Supplemental Materials for ‘Synthesizing data from pretest-posttest-control-group designs in mediation meta-analysis’

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S1 Mathematical Derivations

S1.1 Derivations for Equations Used to Construct Correlation Matrices

S1.1.1 Point-biserial Correlations

In this section, we do not consider the justification for degree of freedom for simplicity.

Considering Equation 1 and 2,

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{N_T\left(\hat{\sigma}_T^2 + \left(\hat{\mu}_T - \frac{N_T\hat{\mu}_T + N_C\hat{\mu}_C}{N_T + N_C}\right)^2\right) + N_C\left(\hat{\sigma}_C^2 + \left(\hat{\mu}_C - \frac{N_T\hat{\mu}_T + N_C\hat{\mu}_C}{N_T + N_C}\right)^2\right)}{N_T + N_C}}}\sqrt{\frac{N_TN_C}{(N_T + N_C)^2}},$$

Equation 3 can be obtained with the following algebraic transformations:

(1) Expanding the denominator:

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{N_T\hat{\sigma}_T^2 + N_C\hat{\sigma}_C^2 + N_T\hat{\mu}_T^2 + N_C\hat{\mu}_C^2 - \frac{(N_T\hat{\mu}_T + N_C\hat{\mu}_C)^2}{N_T + N_C}}{N_T + N_C}\frac{(N_T + N_C)^2}{N_TN_C}}};$$

(2) Simplifying the denominator:

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{(N_T + N_C)(N_T\hat{\sigma}_T^2 + N_C\hat{\sigma}_C^2 + N_T\hat{\mu}_T^2 + N_C\hat{\mu}_C^2) - (N_T\hat{\mu}_T + N_C\hat{\mu}_C)^2}{N_TN_C}}};$$

(3) Expanding the simplified equation:

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{N_T^2\hat{\sigma}_T^2 + N_TN_C\hat{\sigma}_T^2 + N_C^2\hat{\sigma}_C^2 + N_TN_C\hat{\sigma}_C^2 + N_TN_C\hat{\mu}_T^2 + N_TN_C\hat{\mu}_C^2 - 2N_TN_C\hat{\mu}_T\hat{\mu}_C}{N_TN_C}}};$$

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{\frac{N_T\hat{\sigma}_T^2(N_T + N_C) + N_C\hat{\sigma}_C^2(N_T + N_C)}{N_TN_C} + \hat{\mu}_T^2 + \hat{\mu}_C^2 - 2\hat{\mu}_T\hat{\mu}_C}};$$

(4) Obtaining Equation 3 in the main text

$$r_{pb,cs/ps} = \frac{\hat{\mu}_T - \hat{\mu}_C}{\sqrt{(\hat{\mu}_T - \hat{\mu}_C)^2 + (N_T\hat{\sigma}_T^2 + N_C\hat{\sigma}_C^2)\frac{N_T + N_C}{N_T N_C}}}.$$

S1.1.2 Converting from Paired-sample t Values to Bivariate Correlations

Equation 4 in the main text can be obtained by multiplying the *t* value and dividing the SD in both sides of the regular *t*-test equation $t = \frac{\hat{\mu}_{cs}}{\frac{\hat{\sigma}_{cs}}{\sqrt{N}}}$.

S1.1.3 Converting from Confidence Intervals to Bivariate Correlations

The upper or lower bound of the confidence interval of change scores can be computed using $CI_{upper/lower} = \hat{\mu}_{cs} \pm t_{crit}\frac{\hat{\sigma}_{pl,cs}}{\sqrt{N}}$. Therefore, half of the confidence interval can be obtained: $\frac{CI_{upper} - CI_{lower}}{2} = \frac{\hat{\mu}_{cs} + t_{crit}\frac{\hat{\sigma}_{pl,cs}}{\sqrt{N}} - \hat{\mu}_{cs} + t_{crit}\frac{\hat{\sigma}_{pl,cs}}{\sqrt{N}}}{2} = t_{crit}\frac{\hat{\sigma}_{pl,cs}}{\sqrt{N}}$. Then Equation 5 in the main text can be obtained by dividing t_{crit} and multiplying $\sqrt{N}$ in both sides.

S1.1.4 Converting from Regression Coefficients to Bivariate Correlations

The standardized a and c coefficients are in nature bivariate correlations between X and M and between X and Y, respectively. The correlation between M and Y, on the other hand, can be converted from the regression coefficient b using the equation in the main text, which comes from $b_s = \frac{r_{MY} - r_{XM}r_{XY}}{1 - r_{XM}^2}$, the regular equation for converting between regression coefficients and Pearson's correlations.

S1.2 Data-generating Mechanisms

S1.2.1 Change-score Group Variances

$$var(M_{\text{cs}}) = cov(M_2 - M_1, M_2 - M_1)$$

$$= var(M_1) - 2 \times cov(M_2, M_1) + var(M_2)$$

$$var(Y_{\text{cs}}) = cov(Y_2 - Y_1, Y_2 - Y_1)$$

$$= var(Y_1) - 2 \times cov(Y_2, Y_1) + var(Y_2)$$

S1.2.2 Posttest Means of M and Y in the Treatment Group

For each individual study, considering that the pretest both groups and posttest of the control group were fixed at 0, posttest means in the treatment group would be the mean difference of change scores $MD_k = \hat{\mu}_{T,cs} - \hat{\mu}_{C,cs}$, which applies to both M and Y. Considering $d_{cs} = \frac{MD_k}{\sqrt{\frac{\sigma^2_{T,cs}+\sigma^2_{C,cs}}{2}}}$ and $d_{cs} = \frac{2r}{\sqrt{1-r^2}}$, we can obtain $MD_k^M = \frac{2r_{XM,k}}{\sqrt{1-r^2_{XM,k}}} \times \sqrt{\frac{\sigma^2_{M_{cs,T}}+\sigma^2_{M_{cs,C}}}{2}}$, and $MD_k^Y = \frac{2r_{XY,k}}{\sqrt{1-r^2_{XY,k}}} \times \sqrt{\frac{\sigma^2_{M_{cs,T}}+\sigma^2_{M_{cs,C}}}{2}}$.

S1.2.3 Combined Variances of Change Scores of M and Y

Given the fixed pretest and posttest means and variances, $D_T = D_C = \frac{MD_k^{M/Y}}{2}$ in the population when $\pi = 0.5$. Therefore, $\sigma^2_{cb,cs} = \frac{\sigma^2_{\mathrm{C,cs}}+\sigma^2_{\mathrm{T,cs}}}{2} + \frac{(\mathrm{MD_k})^2}{4}$, which applies to both M and Y.

S1.2.4 Generating Pretest Data of Y

We generated pretest data of Y based on change scores of Y using:

$$Y_{1,T/C} = i_{Y_{1,T/C}} + b_{Y_{cs,1,T/C}} Y_{cs,T/C} + e_{Y_{1,T/C}},$$

where $b_{Y_{cs,1,T/C}}$ is the unstandardized regression coefficient when regressing $Y_{cs,T/C}$ on $Y_{1,T/C}$, and the subscript T/C represent the treatment group OR the control group.

Considering the correlation between pretest and posttest in each group is set as ρ_{12}:

$$cor\left(Y_{1,T/C}, Y_{2,T/C}\right) = \rho_{12},$$

the correlation between pretest and change-score in each group can be computed using

$$cor(Y_{1,T/C}, Y_{cs,T/C}) = \frac{cov(Y_{1,T/C}, Y_{2,T/C}) - cov(Y_{1,T/C}, Y_{1,T/C})}{\sigma_{Y_{1,T/C}} \sigma_{Y_{cs,T/C}}} = \frac{\rho_{12}\sigma_{Y_{1,T/C}}\sigma_{Y_{2,T/C}} - \sigma^2_{Y_{1,T/C}}}{\sigma_{Y_{1,T/C}} \sigma_{Y_{cs,T/C}}}.$$

Next, the unstandardized regression coefficient ($b_{Y_{cs,1,T/C}}$) can be obtained:

$$b_{Y_{cs,1,T/C}} = cor(Y_{1,T/C}, Y_{cs,T/C}) \times (\frac{\sigma_{Y_{1,T/C}}}{\sigma_{Y_{cs,T/C}}}) = \frac{\rho_{12}\sigma_{Y_{1,T/C}}\sigma_{Y_{2,T/C}} - \sigma^2_{Y_{1,T/C}}}{\sigma^2_{Y_{cs,T/C}}}.$$

S2 Additional Results

Here, complete results regarding the indirect, direct, moderating effects are shown, including EBIAS, CR, type I error rates, and statistical power. In all figures, "PostVar" represents the posttest variance in the treatment group.

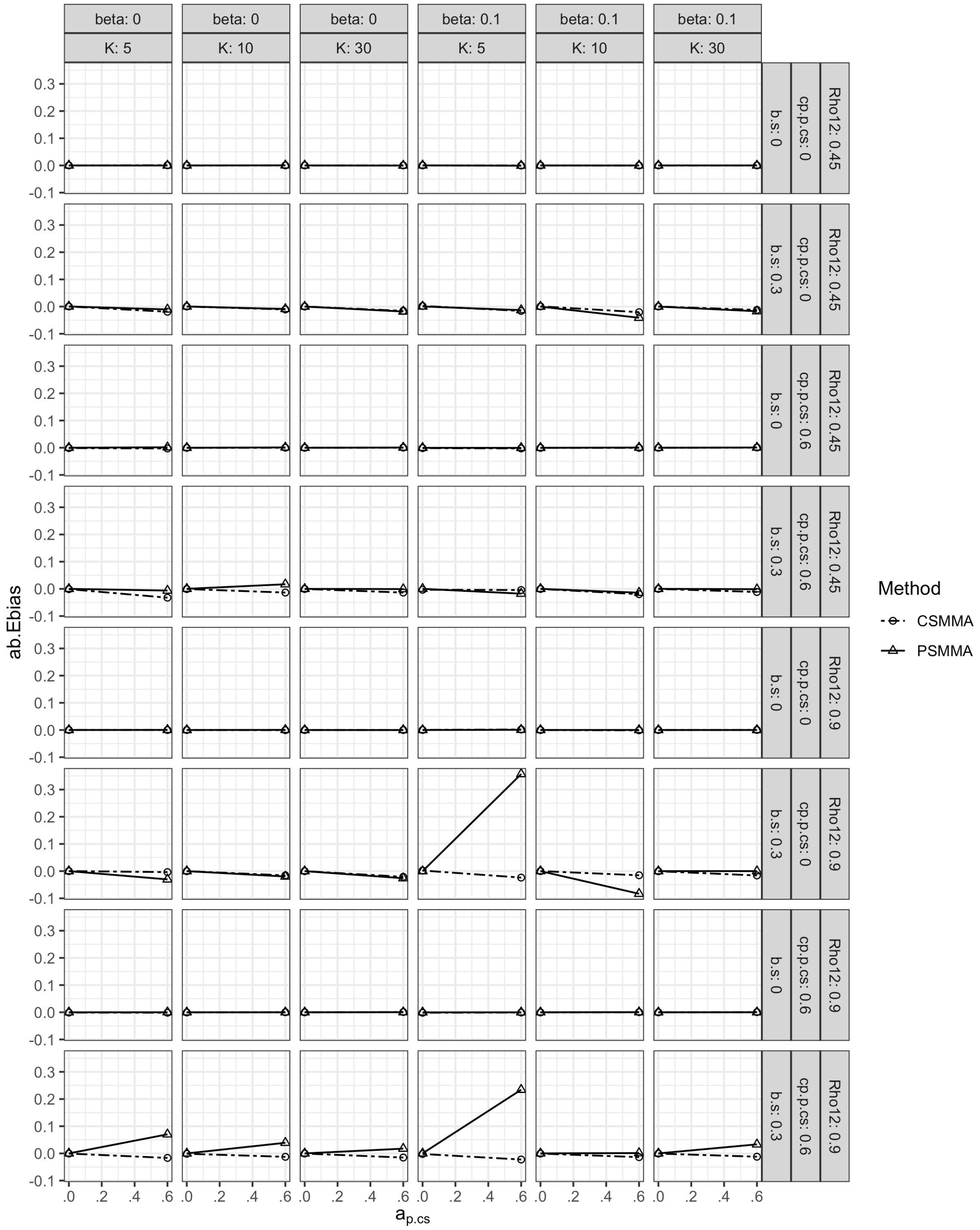
S2.1 Study 1

S2.1.1 Indirect Effect

EBIAS. As shown in Figure S1, EBIAS of CSMMA and PSMMA remained acceptable when K was 10 and 30. However, EBIAS of PSMMA inflated when $K = 5$ and ρ_{12} was 0.9. This inflation was because the true posttest-score parameters were too small.

Figure S1.

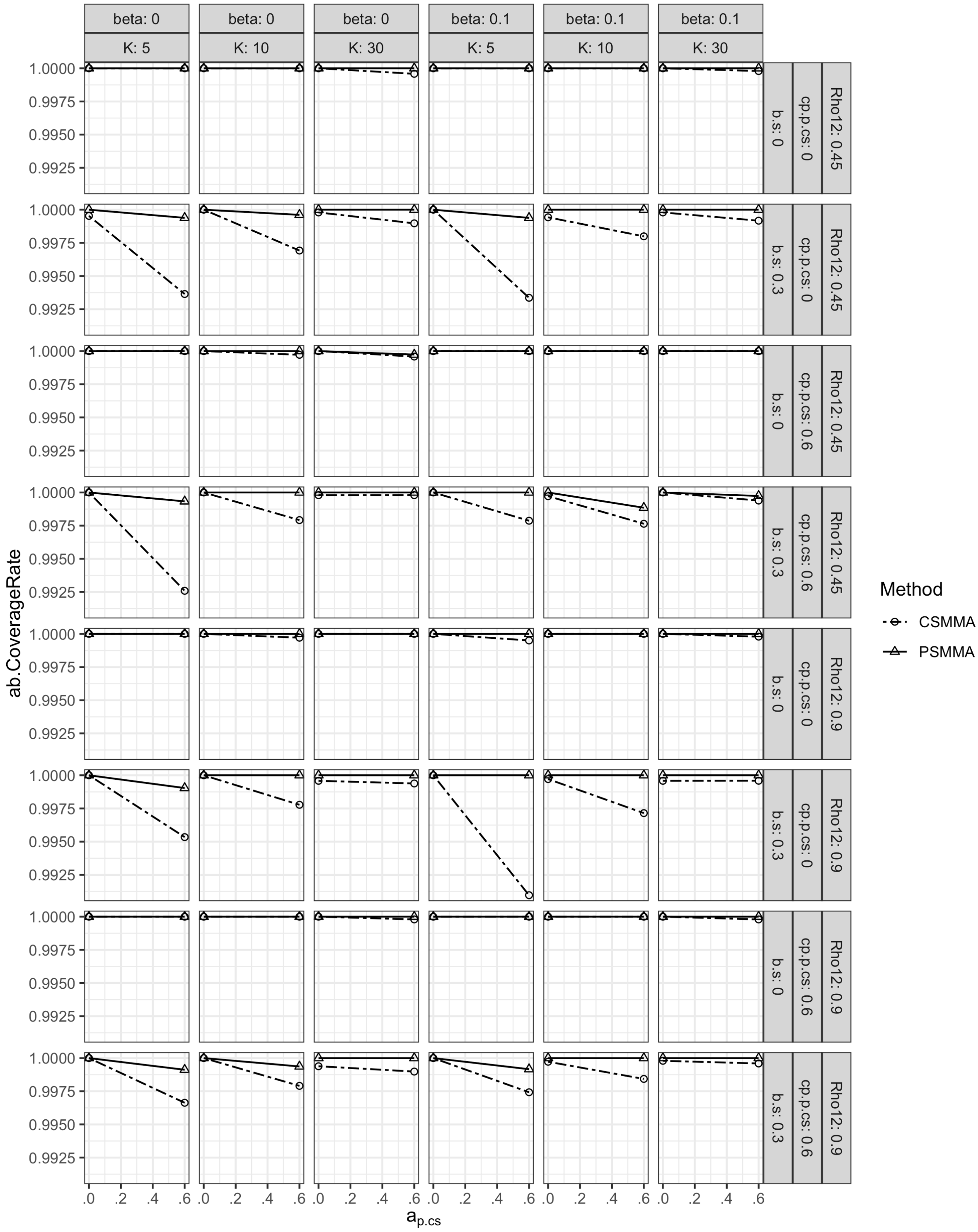
Estimation Bias of the Indirect Effect under All Conditions in Study 1



CR. The CR of both approaches remained above 0.95 (Figure S2).

Figure S2.

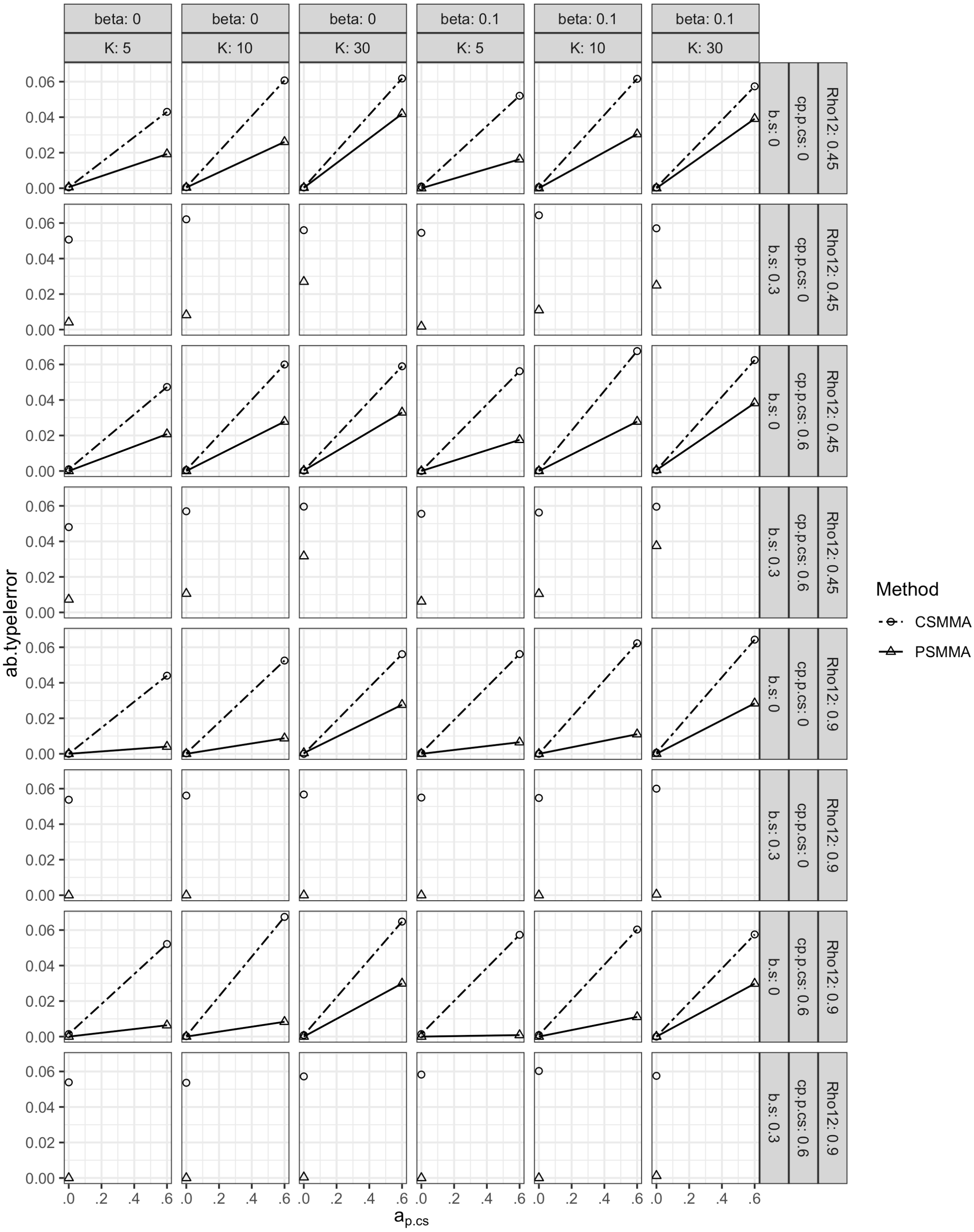
Coverage Rates of the Indirect Effect under All Conditions in Study 1



Type I error Rates. The type I error rates of CSMMA and PSMMA remained below 0.07 (Figure S3).

Figure S3.

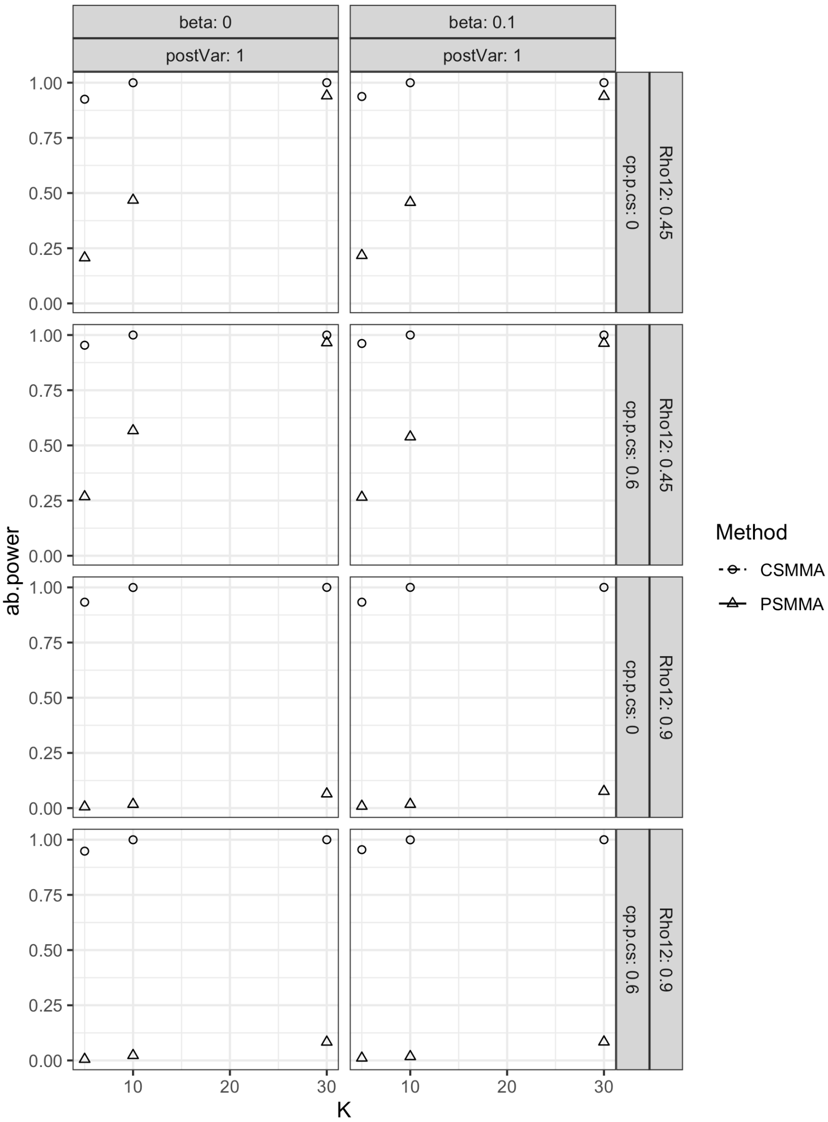
Type I Error Rates of the Indirect Effect under All Conditions in Study 1



Statistical Power. The pattern of statistical power of CSMMA and PSMMA was demonstrated in the main text. The magnitude of the moderating effect $\beta_{c'_{s,cs}}$ and $c'_{s,cs}$ had no apparent effect on statistical power when estimating the indirect effect (Figure. S4).

