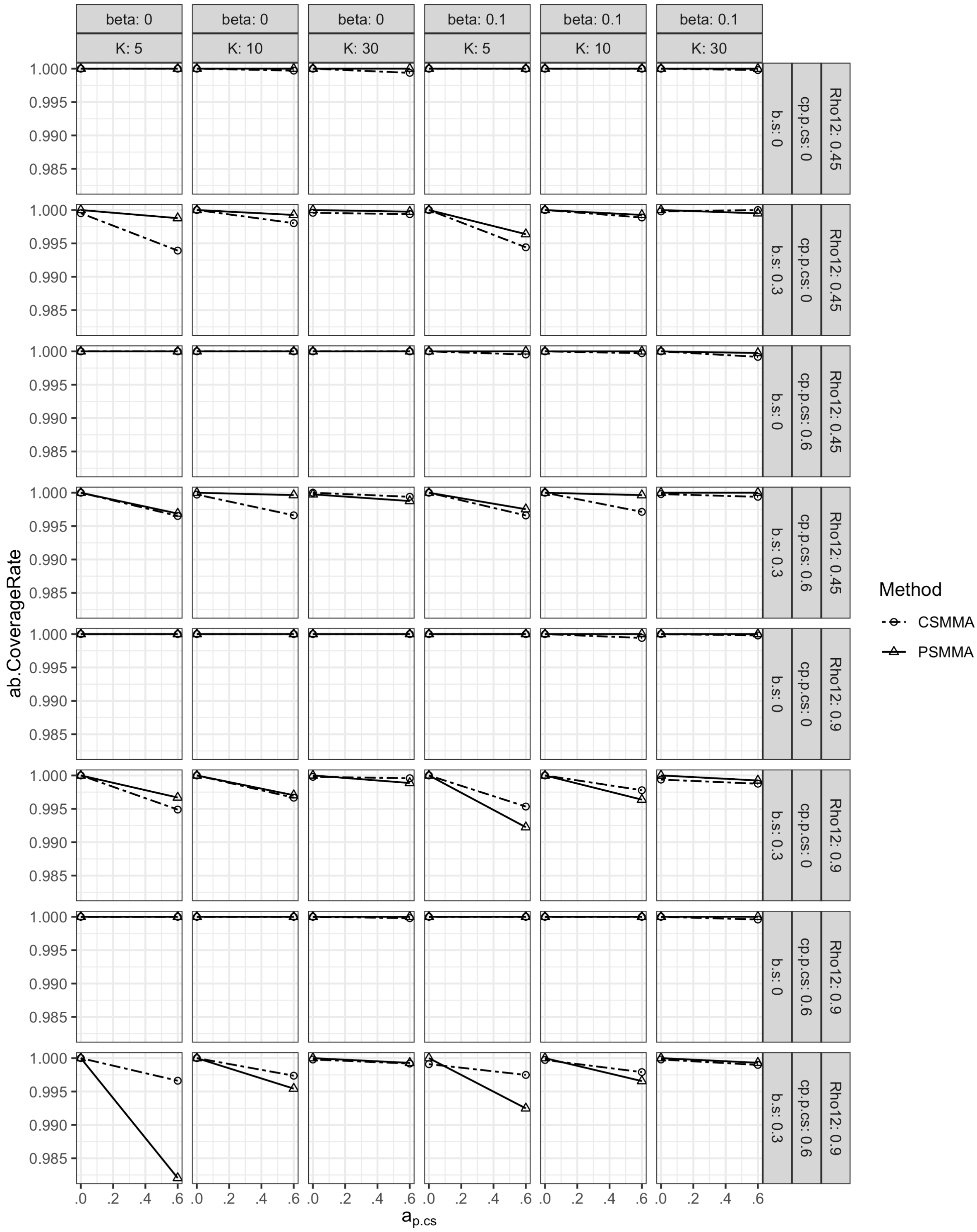


Figure S23.

Type I Error Rates of the Indirect Effect with Unequal Posttest Variances

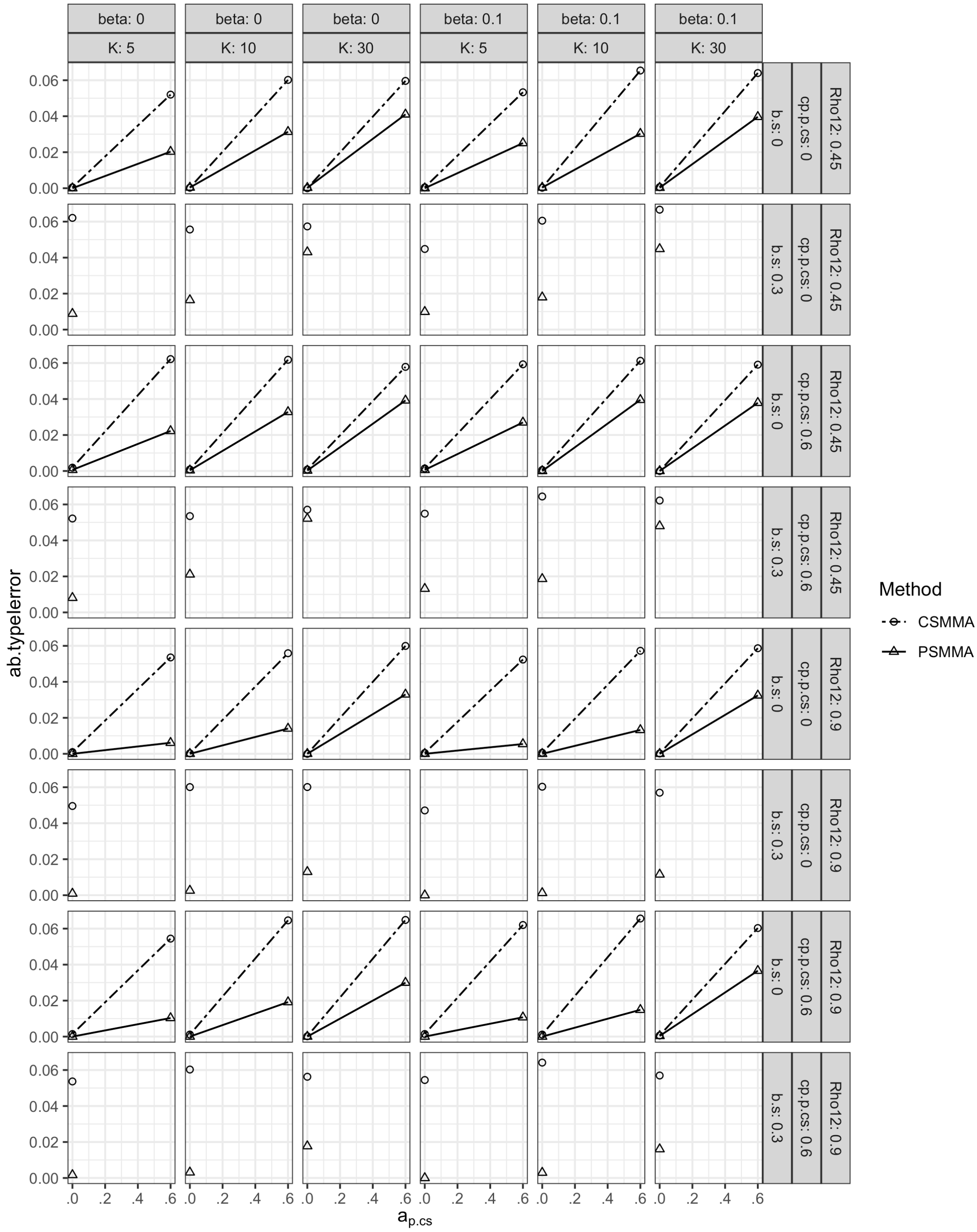
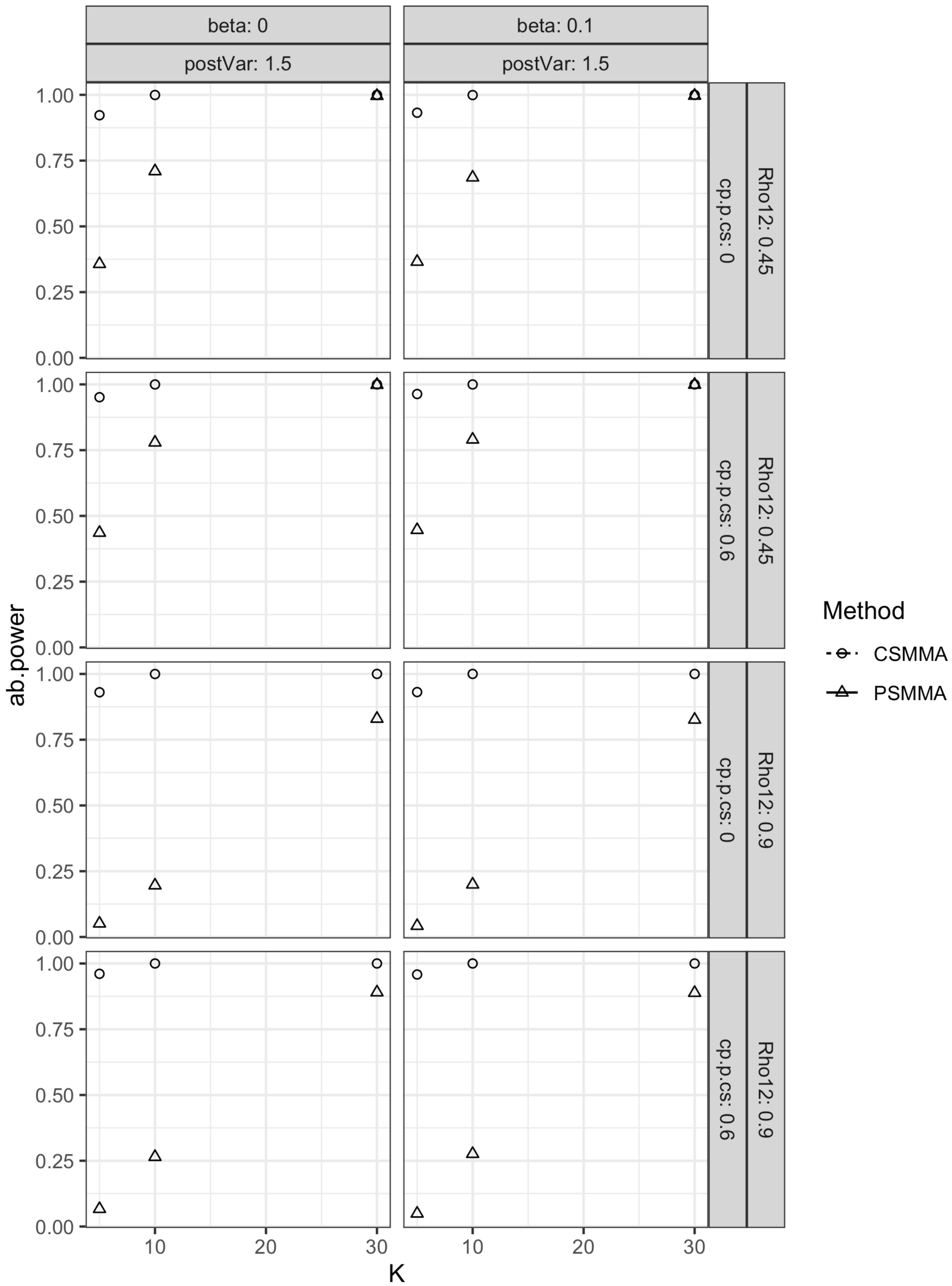


Figure S24.

Statistical Power of the Indirect Effect with Unequal Posttest Variances



The Direct Effect. Similarly, results regarding the direct effect had the same pattern as those under equal posttest variances (Figure S25-28).

Figure S25.

Bias of the Direct Effect with Unequal Posttest Variances

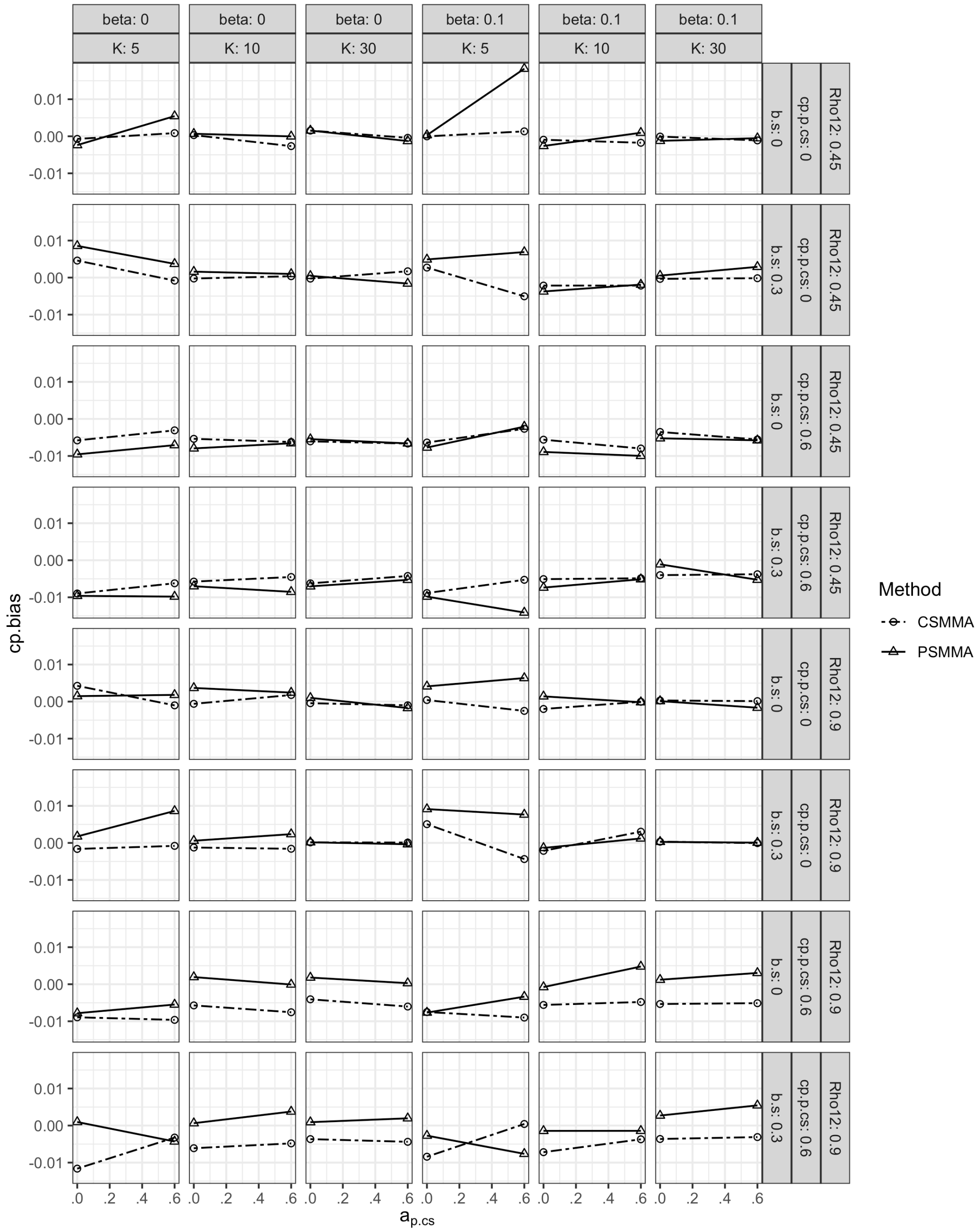


Figure S26.

Coverage Rates of the Direct Effect with Unequal Posttest Variances

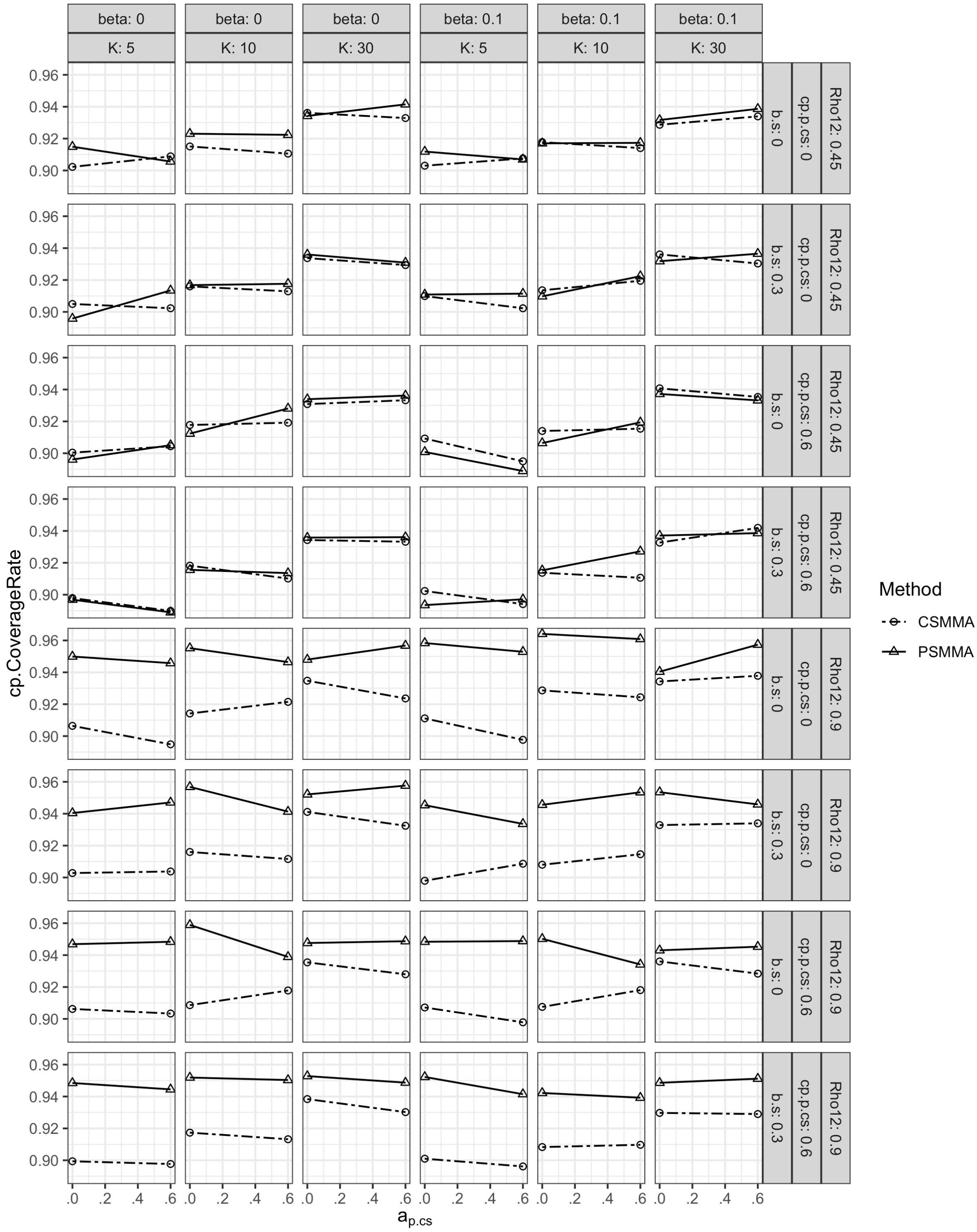


Figure S27.

Type I Error Rates of the Direct Effect with Unequal Posttest Variances

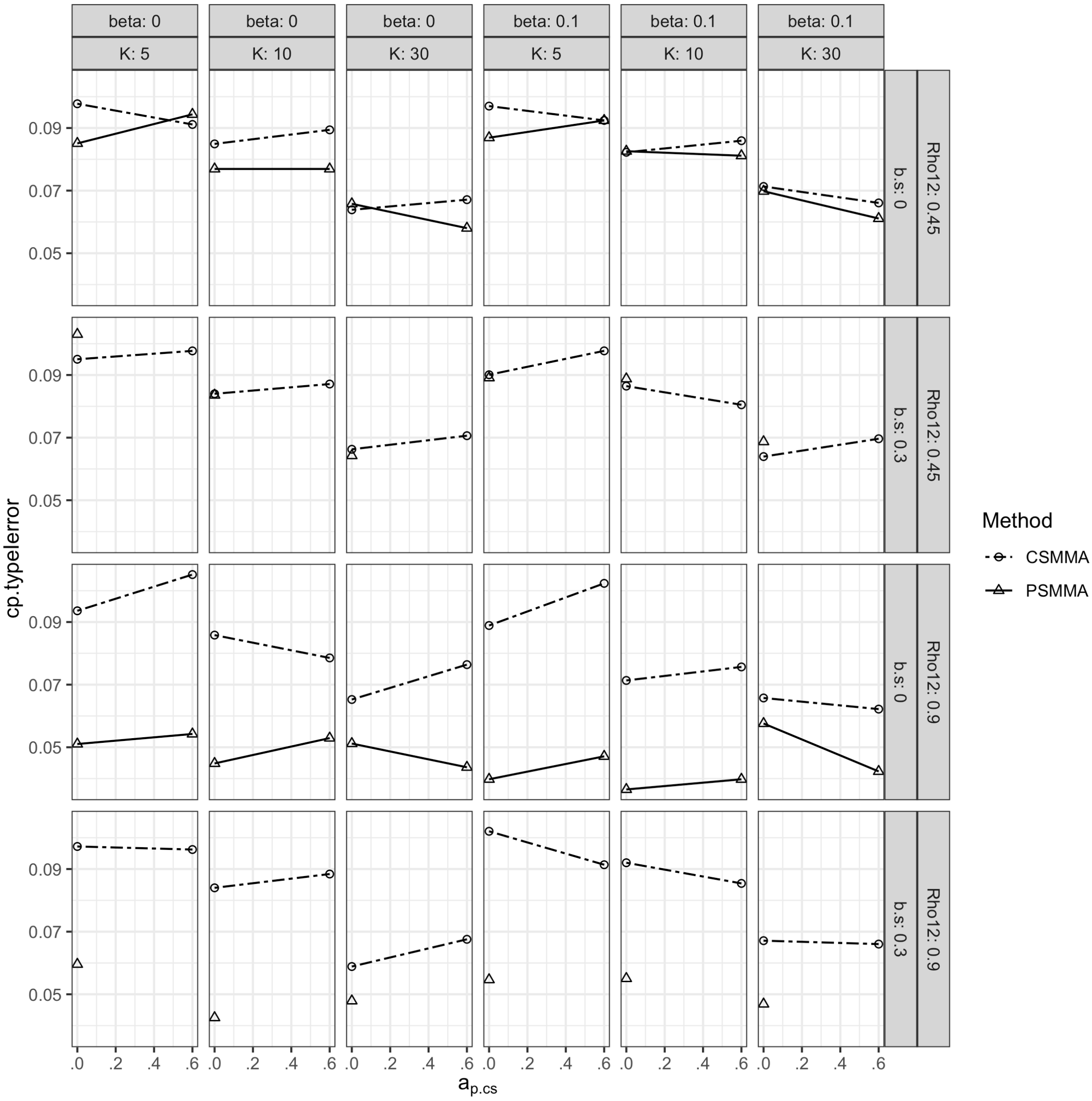
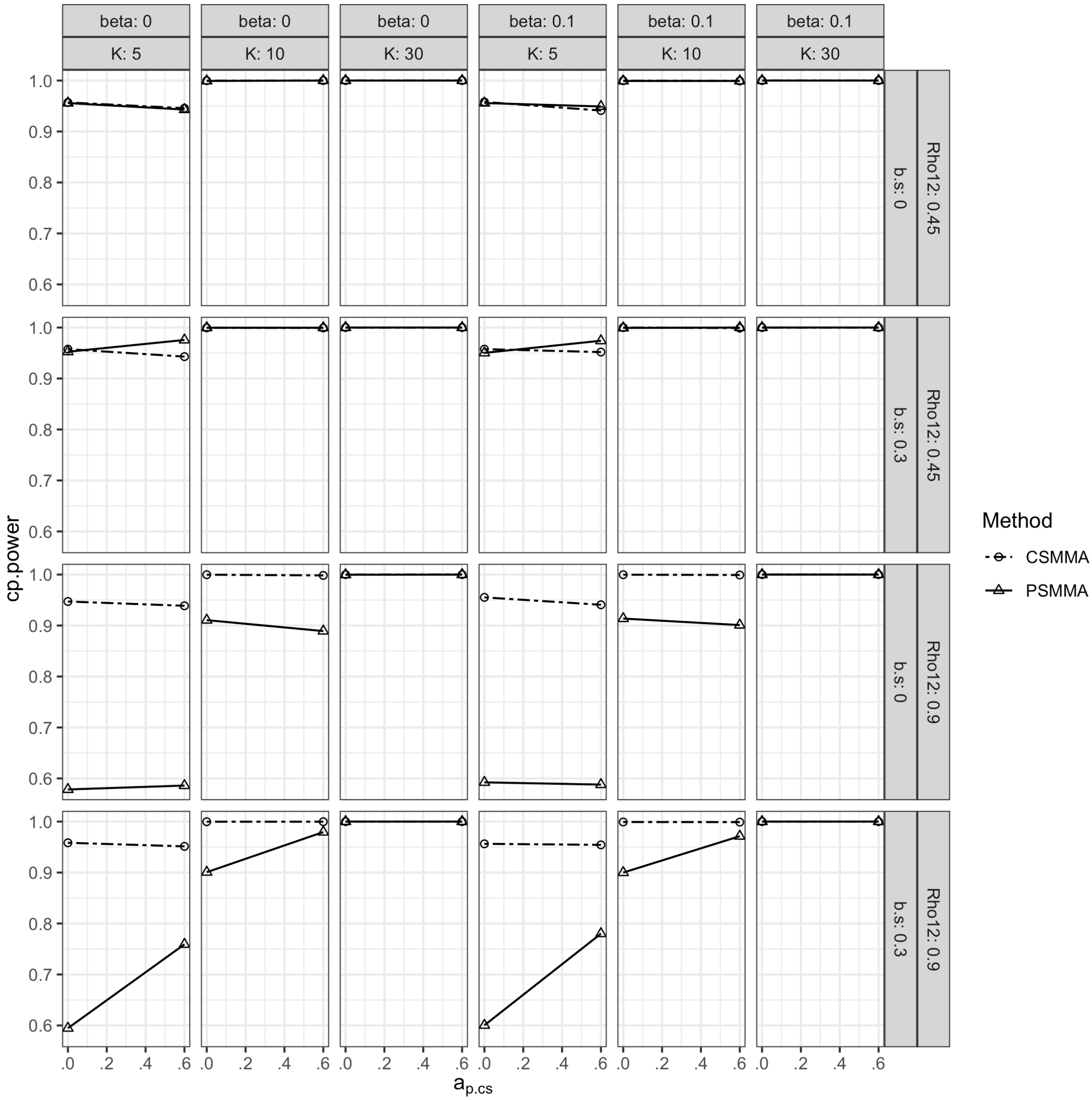


Figure S28.

Statistical Power of the Direct Effect with Unequal Posttest Variances



The Moderating Effect. The bias, CR, type I error rates and power of the moderating effect had similar patterns with the results with equal posttest variances (Figure S29-32).

Figure S29.