Figure S4.

Statistical Power of the Indirect Effect under All Conditions in Study 1

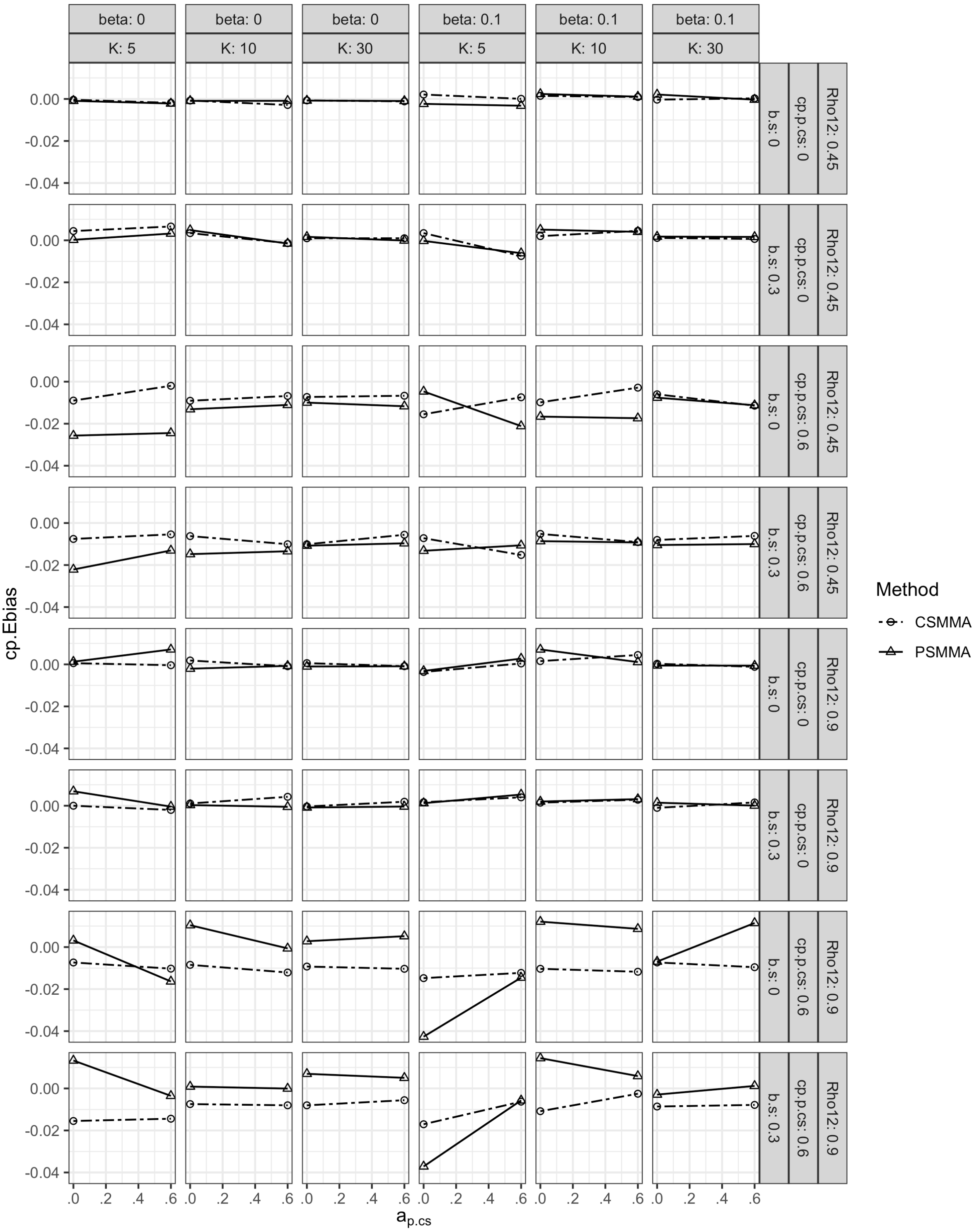


S2.1.2 Direct Effect

EBIAS. As shown in Figure S5, EBIAS of both approaches were ignorable.

Figure S5.

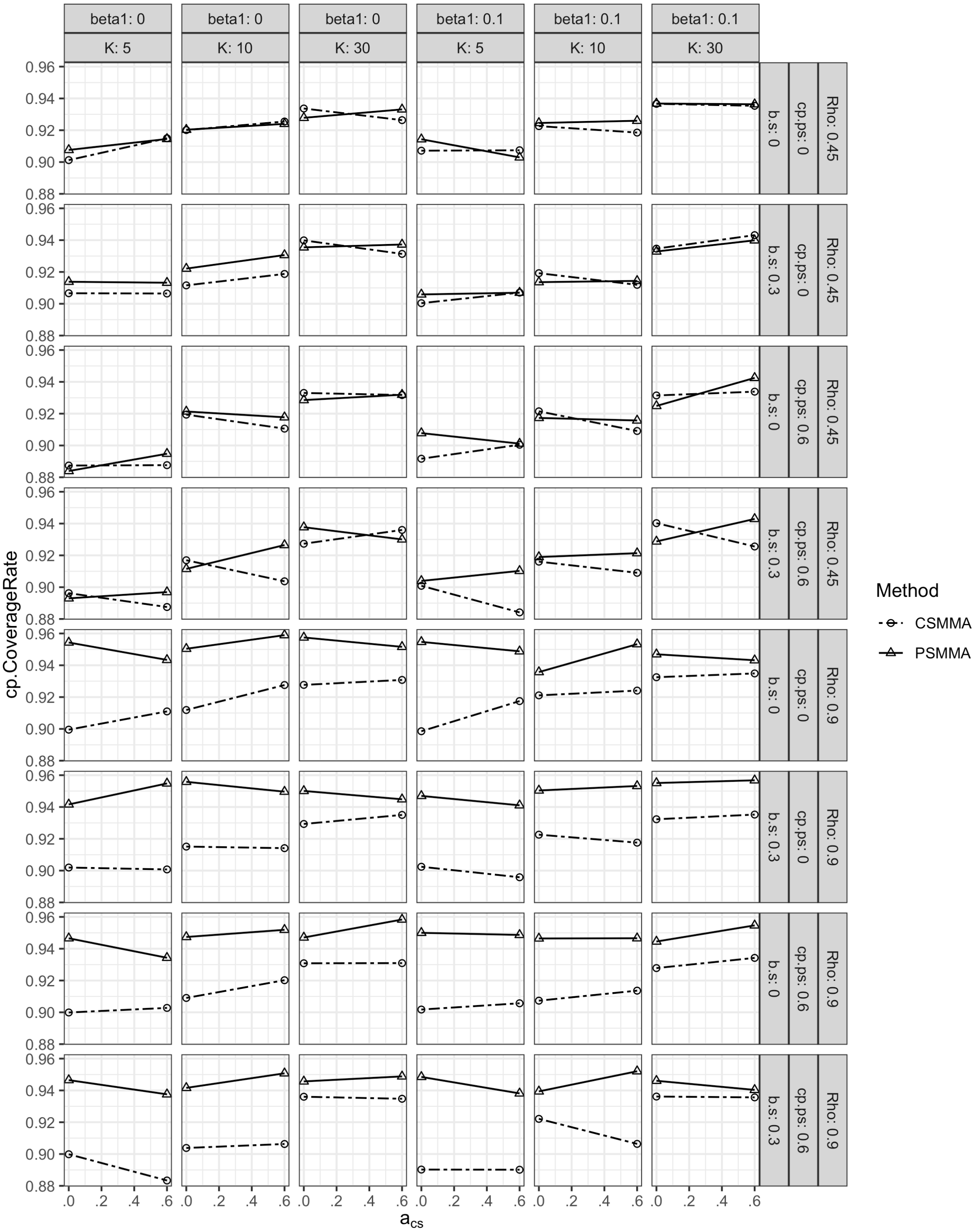
EBIAS of the Direct Effect under All Conditions in Study 1



CR. The CR of both CSMMA and PSMMA remained above 0.85 (Figure S6).

Figure S6.

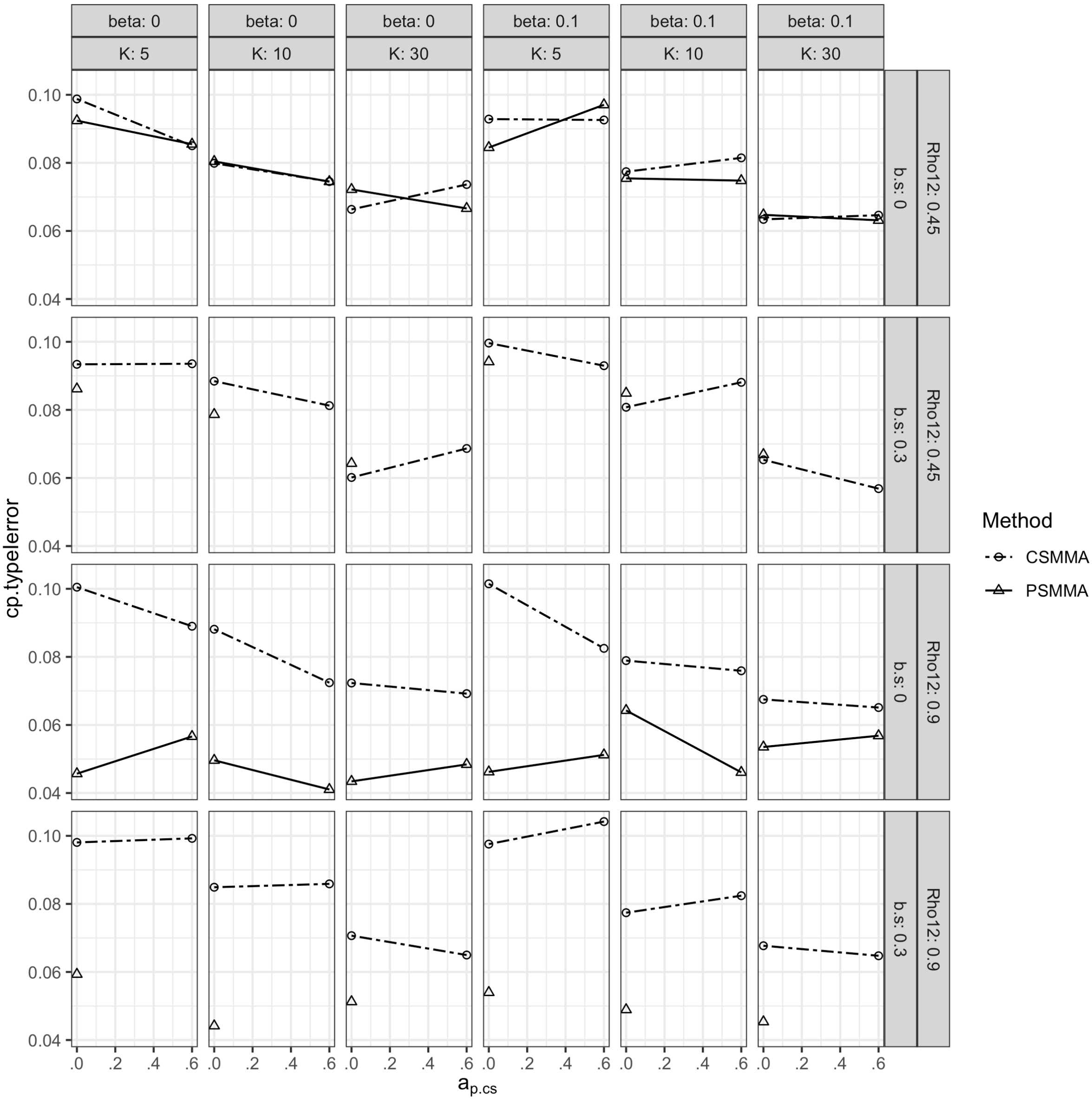
Coverage Rates of the Direct Effect under all Conditions in Study 1



Type I Error Rates. The type I error rates of both approaches remained below 0.1 (Figure S7).

Figure S7.

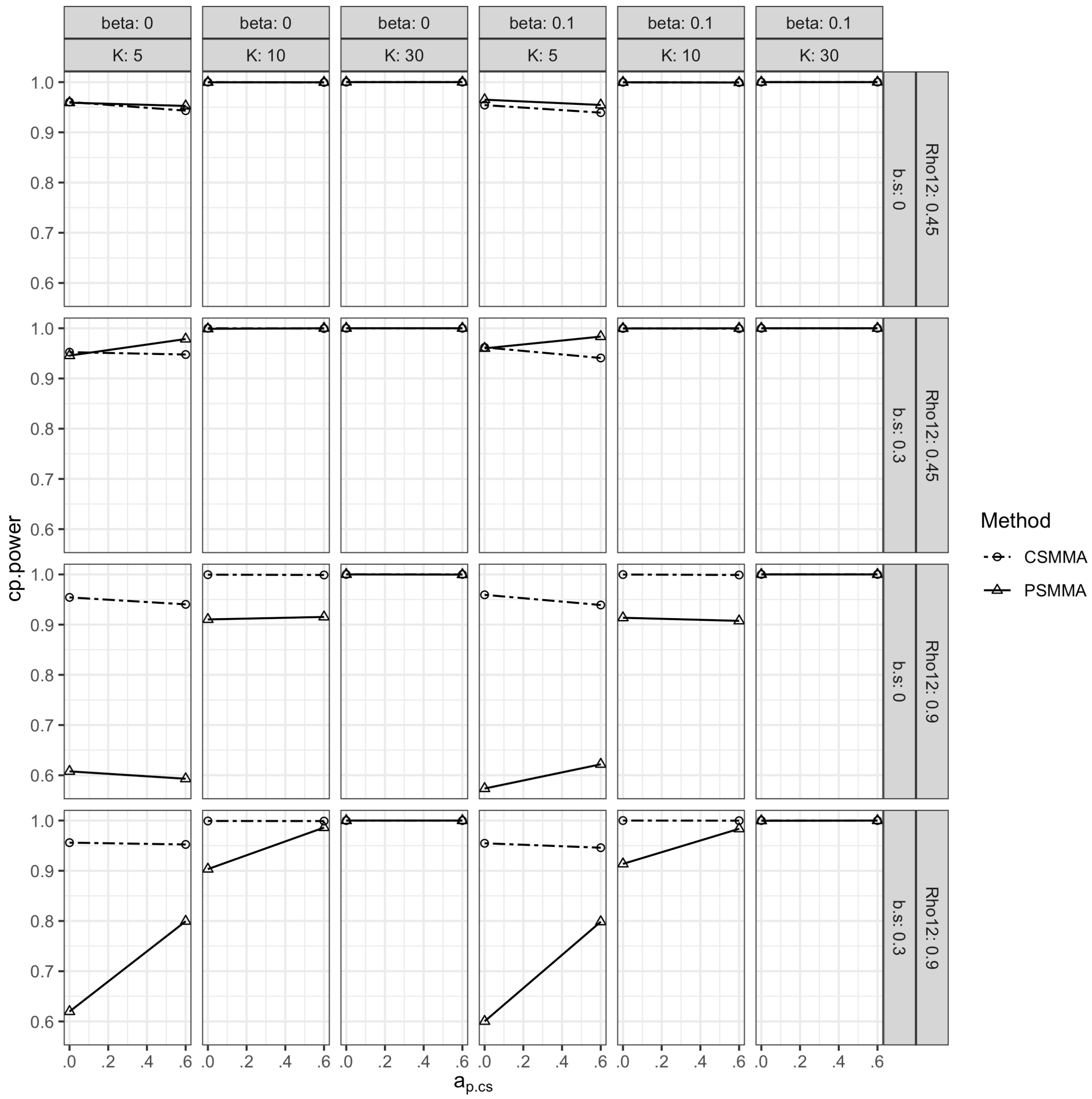
Type I Error Rates of the Direct Effect under All Conditions in Study 1



Statistical Power. The pattern of power of CSMMA and PSMMA was demonstrated in the main text. The size of $\beta_{c'_{s,cs}}$ and $c'_{s,cs}$ had no apparent effect on power (Figure S8).

Figure S8.

Statistical Power of the Direct Effect under All Conditions in Study 1

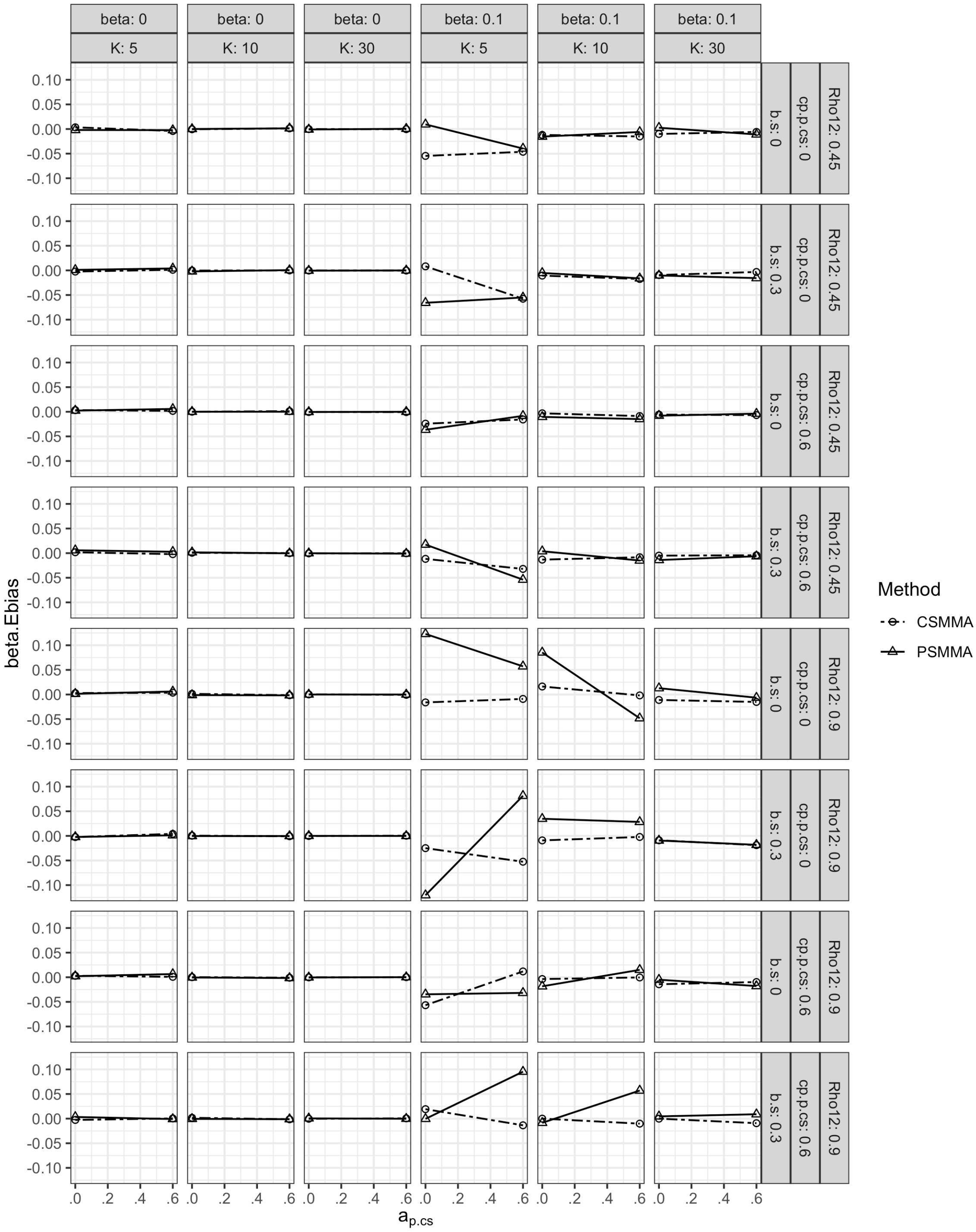


S2.1.3 Moderating Effect

EBIAS. As shown in Figure S9, the EBIAS of both CSMMA and PSMMA were acceptable when K was 10 and 30, but inflated to ± 0.1 when K was 5.

Figure S9.

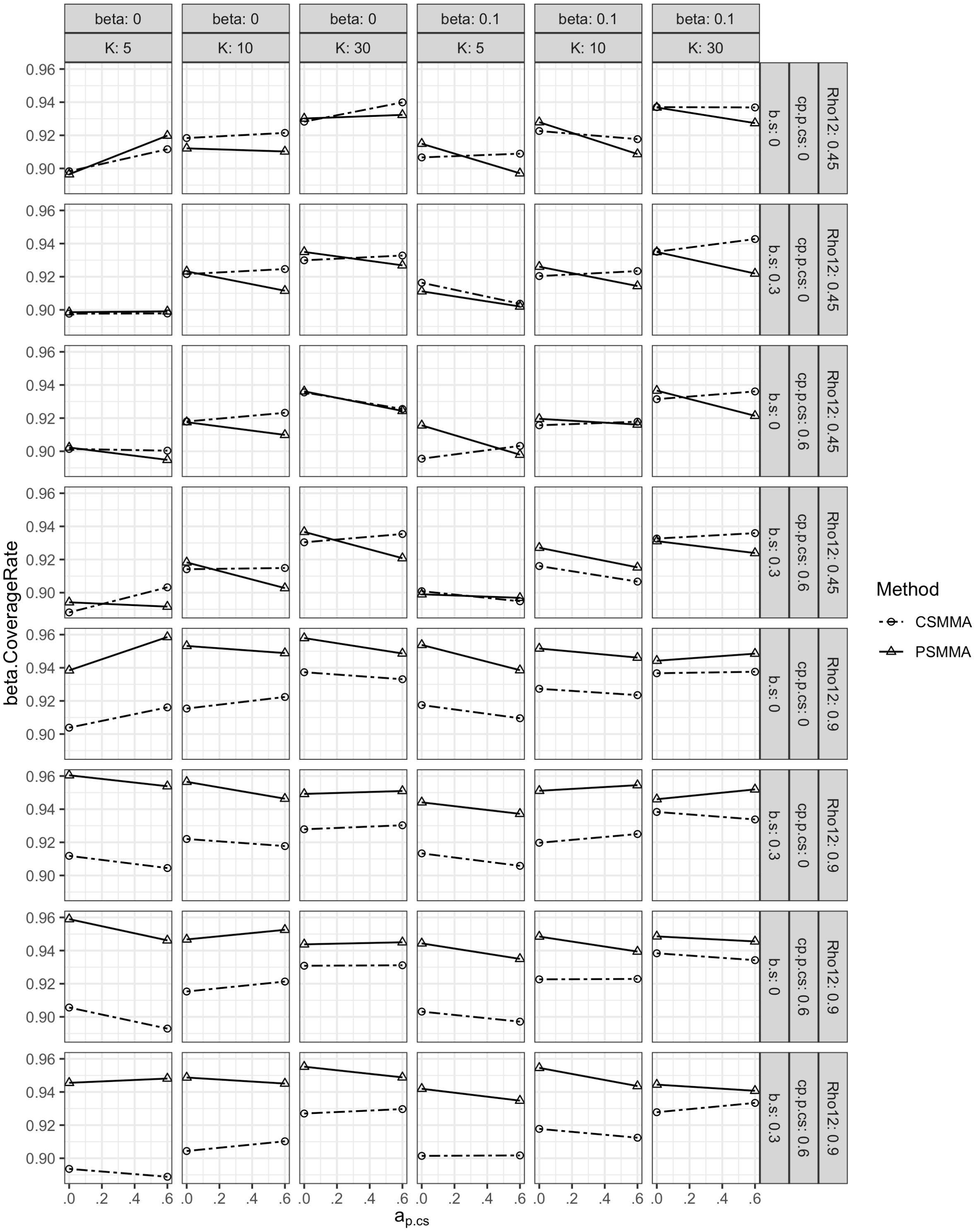
EBIAS of the Moderating Effect under All Conditions in Study 1



CR. The CR of both CSMMA and PSMMA remained above 0.85 (Figure S10).