Bias of the Moderating Effect with Unequal Posttest Variances

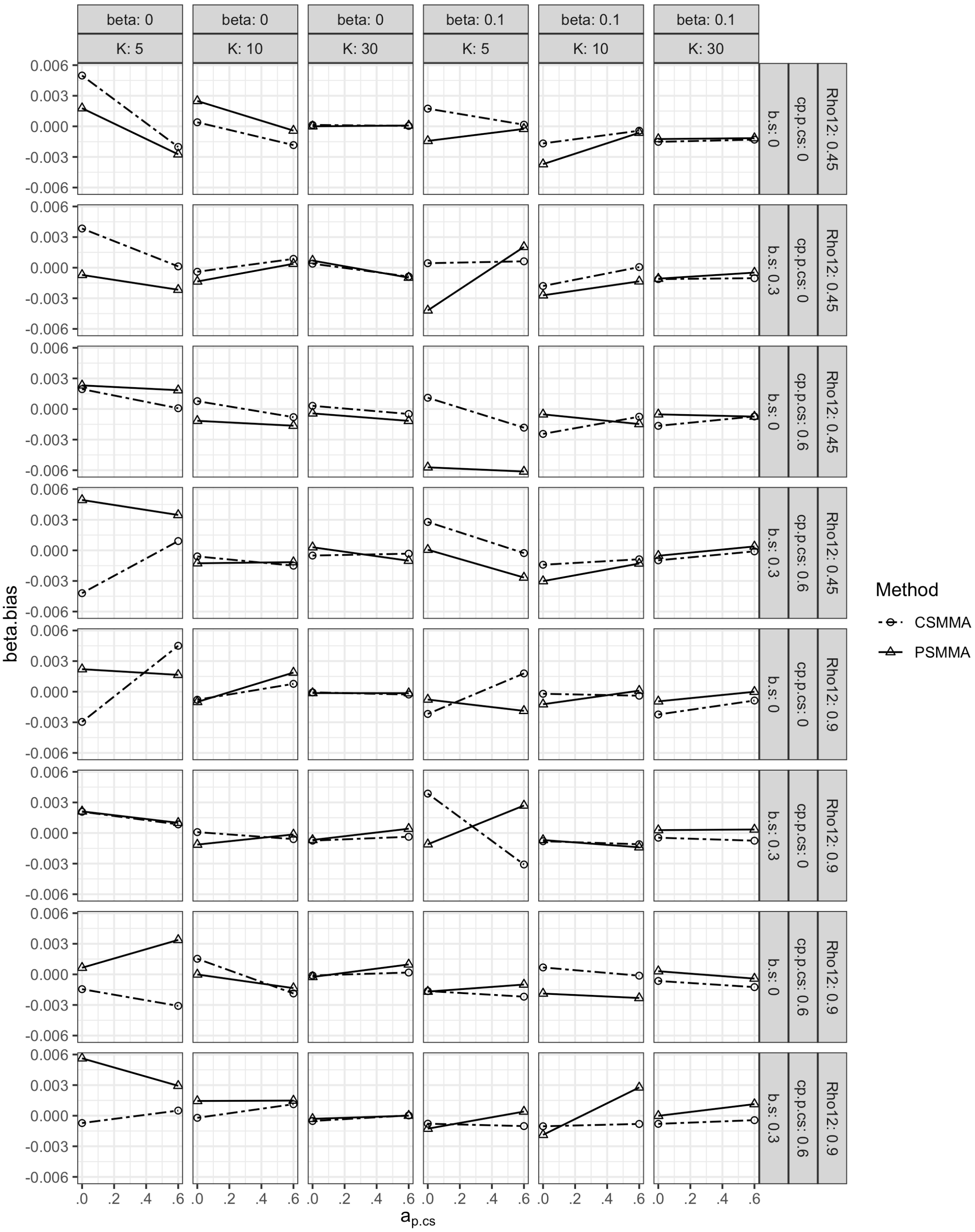


Figure S30.

Coverage Rates of the Moderating Effect with Unequal Posttest Variances

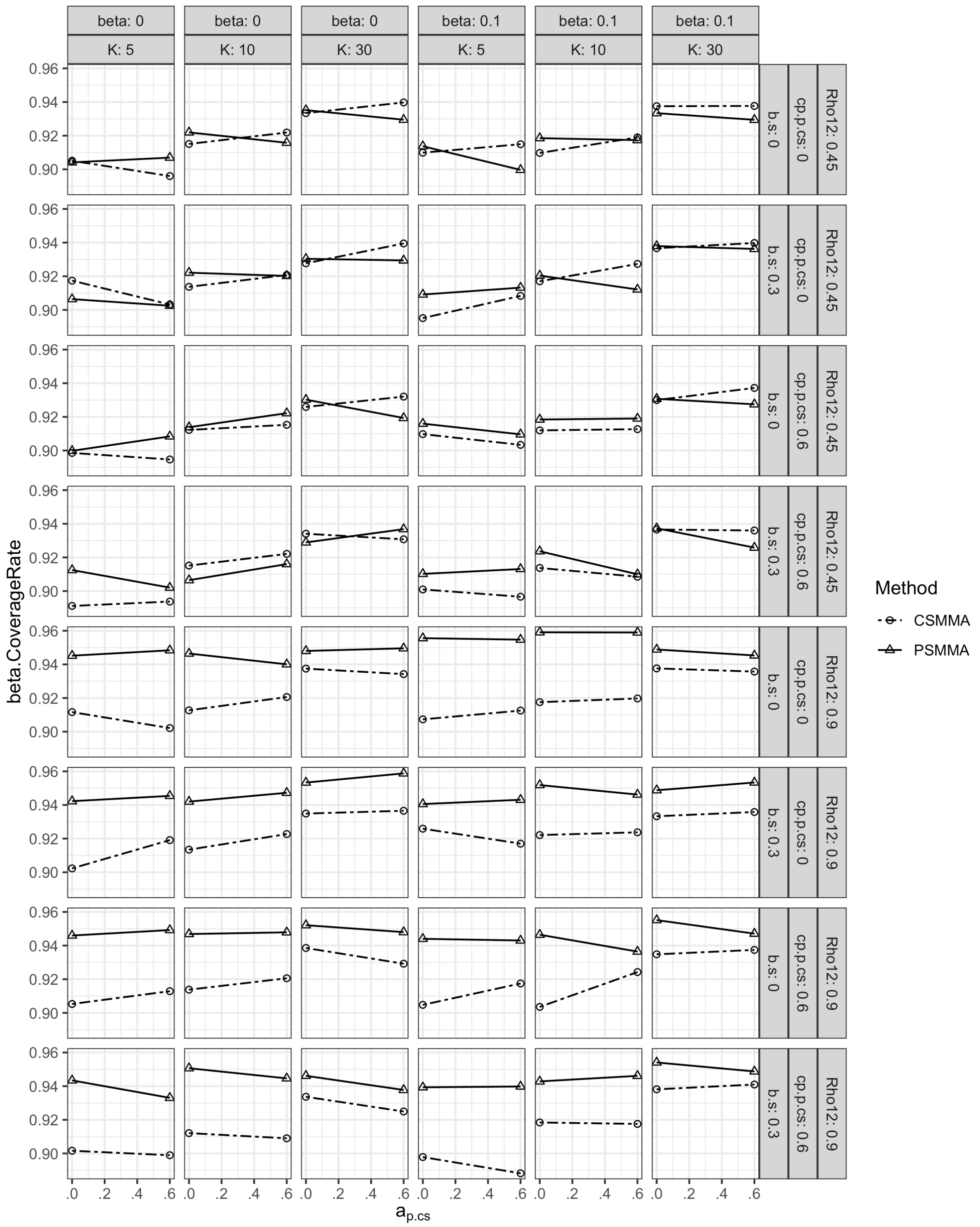


Figure S31.

Type I Error Rates of the Moderating Effect with Unequal Posttest Variances

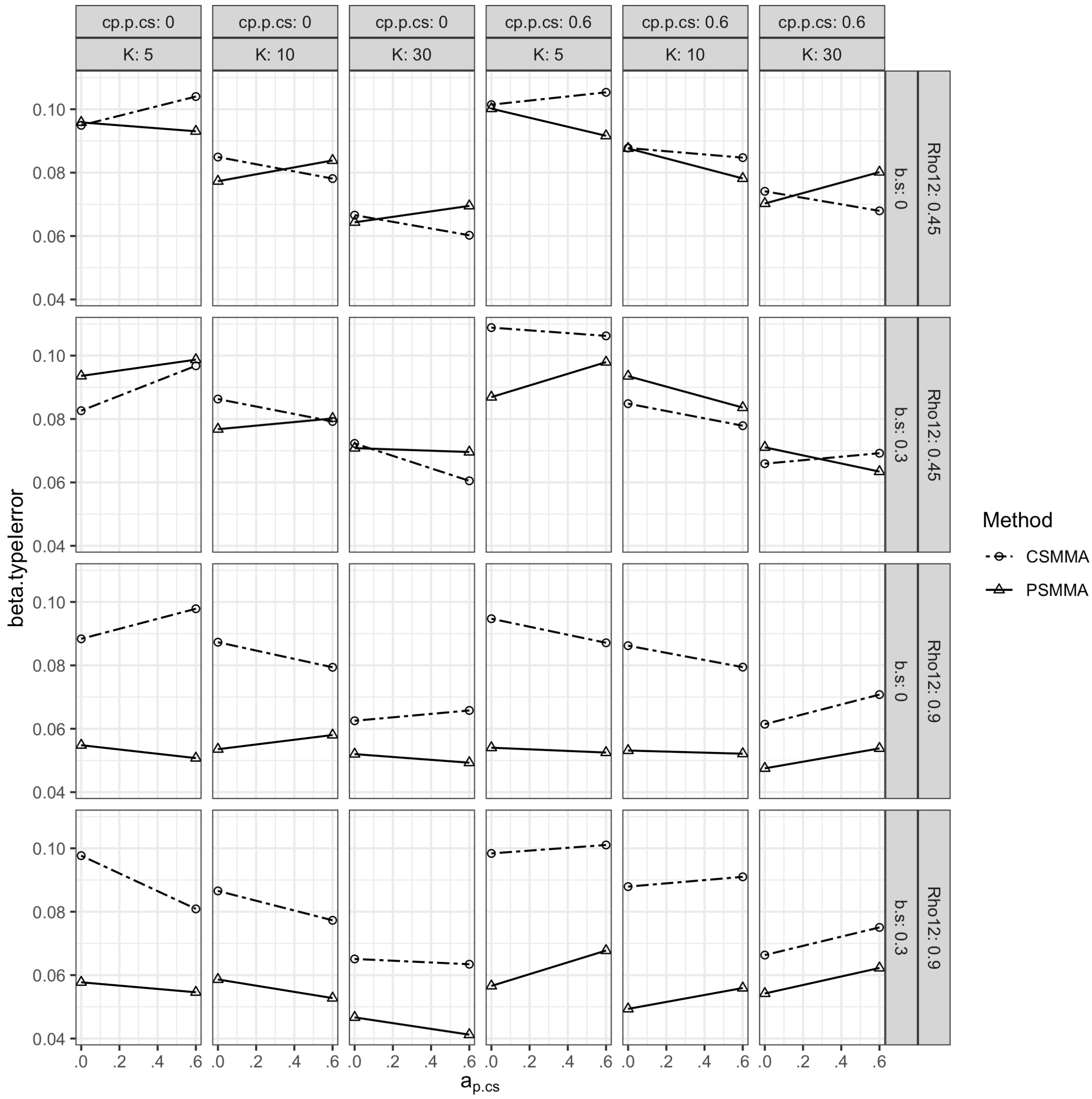
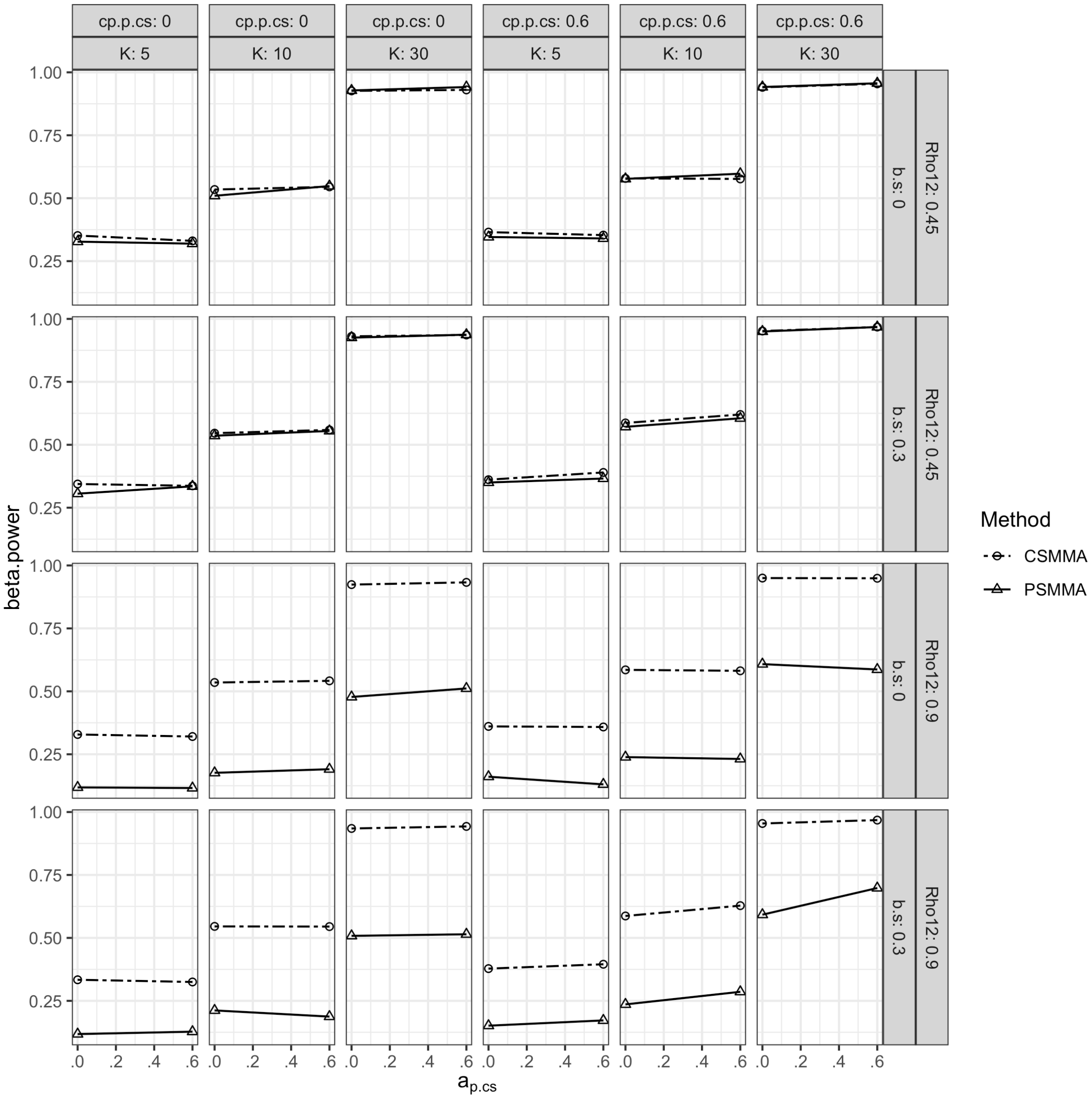


Figure S32.

Statistical Power of the Moderating Effect with Unequal Posttest Variances



Coefficients of the $\boldsymbol{a}$ Path and the $\boldsymbol{b}$ Path. As shown in Figure S33-35 and in Figure S37-39, the patterns of bias, CR, and type I error rates of CSMMA and PSMMA when estimating the a and b paths were similar as those with equal posttest variances. However, the statistical power of PSMMA when estimating the a and b paths increased in the presence of the inflation of posttest variances (Figure S36 and S40).

Figure S33.

Bias of the a path Coefficient with Uneuqal Posttest Variances

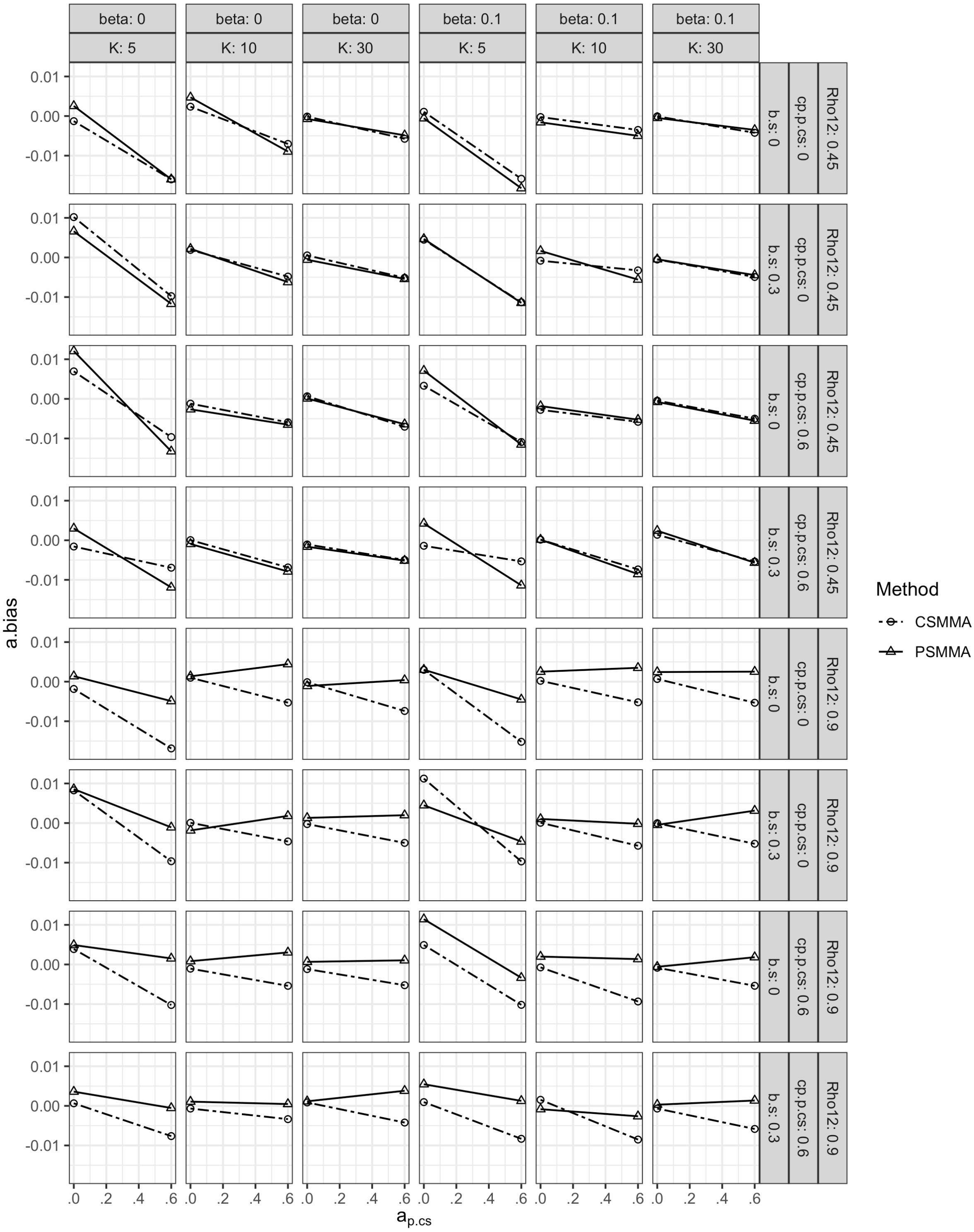


Figure S34.

Coverage Rates of the a path Coefficient with Uneuqal Posttest Variances

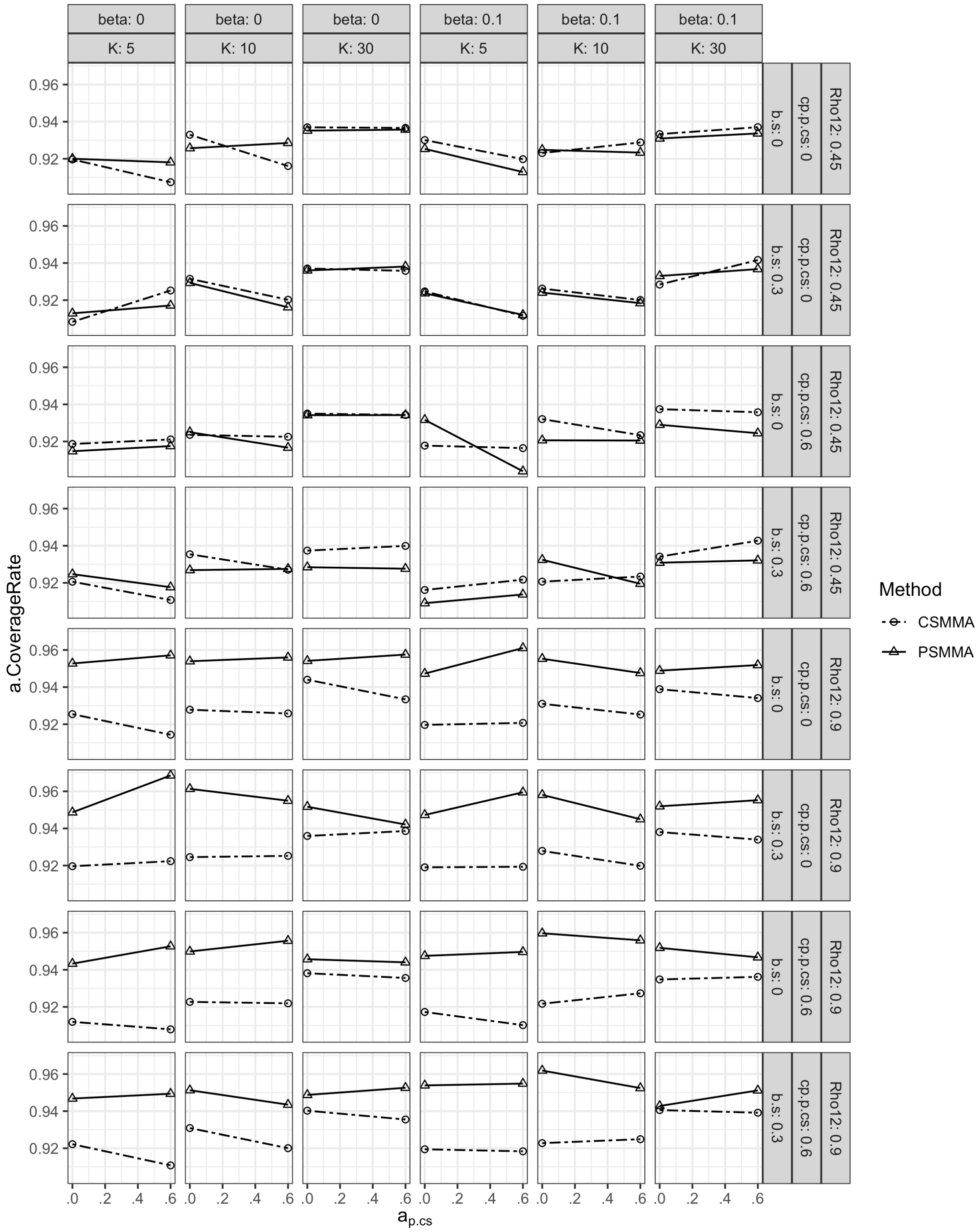


Figure S35.

Type I Error Rates of the a path Coefficient with Uneuqal Posttest Variances

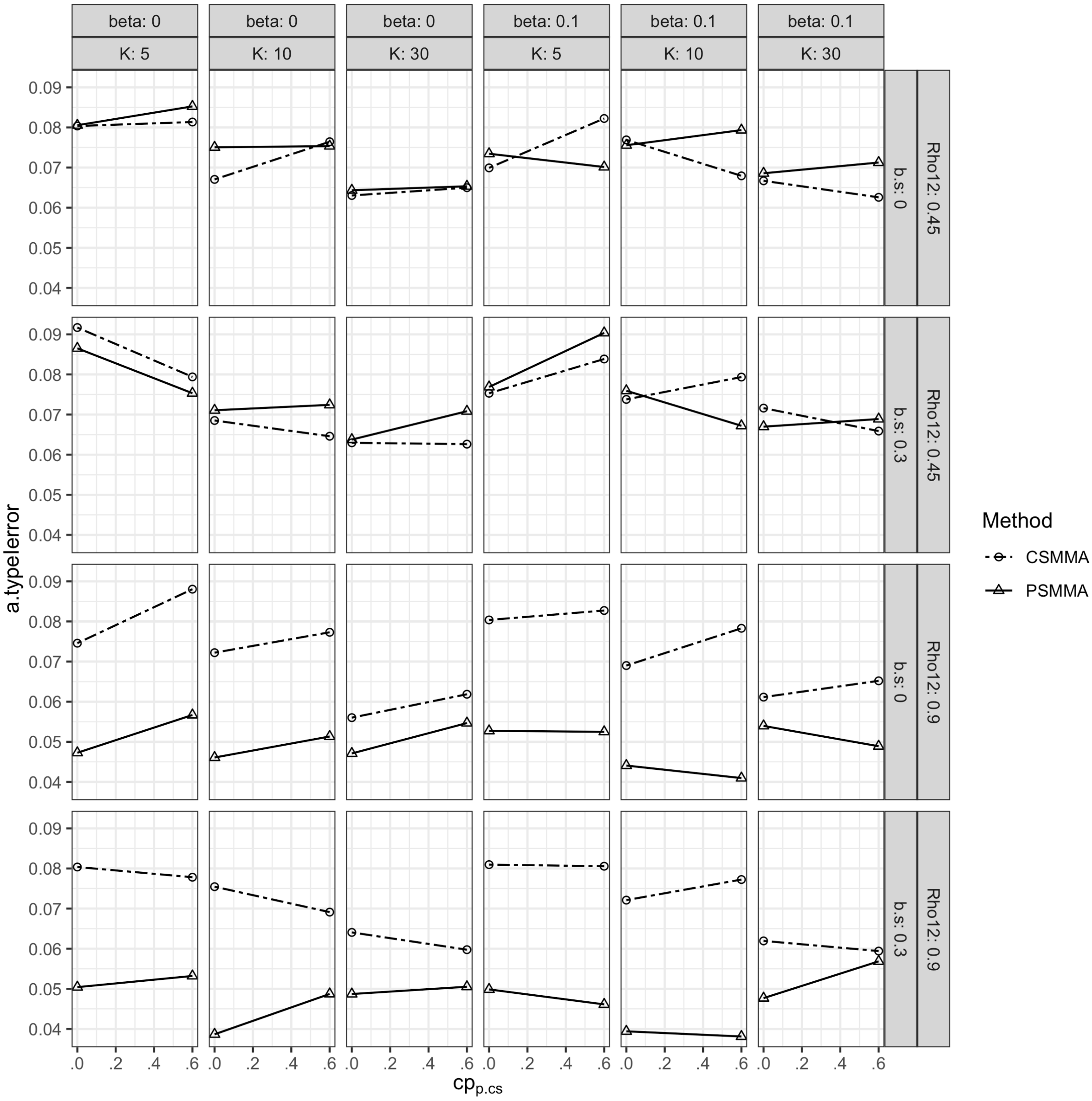


Figure S36.

Statistical Power of the a path Coefficient with Uneuqal Posttest Variances

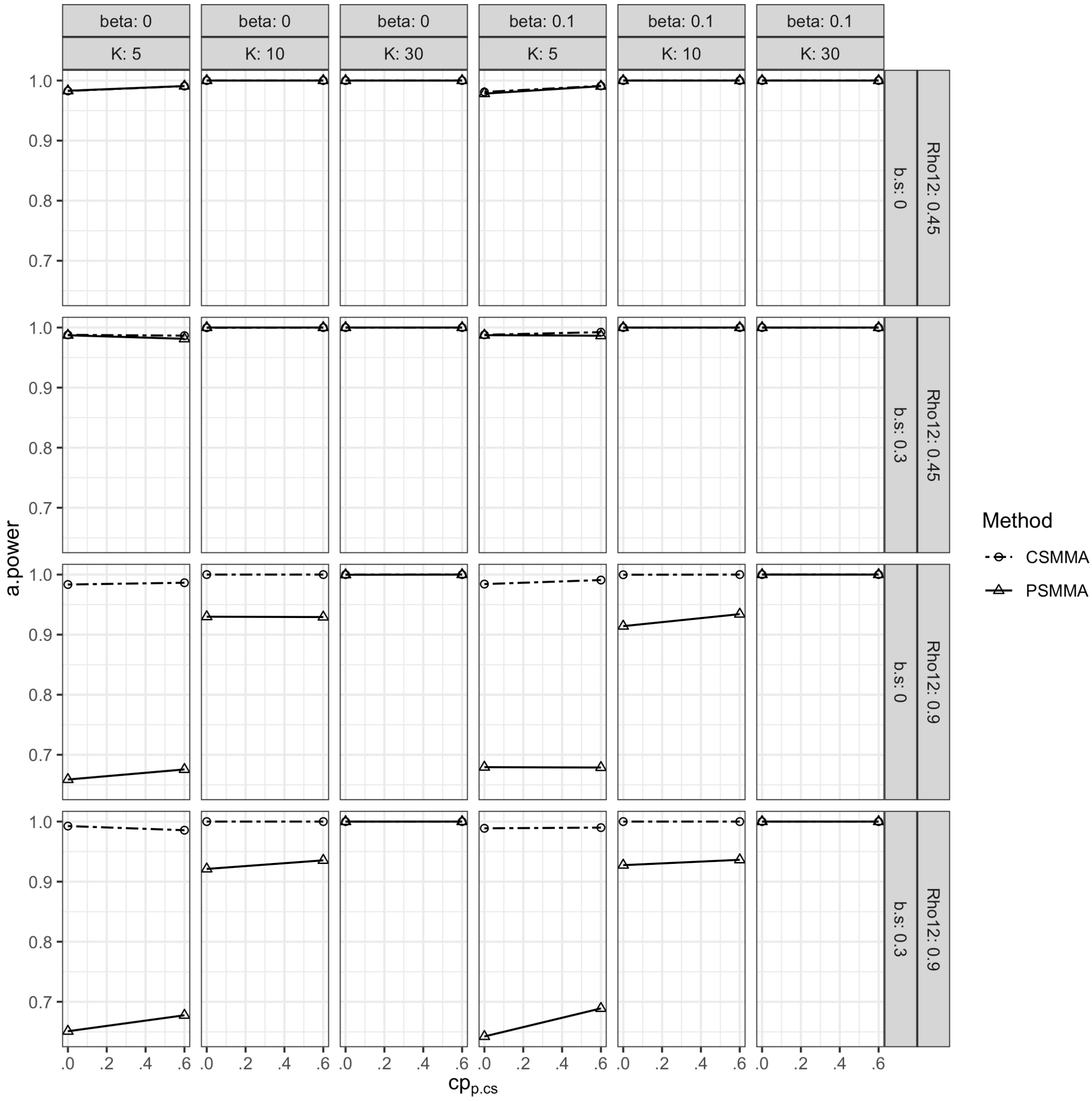


Figure S37.

Bias of the b path Coefficient with Uneuqal Posttest Variances

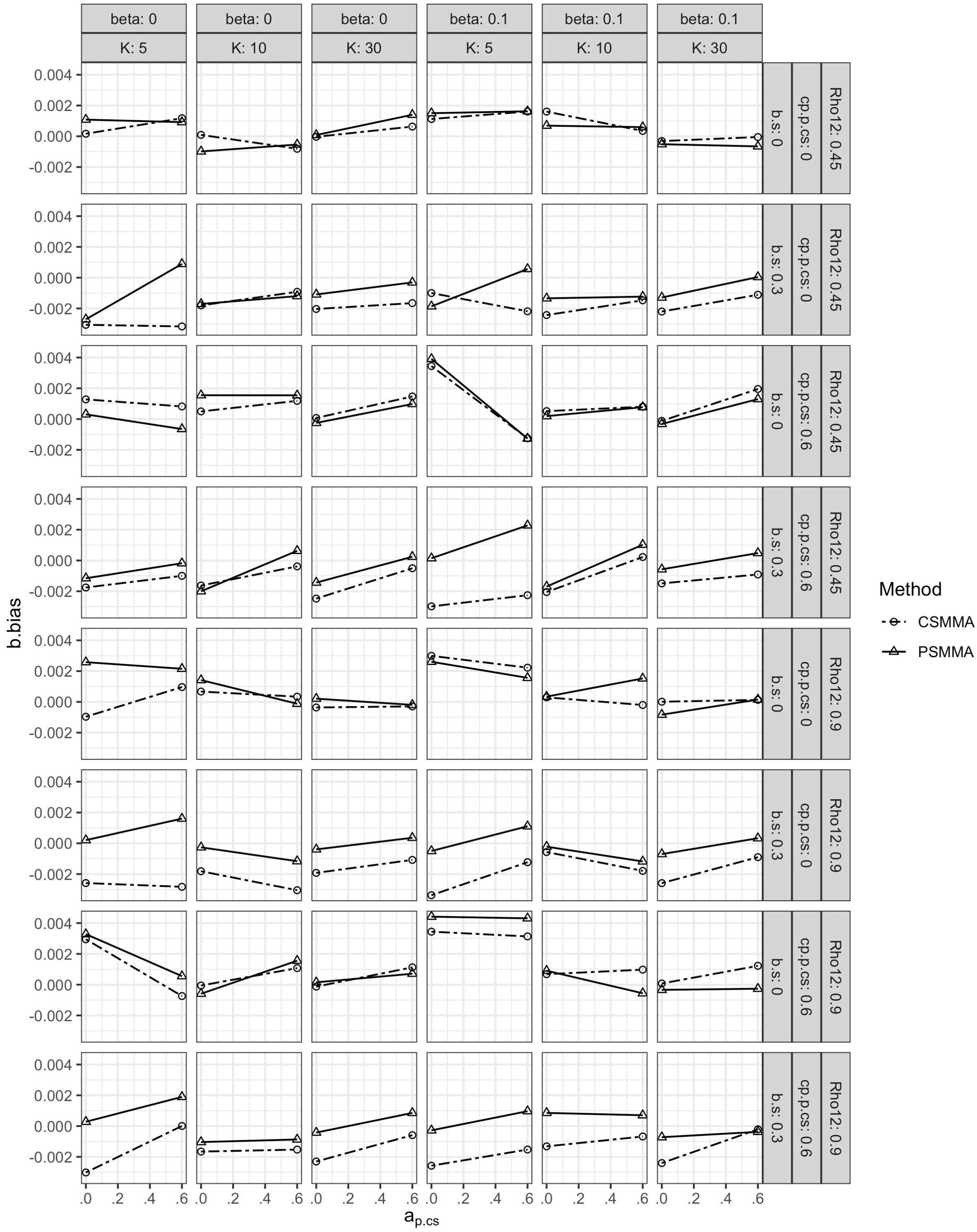


Figure S38.

Coverage Rates of the b path Coefficient with Uneuqal Posttest Variances

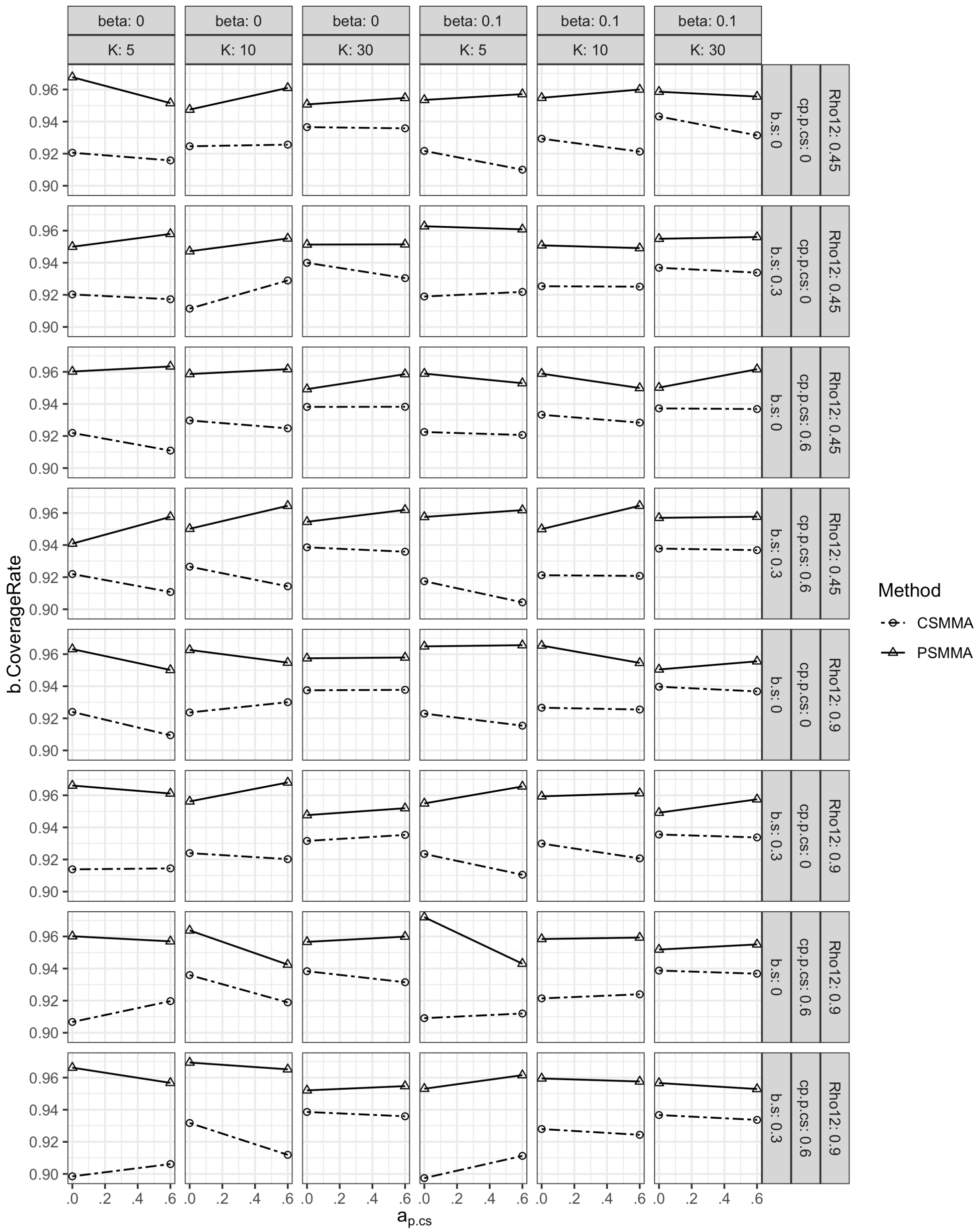


Figure S39.

Type I Error Rates of the b path Coefficient with Uneuqal Posttest Variances

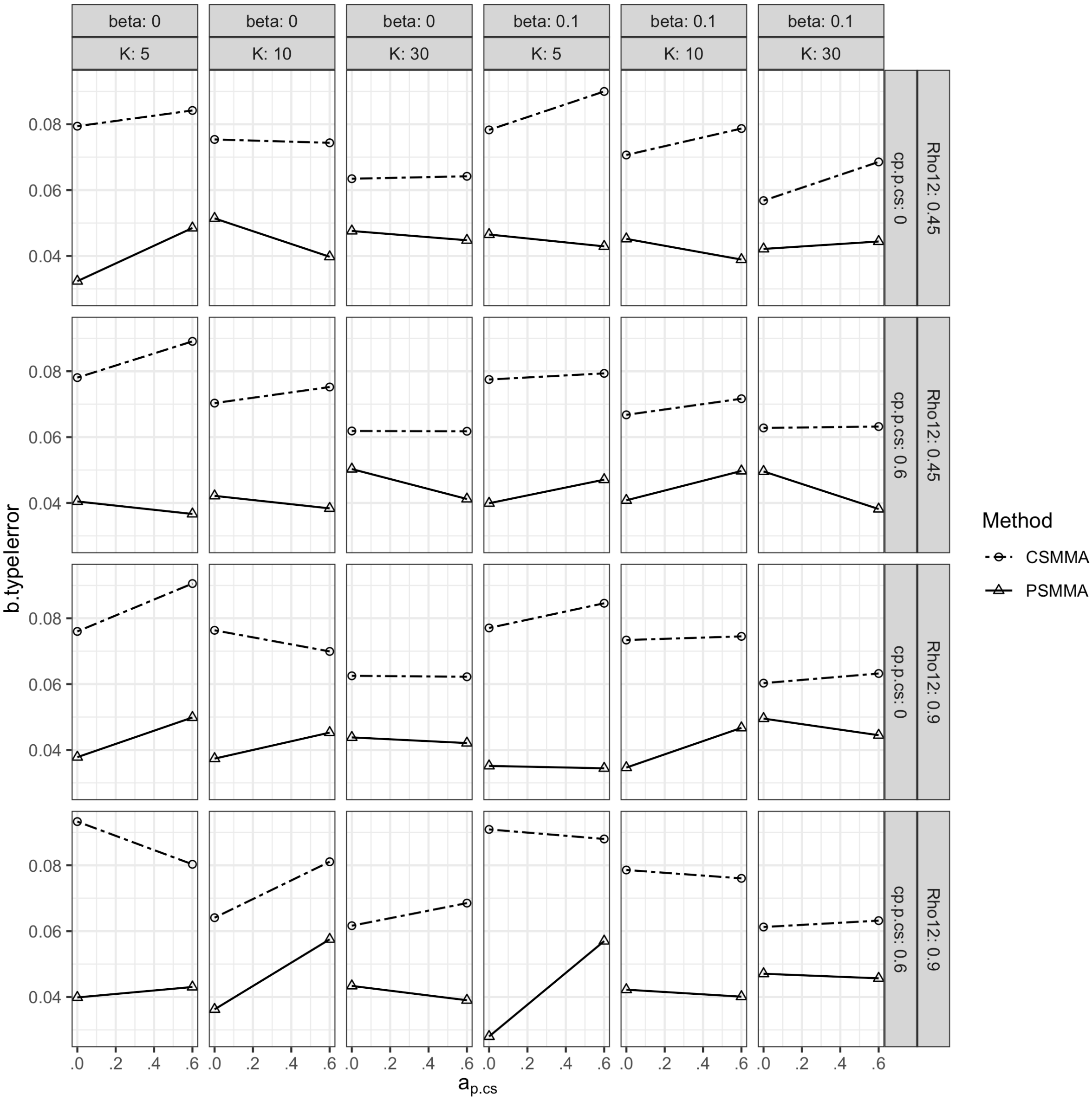
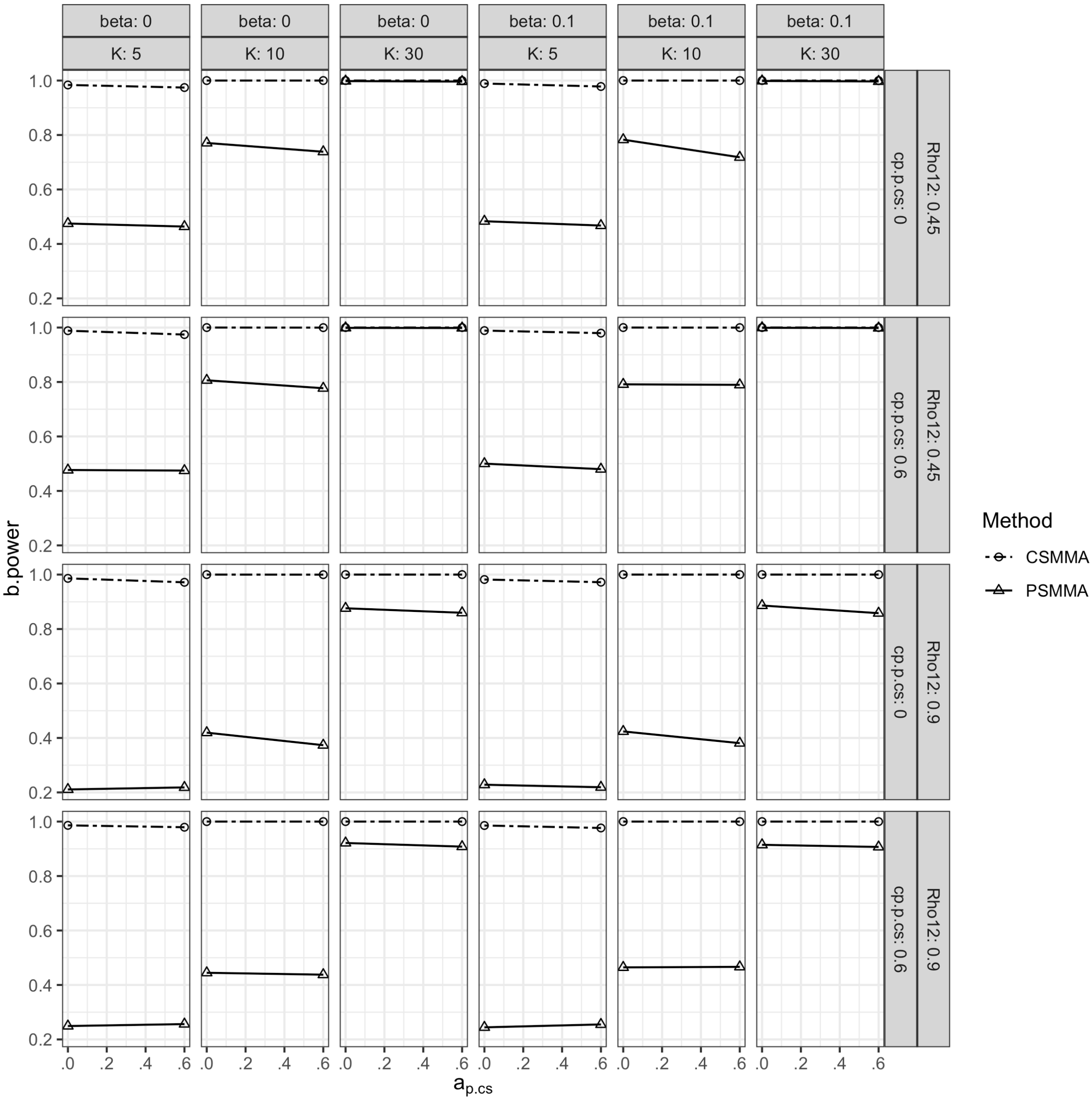


Figure S40.

Statistical Power of the b path Coefficient with Uneuqal Posttest Variances



Positive Definite Rates. As shown in Figure S41, the positive definite rate of CSMMA and PSMMA with equal posttest varianes decreased with a smaller K. Similar patterns were observed when the posttest variances in the treatment group were inflated (Figure S42).

Figure S41.

Positive Definite Rates Reported by OSMASEM (with Equal Posttest Variances)

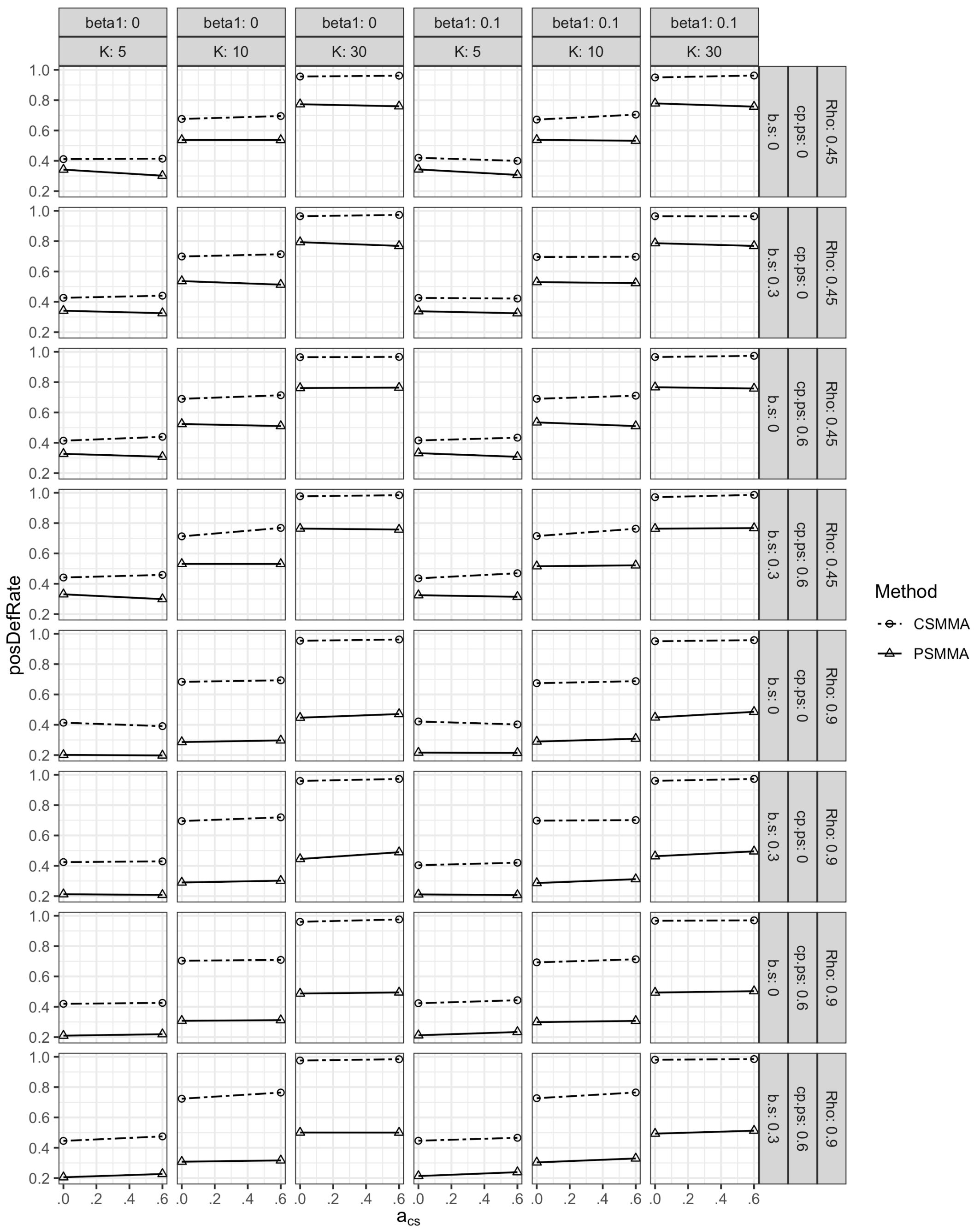
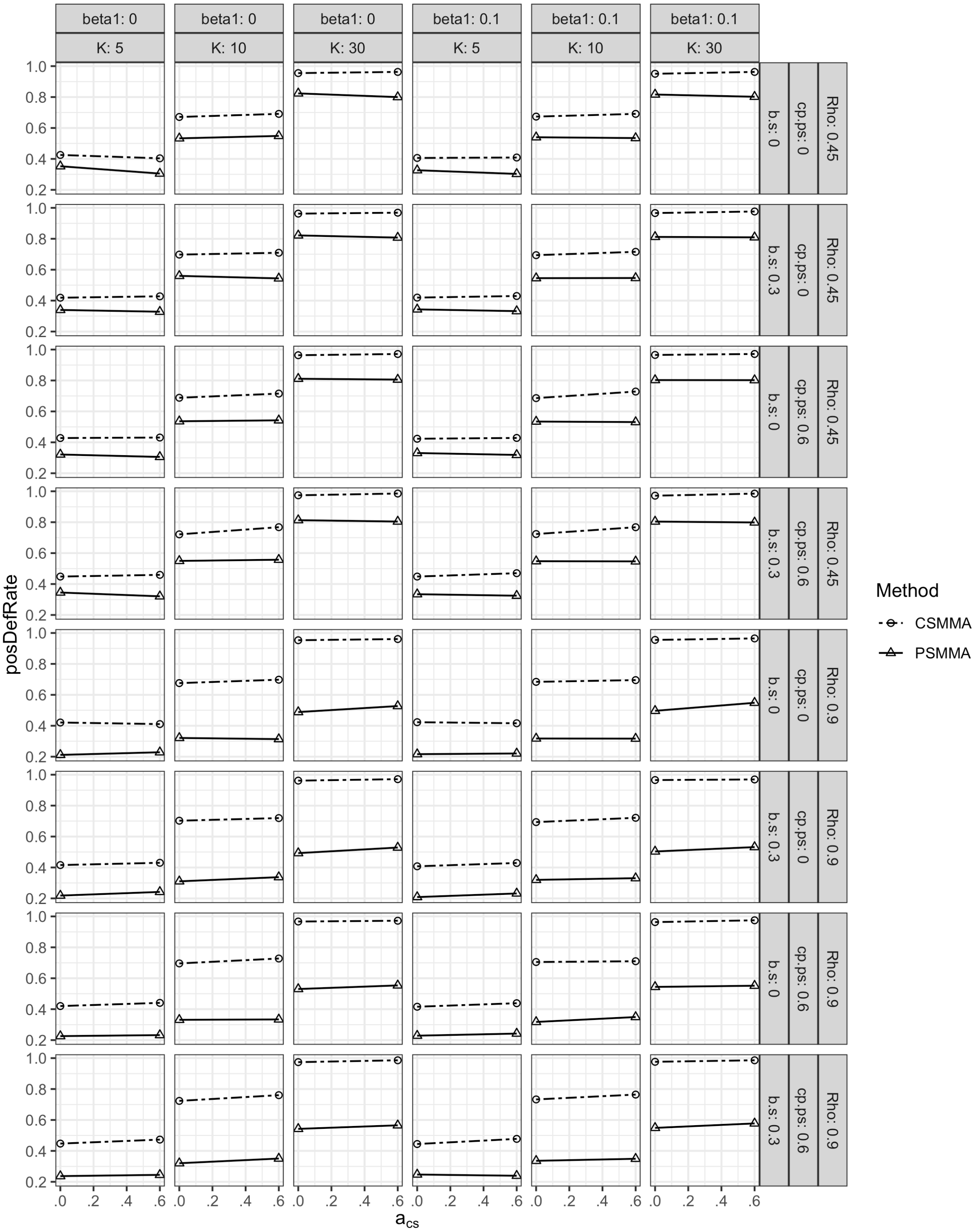


Figure S42.

Positive Definite Rates Reported by OSMASEM (with Unequal Posttest Variances)



S3 Additional Results in the Main Simulations

S3.1 Convergence Rate

The probabilities of reporting negtive definite information matrices are shown in Figures S43 and S44.

Figure S43.

Negative Definite Rates Reported by OSMASEM under Conditions with Between-study Heterogeneity Generated on the Indirect Effect

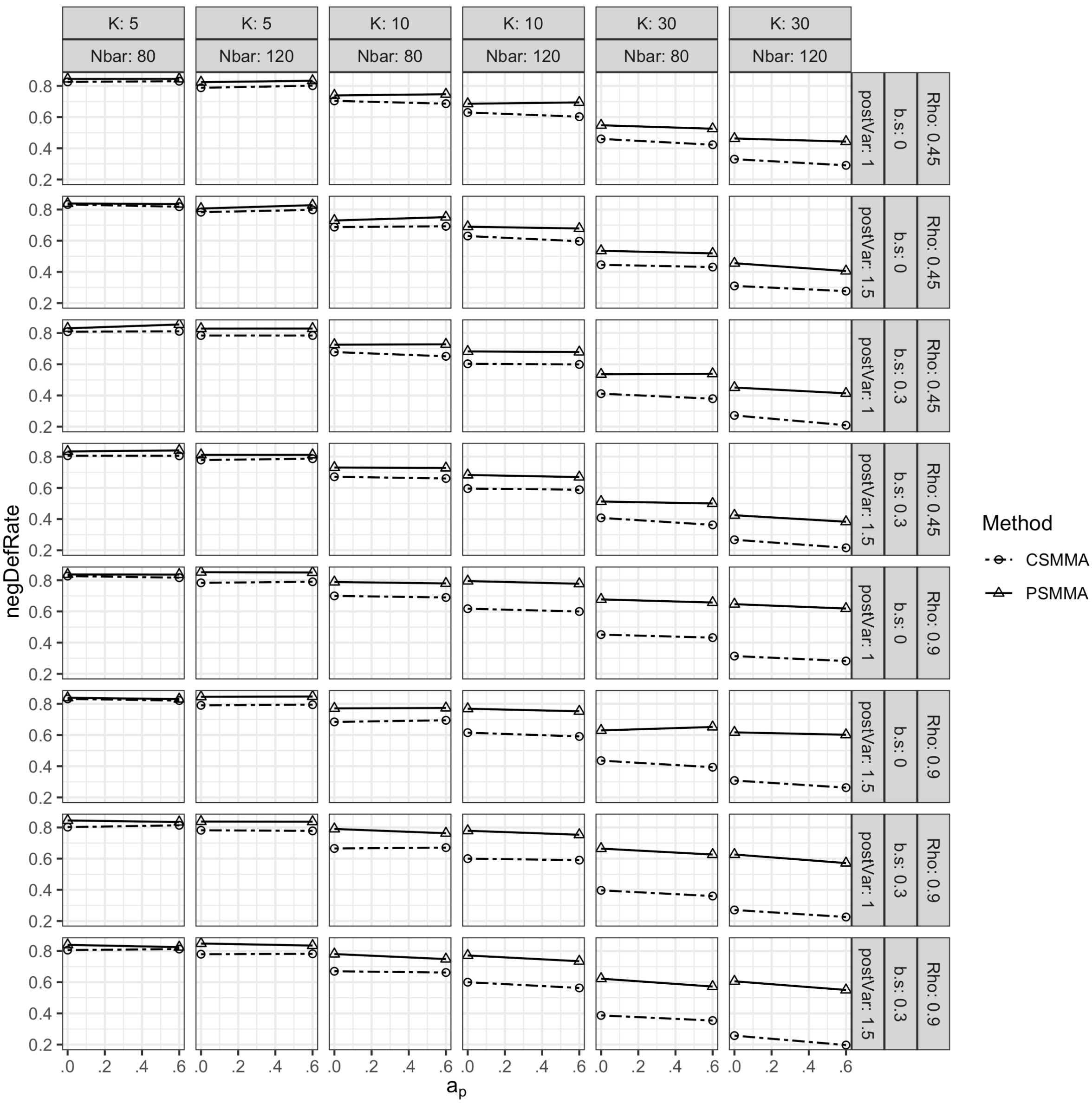
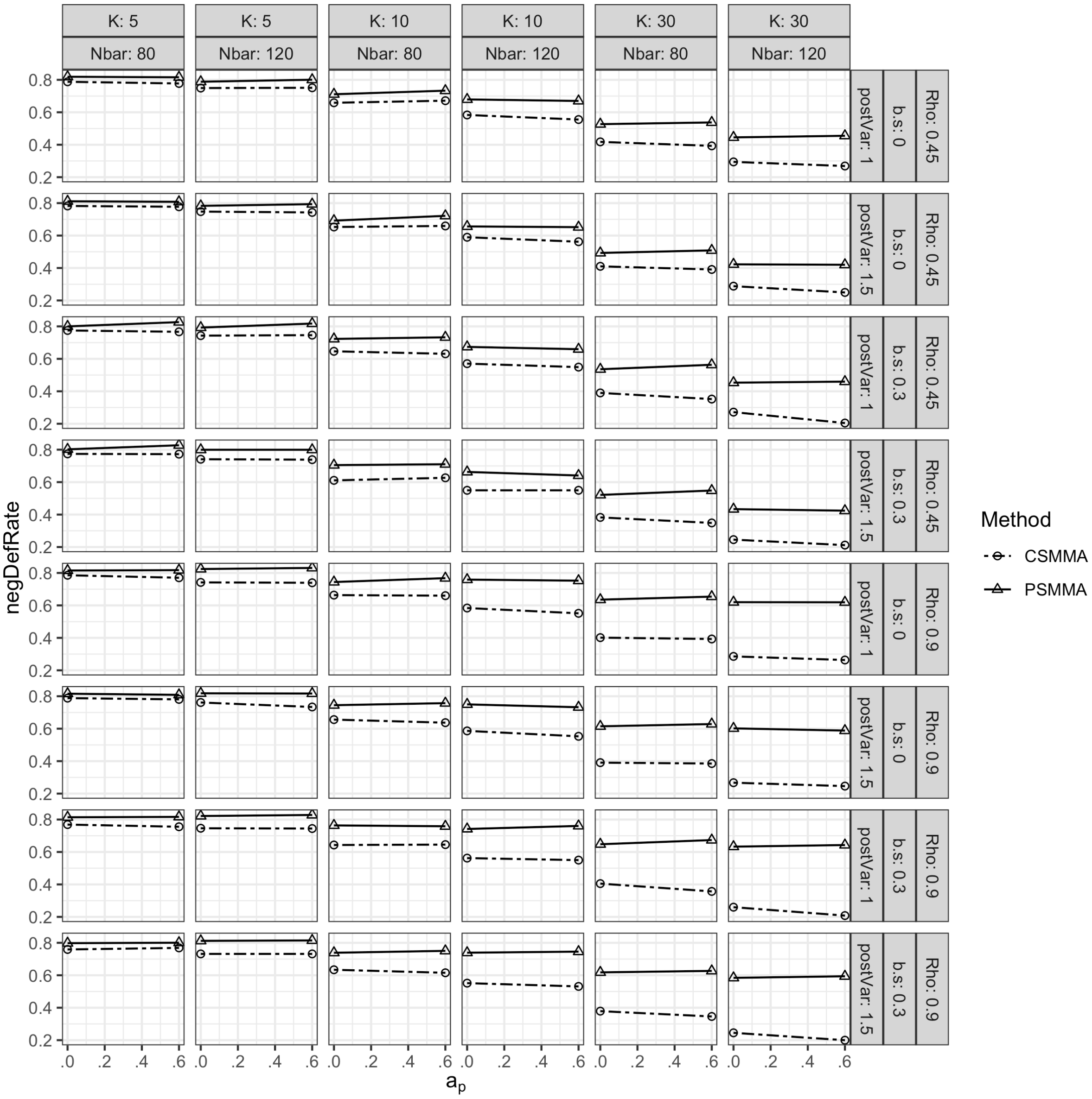


Figure S44.