Figure S10.

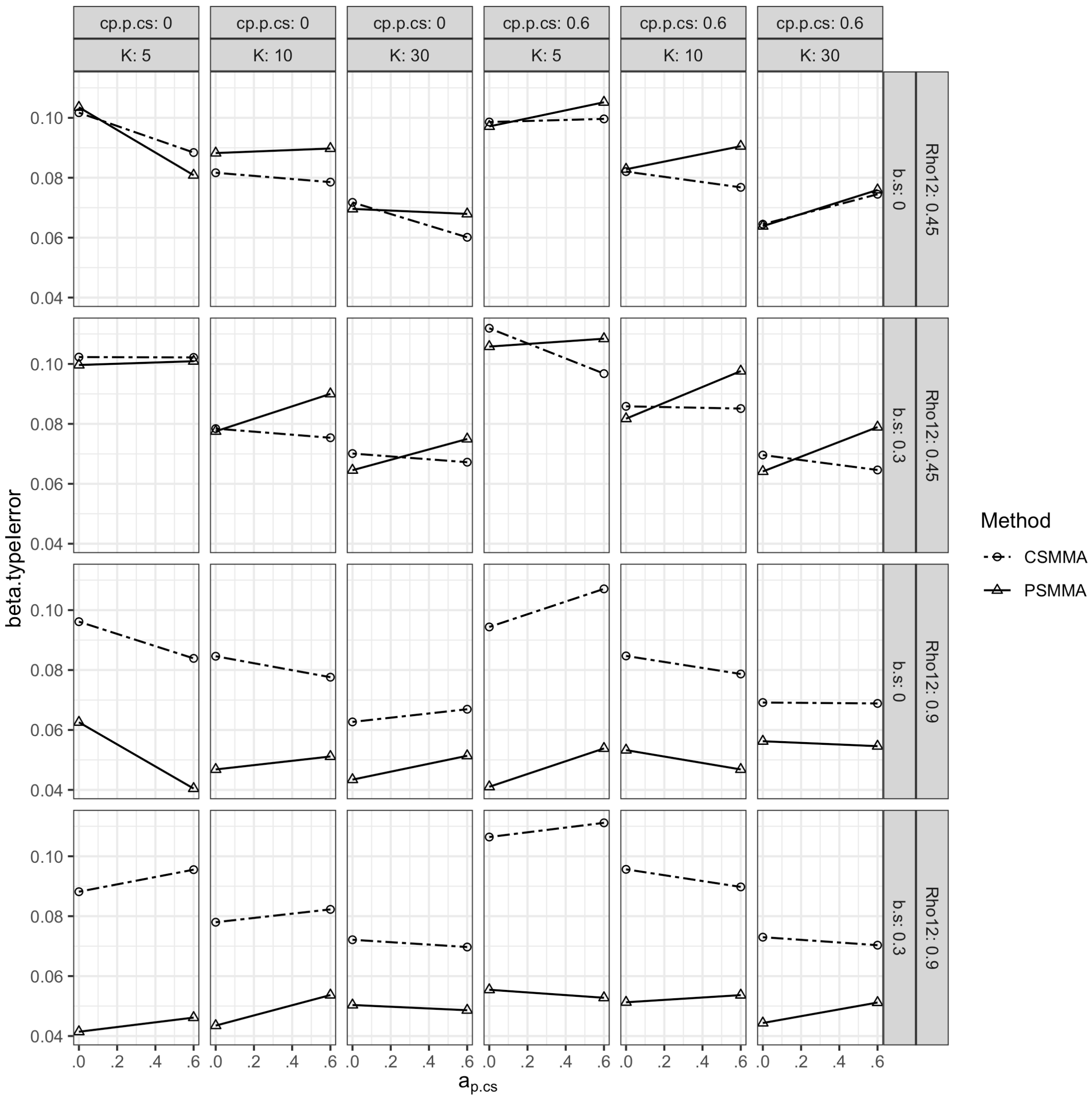
Coverage Rates of the Moderating Effect under All Conditions in Study 1



Type I Error Rates. The pattern of type I error rates was demonstrated in the main text. The magnitude of $a_{s,cs}$, $b_{s,cs}$ and $c'_{s,cs}$ had no apparent effect (Figure S11).

Figure S11.

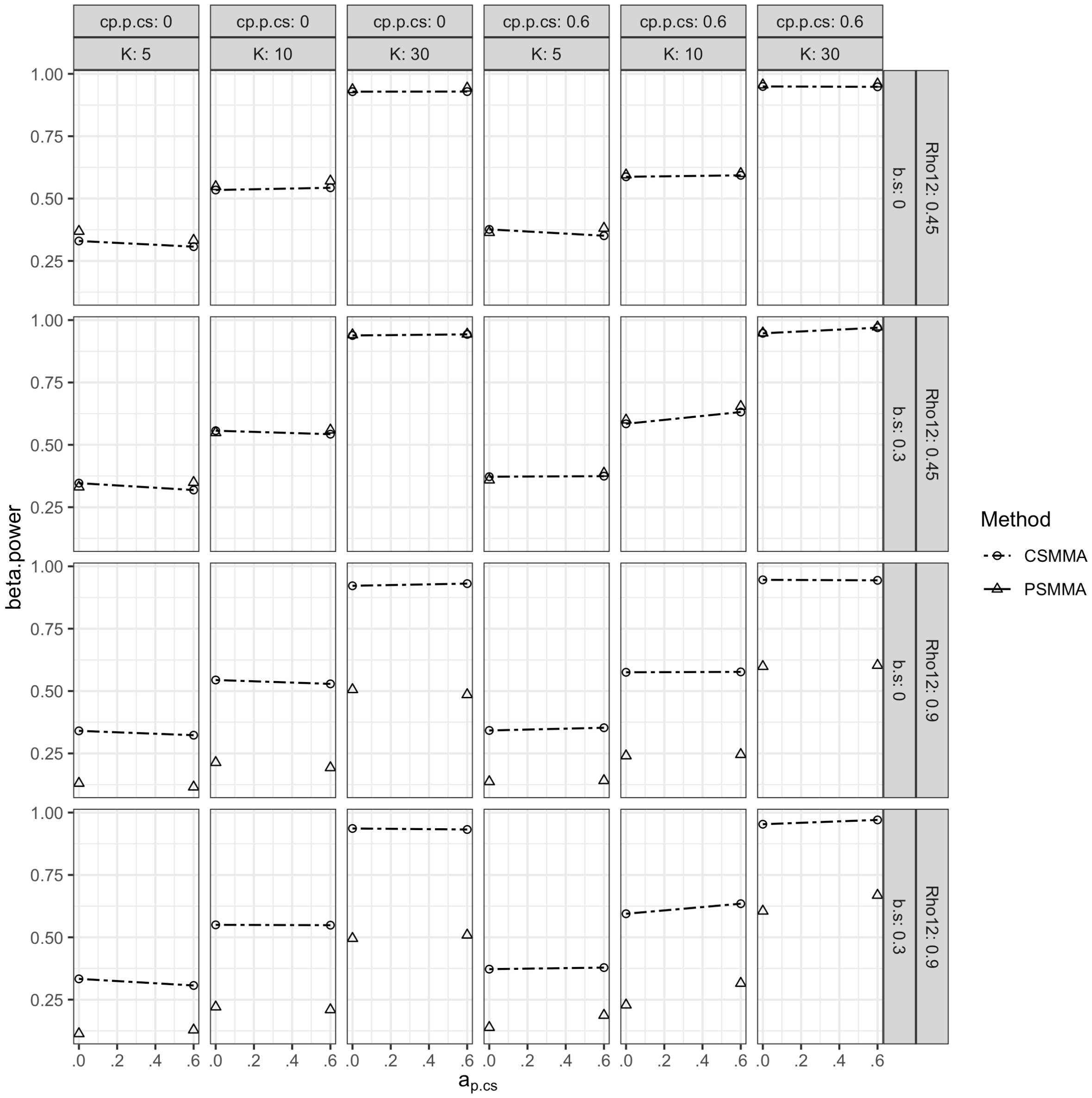
Type I Error Rates of the Moderating Effect under All Conditions in Study 1



Statistical Power. The patter of statistical power regarding the moderating effect was demonstrated in the main text. The magnitude of $a_{s,cs}$, $b_{s,cs}$ and $c'_{s,cs}$ had no apparent effect on statistical power regarding the moderating effect (Figure S12).

Figure S12.

Statistical Power of the Moderating Effect under All Conditions in Study 1



S2.1.4 Coefficients of the a Path and the b Path

As shown in Figure S13-15, the EBIAS, CR and type I error rates of a path estimates of CSMMA and PSMMA both remained favorable under all conditions. The statistical power when estimating the a path (Figure S16), on the other hand, decreased with a larger ρ_{12}, due to small true posttest score coefficient on the a path.

Figure S13.

EBIAS of the a Path under All Conditions in Study 1

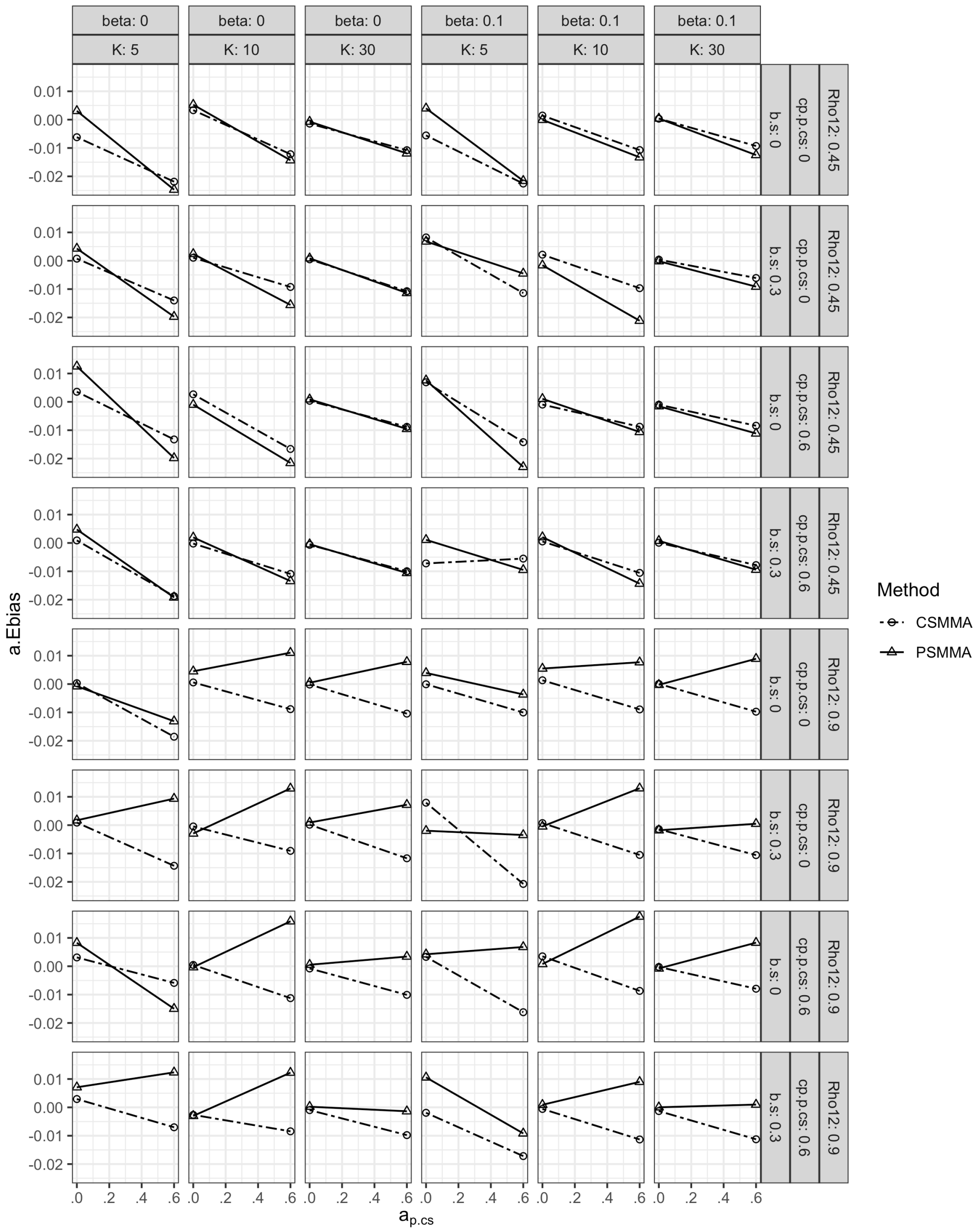


Figure S14.

Coverage Rates of the a Path under All Conditions in Study 1

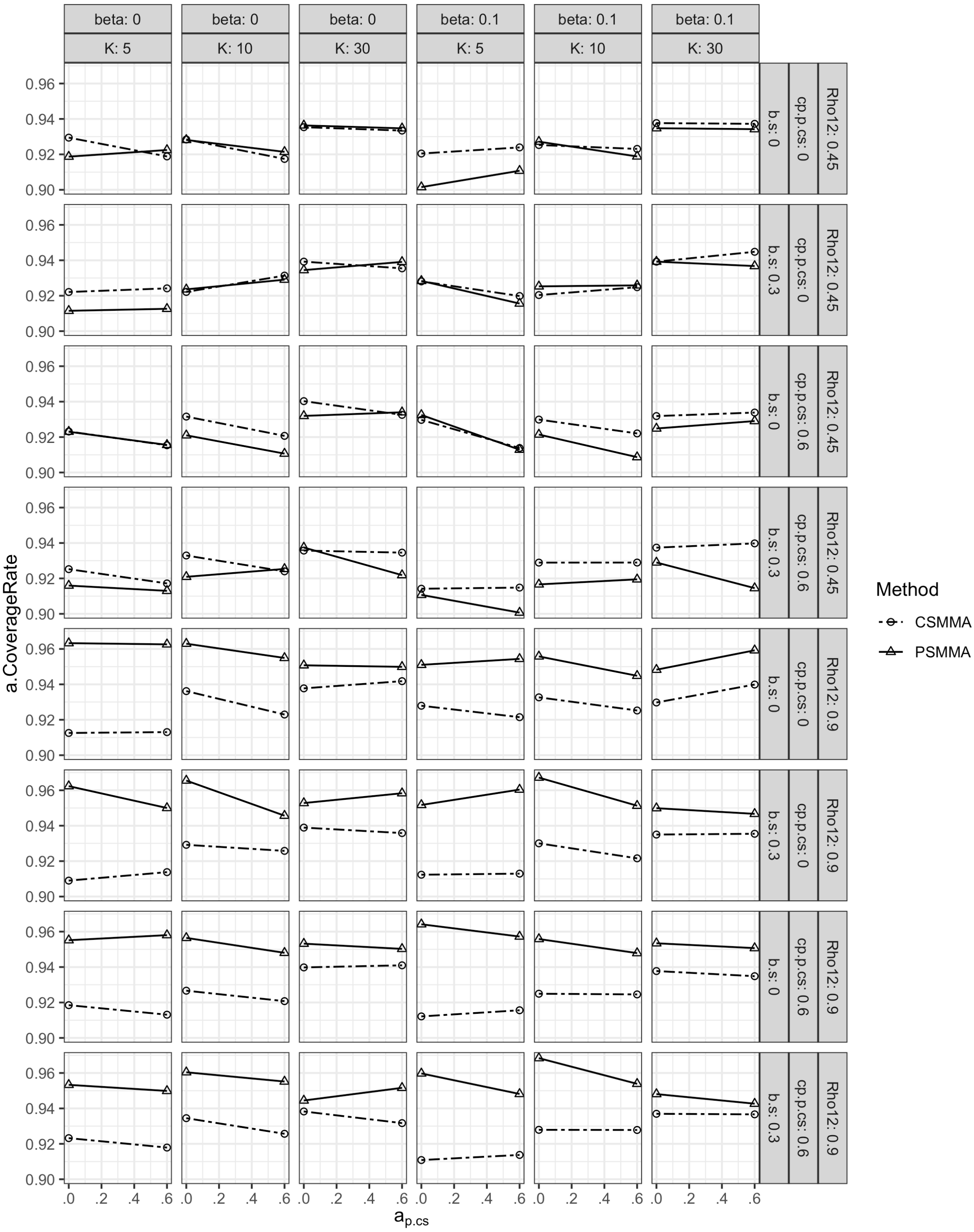


Figure S15.

Type I Error Rates of the a Path under All Conditions in Study 1

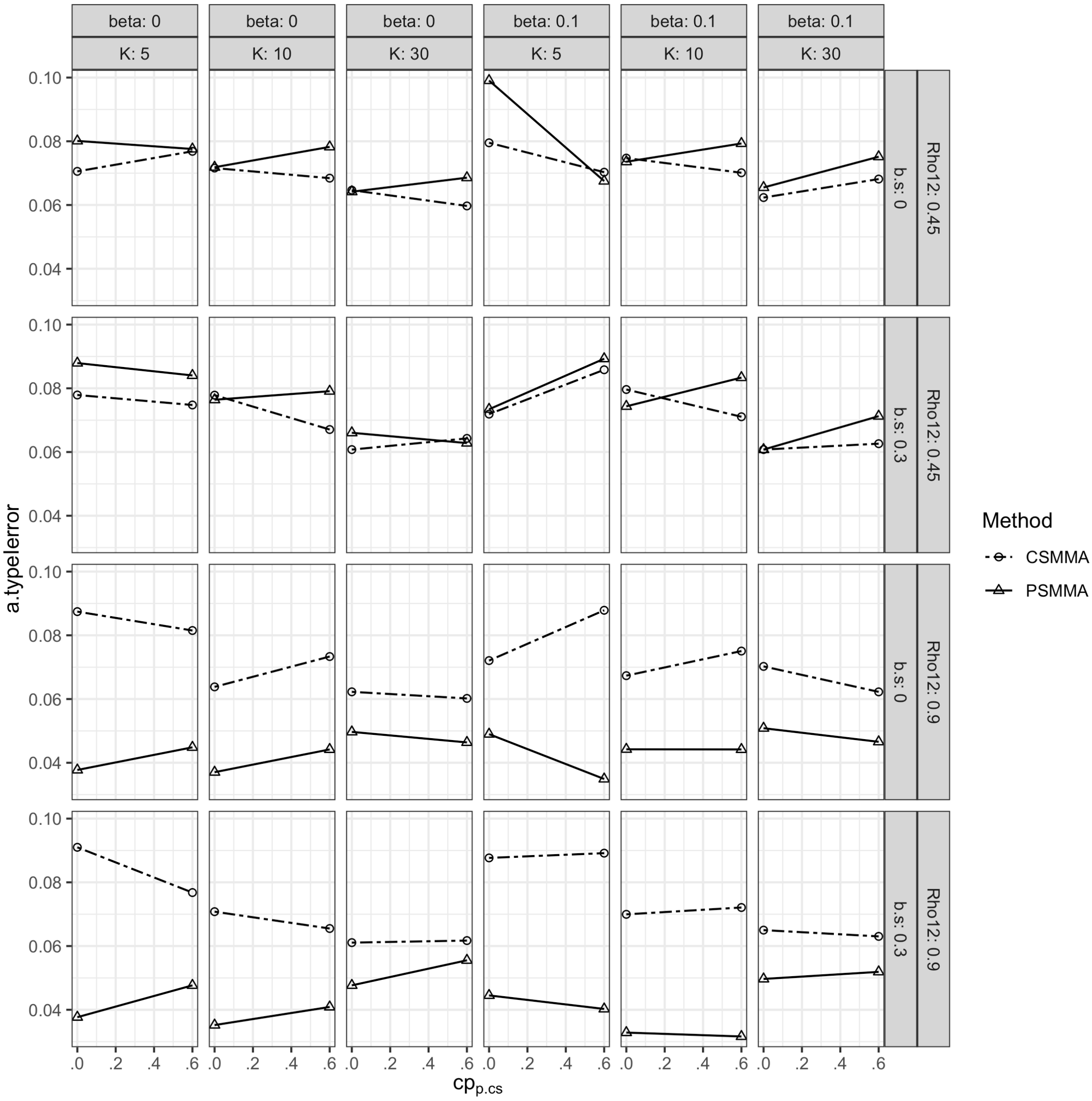
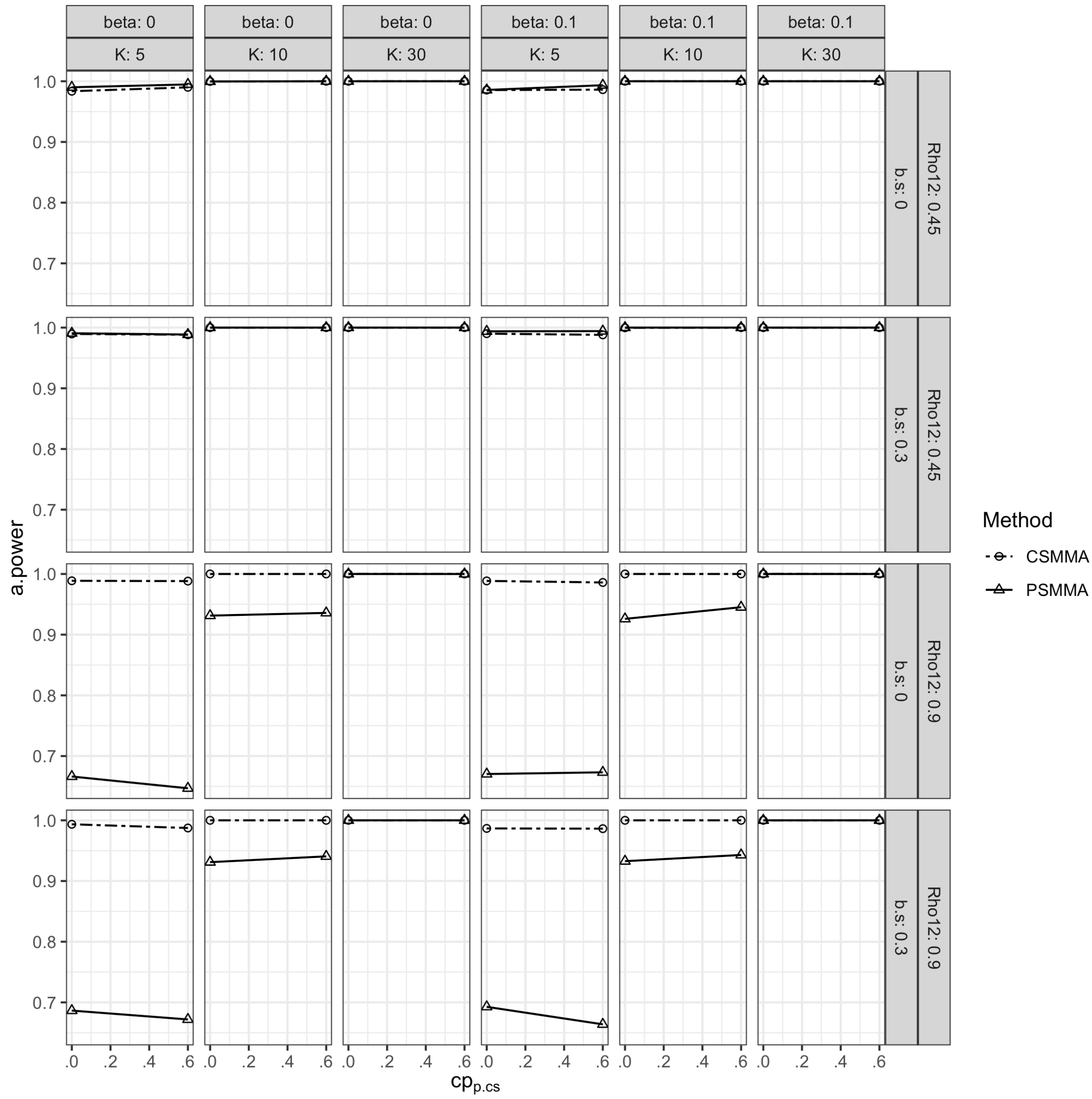


Figure S16.