Negative Definite Rates Reported by OSMASEM under Conditions with Between-study Heterogeneity Generated on the Direct effect



S3.2 Indirect Effect

As shown in Figures S45 and S46, bias of the indirect effect using CSMMA and PSMMA remained acceptable. The CR of both approaches remained above 0.95 (Figures S47 and S48). The type I error rates of CSMMA and PSMMA remained below 0.04 (Figures S49 and S50). As shown in Figures S51 and S52, while CSMMA had favorable power (0.9~1) in all conditions, the power of PSMMA decreased with a smaller K, a smaller $\bar{N}$, or a larger ρ_{12}. In addition, under conditions with inflated posttest variances in the treatment group, PSMMA had larger power than those under conditions with equal variances.

Figure S45.

Bias of the Indirect Effect under Conditions with Between-study Heterogeneity Generated on the Indirect Effect

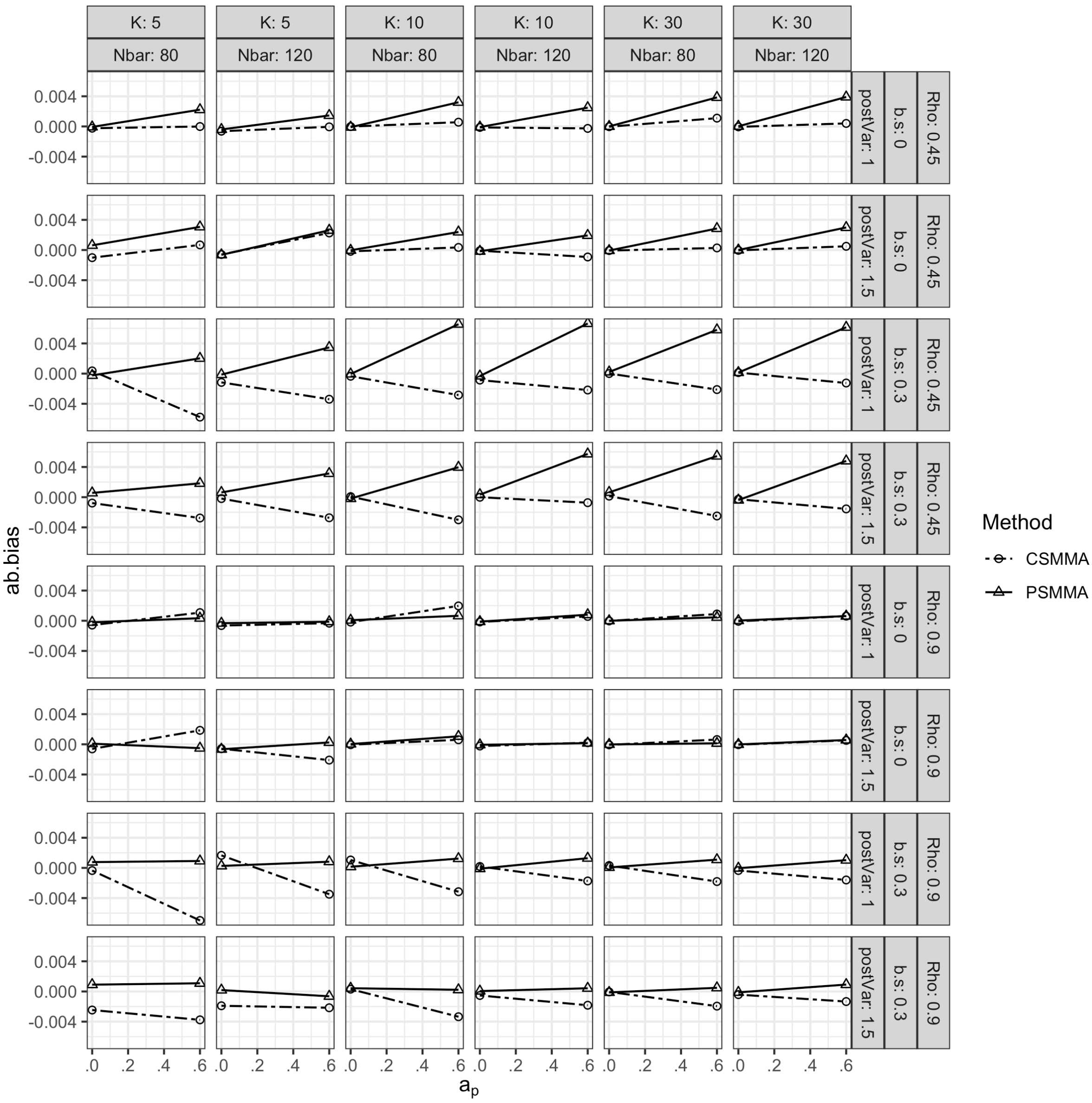


Figure S46.

Bias of the Indirect Effect under Conditions with Between-study Heterogeneity Generated on the Direct Effect

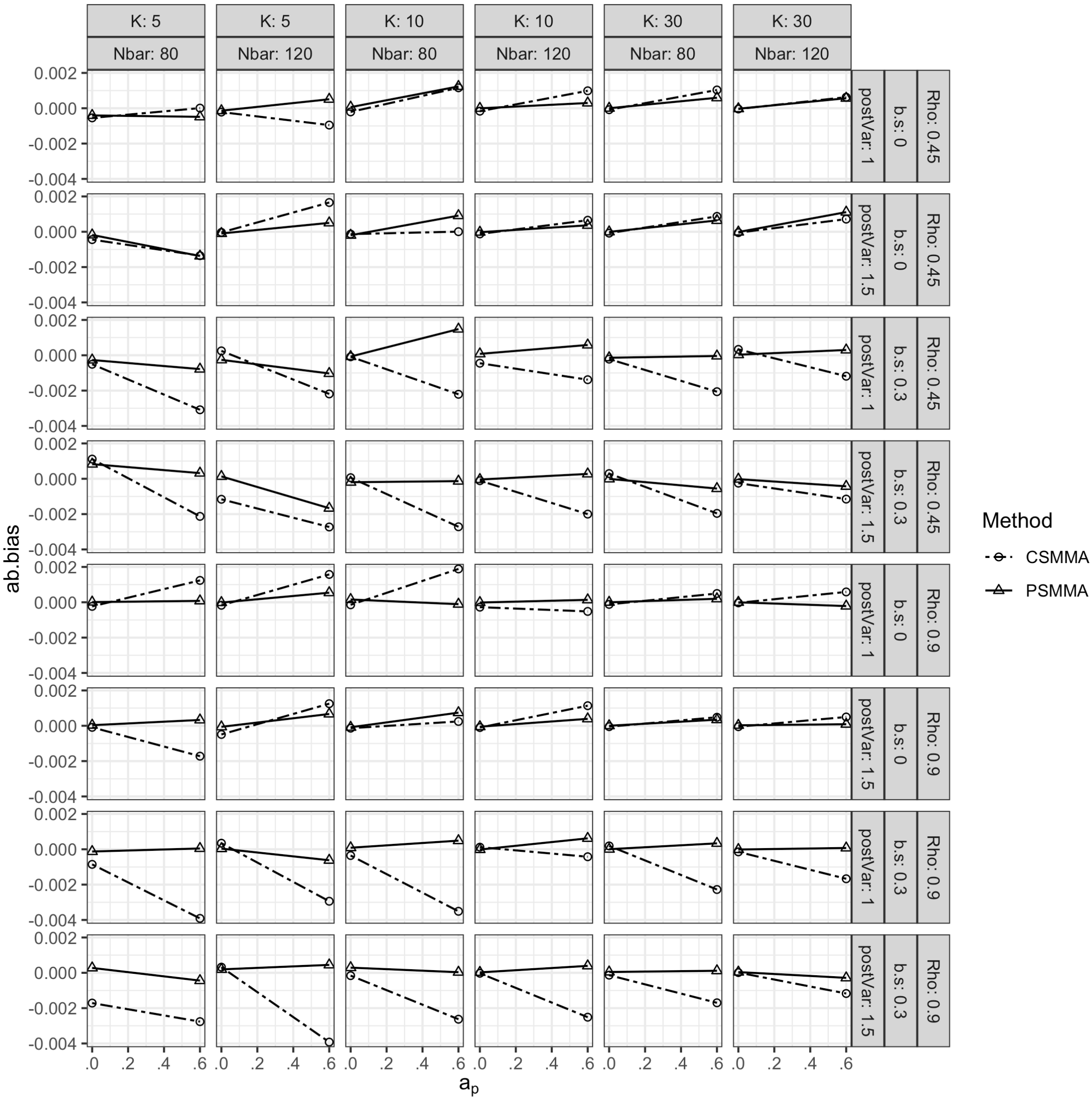


Figure S47.

Coverage Rate of the Indirect Effect with Between-study Heterogeneity Generated on the Indirect Effect

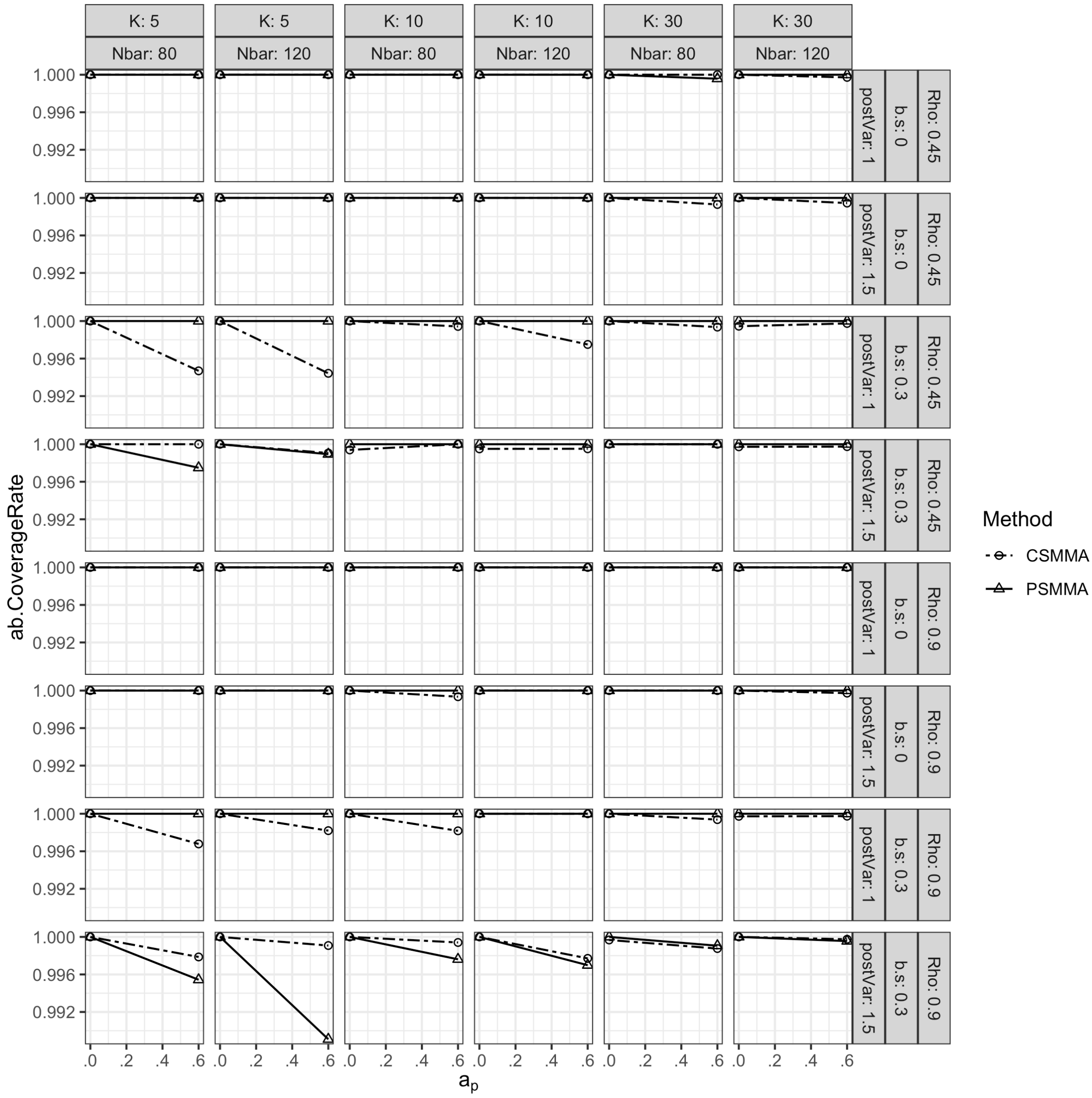


Figure S48.

Coverage Rate of the Indirect Effect with Between-study Heterogeneity Generated on the Direct Effect

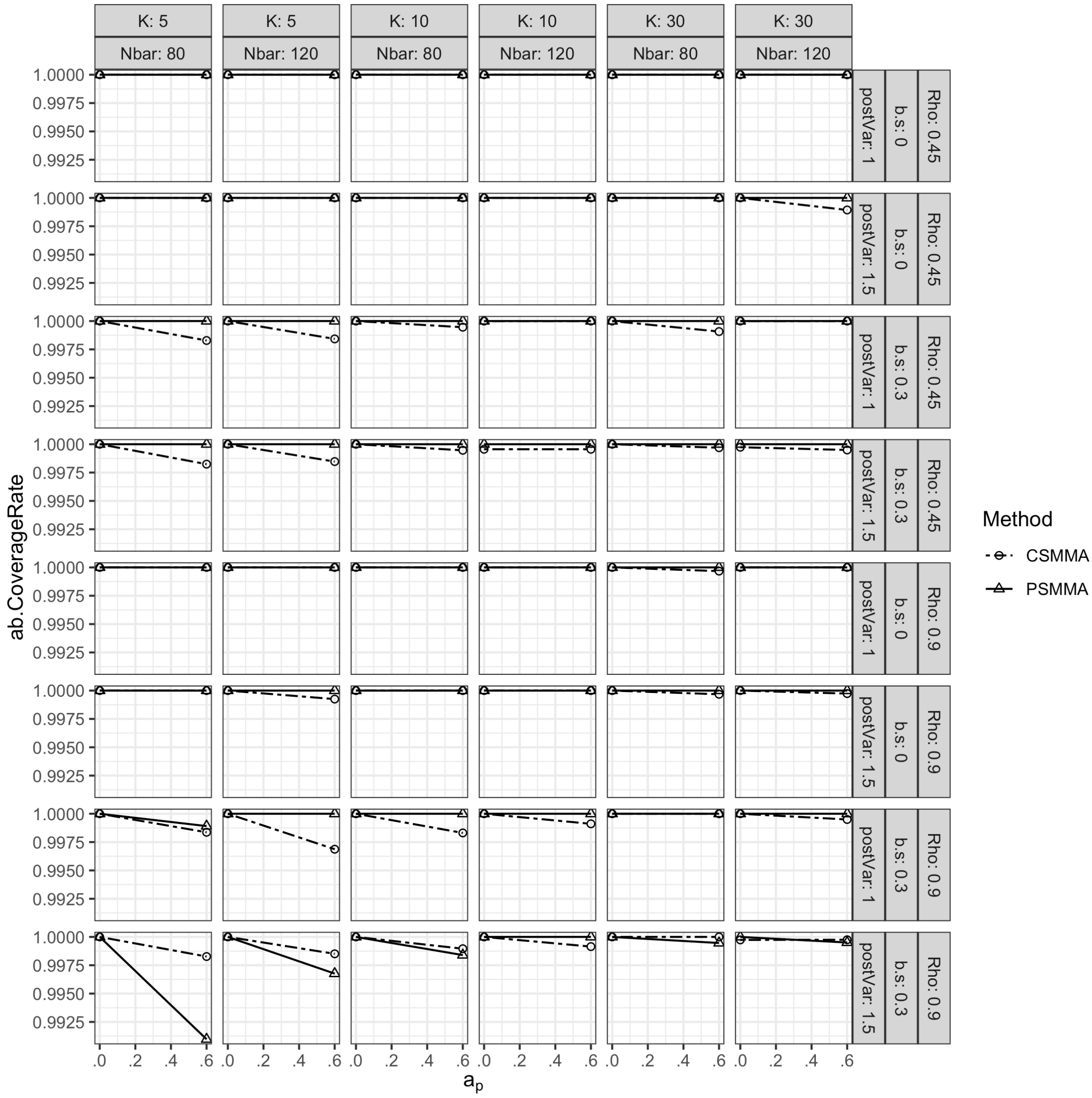


Figure S49.

Type I Error Rate of the Indirect Effect with Between-study Heterogeneity Generated on the Indirect Effect

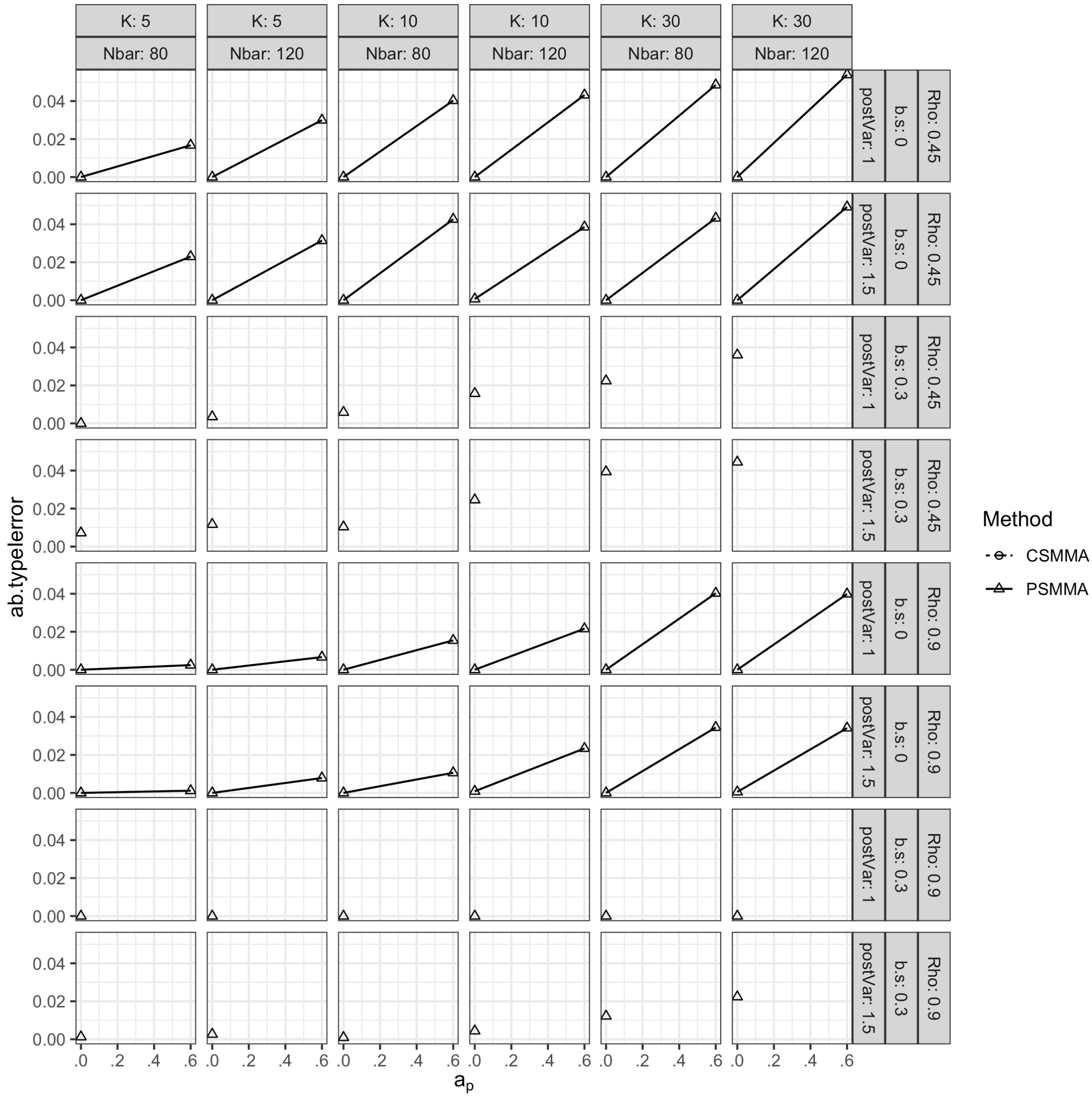
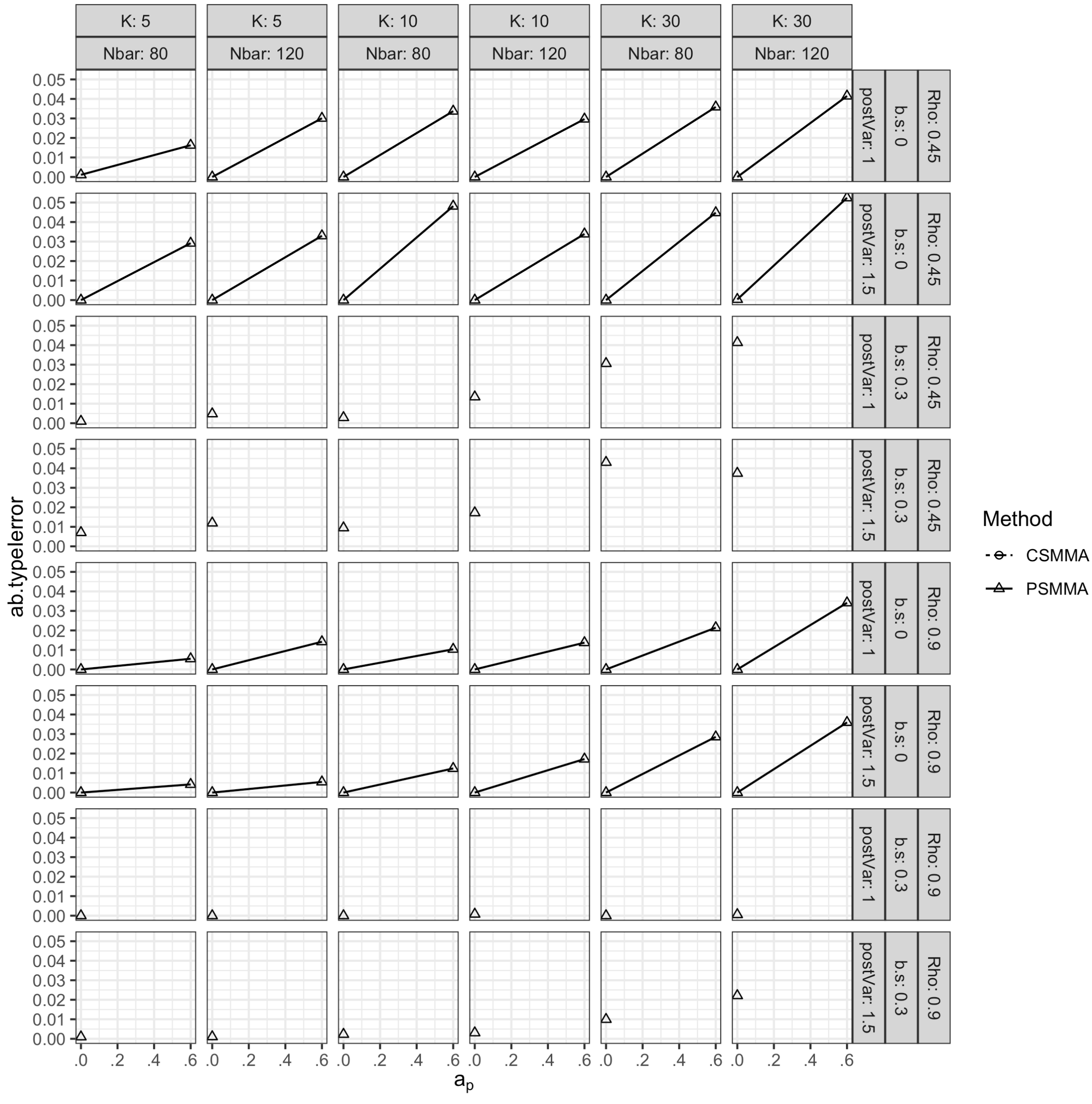


Figure S50.

Type I Error Rate of the Indirect Effect with Between-study Heterogeneity Generated on the Direct Effect



S3.3 Direct Effect

As shown in Figures S52 and S53, bias of the direct effect using CSMMA and PSMMA remained acceptable. The CR of both approaches remained above 0.9 (Figures S54 and S55). As shown in Figures S56 and S57, while CSMMA had favorable power (0.9~1) in all conditions, the power of PSMMA decreased with a smaller K, a smaller $\overline{N}$ when $\rho_{12} = 0.9$.

Figure S52.

Bias of the Direct Effect with Between-study Heterogeneity Generated on the Indirect Effect

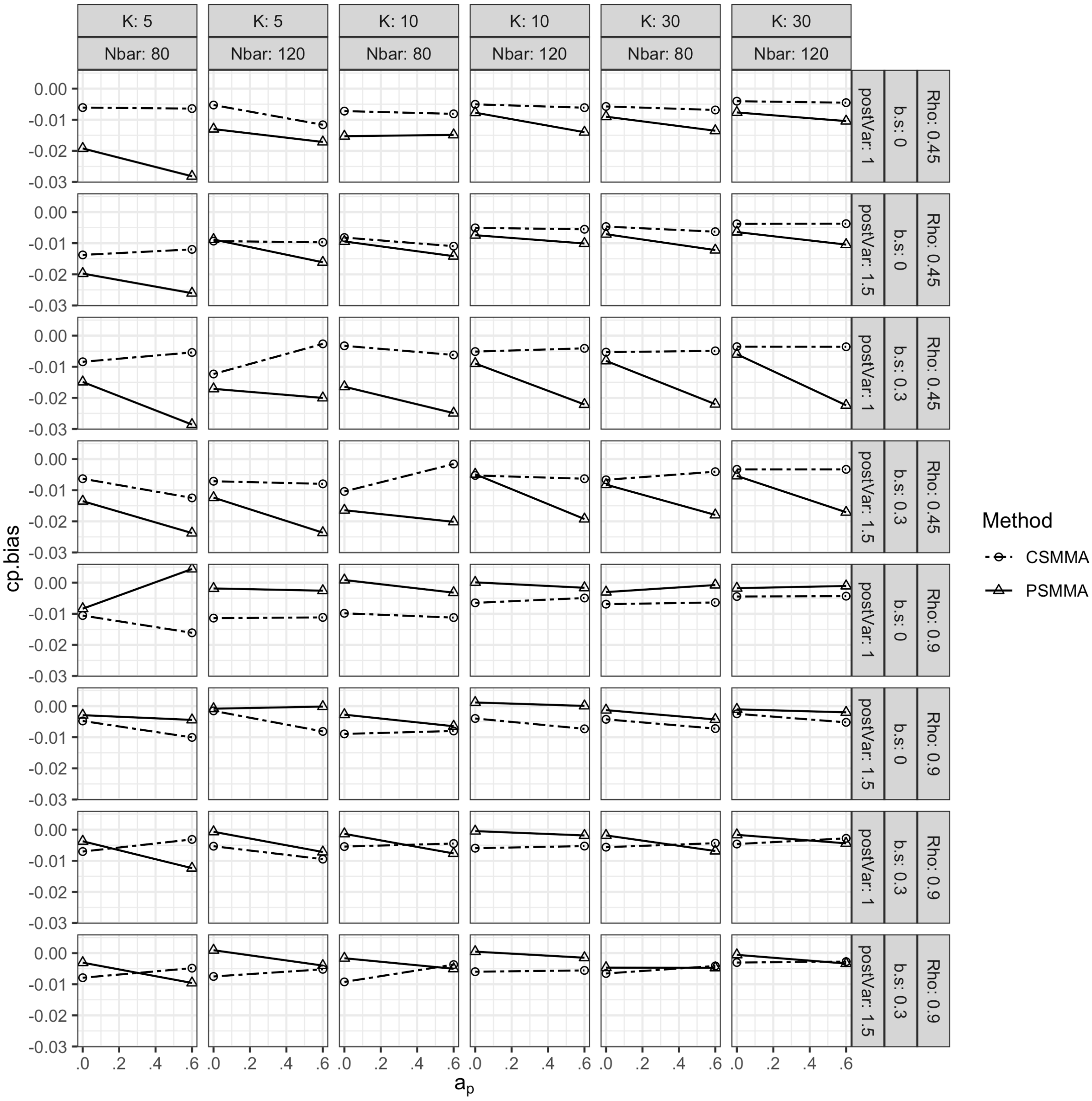


Figure S53.

Bias of the Direct Effect with Between-study Heterogeneity Generated on the Direct Effect

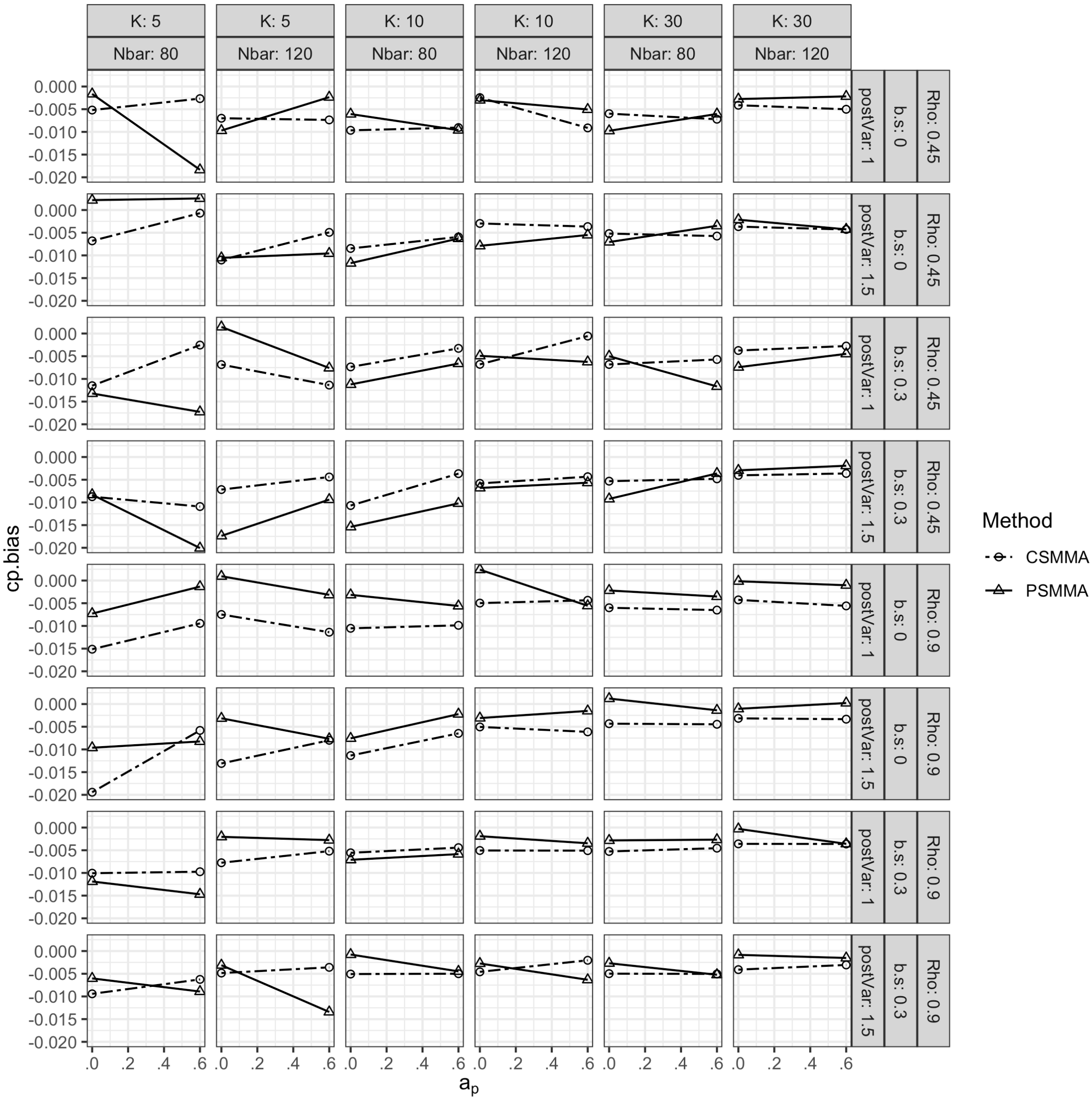


Figure S54.

Coverage Rate of the Direct Effect with Between-study Heterogeneity Generated on the Indirect Effect

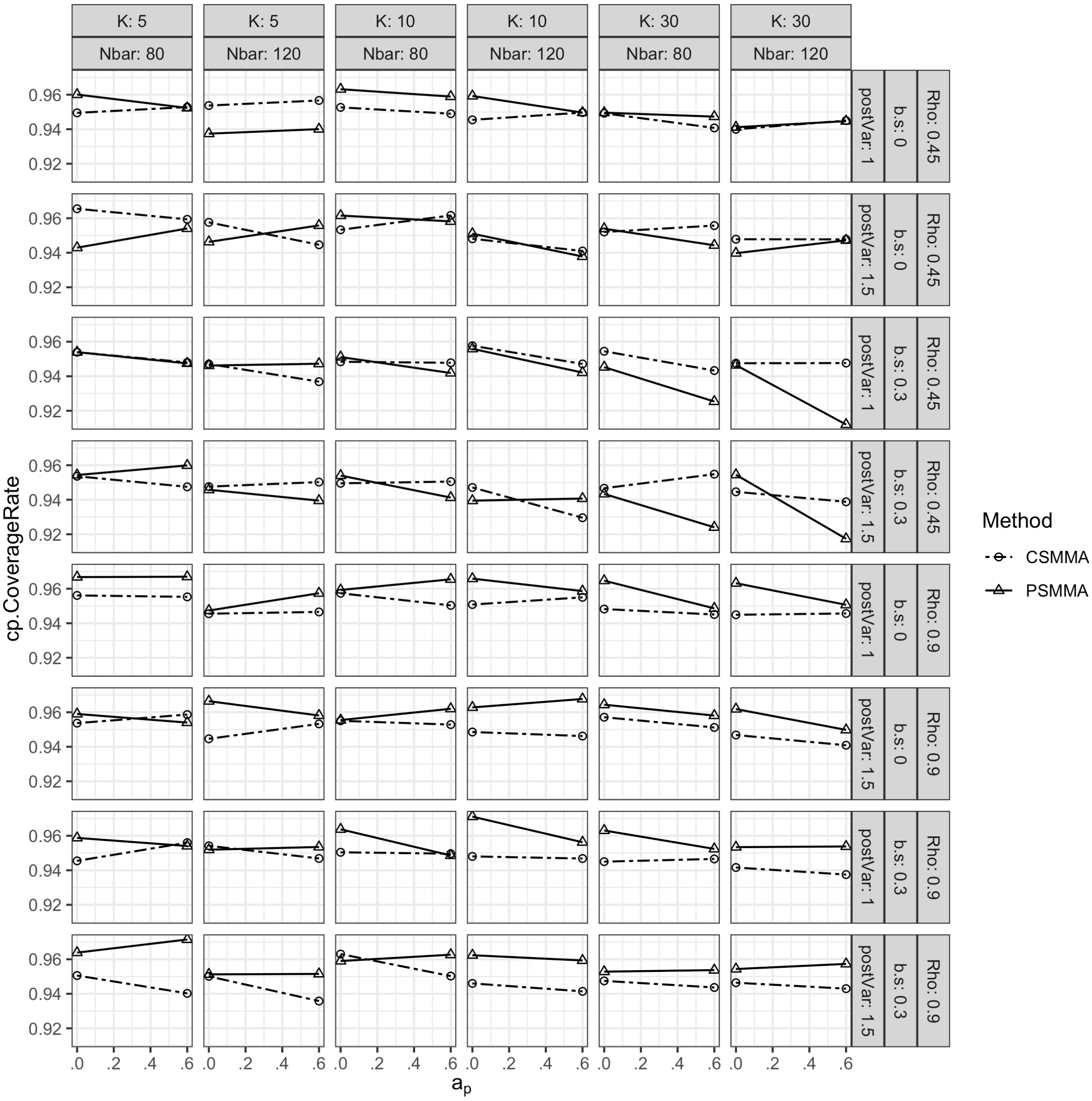


Figure S55.

Coverage Rate of the Direct Effect with Between-study Heterogeneity Generated on the Direct Effect

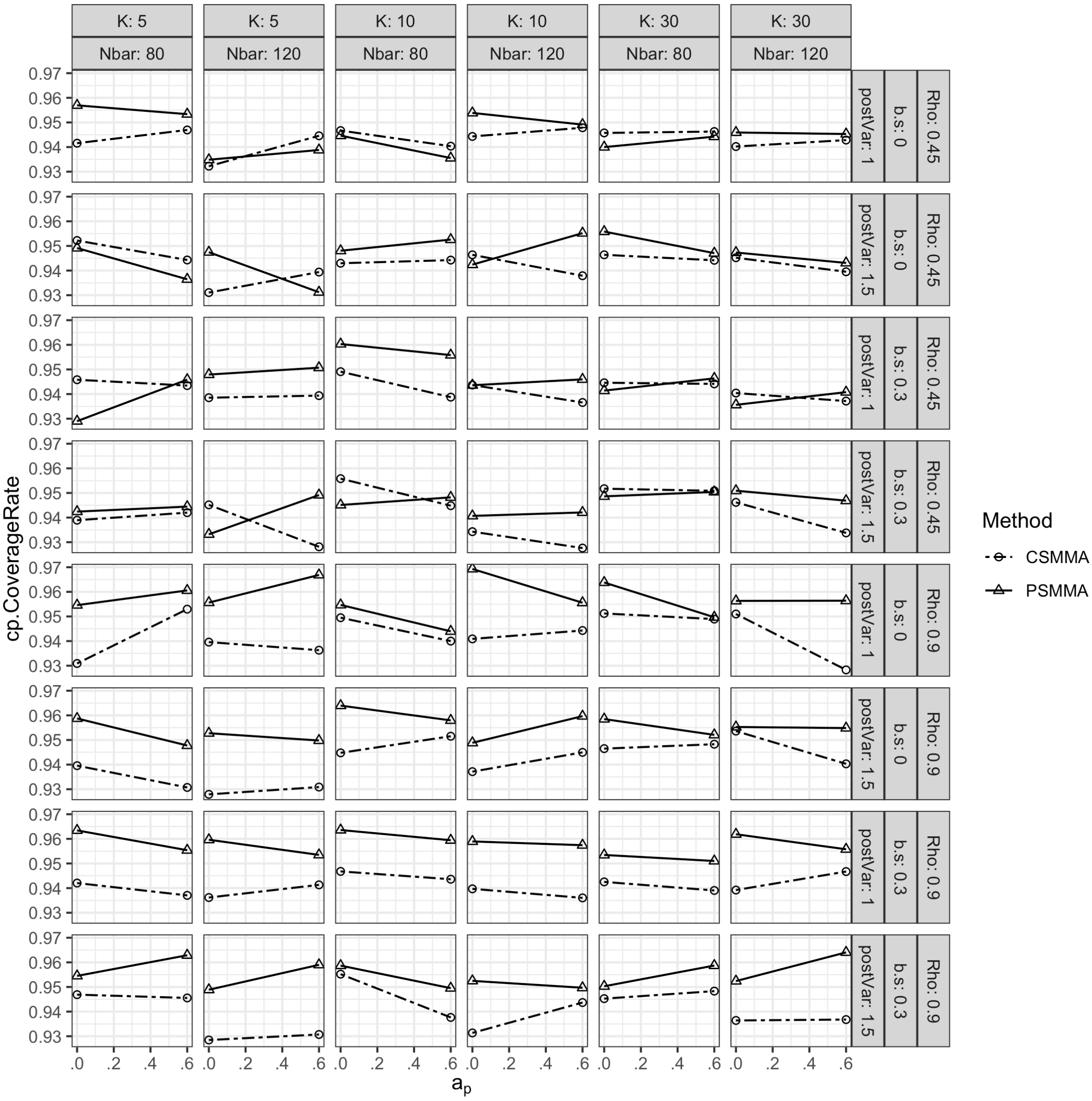


Figure S56.

Statistical Power of the Direct Effect with Between-study Heterogeneity Generated on the Indirect Effect

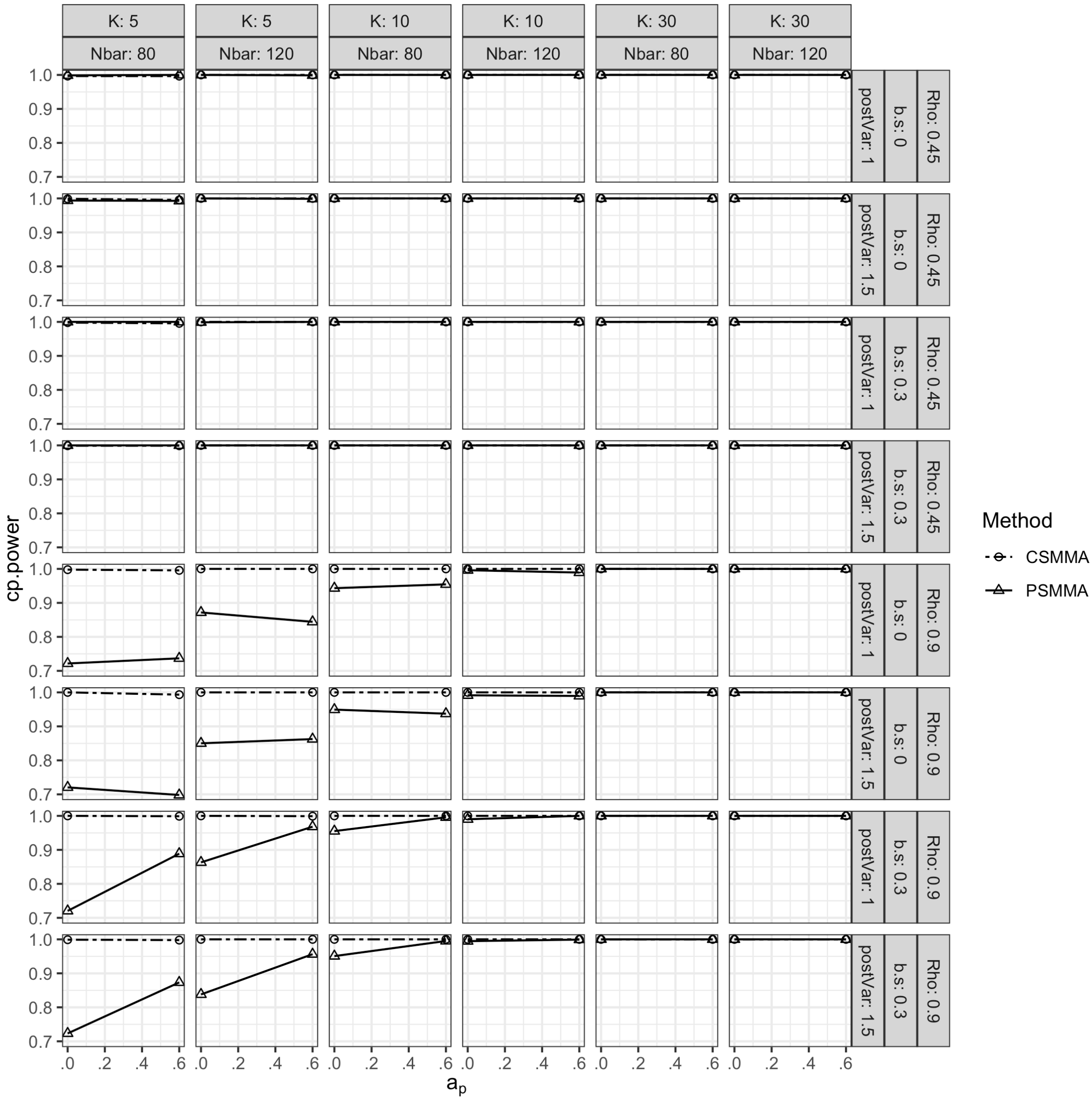
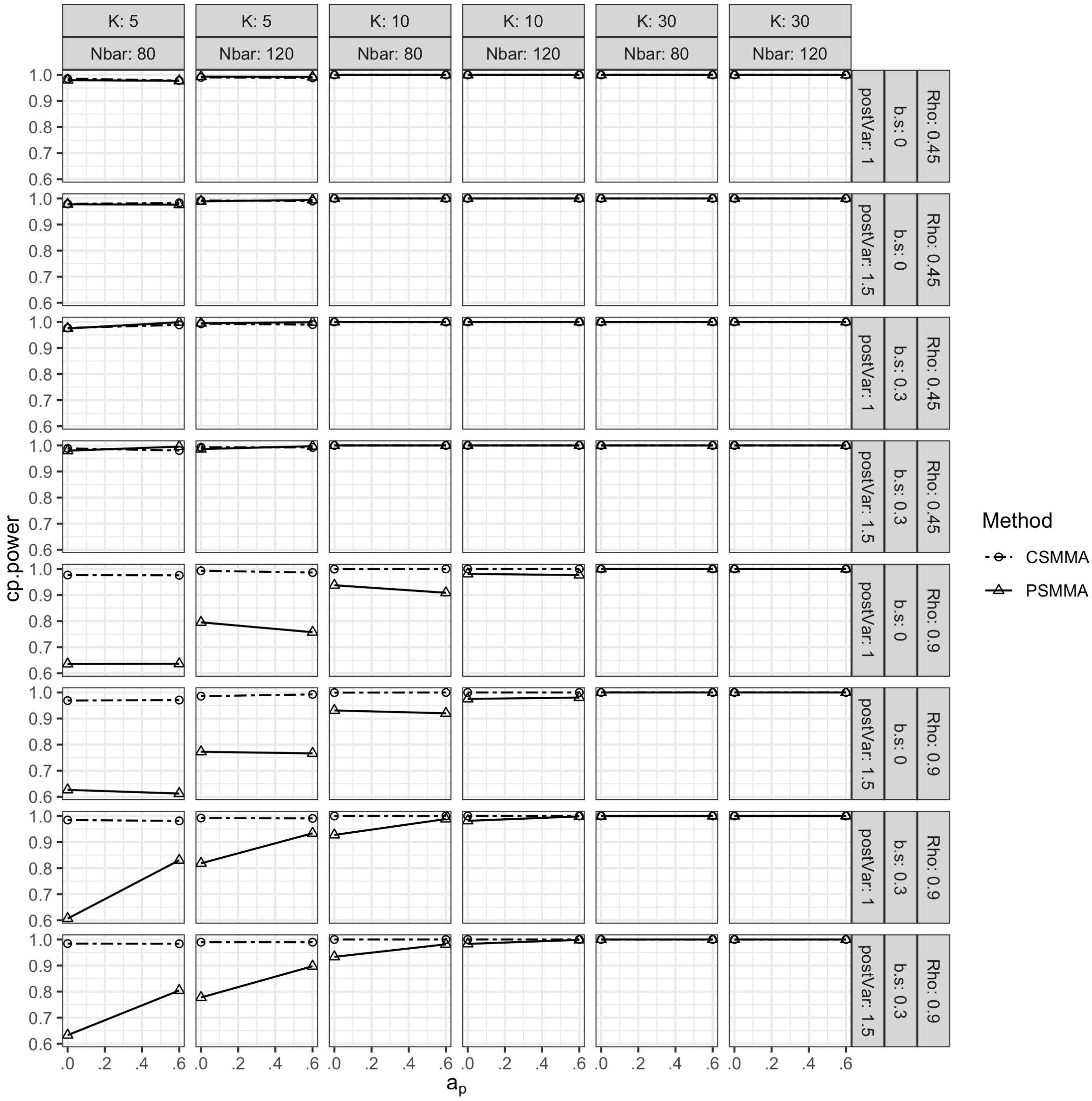


Figure S57.

Statistical Power of the Direct Effect with Between-study Heterogeneity Generated on the Direct Effect



S3.4 Moderating Effect

As shown in Figures S58 ~ S60, bias of the direct effect using CSMMA and PSMMA remained acceptable. The CR of both approaches remained above 0.9 (Figures S61 ~ S63). As shown in Figures S64 ~ S66, when $\rho_{12} = 0.45$, CSMMA and PSMMA had comparable power under all conditions. However, when $\rho_{12} = 0.9$, the power of PSMMA was much smaller than that of CSMMA under all conditions.

Figure S58.

Bias of the Moderating Effect Generated on the a Path under Conditions with Between-study Heterogeneity Generated on the Indirect Effect

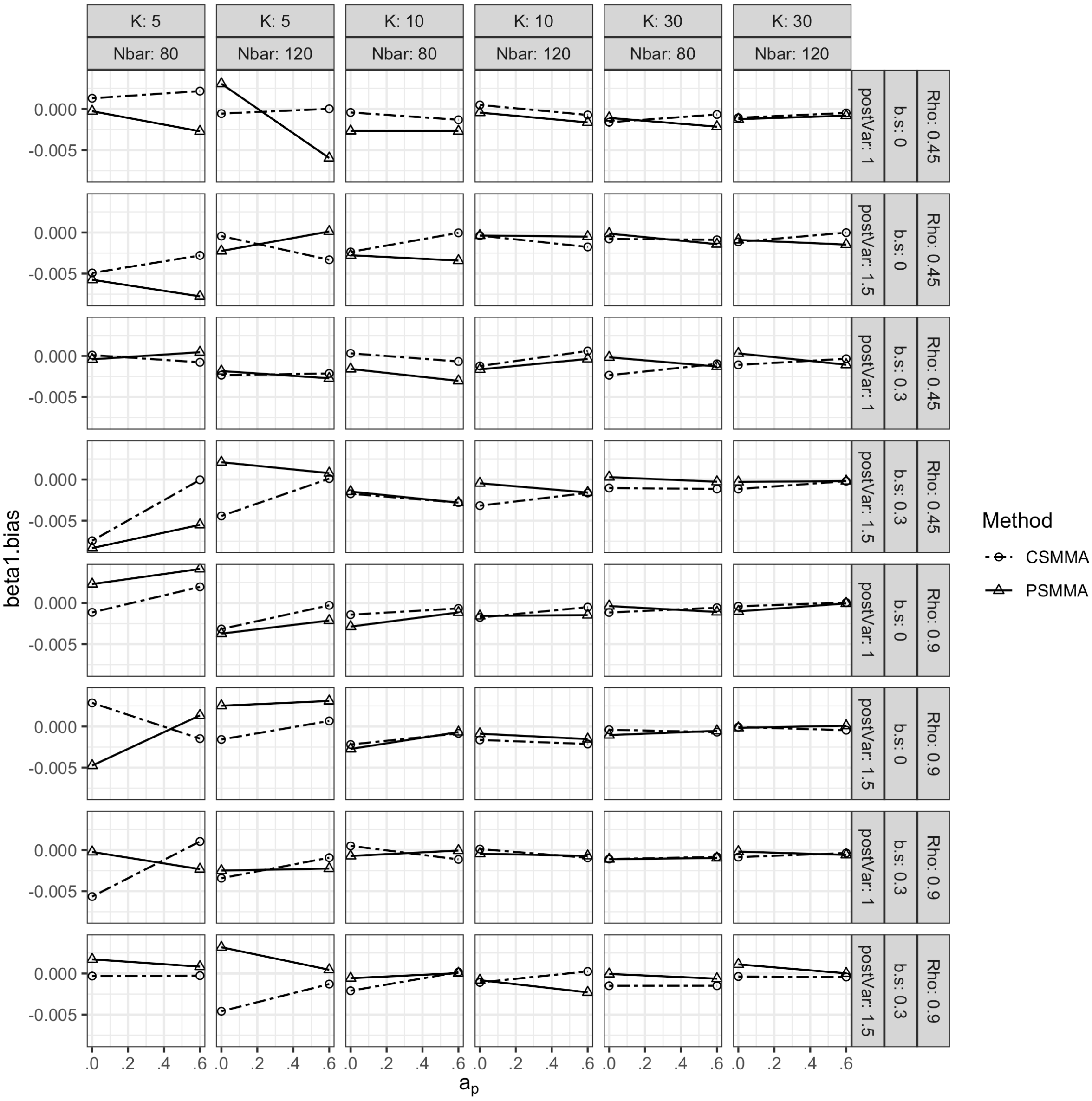


Figure S59.

Bias of the Moderating Effect Generated on the b Path under Conditions with Between-study Heterogeneity Generated on the Indirect Effect

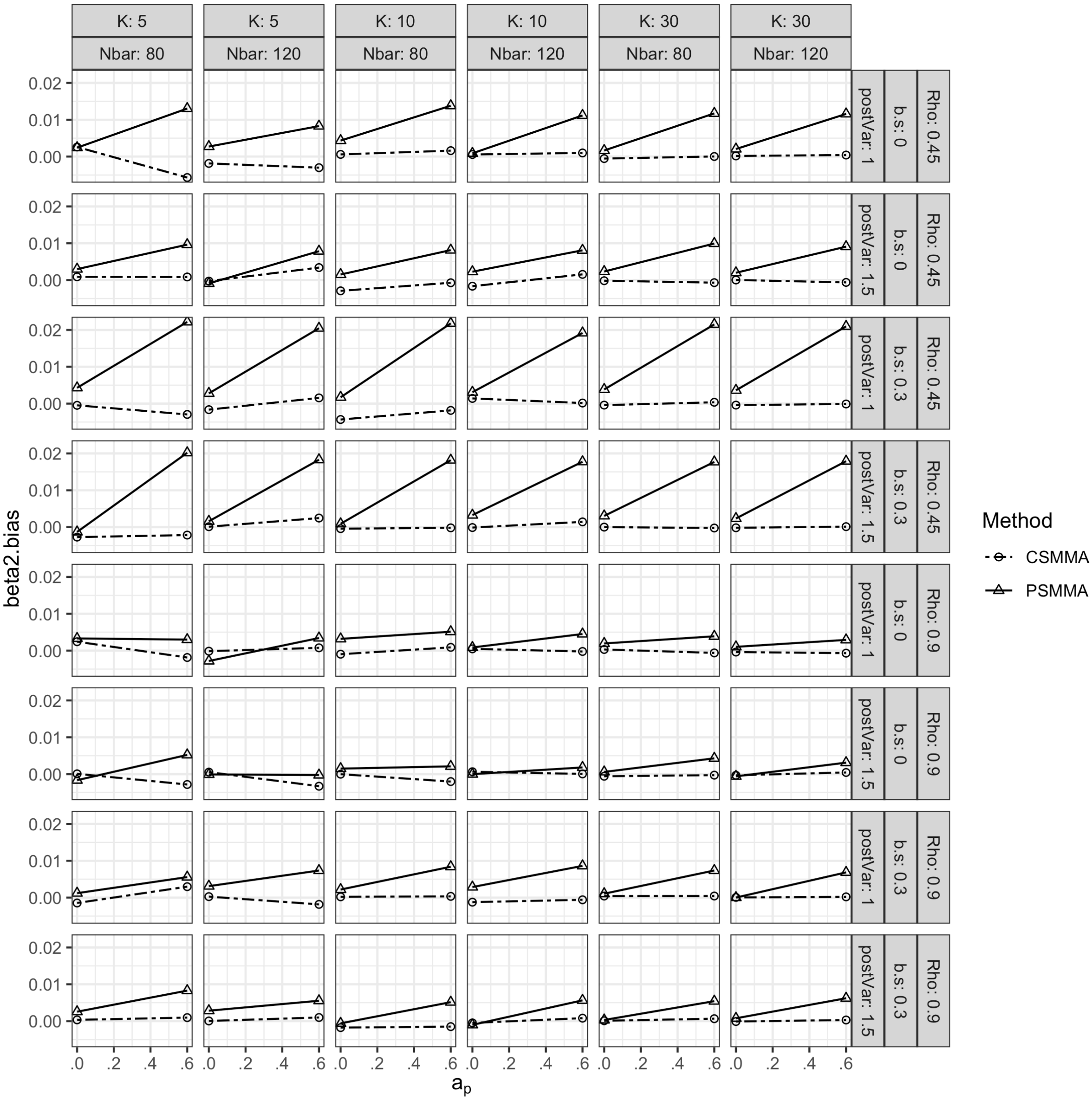


Figure S60.