Statistical Power of the a Path under All Conditions in Study 1



When estimating the b path, while the EBIAS of CSMMA remained acceptable under all conditions, EBIAS of PSMMA fluctuated when ρ_{12} was 0.9 (Figure S17) because the true path coefficients (the denominator of EBIAS) were too small. The CR of both CSMMA and PSMMA remained above 0.9 (Figure S18). The type I error rates of CSMMA and PSMMA fluctuated from 0.02 to 0.1 (Figure S19). While the power of CSMMA remained above 0.9, that of PSMMA dropped with a smaller K and a larger ρ_{12} (Figure S20).

Figure S17.

EBIAS of the b Path Coefficient under All Conditions in Study 1

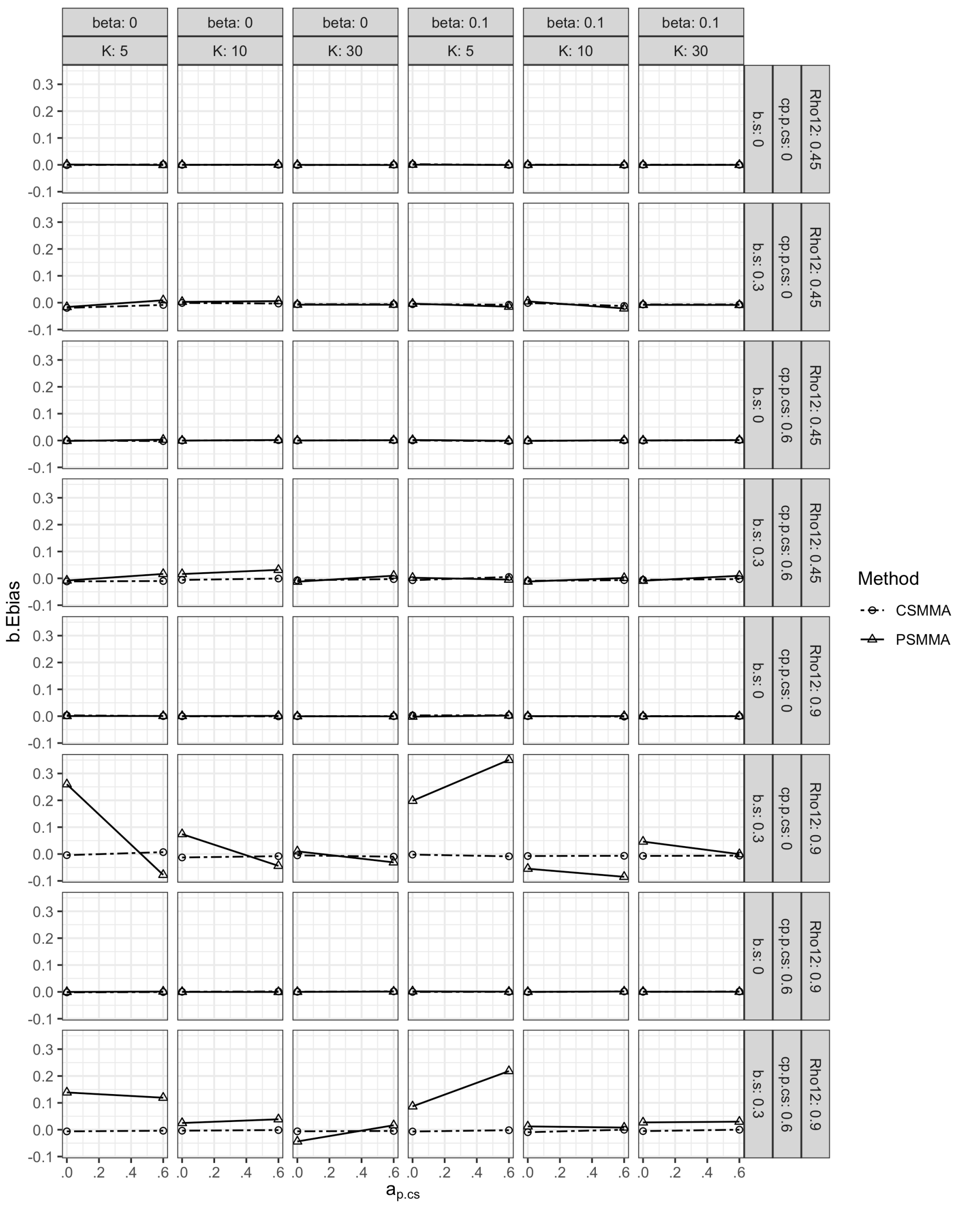


Figure S18.

Coverage Rates of the b Path Coefficient under All Conditions in Study 1

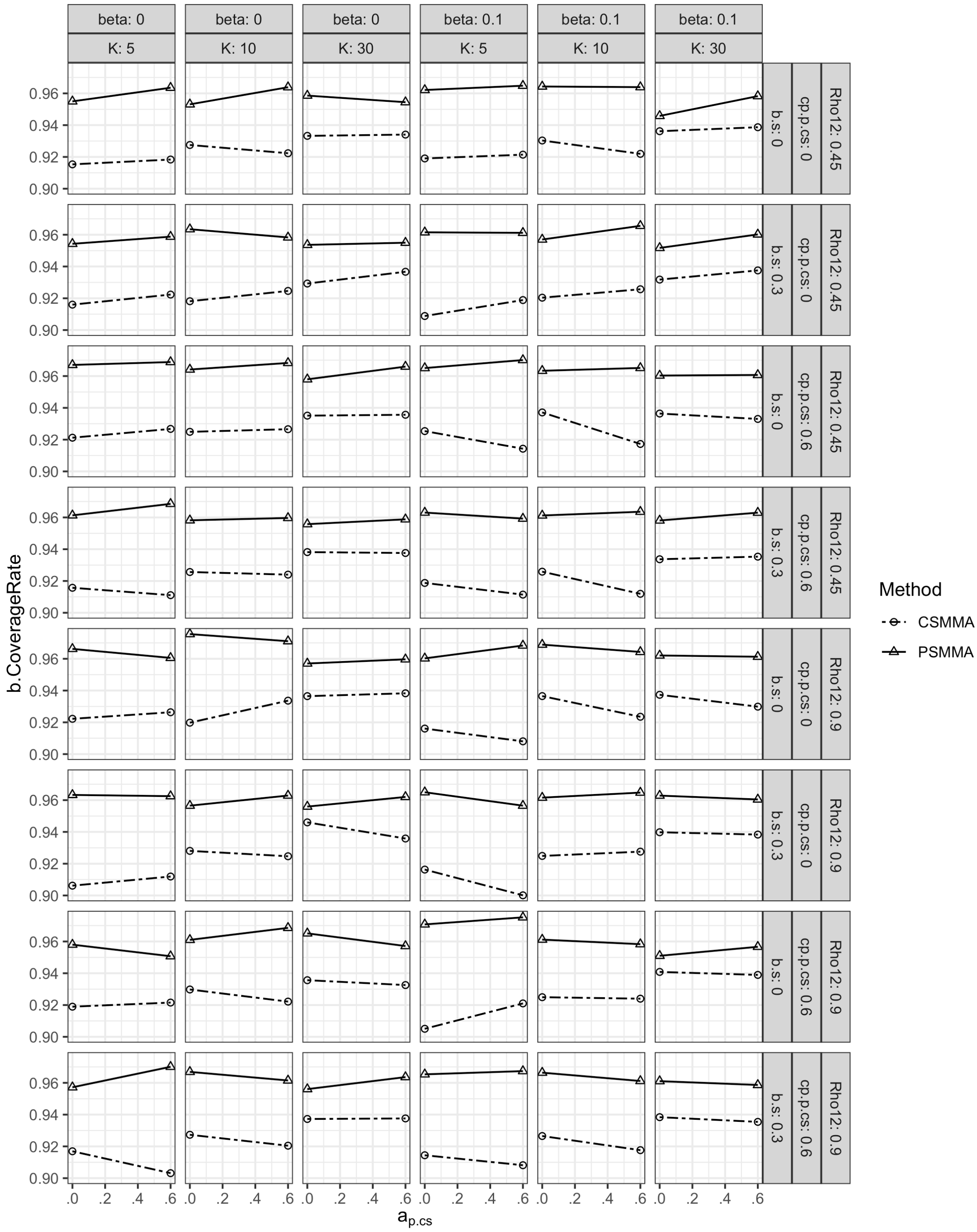


Figure S19.

Type I Error Rates of the b Path Coefficient under All Conditions in Study 1

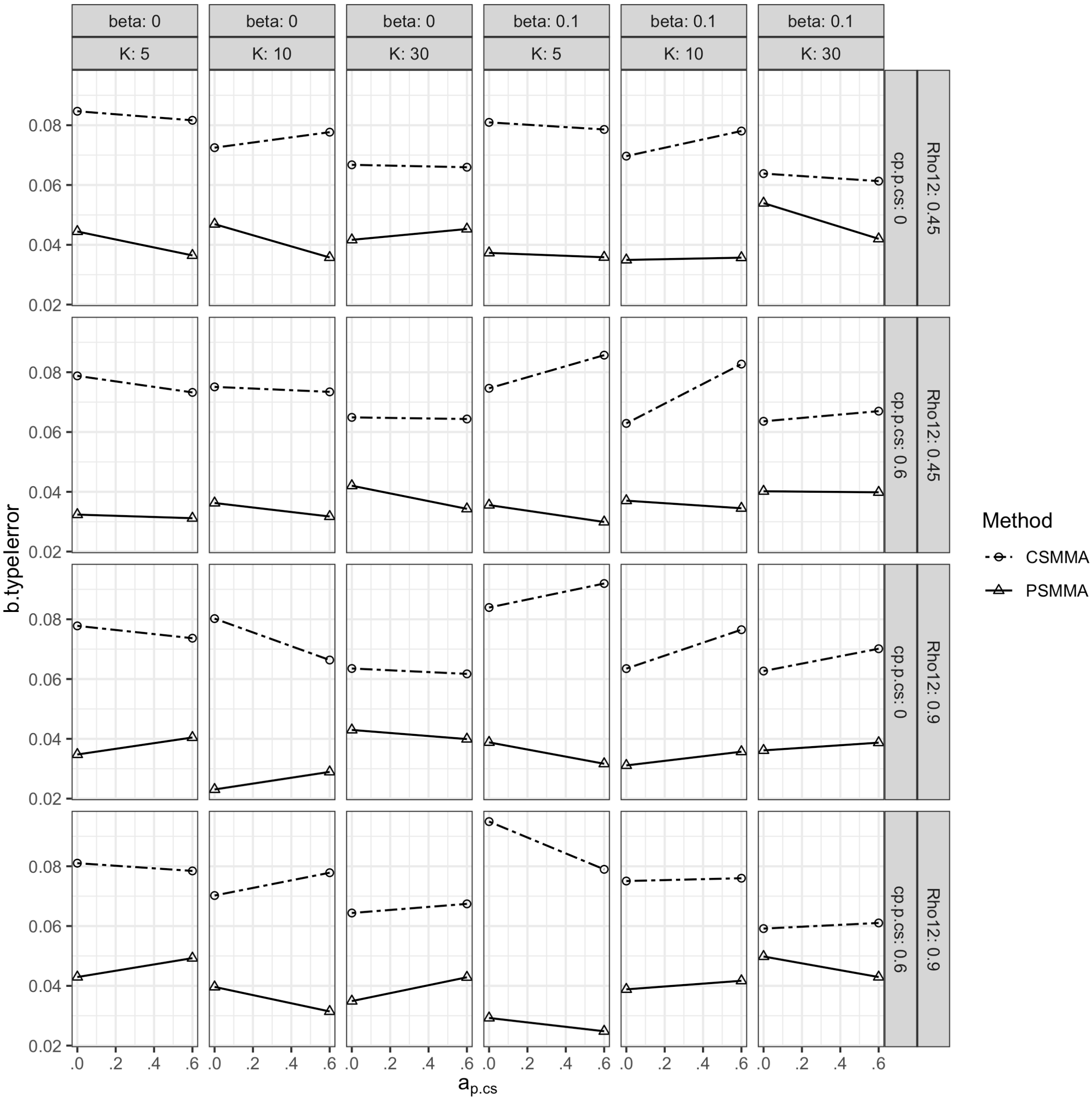
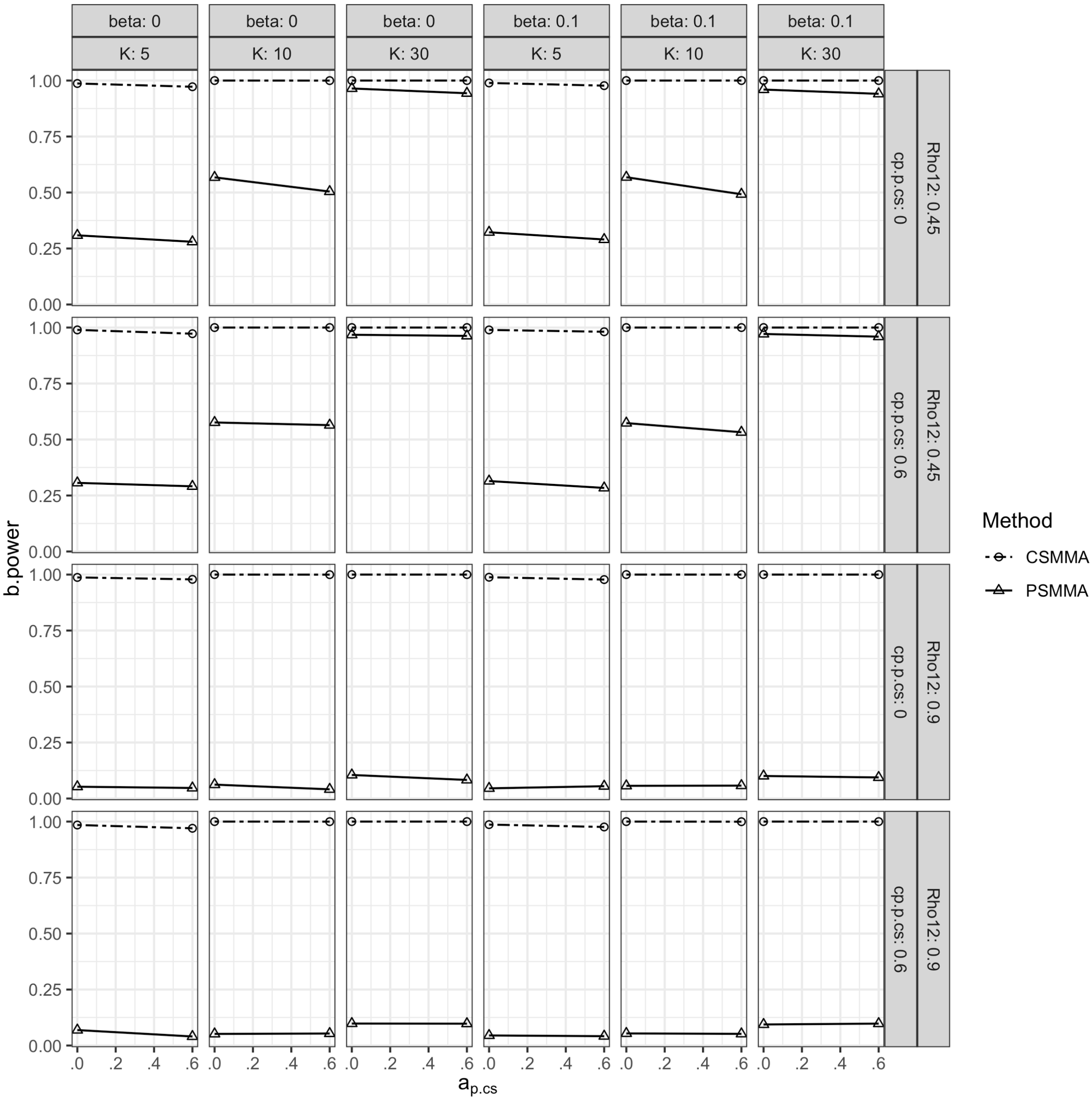


Figure S20.

Statistical Power of the b Path Coefficient under All Conditions in Study 1



S2.2 Study 2

S2.2.1 Indirect Effect

As shown in Figure S21-23, the patterns of EBIAS, CR, type I error rates and power of CSMMA and PSMMA in Study 2 were similar to Study 1. However, as illustrated in the main text, the inflation of posttest variances increased the power of PSMMA, and $\beta_{c'_{s,cs}}$ and $c'_{s,cs}$ had no apparent effect on this pattern (Figure S24).

Figure S21.

Estimation Bias of the Indirect Effect under All Conditions in Study

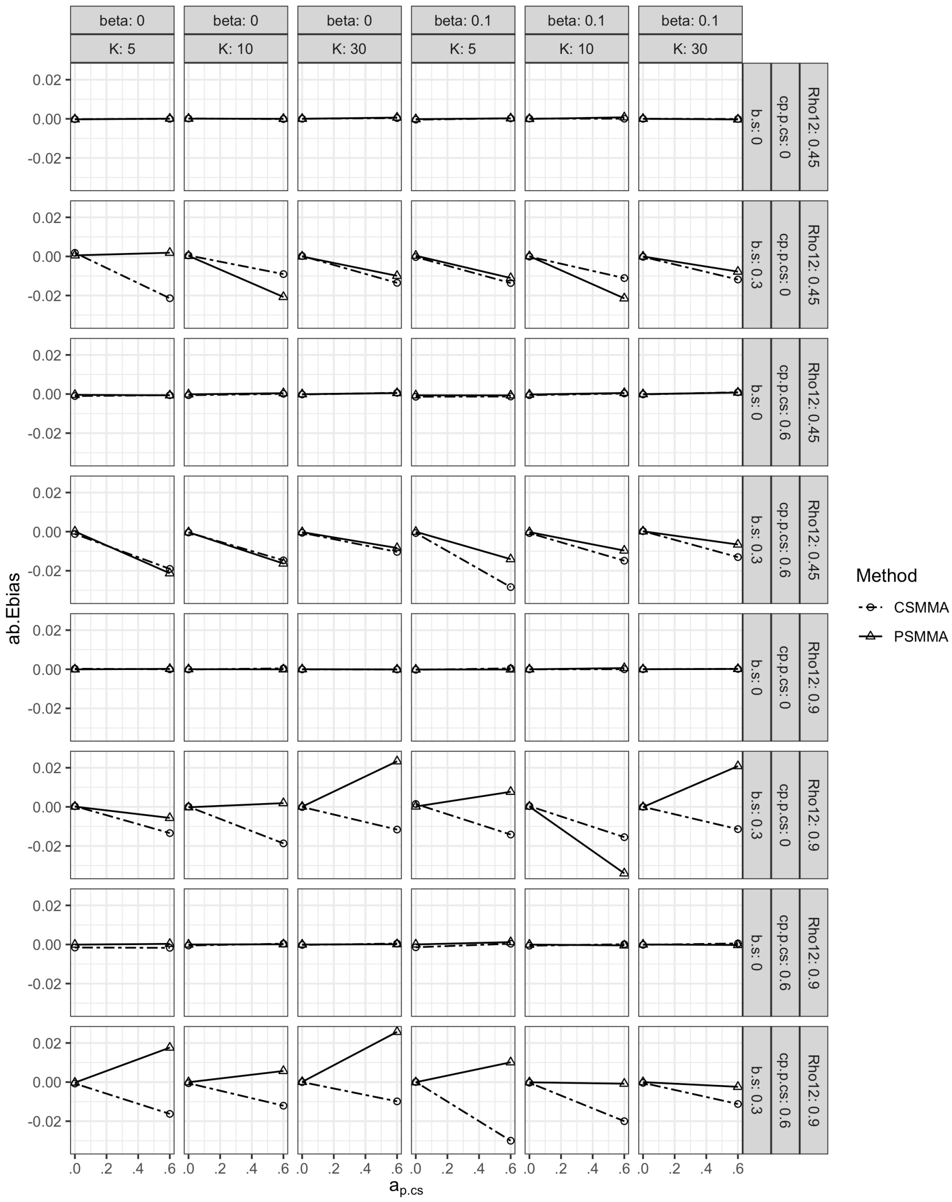


Figure S22.

Coverage Rates of the Indirect Effect under All Conditions in Study 2

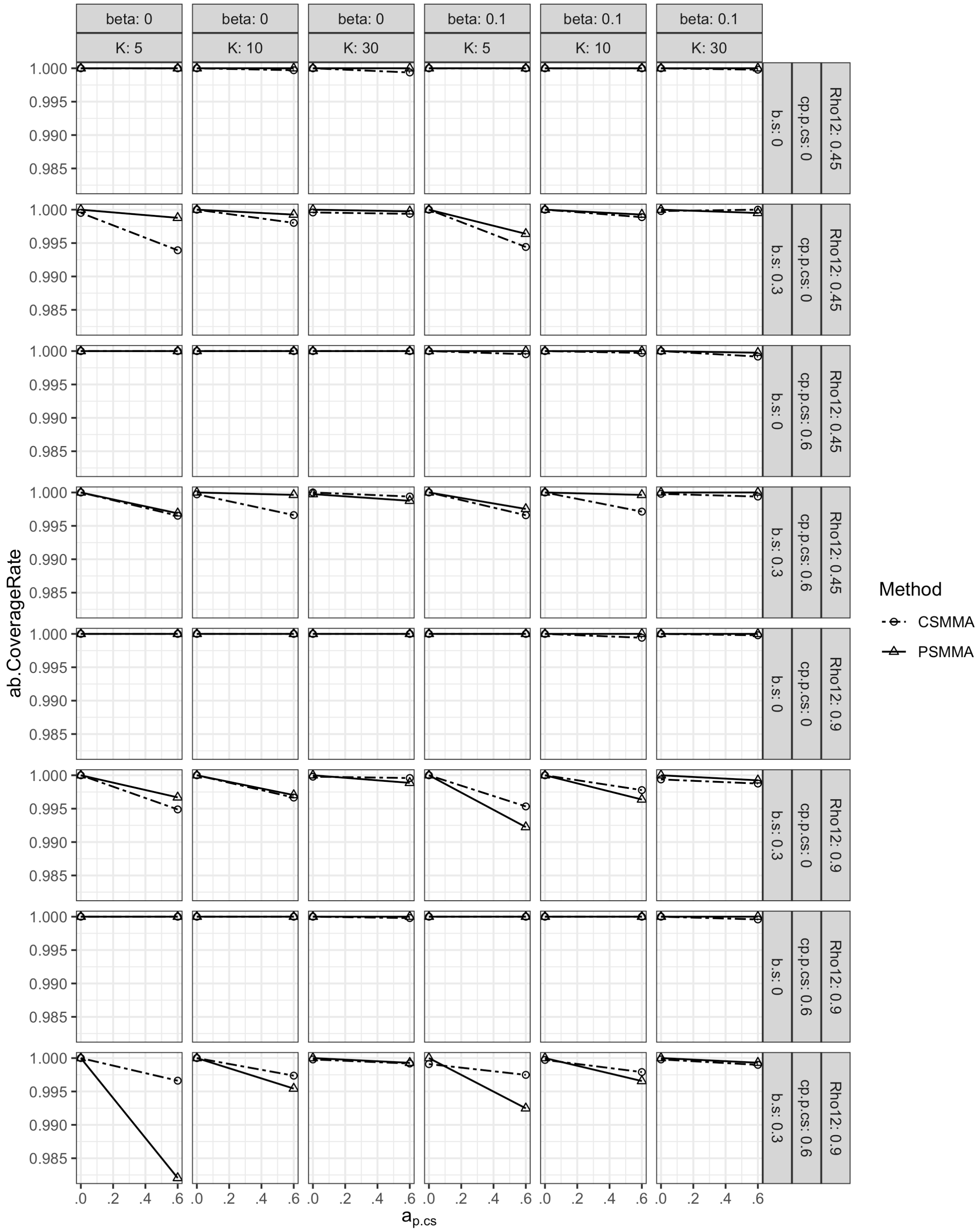


Figure S23.