Bias of the Moderating Effect Generated on the c' *Path*

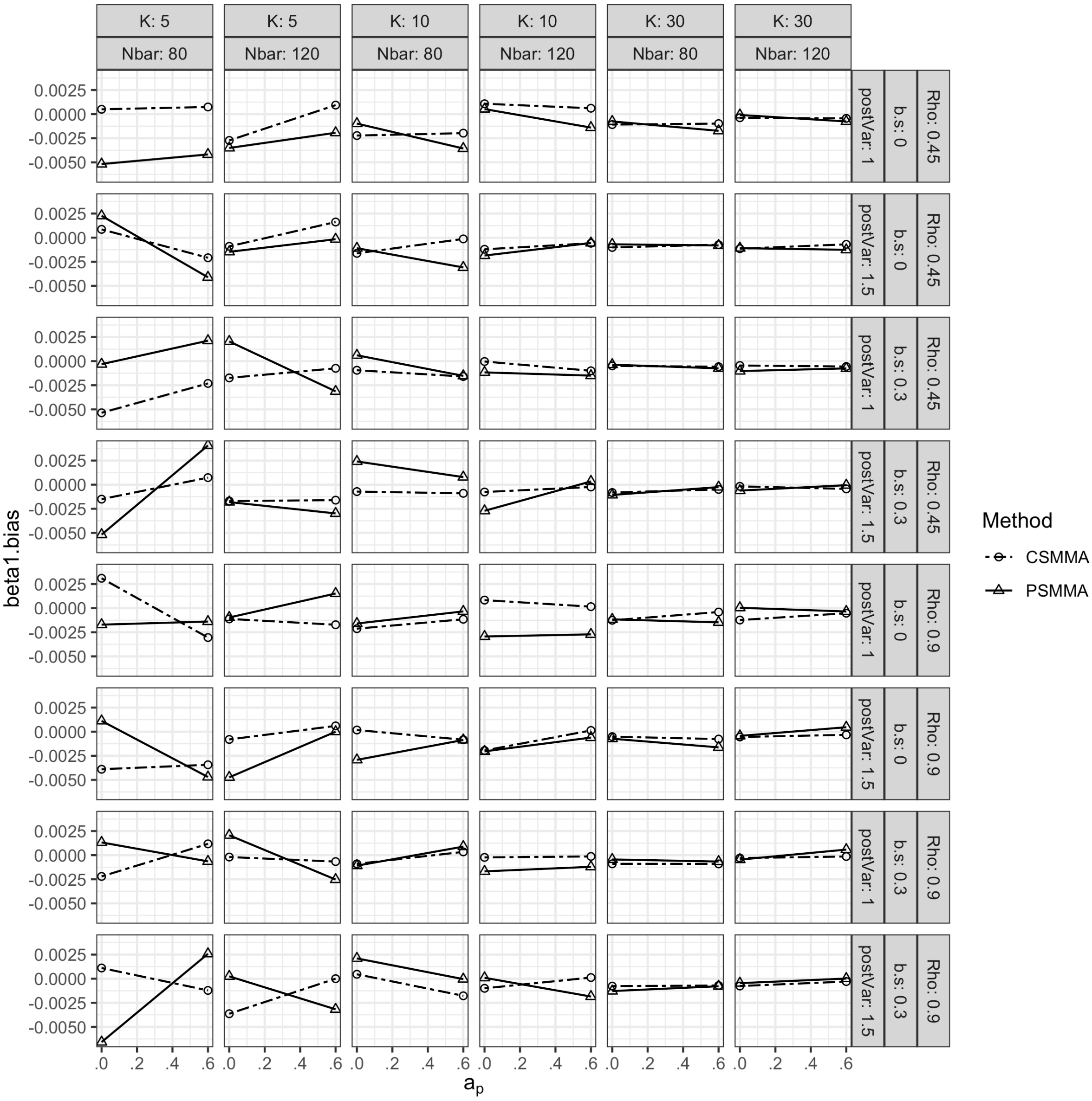


Figure S61.

Coverage Rate of the Moderating Effect Generated on the a Path under Conditions with Between-study Heterogeneity Generated on the Indirect Effect

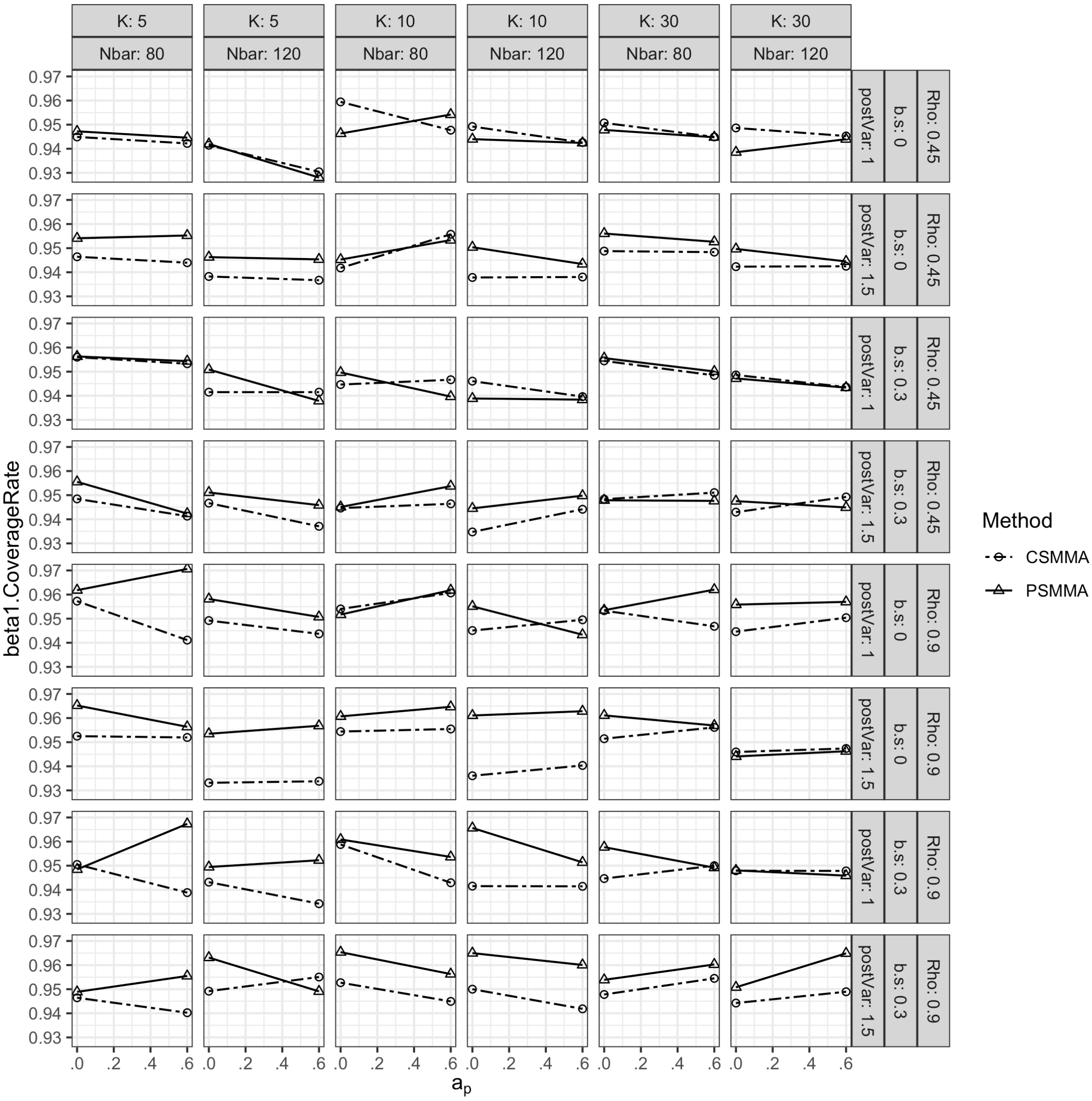


Figure S62.

Coverage Rate of the Moderating Effect Generated on the b Path under Conditions with Between-study Heterogeneity Generated on the Indirect Effect

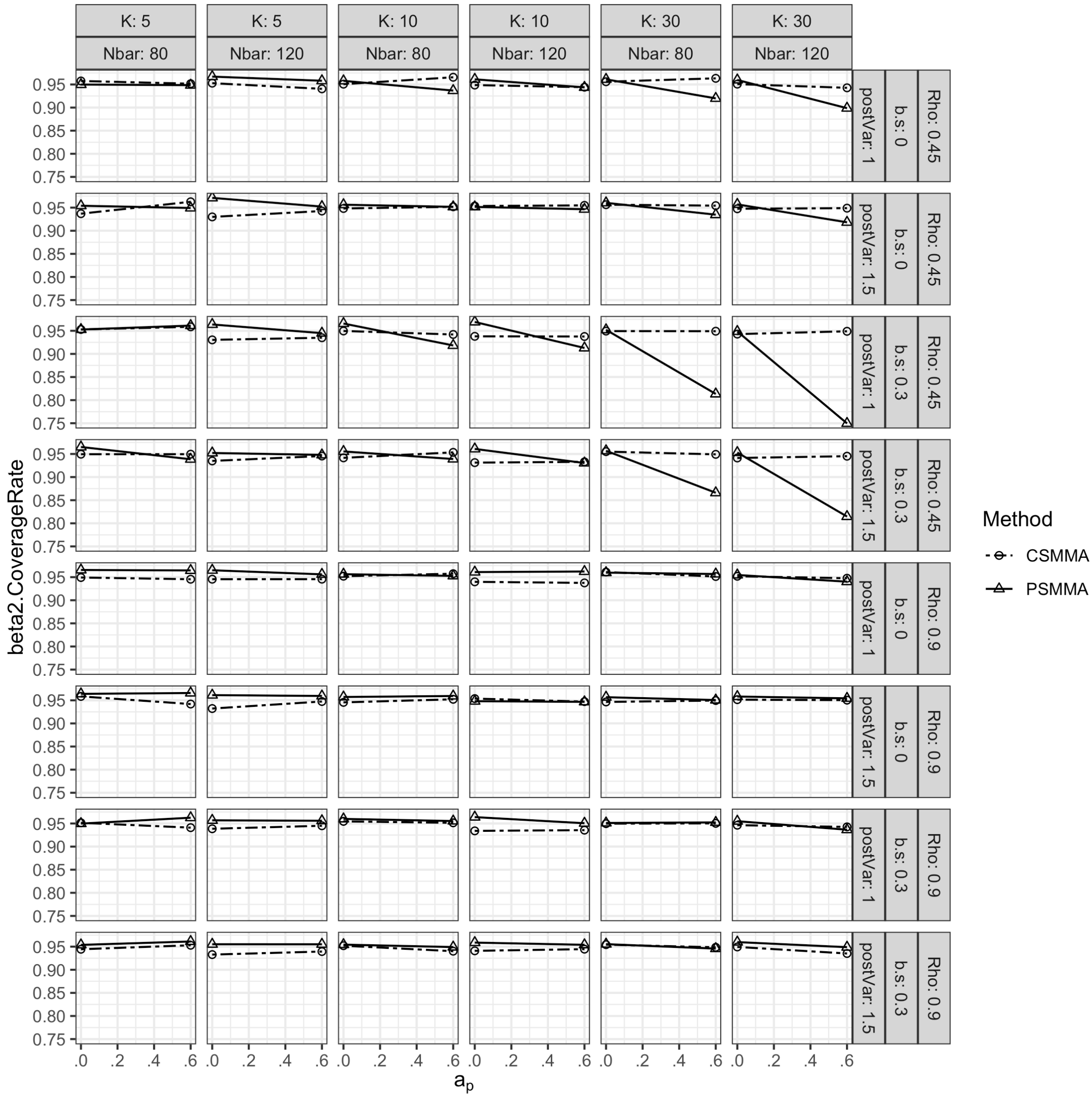


Figure S63.

Coverage Rate of the Moderating Effect Generated on the c' *Path*

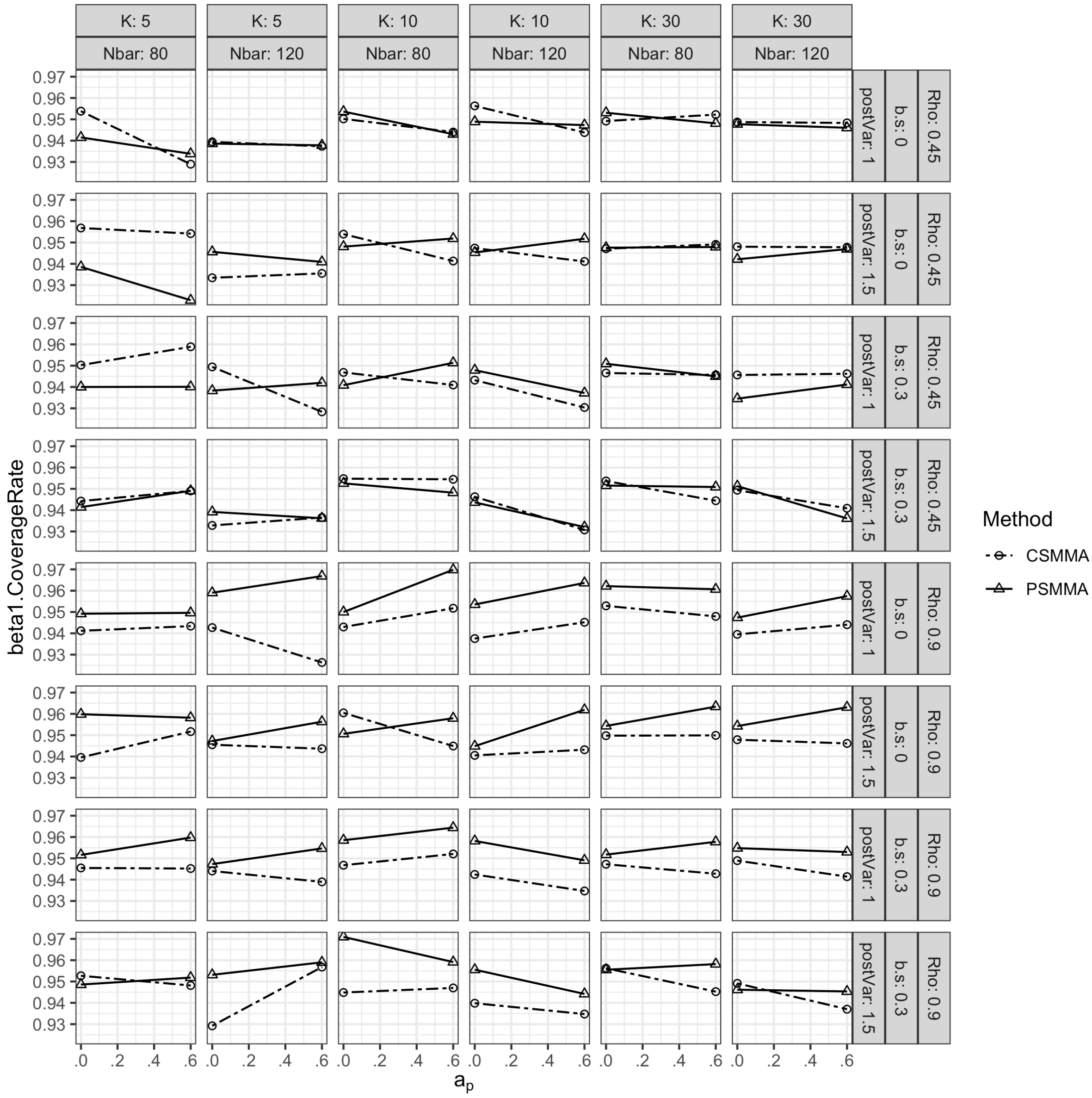


Figure S64.

Statistical Power of the Moderating Effect Generated on the a Path under Conditions with Between-study Heterogeneity Generated on the Indirect Effect

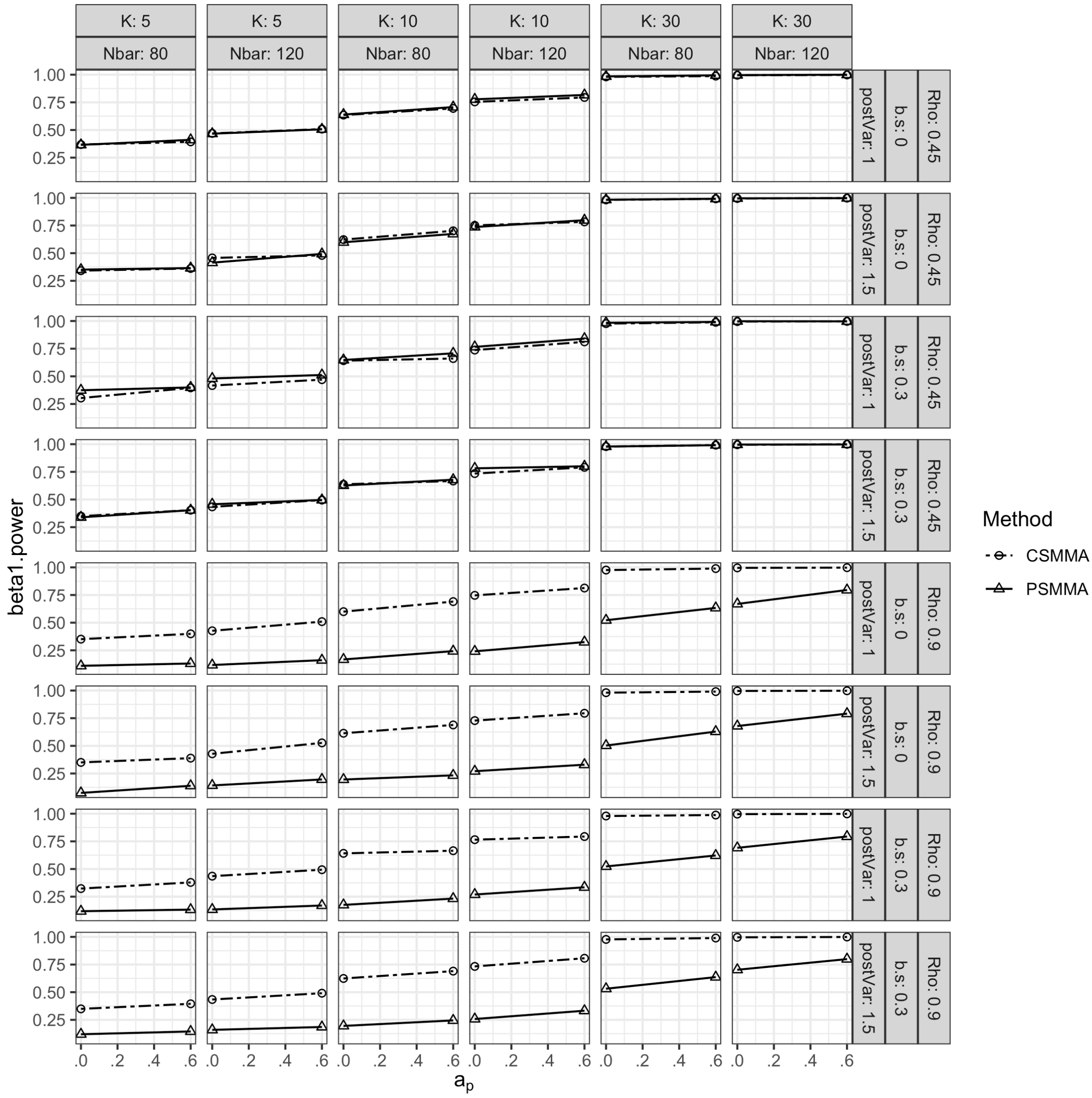


Figure S65.

Statistical Power of the Moderating Effect Generated on the b Path under Conditions with Between-study Heterogeneity Generated on the Indirect Effect

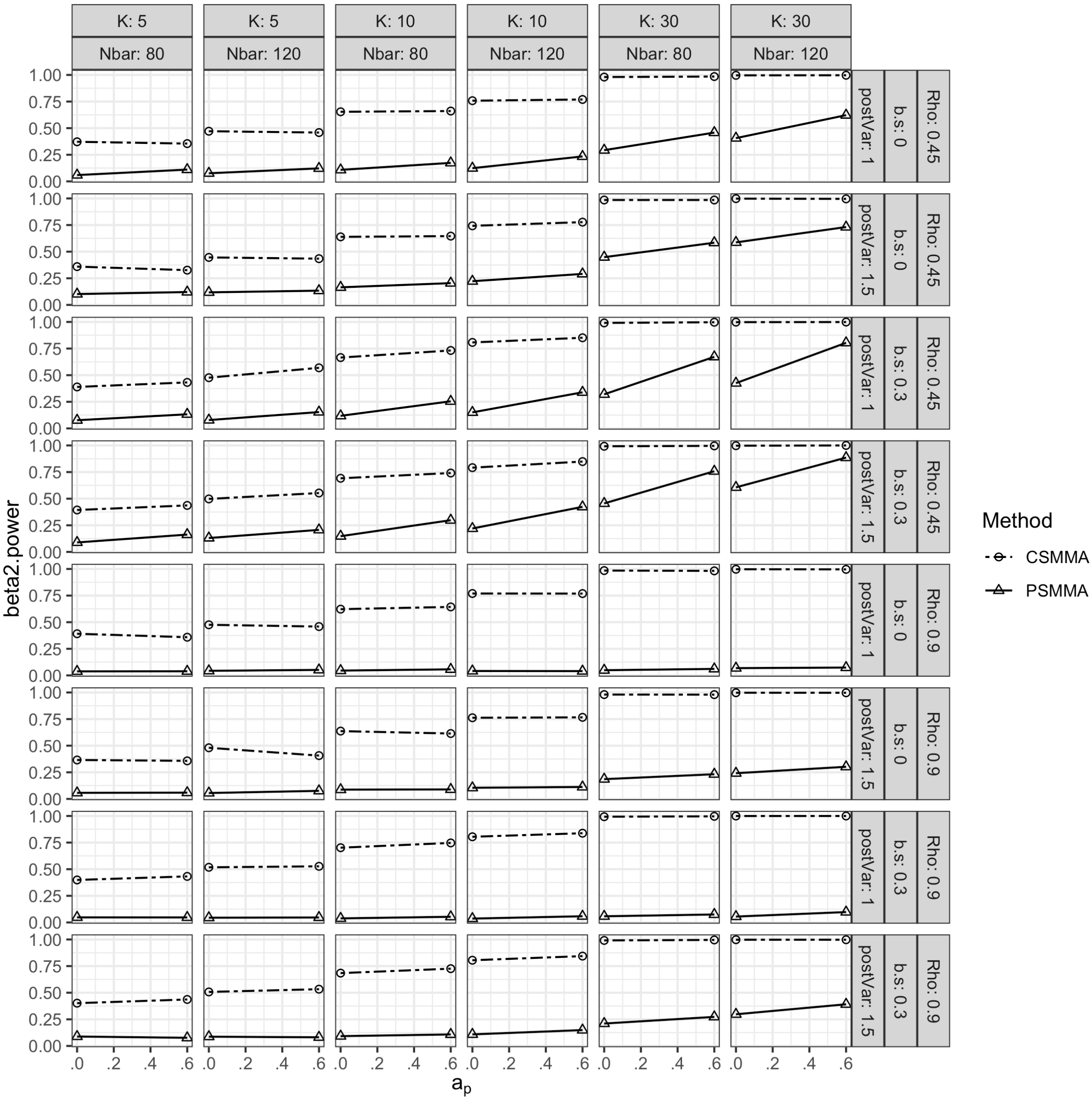
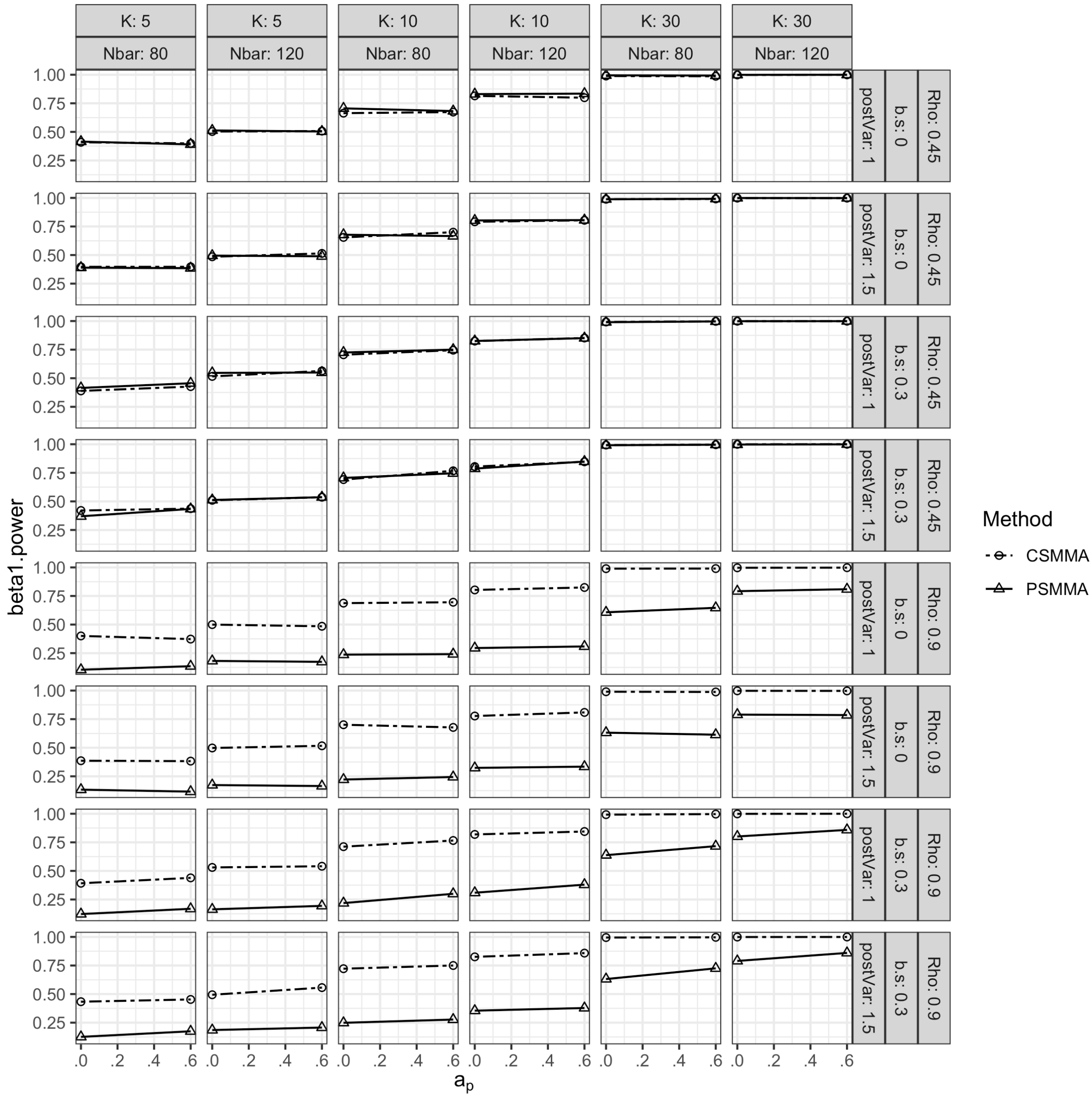


Figure S66.