Type I Error Rates of the Indirect Effect under All Conditions in Study 2

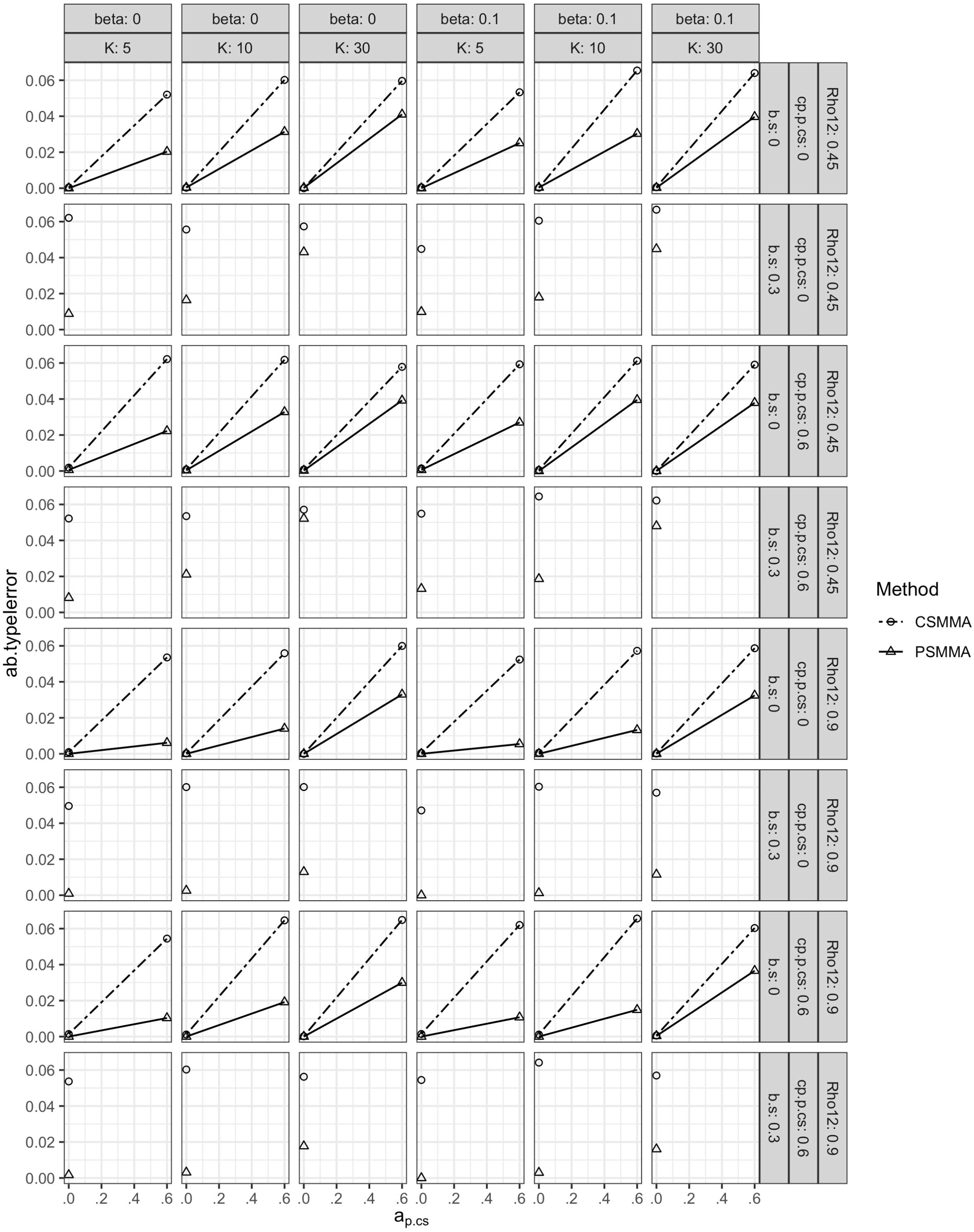
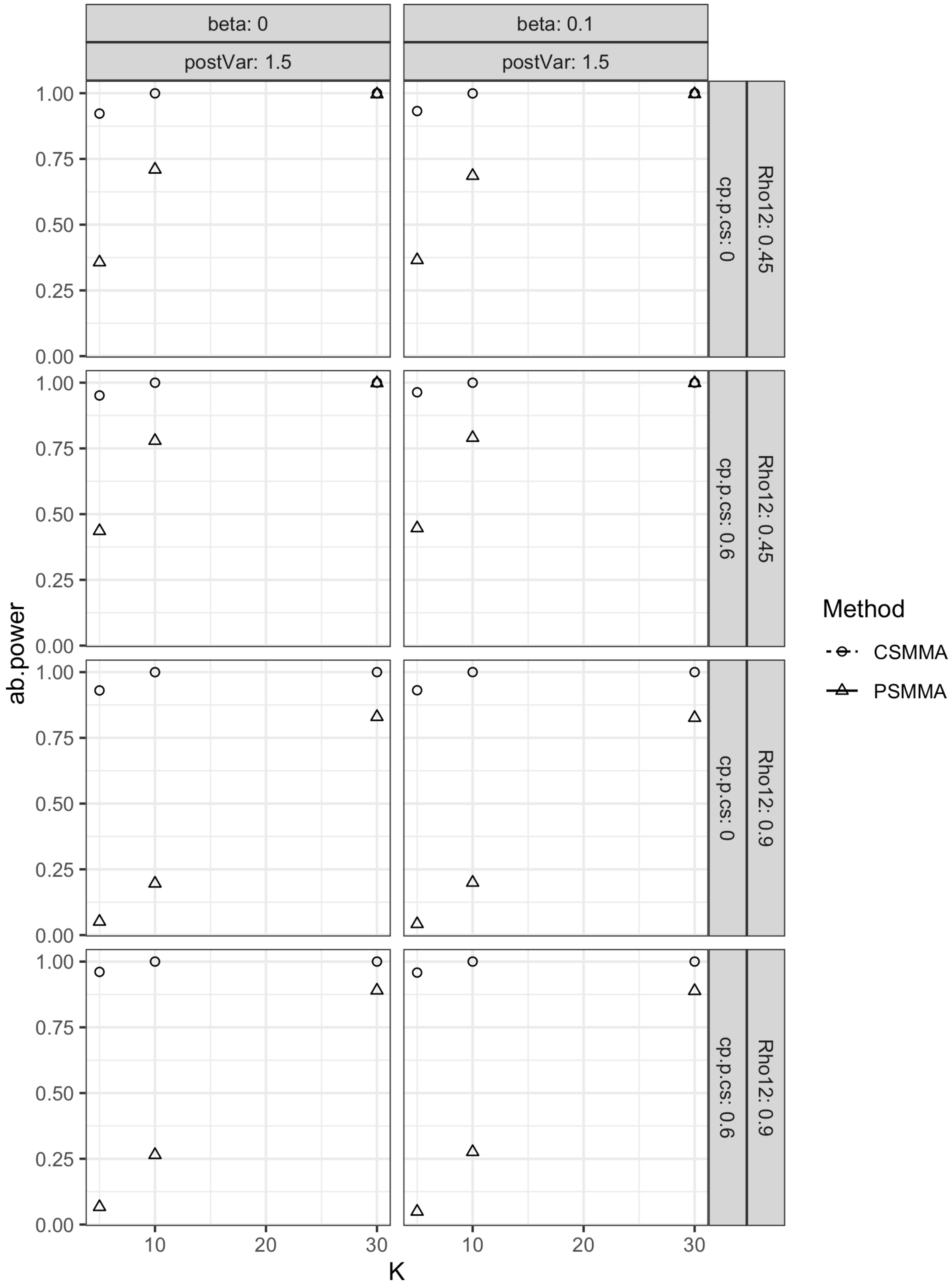


Figure S24.

Statistical Power of the Indirect Effect under All Conditions in Study 2



S2.2.2 Direct Effect

Similarly, results regarding the direct effect had the same pattern as in Study 1 (Figure S25-28).

Figure S25.

EBIAS of the Direct Effect under All Conditions in Study 2

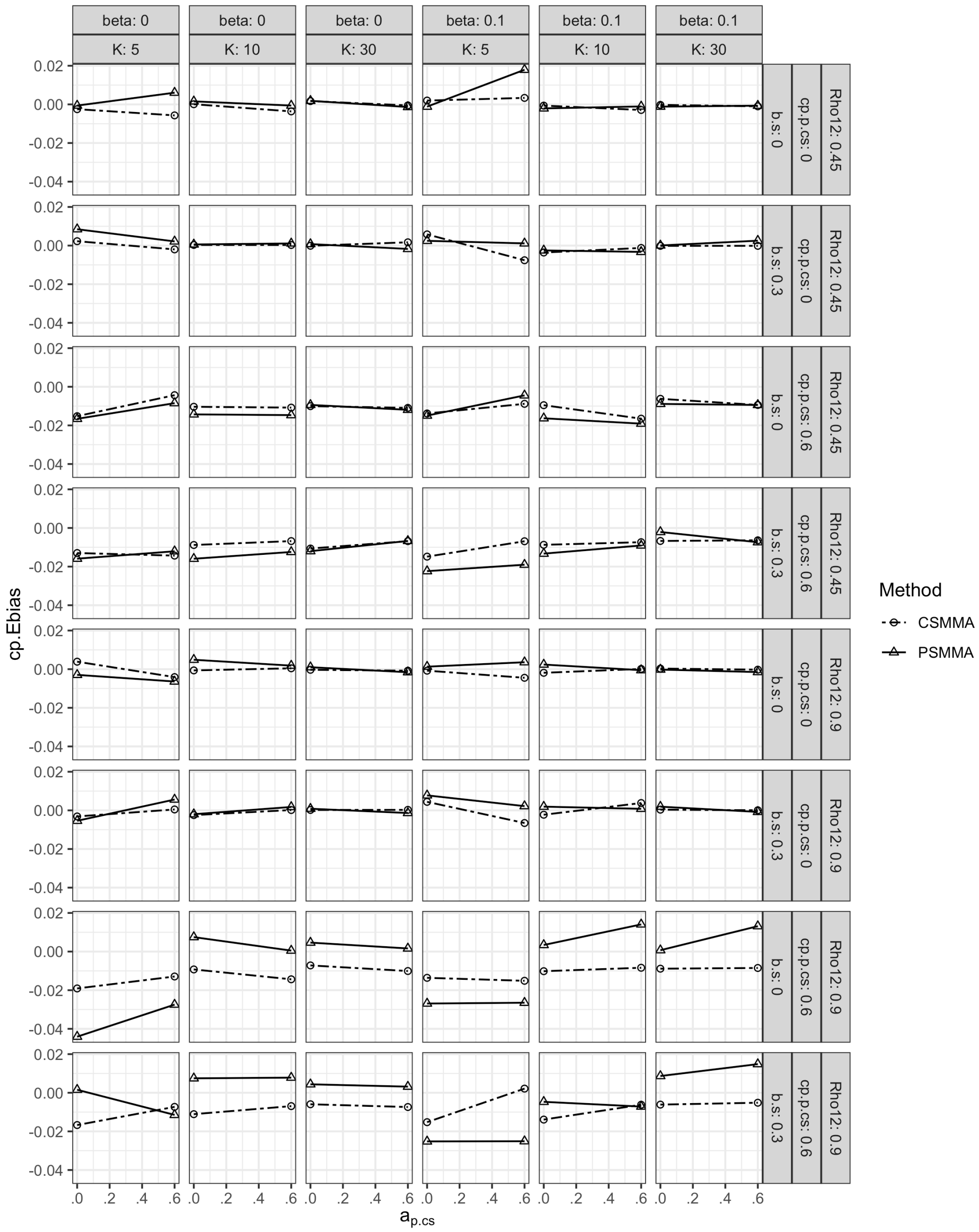


Figure S26.

Coverage Rates of the Direct Effect under All Conditions in Study 2

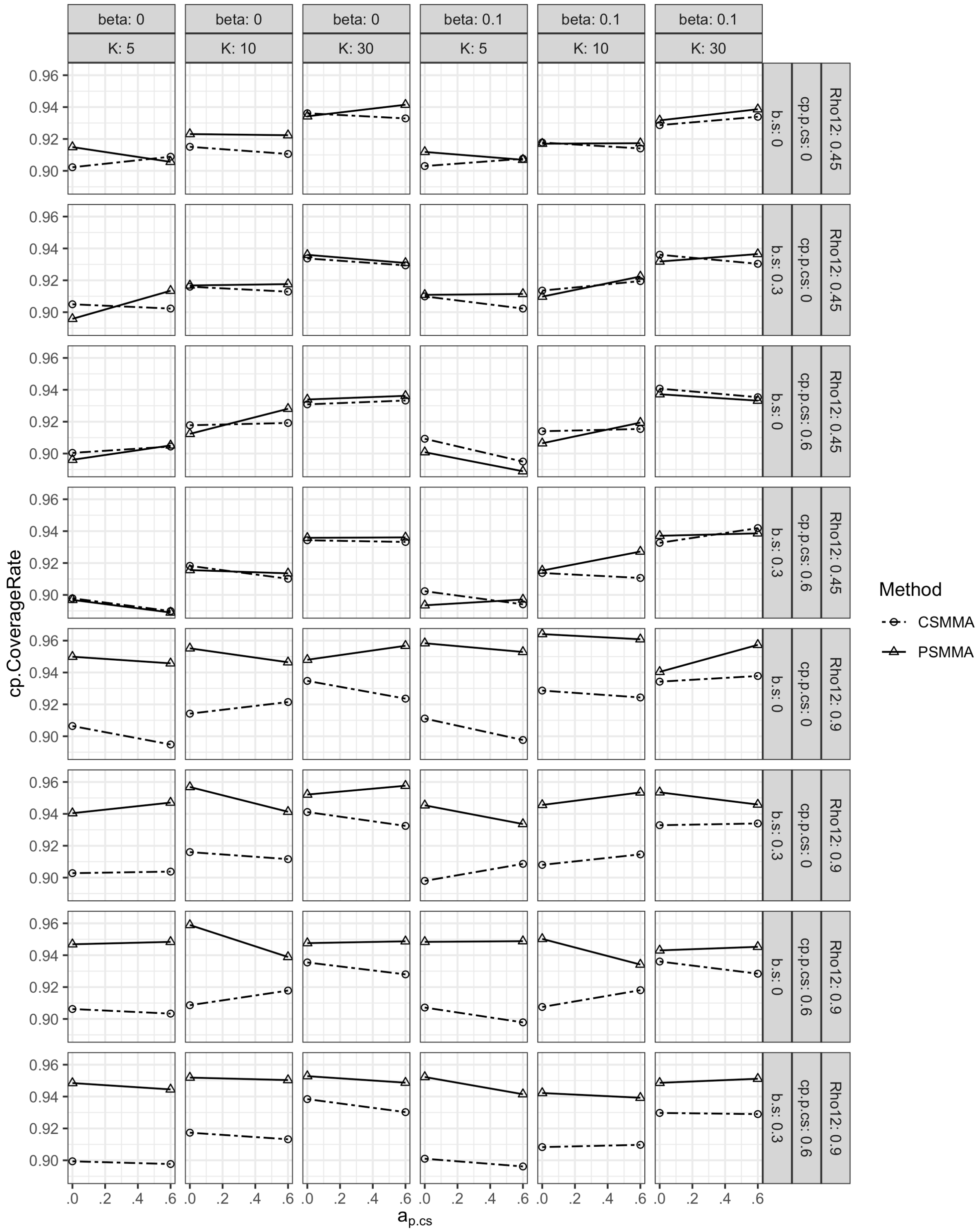


Figure S27.

Type I Error Rates of the Direct Effect under All Conditions in Study 2

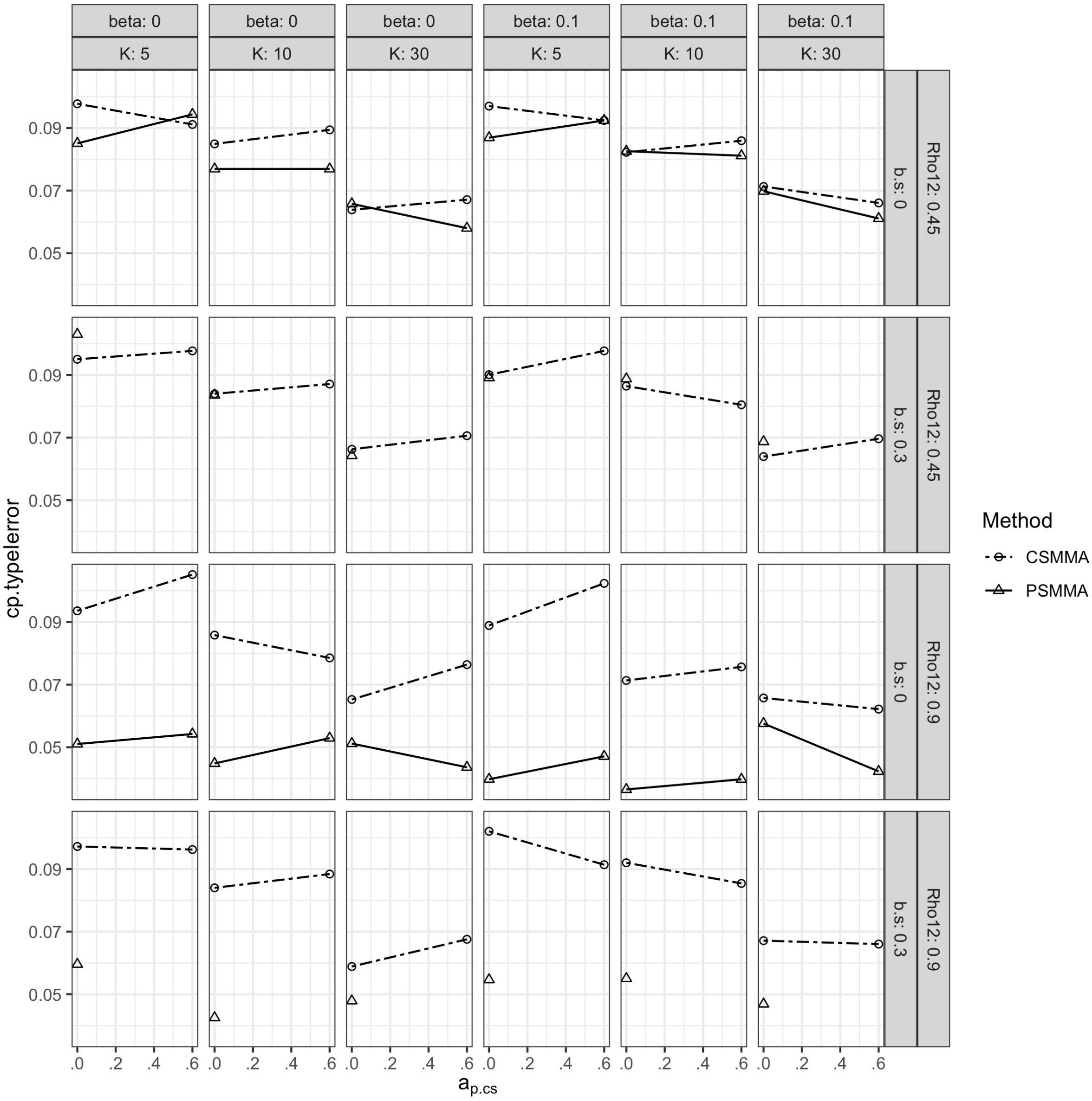
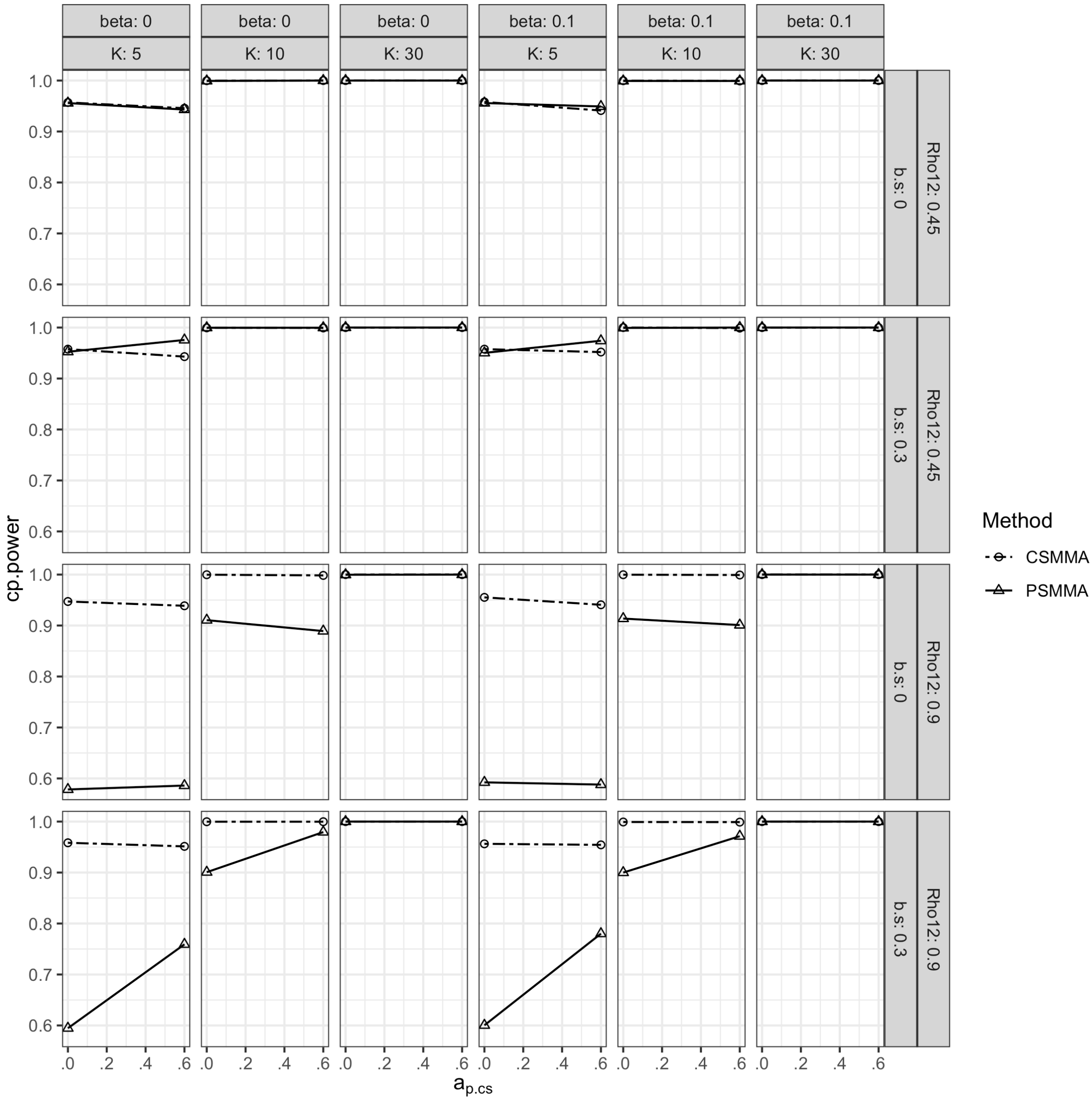


Figure S28.