Statistical Power of the Moderating Effect Generated on the c' Path



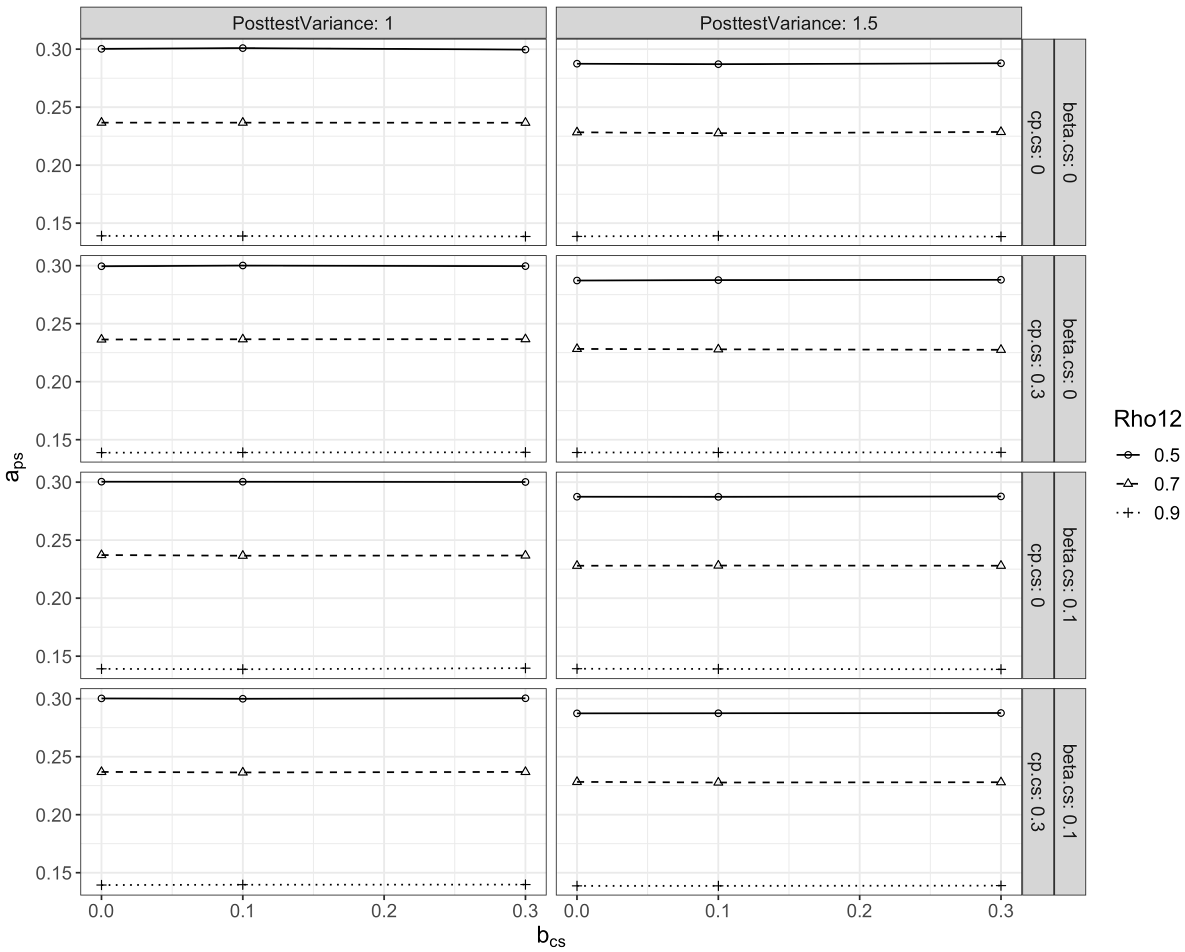
S4 The Relationship between Parameters in PSMMA and CSMMA

S4.1 Parameters in PSMMA with Nonzero Counterparts in CSMMA

As shown in Figure S67, when $a_{s.cs} = 0.3$, $a_{s.ps}$ decreased with a larger pretest-posttest correlation (Rho12) and posttest variance inflation. The size of $b_{s.cs}, cp_{s.cs}$, and $\beta_{c'_{s.cs}}$ did not have apparent effect here.

Figure S67.

Posttest-score Parameter of the a *path (*$a_{s.cs} = 0.3$*)*



Similar patterns have been observed in the c' and $\beta_{c'_{s.cs}}$ paths too (Figure S68 and S69).

Figure S68.

Posttest-score Parameter of the c' *path* ($c'_{s.cs} = 0.3$)

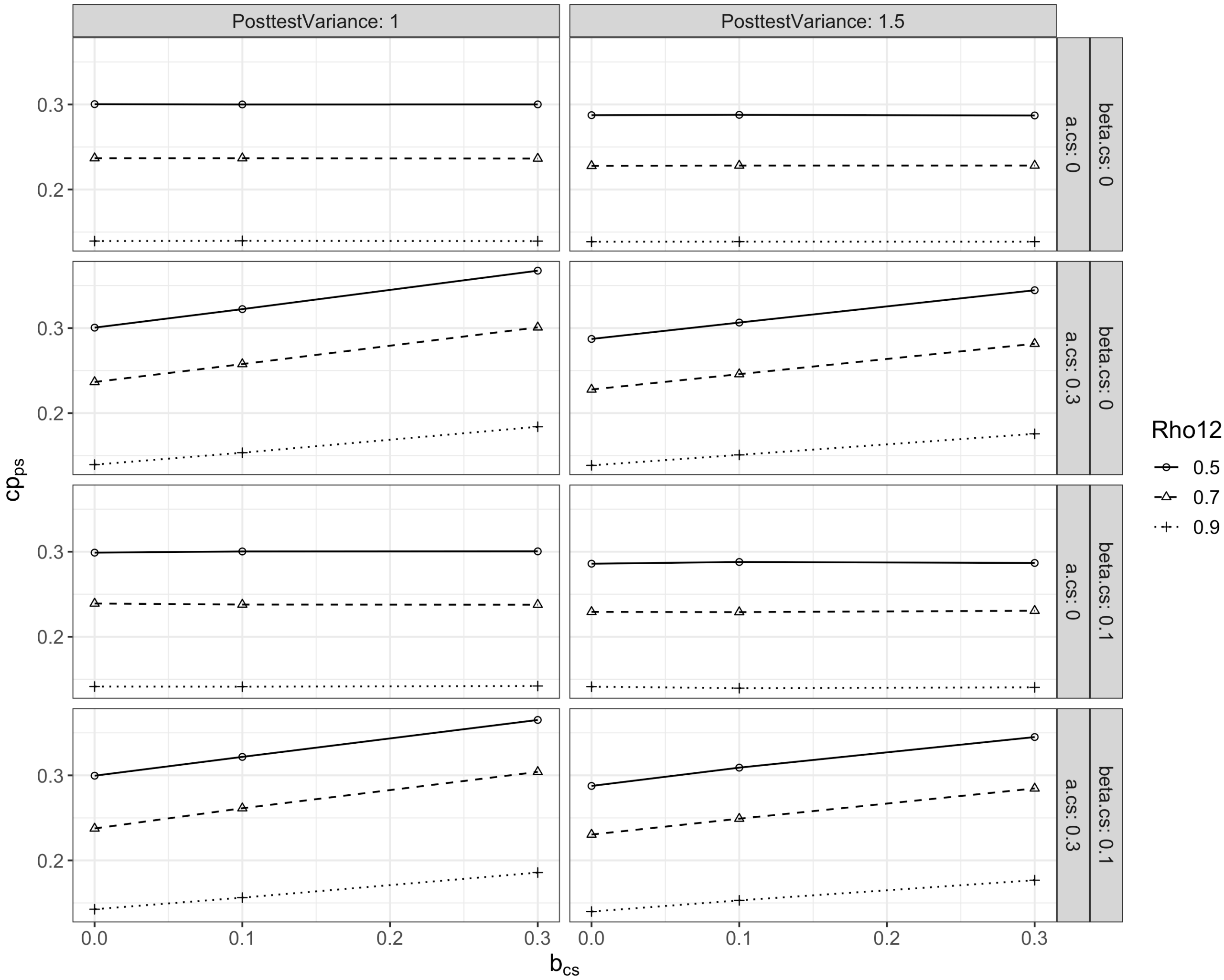
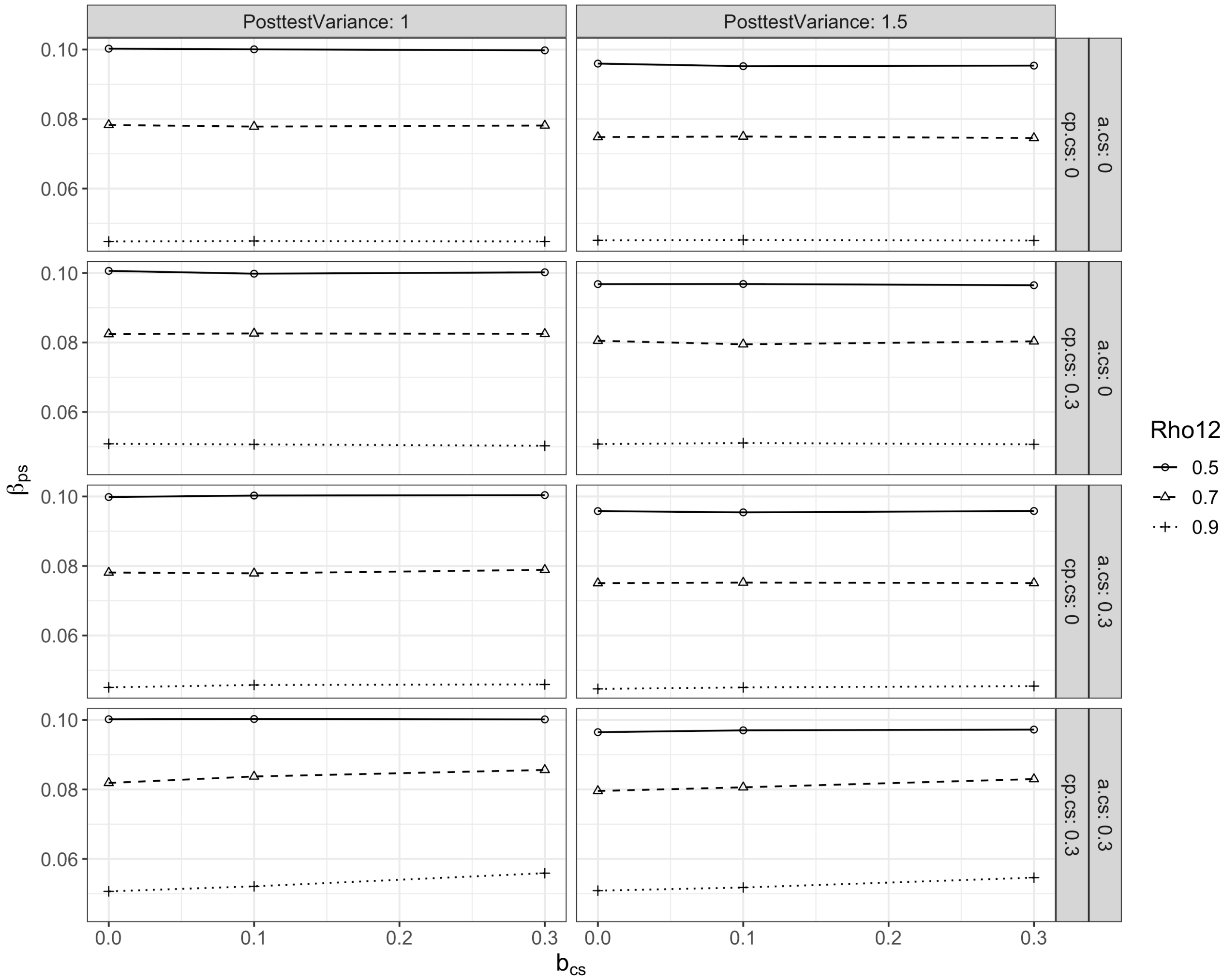


Figure S69.

Posttest-score Parameter of the $\beta_{c'}$ *path (*$\beta_{c'_{s.cs}} = 0.1$*)*

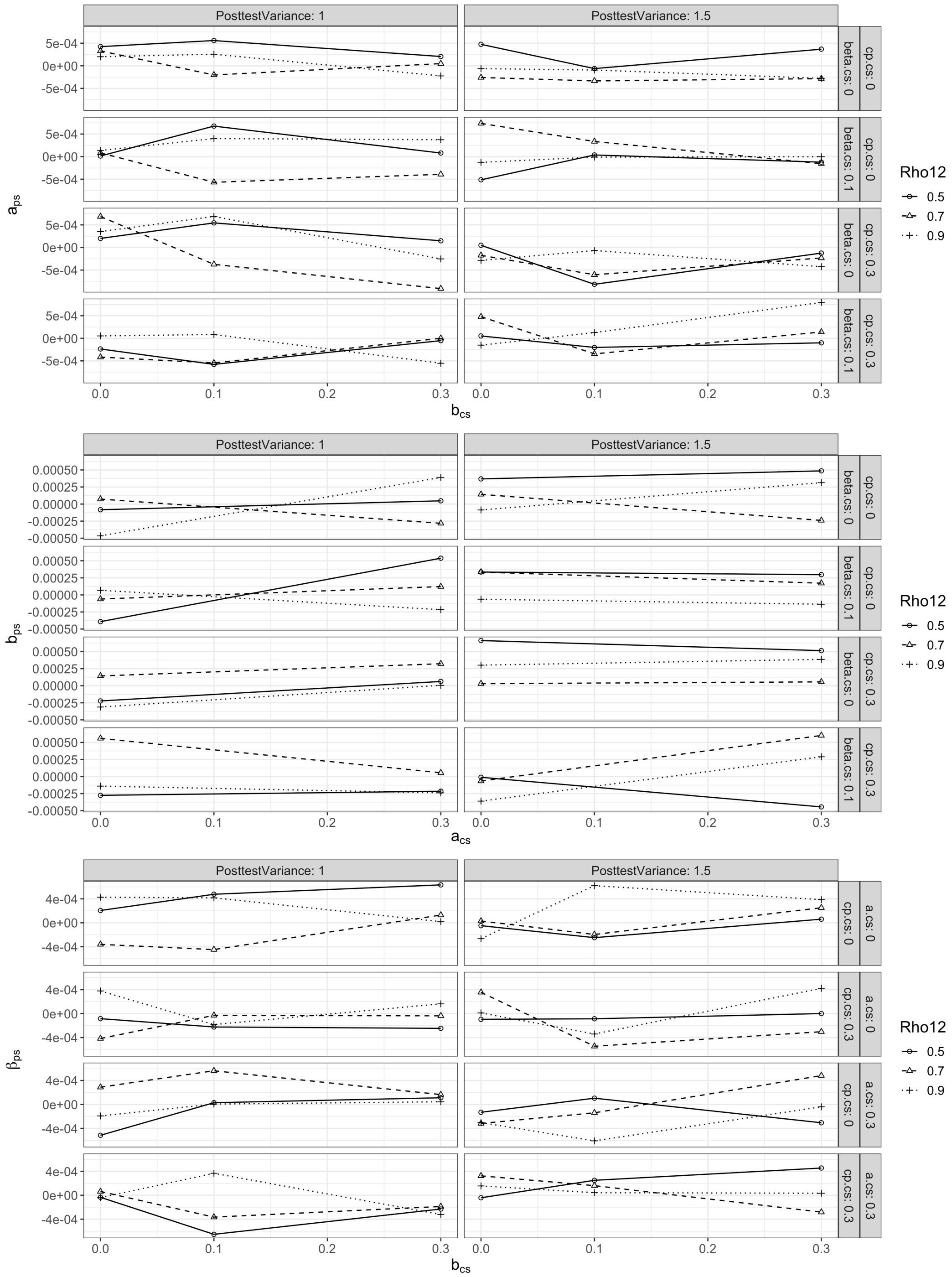


S4.2 Asynchrony in Taking a Value of Zero

As shown in Figure S70, the posttest-score parameters on a, b, and $\beta_{c'}$ paths were consistently zero when the counterparts in CSMMA were zero.

Figure S70.

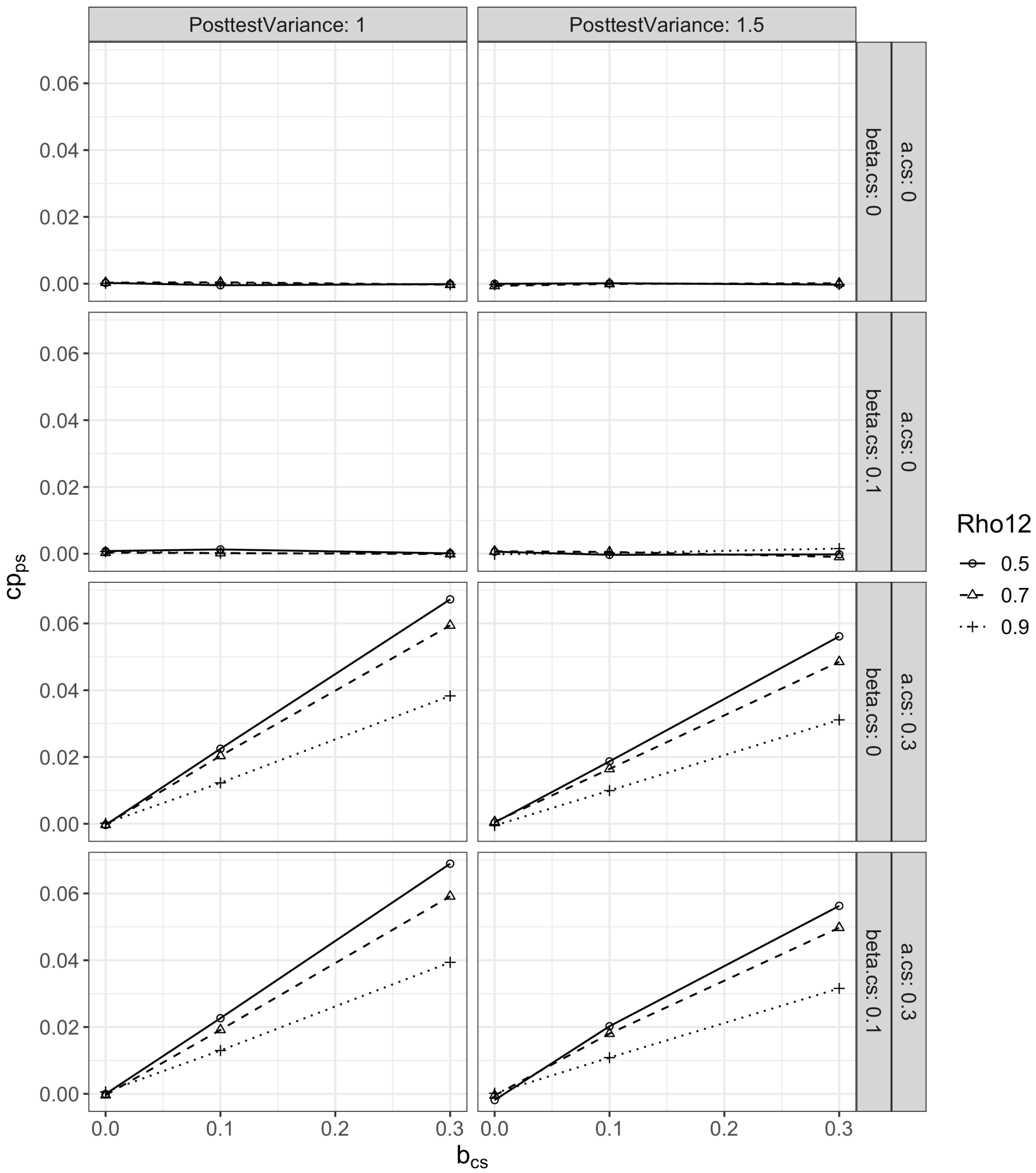
Posttest-score Parameter of the a, b, and $\beta_{c'}$ *paths* ($a_{s.cs} = 0; b_{s.cs} = 0; \beta_{c'_{s.cs}} = 0$)



For a zero change-score c', the corresponding posttest-score c' was also zero when one of the change-score a and b paths was zero (the top two rows in Figure S71). However, when the change-score c' equaled zero but the change-score a and b paths were simutaneously nonzero, the mean c' path in PSMMA took a nonzero value (the bottom two rows in Figure S71).

Figure S71.

Posttest-score Parameter of the c' path ($c' = 0$)



S5 Simulations on MMA with Missing Correlations

Given that missing data are not uncommon in meta-analysis, we conducted simulation studies (Studies 2 and 3) with missing correlations considered (missing completely at random). The data-generating procedure in Studies 2 and 3 was the same as in the main study, except that a proportion of correlations were replaced with "NA". Study 2 assessed MMA with the between-studies heterogeneity generated on the indirect effect (on both a and b paths), whereas Study 3 examined MMA with the between-studies heterogeneity generated in the direct effect (on the c' path). In addition, the implementation of CSMMA and PSMMA, true path coefficients of PSMMA, and performance measures were the same as in the main study. Both simulations were implemented using R Version 4.2.2 (R code team, 2022), with 5,000 repetitions for each condition.

S5.1 Manipulated Factors and Fixed Parameters

In both studies, we investigated the impact of several factors on the performance of CSMMA and PSMMA, including the proportion of missing correlations, standardized change-score (denoted by the subscript cs) path parameters ($a_{s,cs}$ and $b_{s,cs}$), pretest-posttest correlation (ρ_{12}), the posttest variance of M and Y in the treatment group ($\sigma^2_{W_{2,T}}$), mean within-study sample size ($\bar{N}$), and the number of primary studies (K). A $2\ (a_{s,cs})\ \times 2\ (b_{s,cs})\ \times 2\ (\rho_{12}) \times 2\ \left(\sigma^2_{W_{2,T}}\right) \times 2\ (\bar{N}) \times 3\ (K)\ \times 2$ (missing rate) design was used. The settings for $a_{s,cs}$, $b_{s,cs}$, ρ_{12}, $\sigma^2_{W_{2,T}}$, $\bar{N}$ and K were identical as in the main study. Two values of proportion of missing correlations are used (40% and 60%), following the simulation conducted in Jak & Cheung (2020). To simulate situations in reality, we considered relatively high missing rates for the M-Y correlations as opposed to the other two correlations.

Specifically, under the conditions where the missing rate equaled 40% (of all bivariate correlations, r_{XM}, r_{XY}, and r_{MY}), 40% of r_{XM} and 80% of r_{MY} were set as missing. On the other hand, under the conditions where the missing rate equaled 60%, 90% of r_{XM} and 90% of r_{MY} were set as missing. The fixed model parameters were identical as in the main simulation.

S5.2 Simulation Results

S5.2.1 Convergence Rates in Studies 2 and 3

Convergence rate of Study 2 is shown in Figure S72, whereas that of Study 3 is shown in Figure S73.

Figure S72. Convergence Rate in Study 2

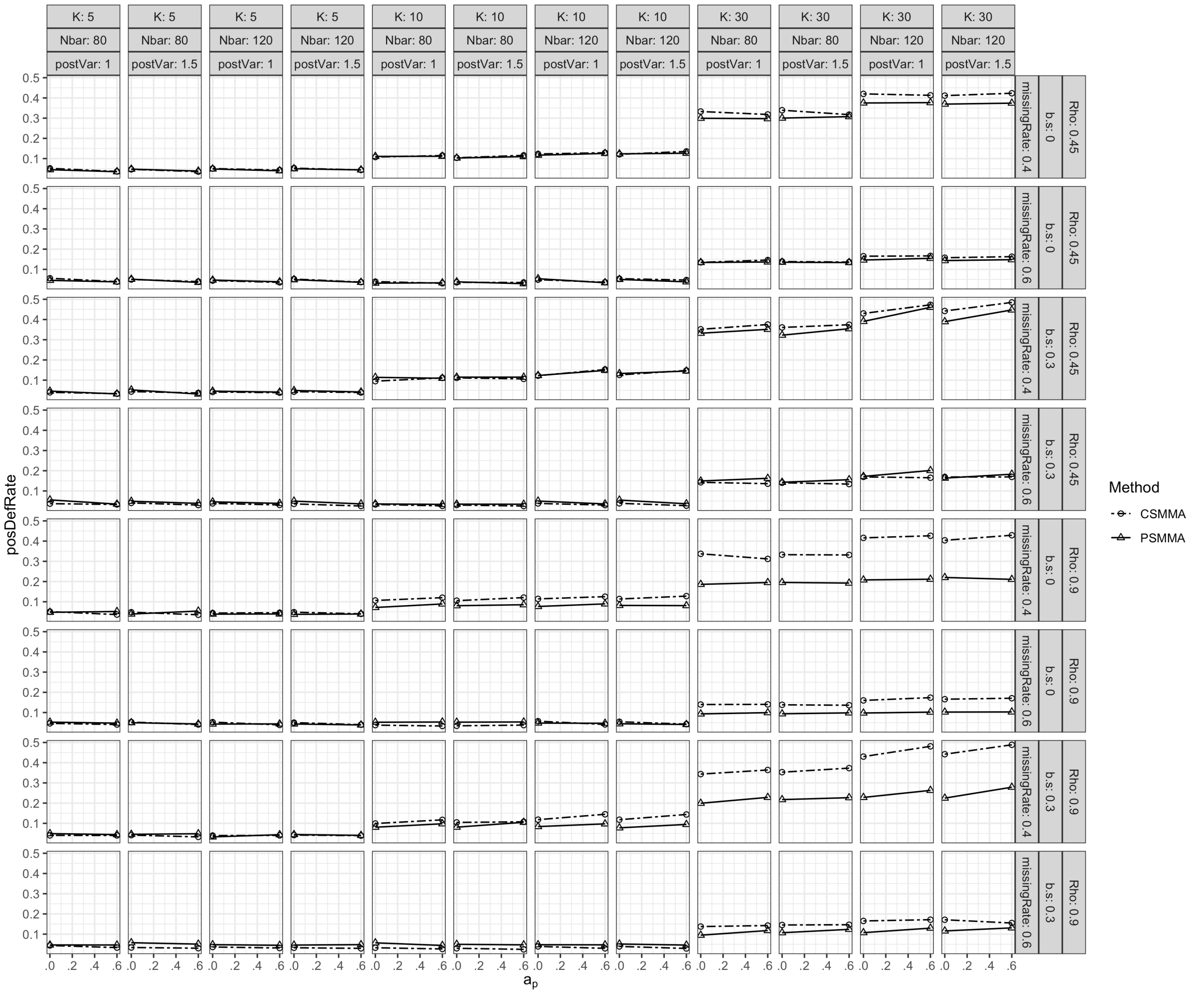
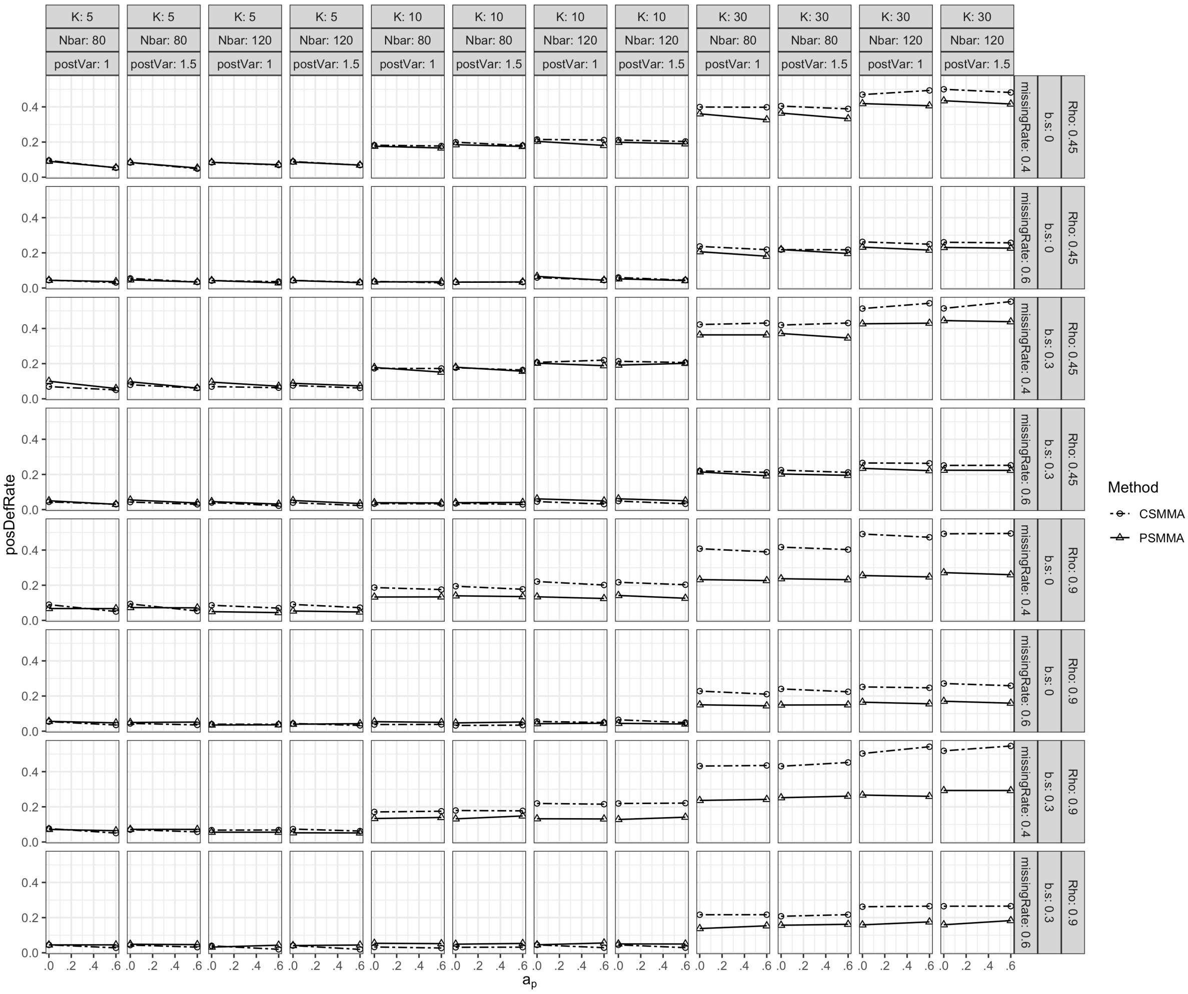


Figure S73. Convergence Rate in Study 3



S5.2.2 Study 2

Indirect Effect. Regarding the point and interval estimation of the indirect effect, CSMMA and PSMMA had comparable bias (around 0) and good coverage rate (>0.9) (Figure S74 and S75). Regarding the testing of the indirect effect, while CSMMA and PSMMA both had well-controlled type I error rates (0~0.1; Figure S76), the statistical power of CSMMA was higher than that of PSMMA (Figure S77).

Figure S74. Bias of the Indirect Effect in Study 2

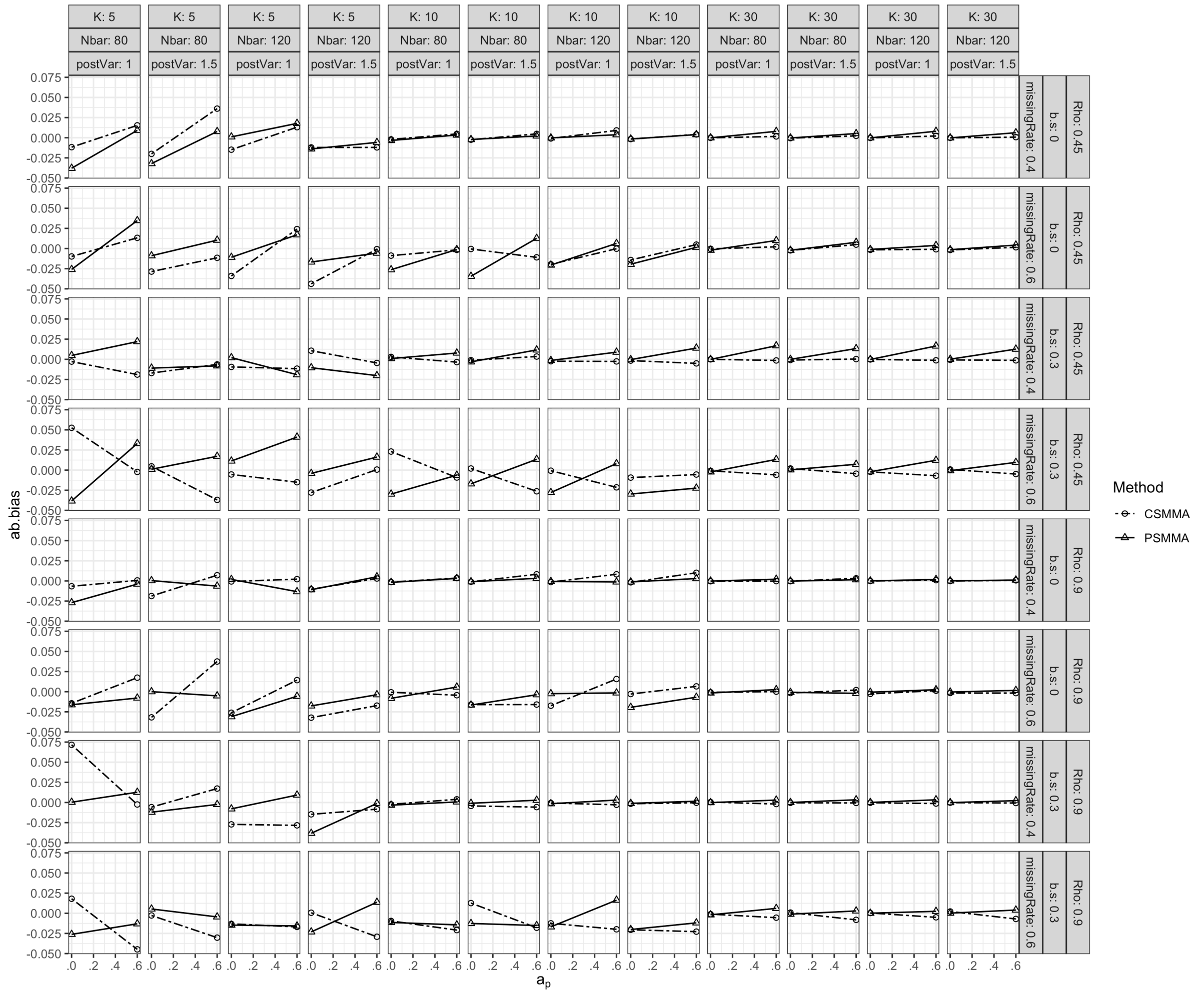


Figure S75. Coverage Rate of the Indirect Effect in Study 2

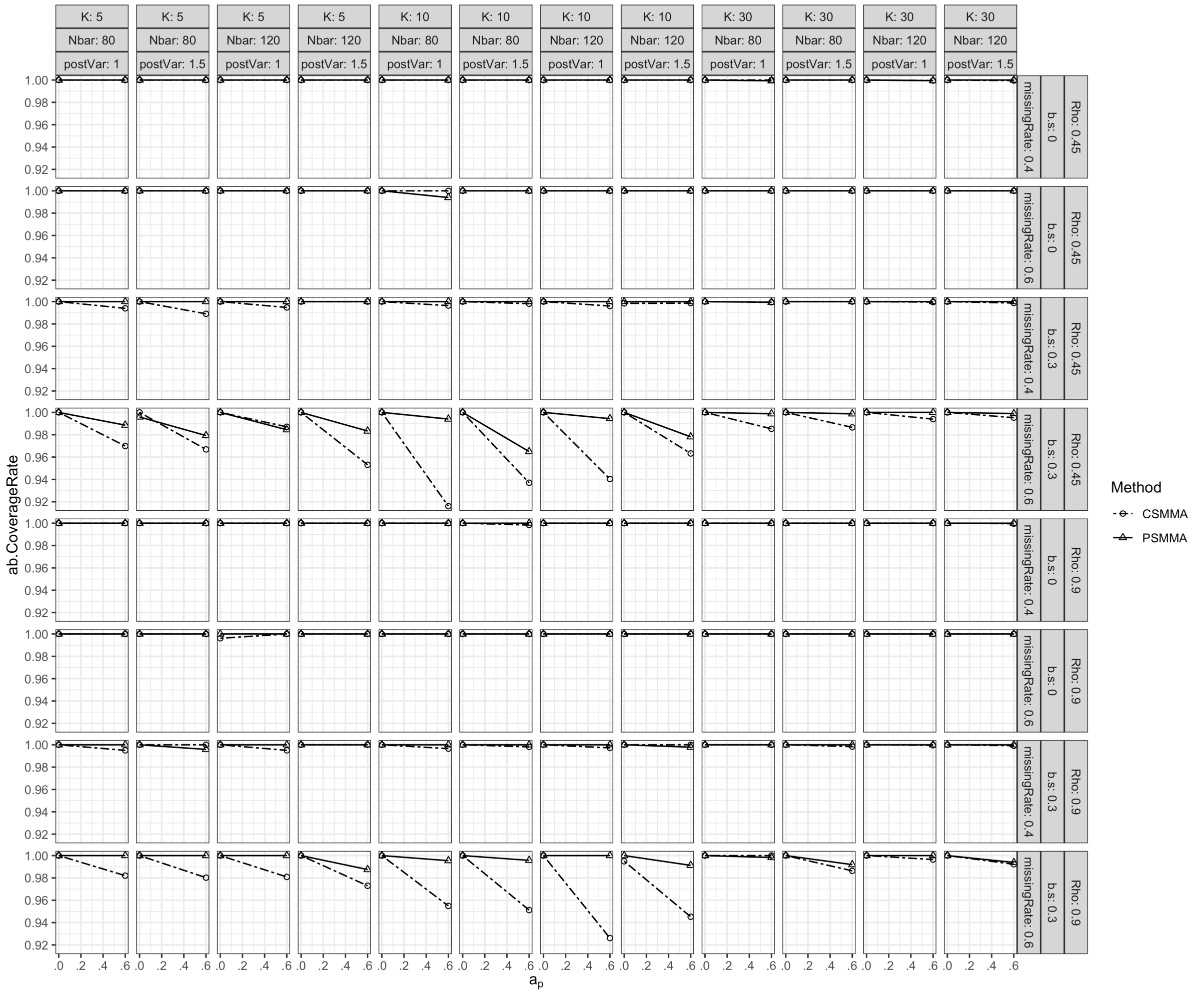


Figure S76. Type I Error Rate of the Indirect Effect in Study 2

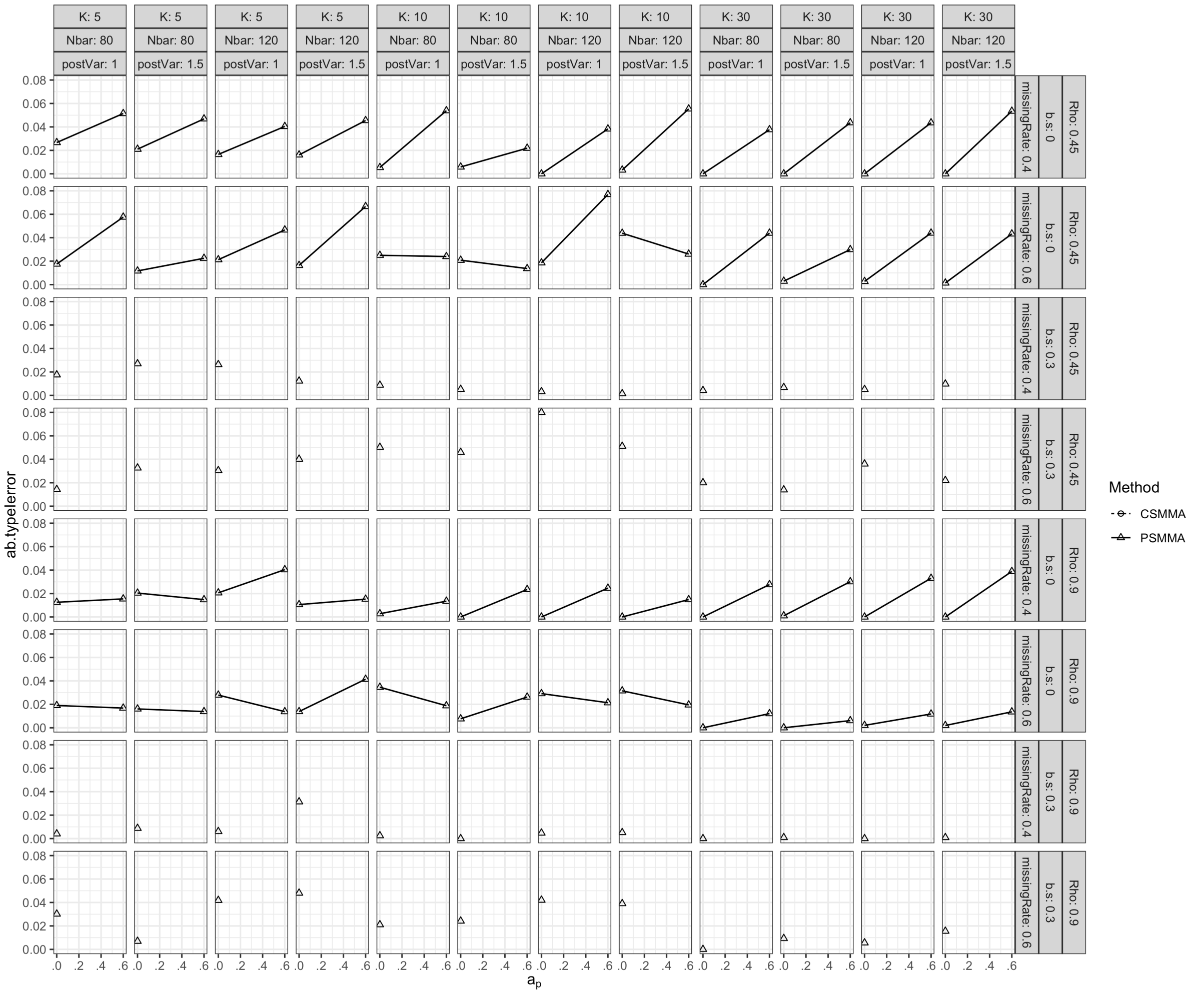
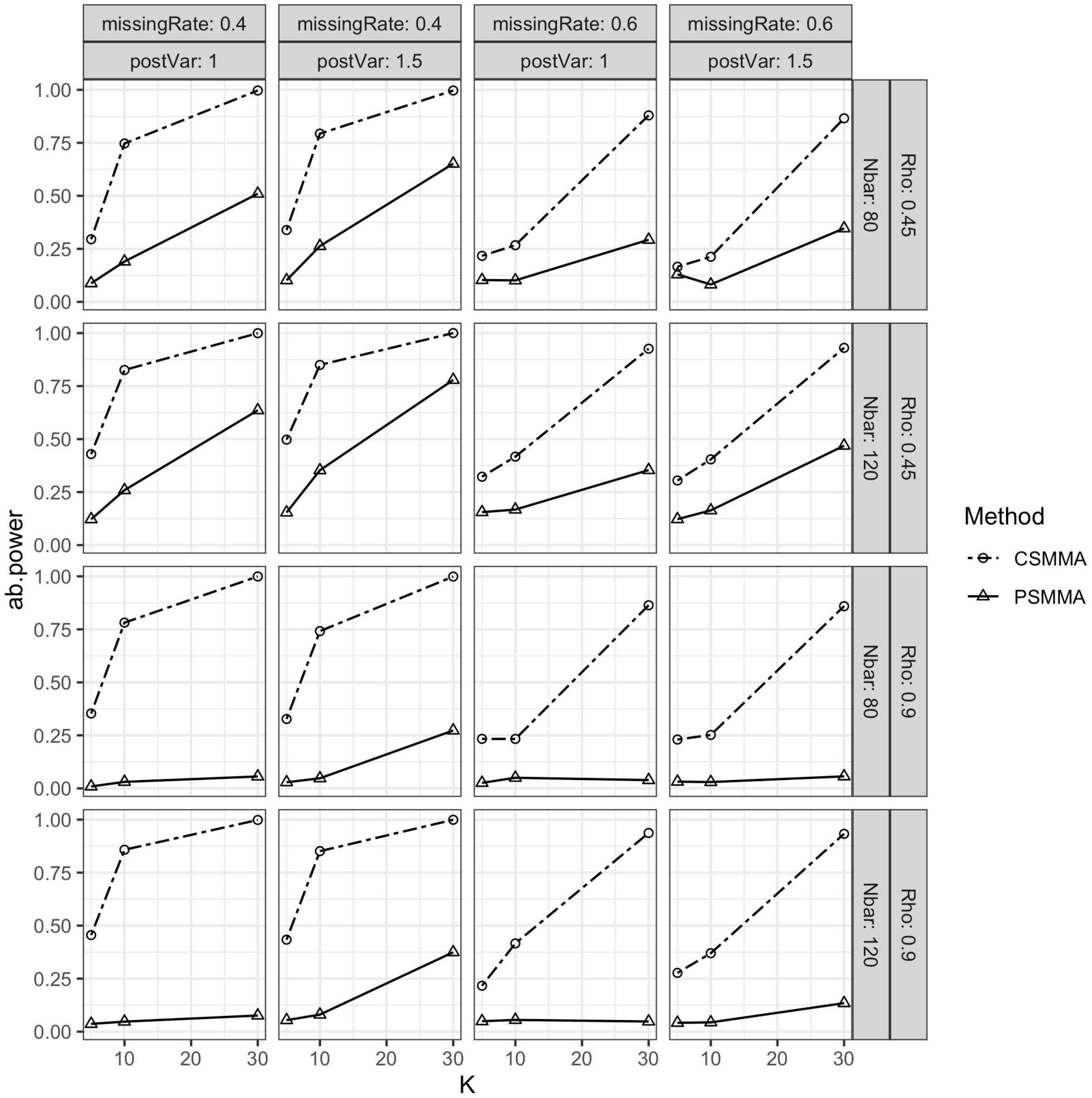


Figure S77. Statistical Power of the Indirect Effect in Study 2



Direct and Moderating Effects. Regarding inferences of the direct and moderating effects, results with missing correlations were similar to the main simulation, except that the statistical power was smaller than that in Study 1 (Figures S78, S79, and S80).

Figure S78. Statistical Power of the Direct Effect in Study 2

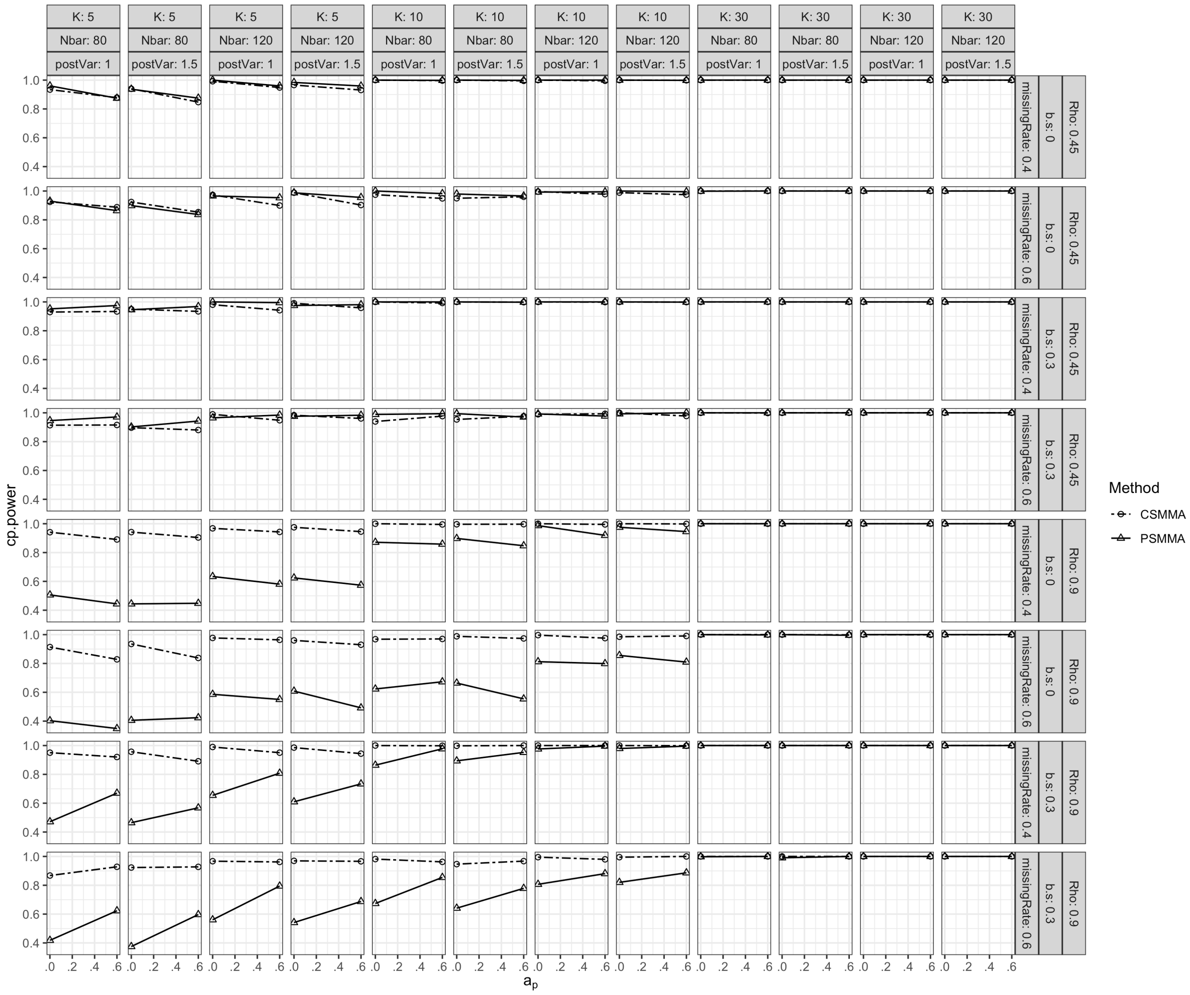


Figure S79. Statistical Power of the Moderating Effect on the a Path in Study 2

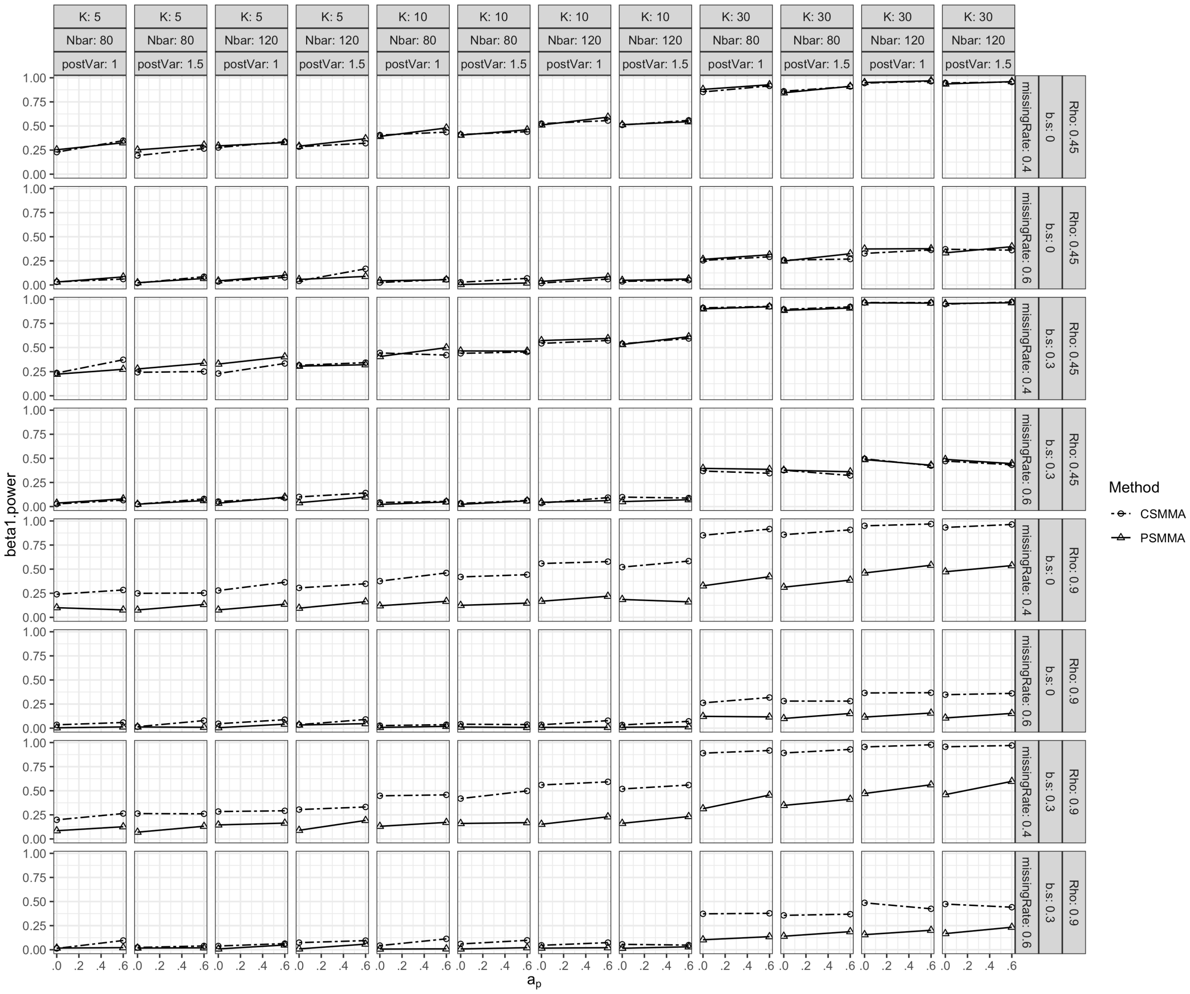
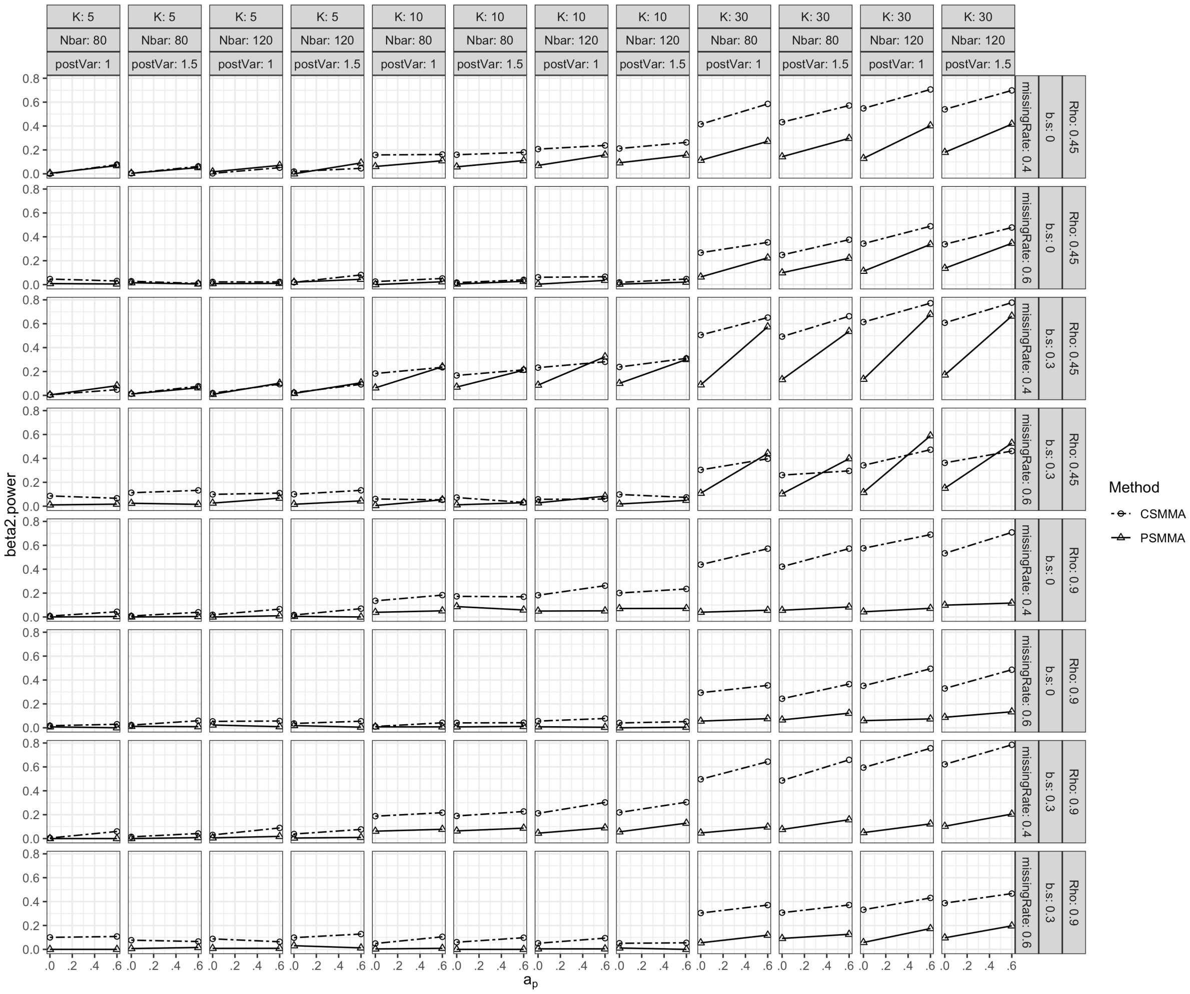


Figure S80. Statistical Power of the Moderating Effect on the *b* Path in Study 2



S5.2.3 Study 3

Regarding inferences of the indirect, direct, and moderating effects, results of Study 3 were similar to Study 2 with one exception about statistical power. The power of CSMMA in Study 3 was slightly higher than that in Study 2 (Figure S81). The comparison between the power of CSMMA and PSMMA followed a patter similar to that in Study 2.

Figure S81. Statistical Power of the Indirect Effect in Study 4