Statistical Power of the Direct Effect under All Conditions in Study 2



S2.2.3 Moderating Effect

The EBIAS, CR, type I error rates and statistical power regarding the moderating effect had similar patterns with Study 1 (Figure S29-32).

Figure S29.

EBIAS of the Moderating Effect under All Conditions in Study 2

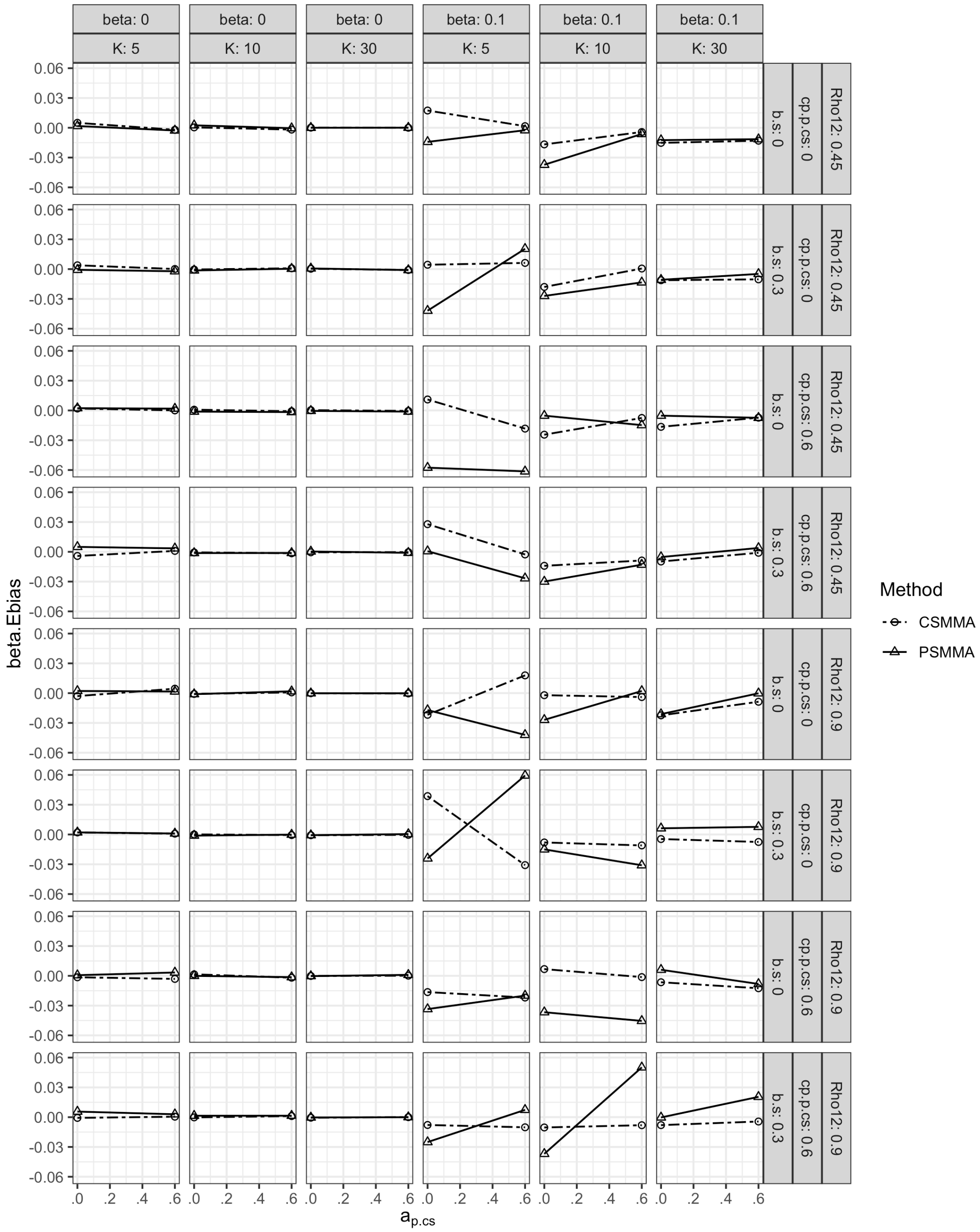


Figure S30.

Coverage Rates of the Moderating Effect under All Conditions in Study 2

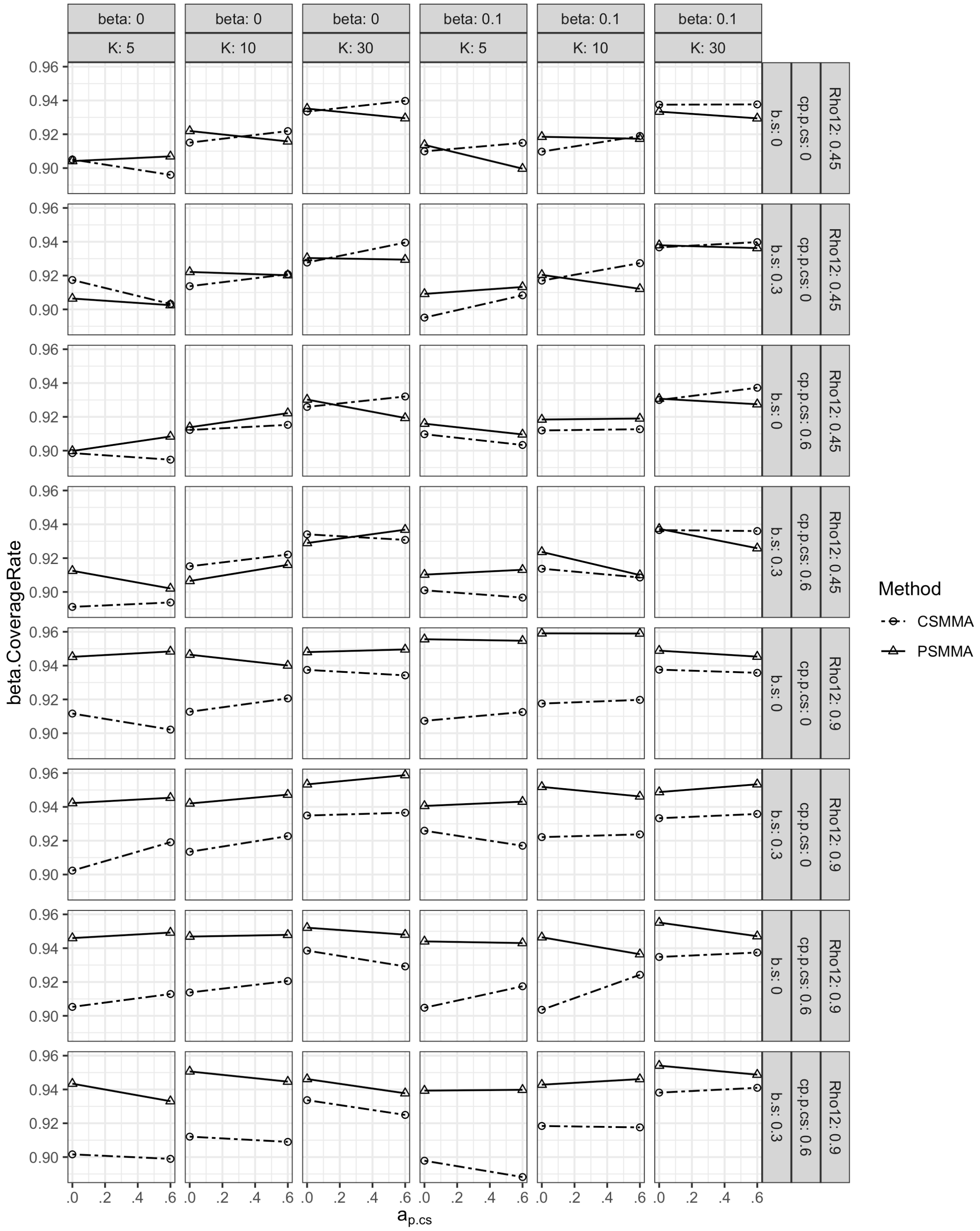


Figure S31.

Type I Error Rates of the Moderating Effect under All Conditions in Study 2

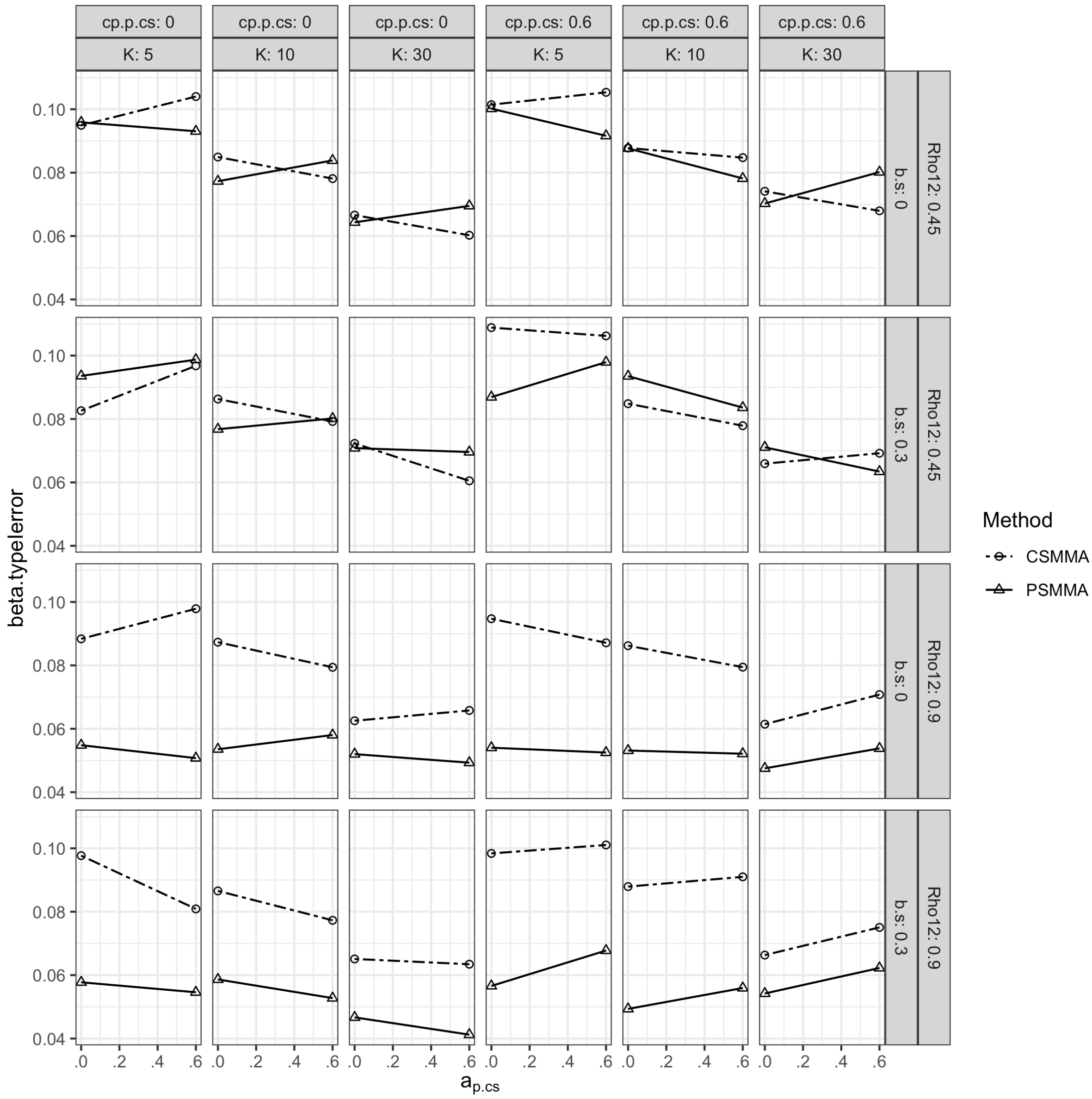
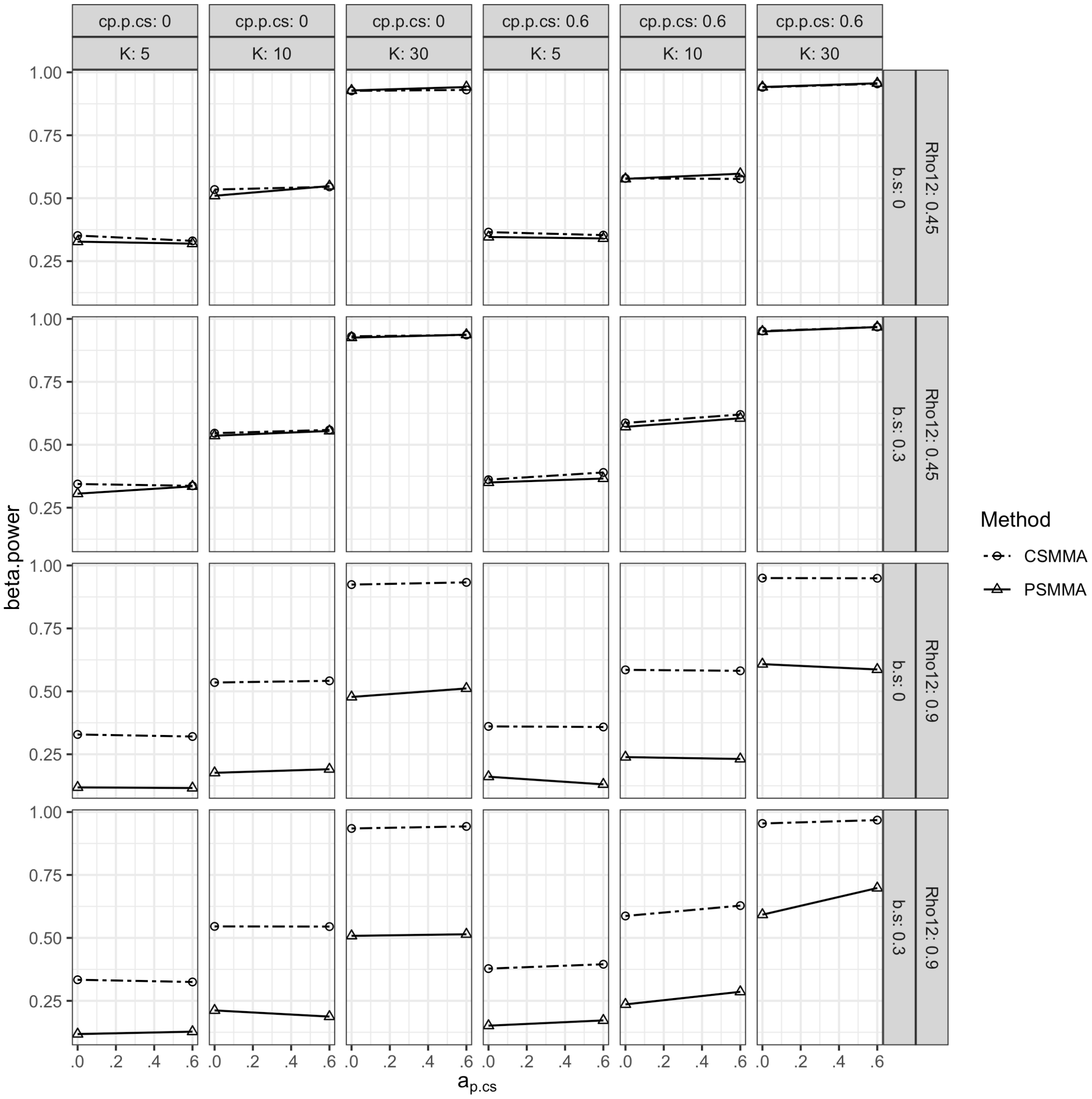


Figure S32.

Statistical Power of the Moderating Effect under All Conditions in Study 2



S2.2.4 Coefficients of the a path and the b path

As shown in Figure S33-35 and in Figure S37-39, the patterns of EBIAS, CR, and type I error rates of CSMMA and PSMMA when estimating the a and b paths were similar as in Study 1. However, the statistical power of PSMMA when estimating the a and b paths increased in the presence of the inflation of posttest variances (Figure S36 and S40).

Figure S33.

EBIAS of the a path Coefficient under All Conditions in Study 2

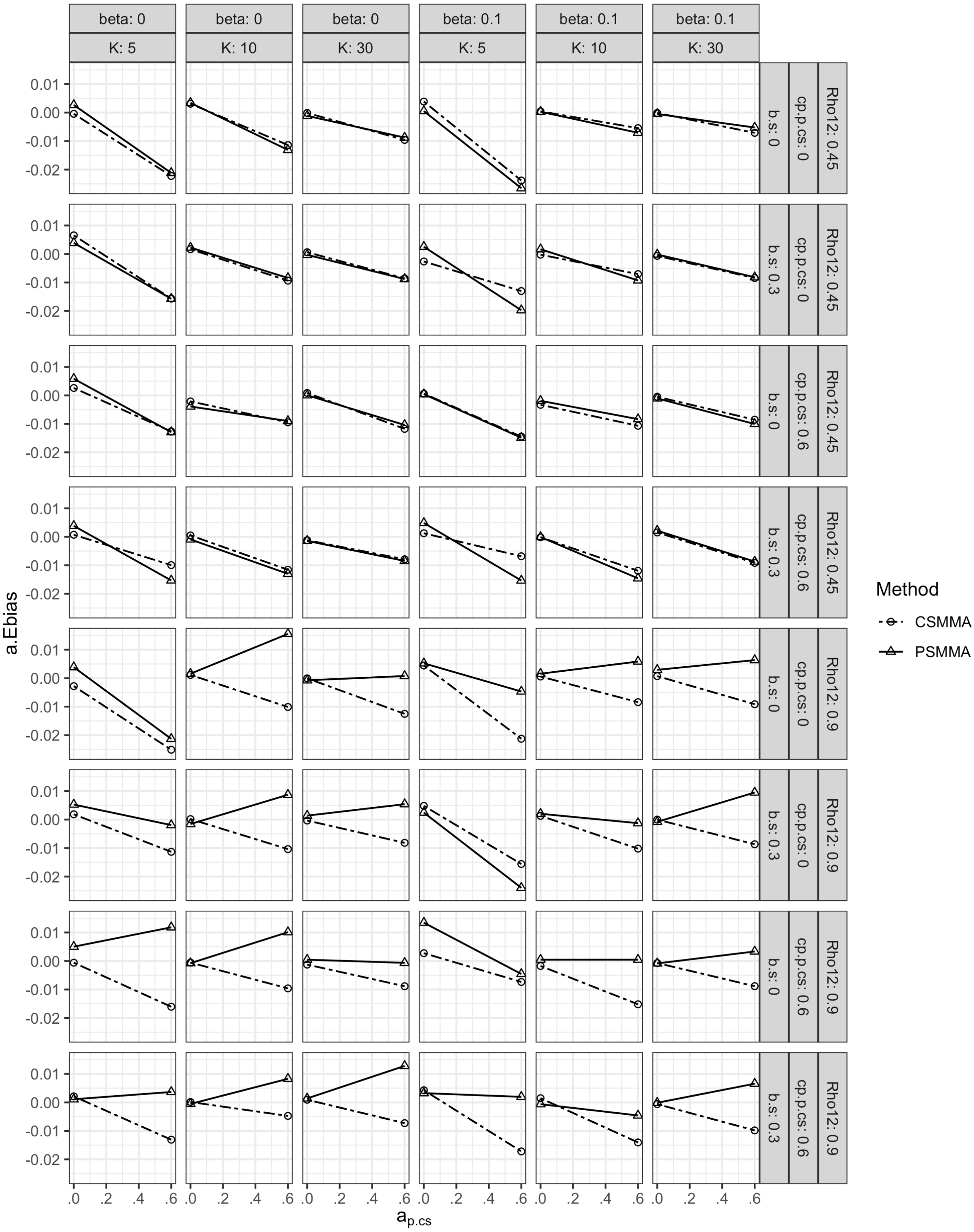


Figure S34.

Coverage Rates of the a path Coefficient under All Conditions in Study 2

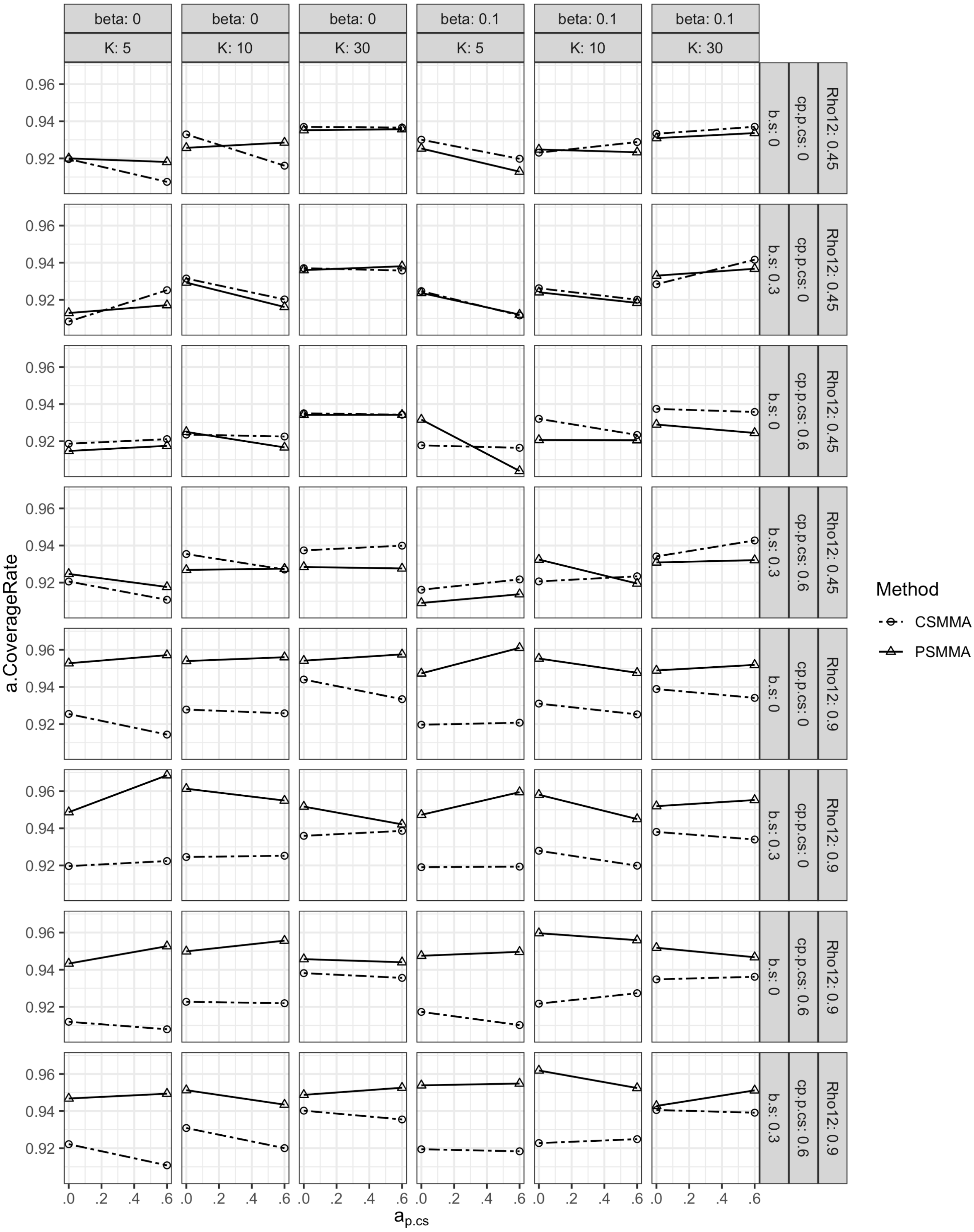


Figure S35.

Type I Error Rates of the a path Coefficient under All Conditions in Study 2

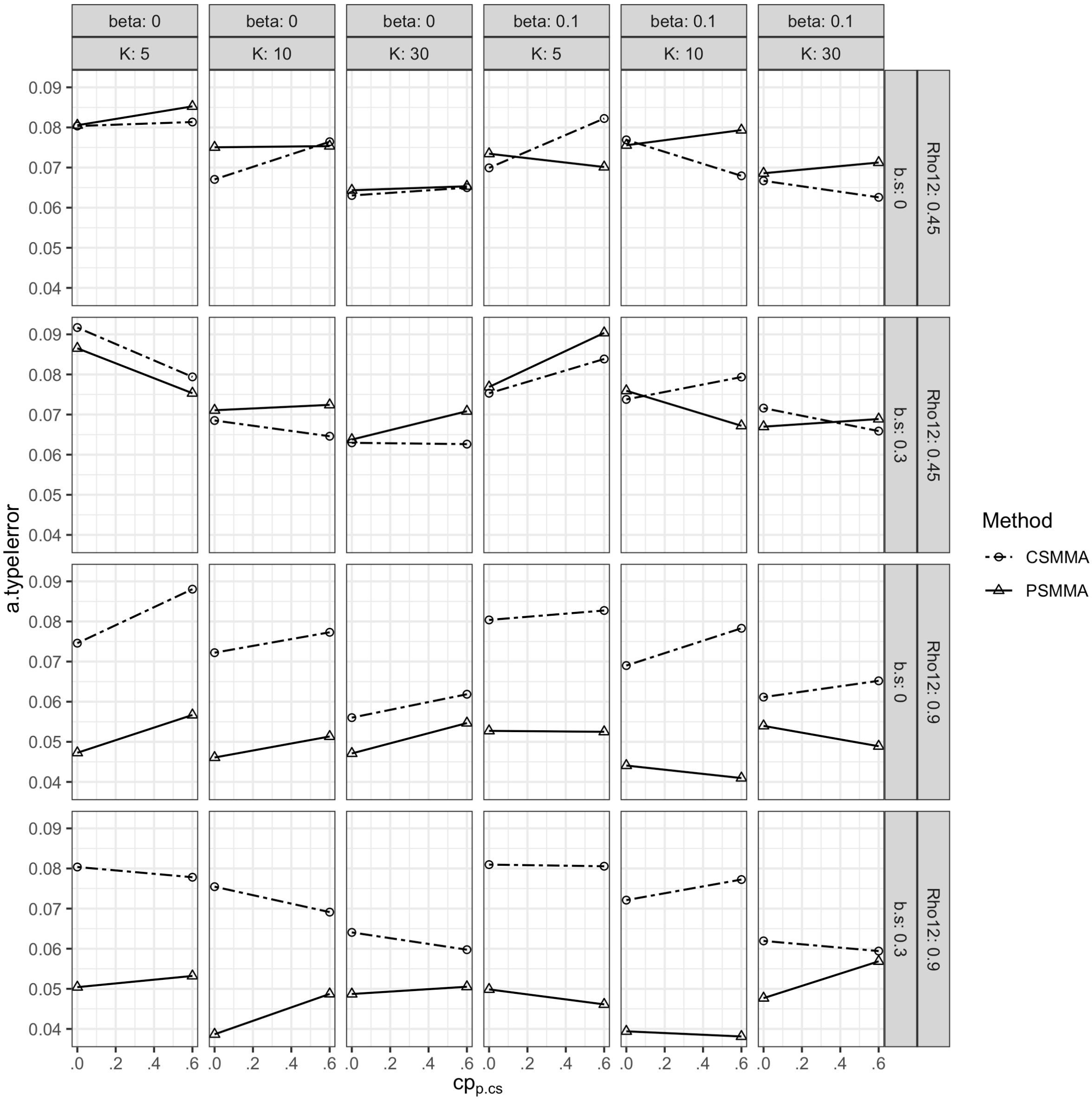


Figure S36.

Statistical Power of the a path Coefficient under All Conditions in Study 2

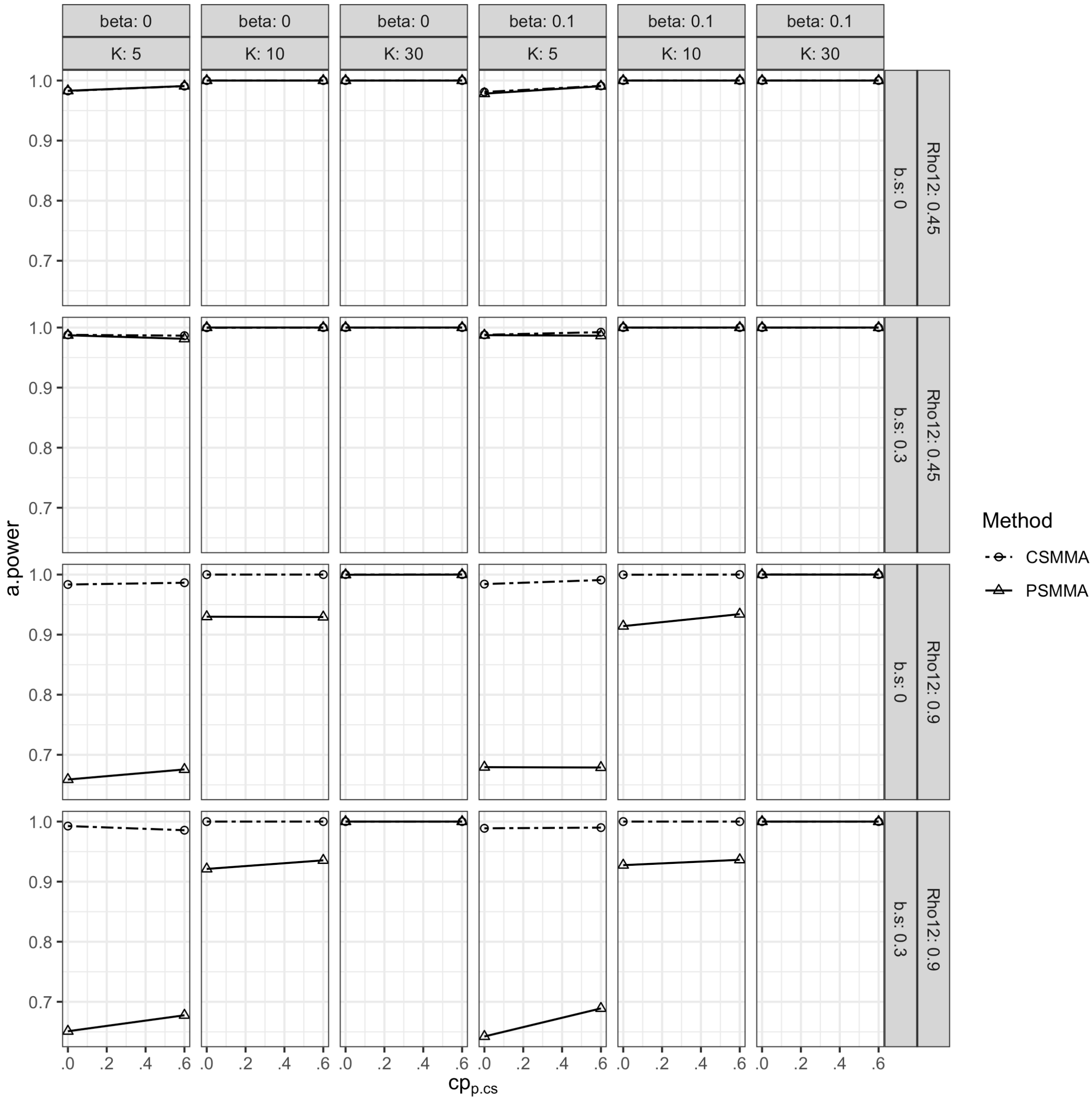


Figure S37.

EBIAS of the b path Coefficient under All Conditions in Study 2

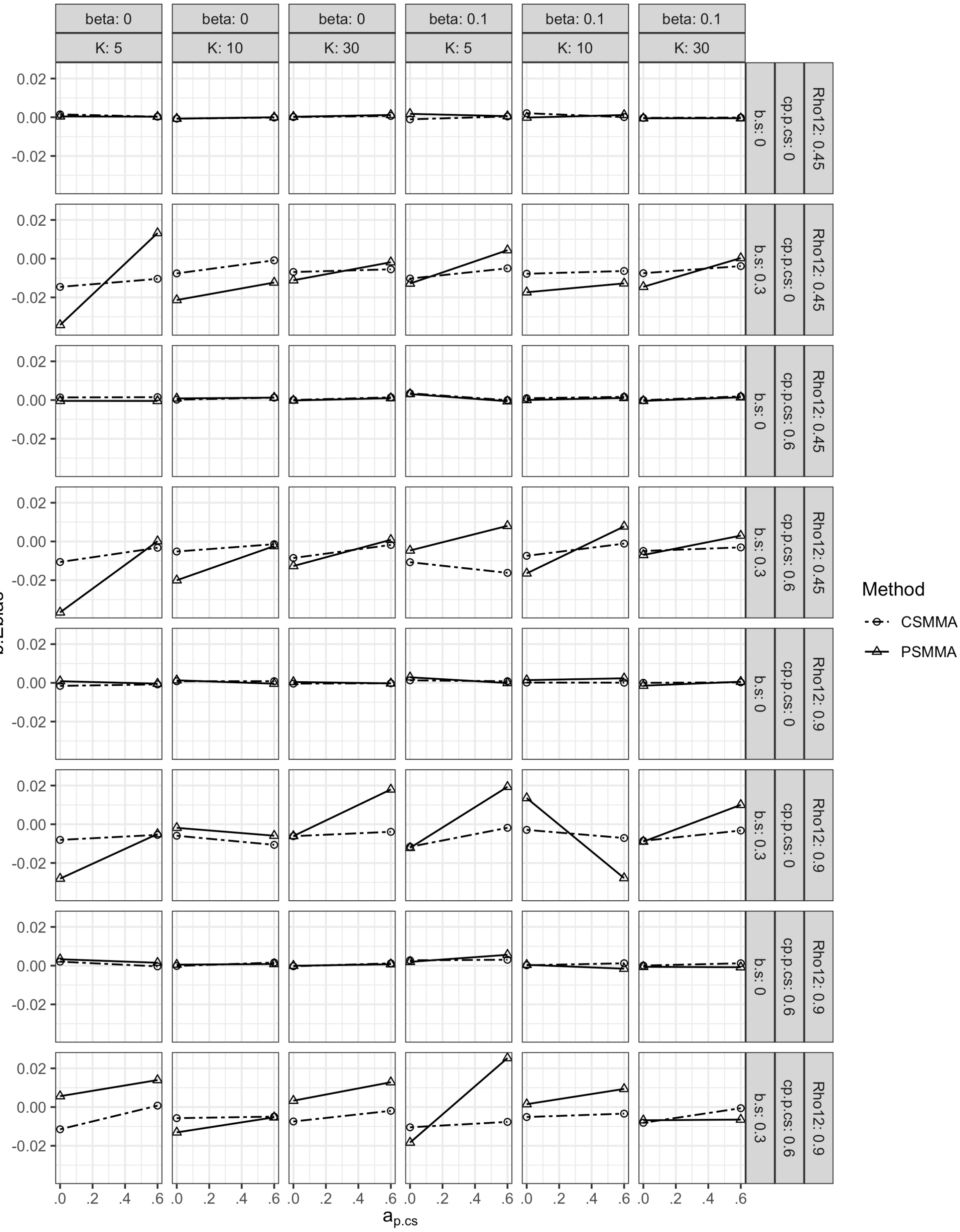


Figure S38.

Coverage Rates of the b path Coefficient under All Conditions in Study 2

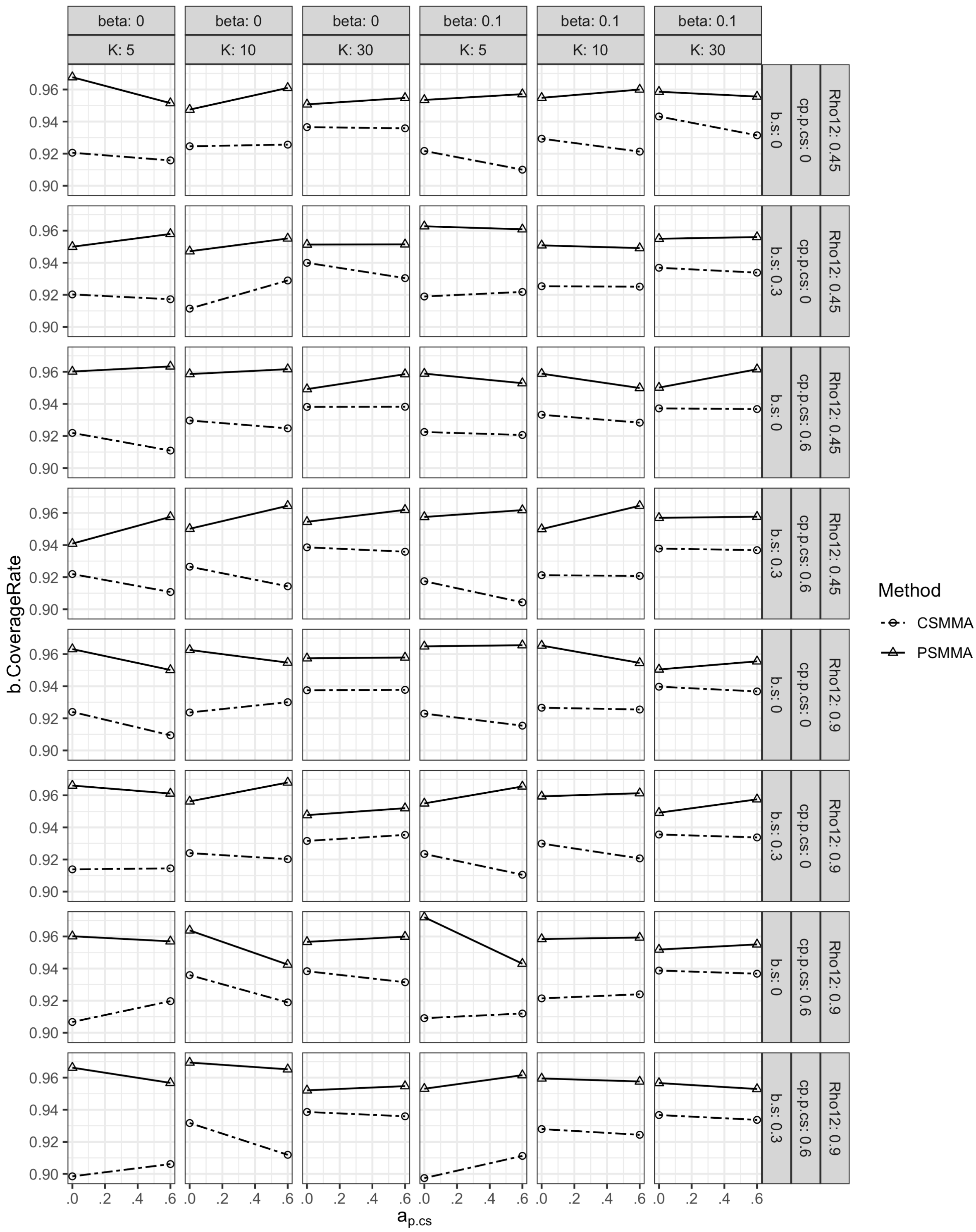


Figure S39.

Type I Error Rates of the b path Coefficient under All Conditions in Study 2

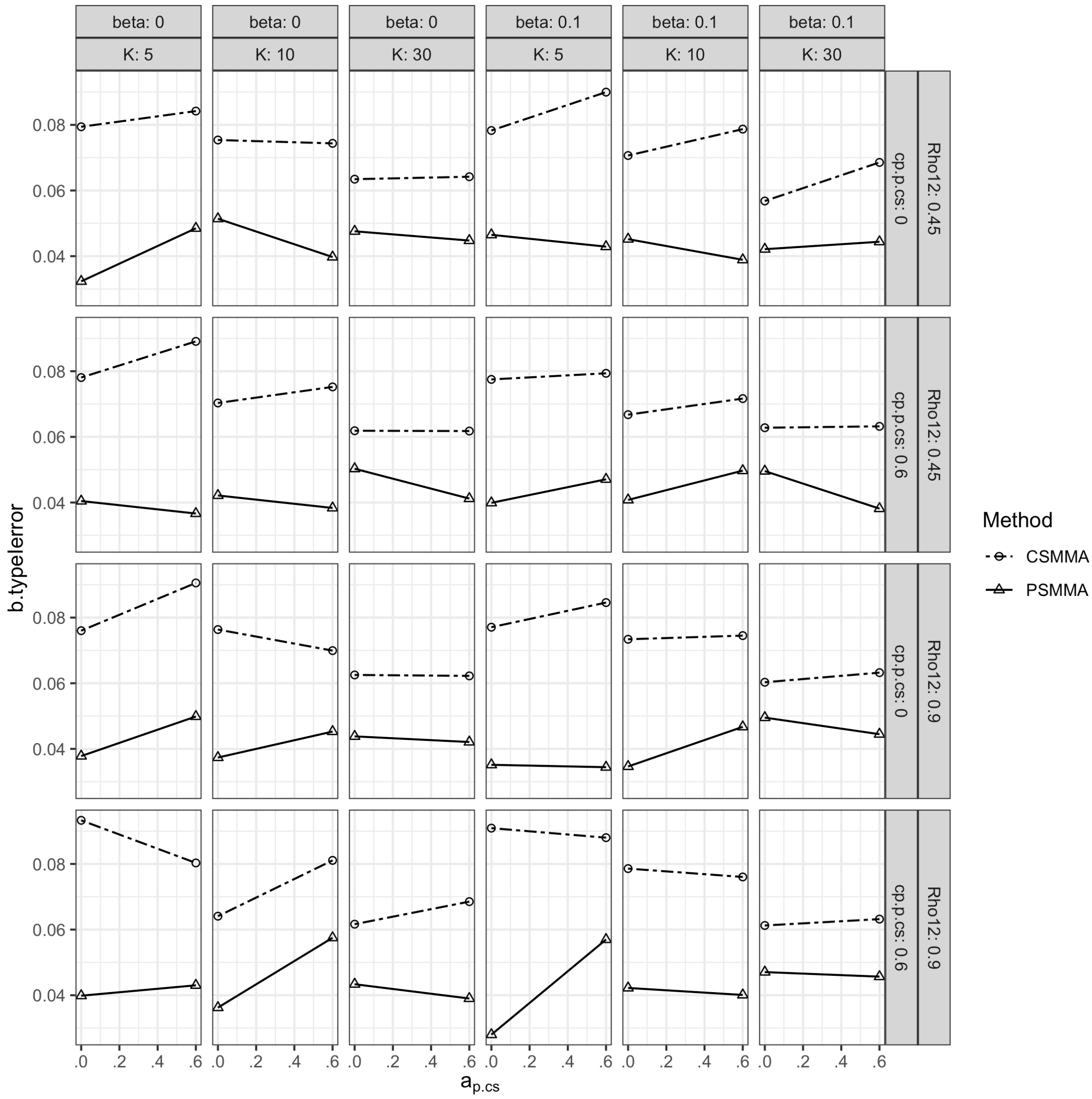
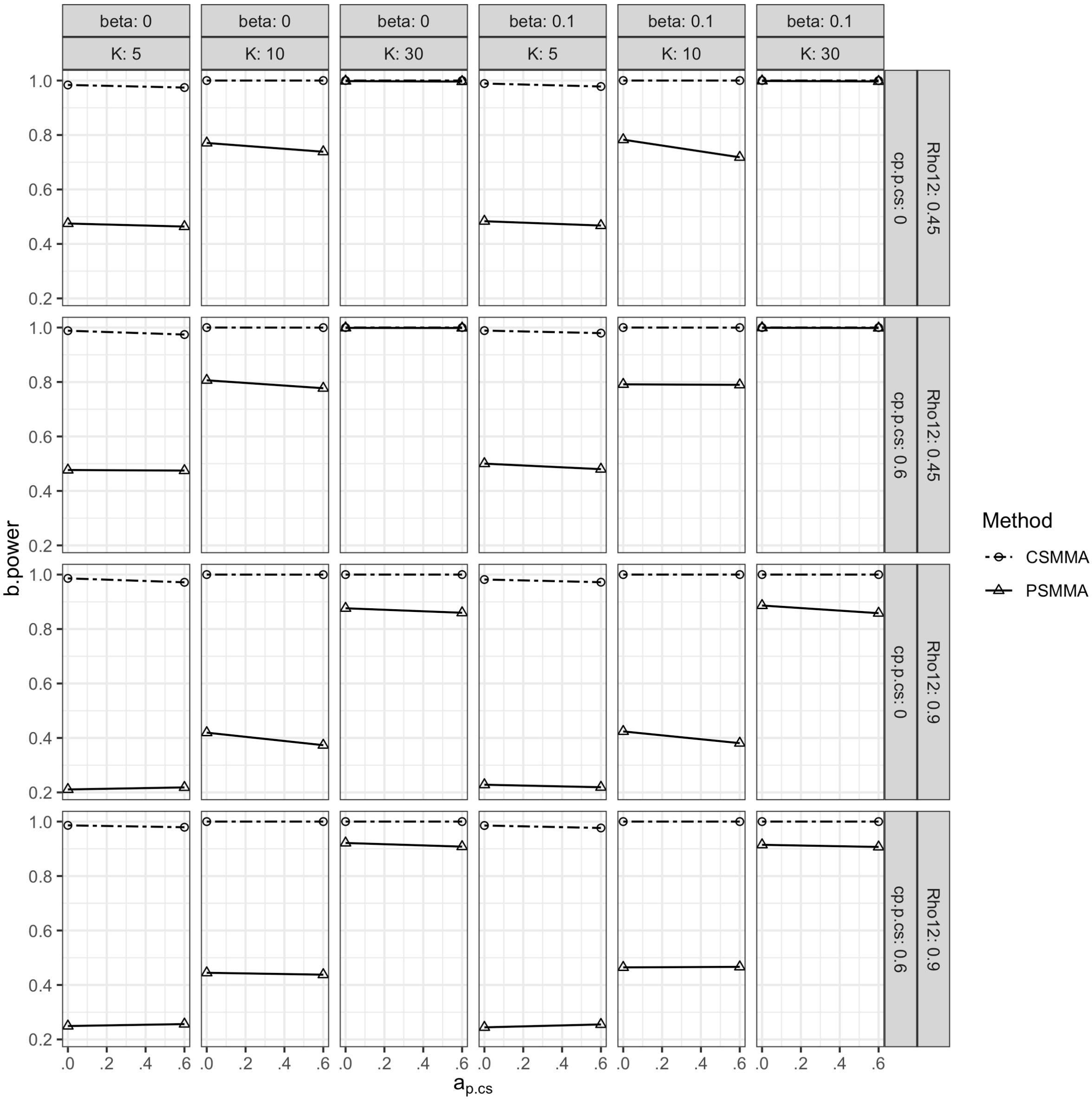


Figure S40.

Statistical Power of the b path Coefficient under All Conditions in Study 2



S2.3 Positive Definite Rates in Study 1 & 2

As shown in Figure S41, the positive definite rate of CSMMA and PSMMA in Study 1 decreased with a smaller K. Similar patterns were observed in Study 2 (Figure S42).

Figure S41.

Positive Definite Rates in Study 1 as Reported by OSMASEM

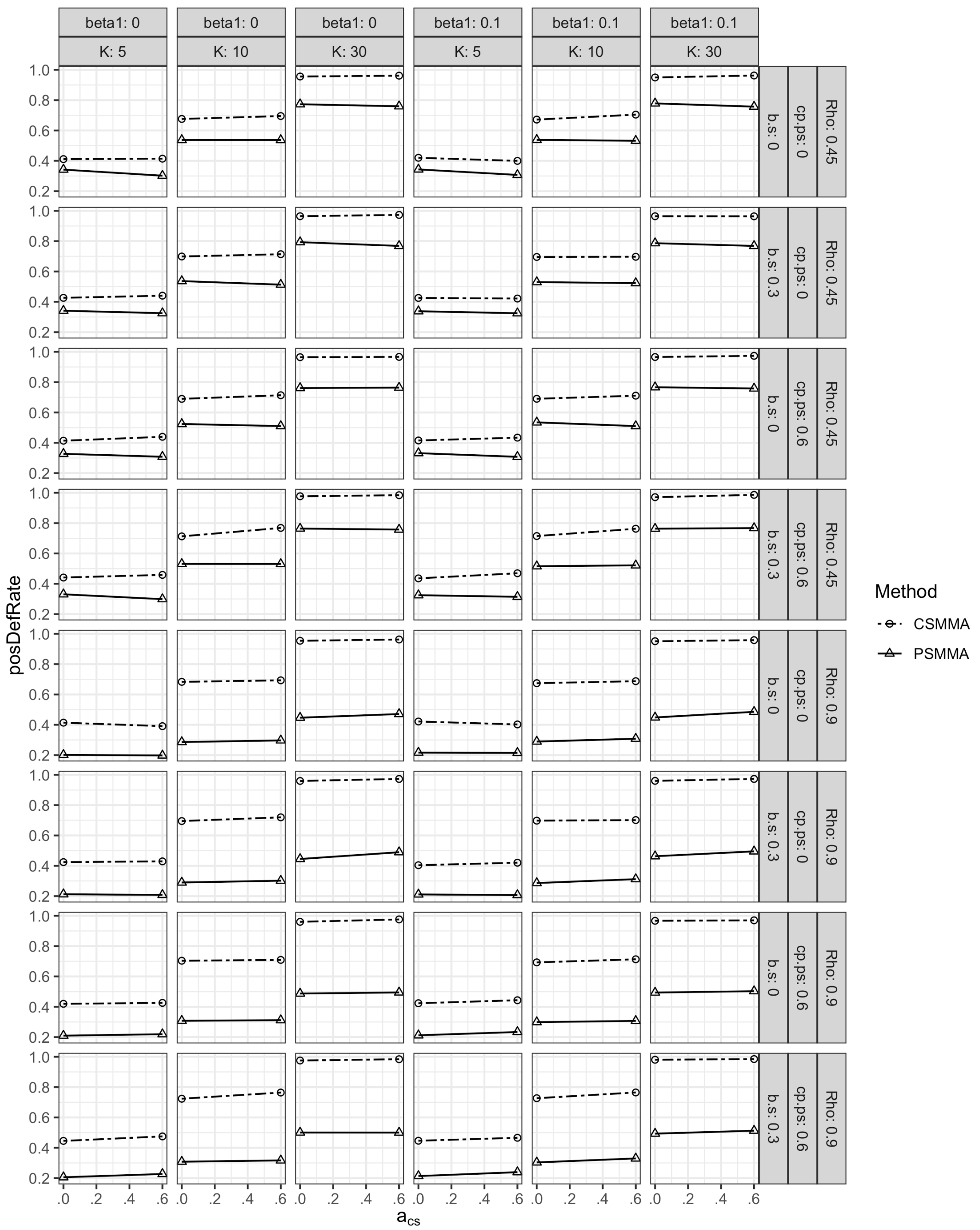
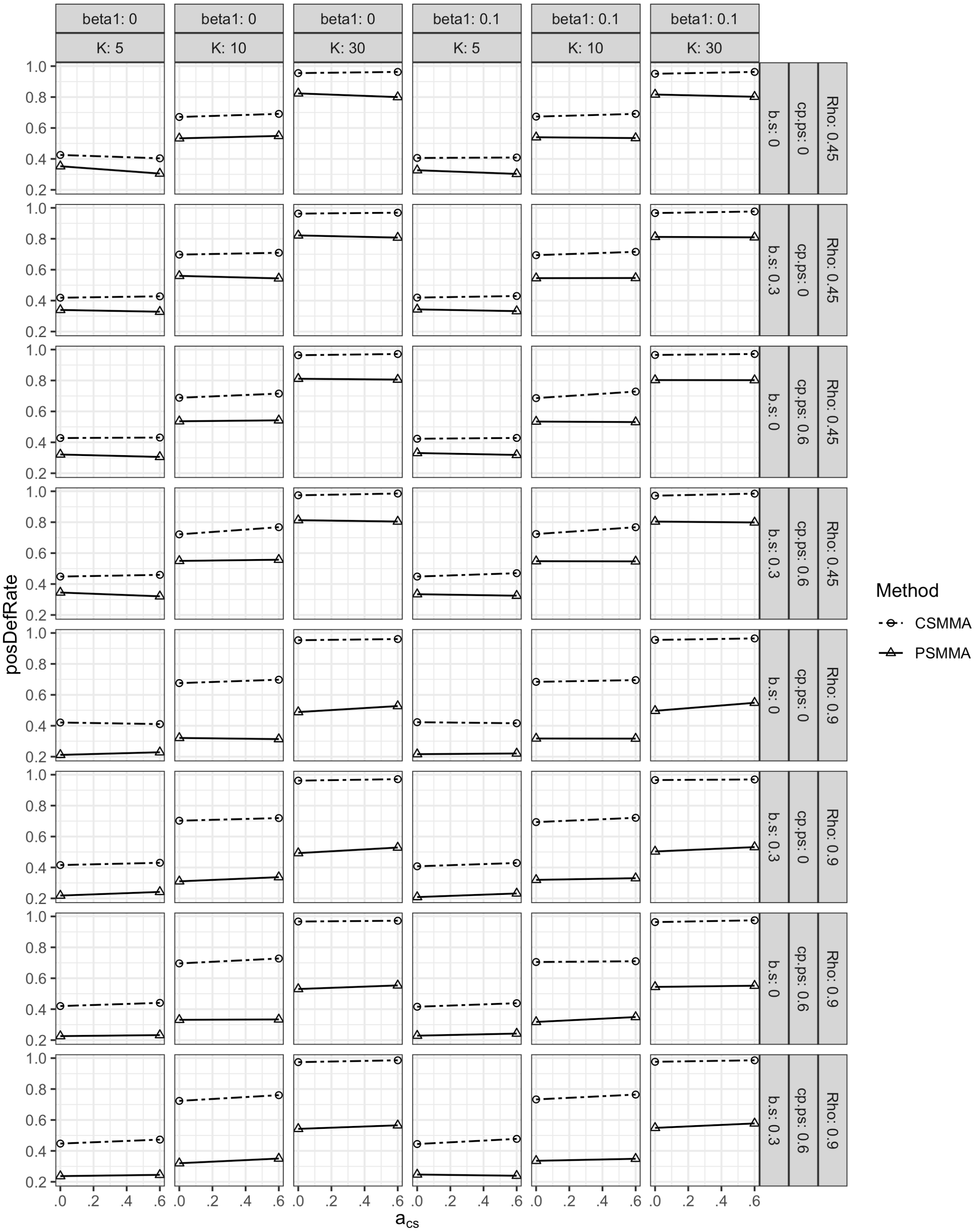


Figure S42.

Positive Definite Rates in Study 2 as Reported by OSMASEM



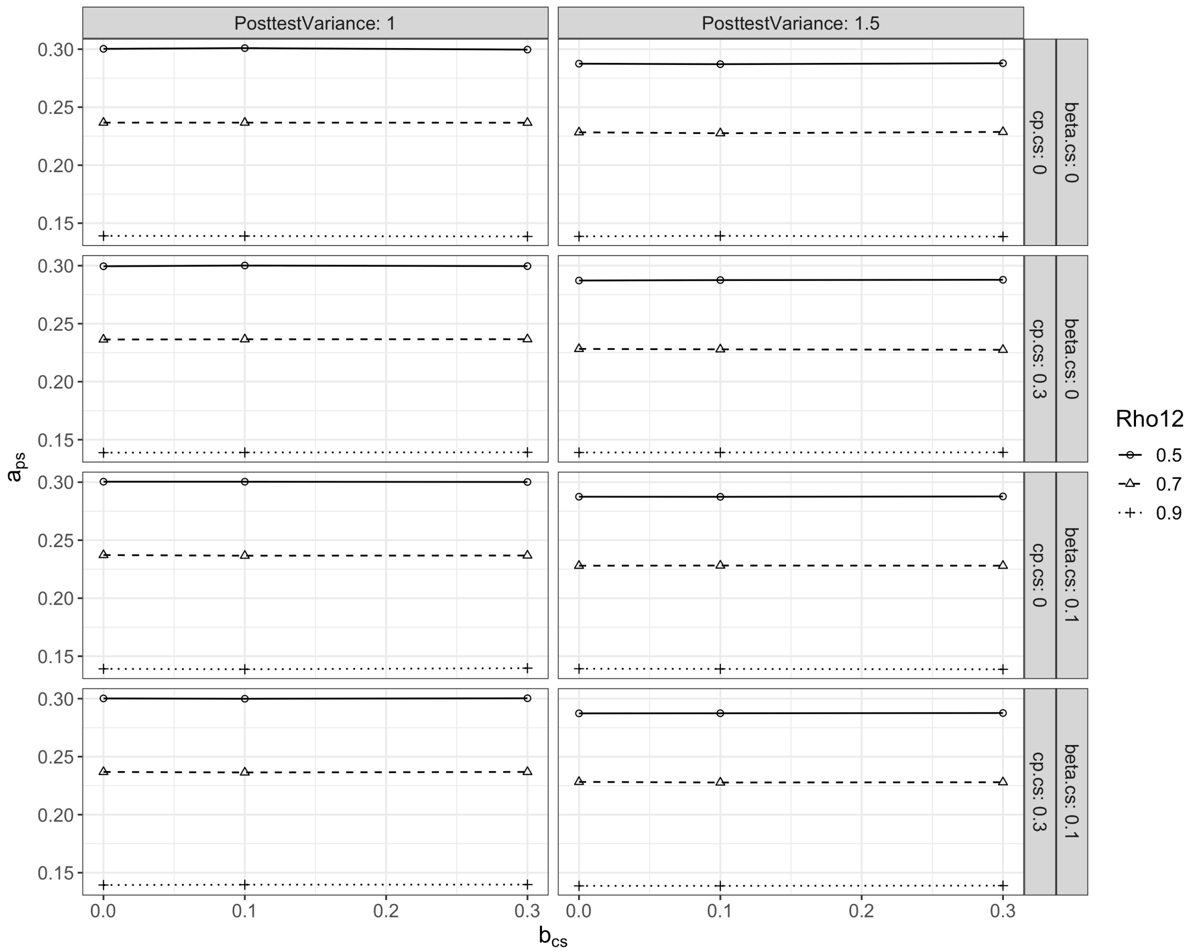
S3 The Relationship between Parameters in PSMMA and CSMMA

S3.1 Parameters in PSMMA with Nonzero Counterparts in CSMMA

As shown in Figure S43, when $a_{s.cs} = 0.3$, $a_{s.ps}$ decreased with a larger pretest-posttest correlation (Rho12) and posttest variance inflation. The size of $b_{s.cs}, cp_{s.cs}$, and $\beta_{c'_{s.cs}}$ did not have apparent effect here.

Figure S43.

Posttest-score Parameter of the a *path (*$a_{s.cs} = 0.3$*)*



Similar patterns have been observed in the c' and $\beta_{c'_{s.cs}}$ paths too (Figure S44 and S45).

Figure S44.

Posttest-score Parameter of the c' *path (*$c'_{s.cs} = 0.3$*)*

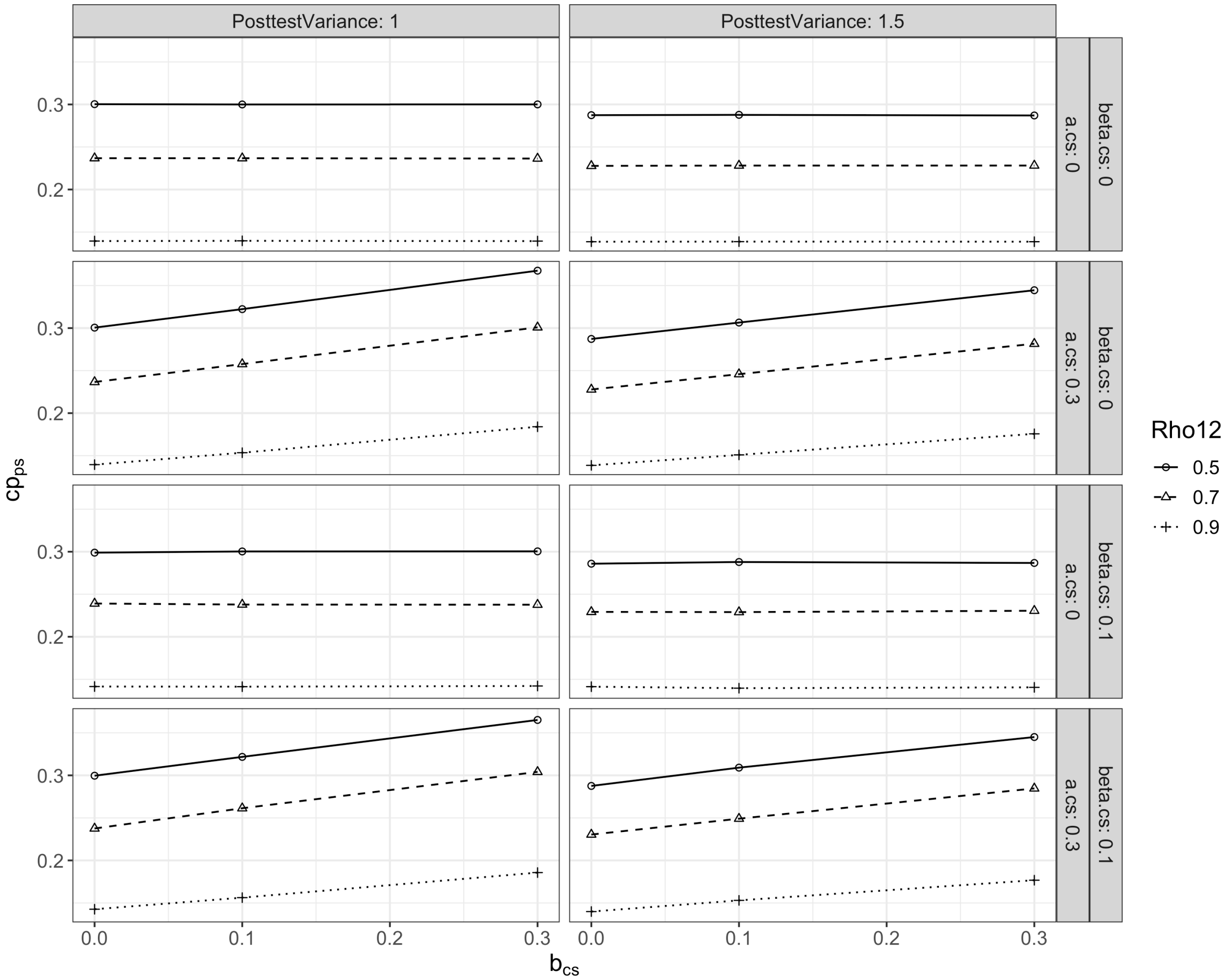
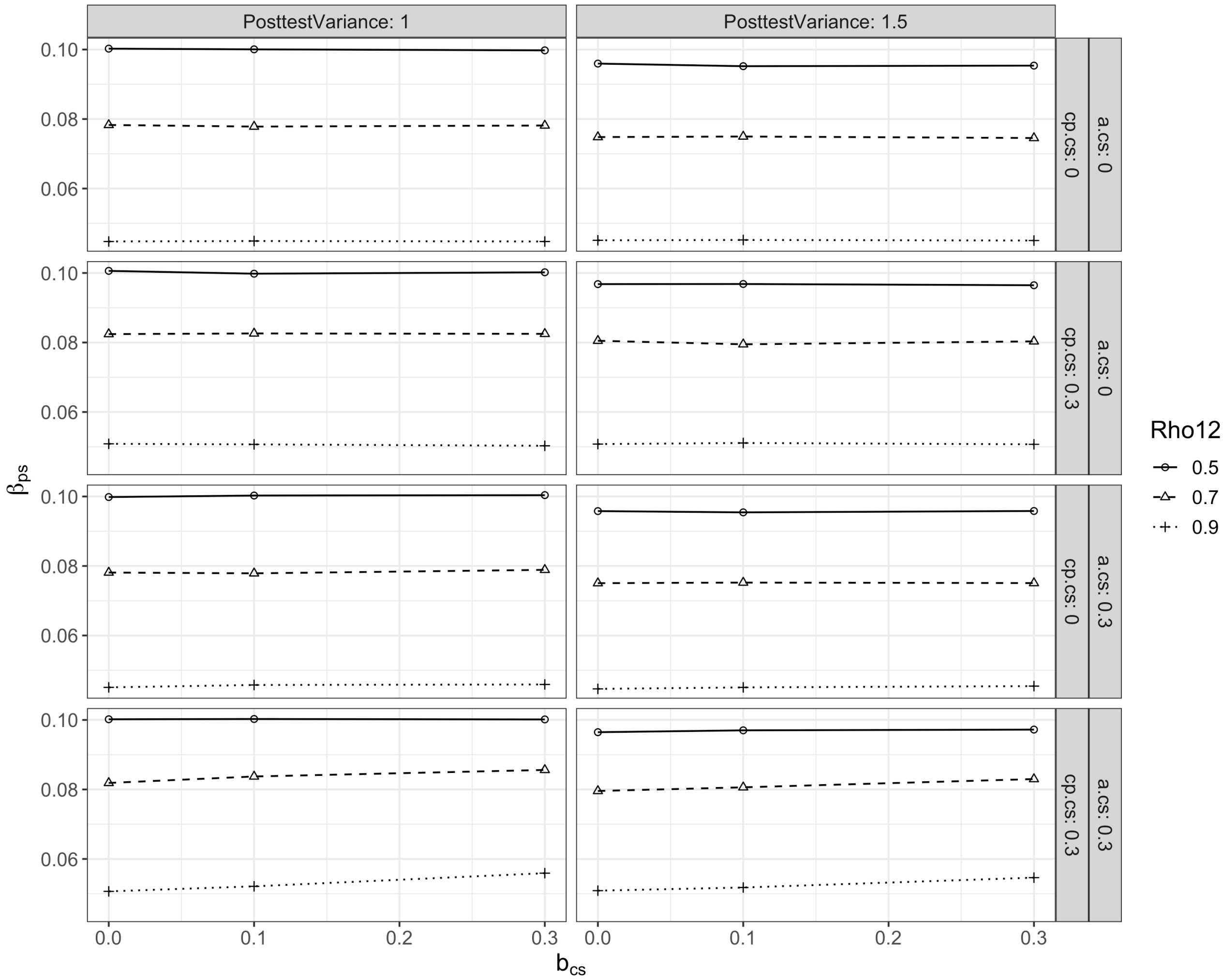


Figure S45.

Posttest-score Parameter of the $\beta_{c'}$ *path* ($\beta_{c'_{s.cs}} = 0.1$)

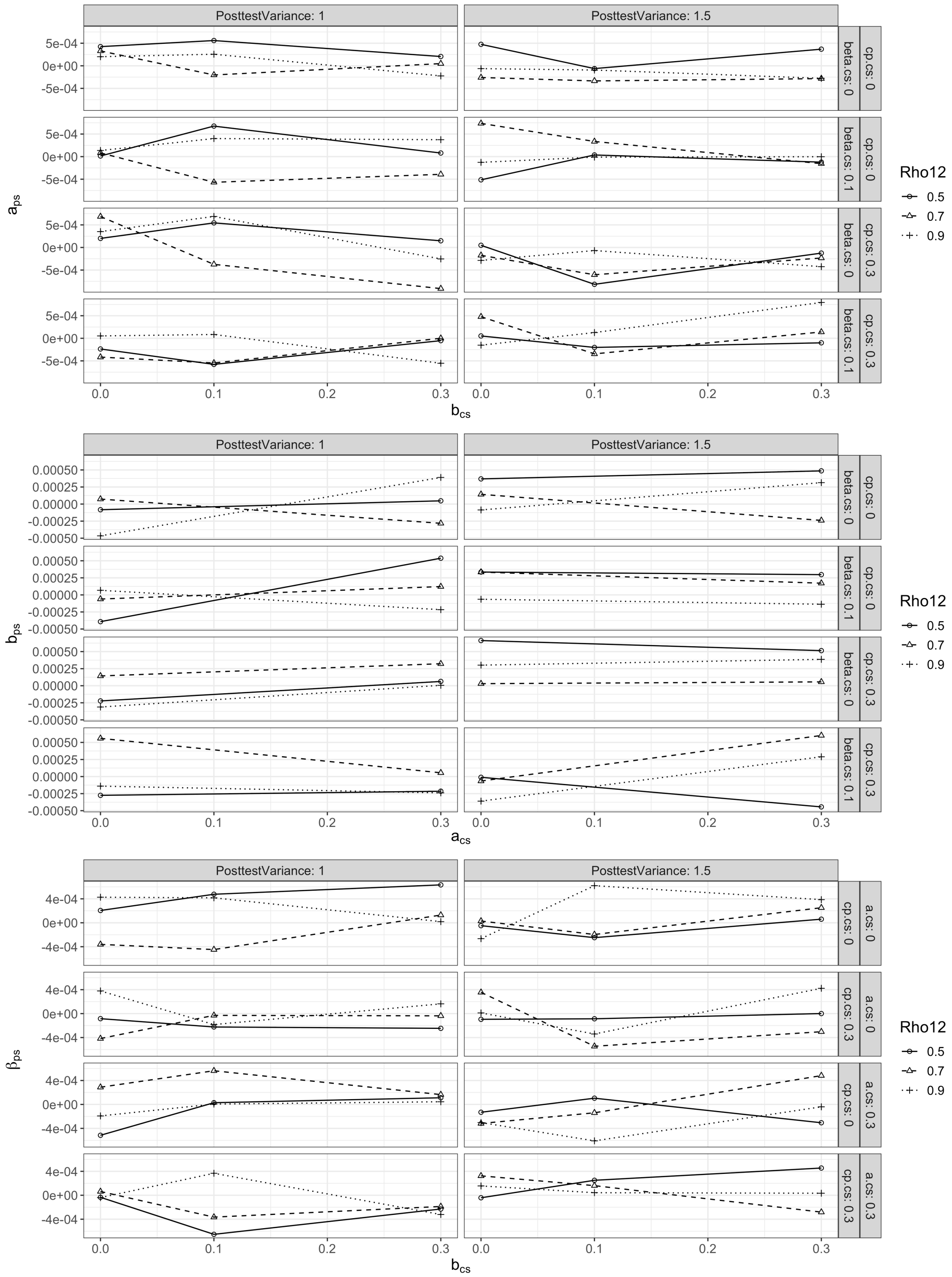


S3.2 Asynchrony in Taking a Value of Zero

As shown in Figure S46, the posttest-score parameters on a, b, and $\beta_{c'}$ paths were consistently zero when the counterparts in CSMMA were zero.

Figure S46.

Posttest-score Parameter of the a, b, and $\beta_{c'}$ *paths (*$a_{s.cs} = 0; b_{s.cs} = 0; \beta_{c'_{s.cs}} = 0$*)*



For a zero change-score c', the corresponding posttest-score c' was also zero when one of the change-score a and b paths was zero (the top two rows in Figure S47). However, when the change-score c' equaled zero but the change-score a and b paths were simutaneously nonzero, the mean c' path in PSMMA took a nonzero value (the bottom two rows in Figure S47).

Figure S47.

Posttest-score Parameter of the c' path ($c' = 0$)