

**A New Causal Mediation Approach  
Based on Observational Mediation Modeling and  
Instrumental Variable Regression**

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Presenter: Zhiming Lu  
07/26/2023

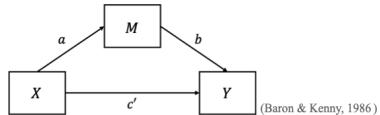
Good afternoon. The research we're gonna present is about causal inferences in mediation analysis, which has long been facing challenges in psychological research.

## Causality in Mediation Analysis

To begin with, let's take a look at the problem with causality in mediation analysis.

## ► Introduction

- Mediation analysis

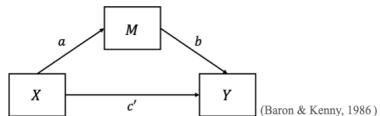


► Causality is implied by definition (MacKinnon & Pirlott, 2014)

In a mediation model, the independent variable (X) impacts the dependent variable (Y) by influencing the mediator (M). So, by definition, causality is implied for all three paths.

## ► Introduction

- Mediation analysis



- Causality is implied by definition (MacKinnon & Pirlott, 2014)

- No confounding assumptions for mediation analysis (VanderWeele & Vansteelandt, 2009)

1. no unmeasured confounding between  $X$  and  $M$
2. no unmeasured confounding between  $X$  and  $Y$
3. no unmeasured confounding between  $M$  and  $Y$
4. no measured or unmeasured confounding between  $M$  and  $Y$  that is affected by  $X$

To establish causality in mediation, these no-confounding assumptions must be satisfied.

The first three assumptions require that there are no omitted common causes in any bivariate relation, and the fourth assumption essentially assumes of no omitted mediators that covary with  $M$ .

To satisfy these assumptions, researchers resort to experimental control and statistical control approaches.



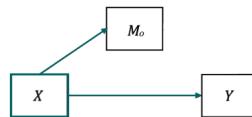
Experimental Control  
for Causality in Mediation

Let's take a look at the experimental control approaches first.

## ► Experimental Control

- Measurement-of-mediation design (Pirlott & MacKinnon, 2016)

Manipulating  $X$  and measuring  $M$  and  $Y$   
Strengthening causality in the  $X\text{-}M$  and  $X\text{-}Y$  relations



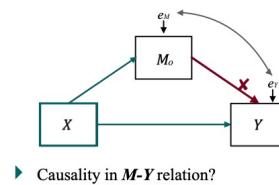
The most commonly used design is called a measurement-of-mediation design, in which  $X$  is manipulated and  $M$  and  $Y$  are measured.

In this way, causality in these two paths can be strengthened.

## ► Experimental Control

- Measurement-of-mediation design (Pirlott & MacKinnon, 2016)

Manipulating  $X$  and measuring  $M$  and  $Y$   
Strengthening causality in the  $X-M$  and  $X-Y$  relations



► Causality in  $M-Y$  relation?

However, because  $M$  and  $Y$  are both observational, the residuals of  $M$  and  $Y$  could be potentially correlated.

Therefore, the relation between  $M$  and  $Y$  cannot be interpreted as causal. And this, is the main problem in the field of causal inference in mediation analysis.

## ► Experimental Control

- Manipulation-of-mediator designs (Pirlott & MacKinnon, 2016)

To resolve this problem, it's been recommended to experimentally manipulate the mediator, using manipulation-of-mediator designs.

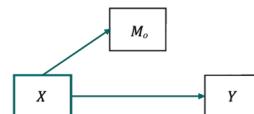
## ► Experimental Control

- Manipulation-of-mediator designs (Pirlott & MacKinnon, 2016)

- Double Randomization Design (DRD, experimental causal chain design)

(Spencer et al., 2005; Stone-Romero & Rosopa, 2008)

Exp 1: Manipulating  $X$  and measuring  $M$  and  $Y$



Exp 2: Manipulating  $M$  and measuring  $Y$  (holding  $X$  constant)



One commonly used design is double randomization design. It's a two-experiment design.

Experiment 1 is a measurement-of-mediation design.

In Experiment 2, the mediator is manipulated and  $Y$  is measured, with  $X$  held constant.

By conducting two experiments, a “causal chain” is established.

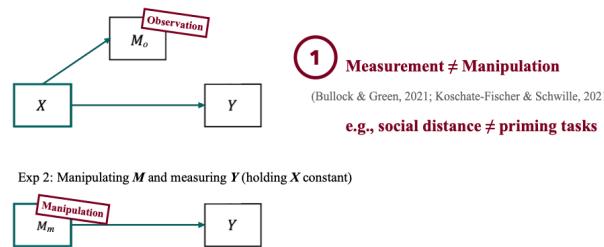
## ► Experimental Control

- Manipulation-of-mediator designs (Pirlott & MacKinnon, 2016)

- Double Randomization Design (DRD, experimental causal chain design)

(Spencer et al., 2005; Stone-Romero & Rosopa, 2008)

Exp 1: Manipulating  $X$  and measuring  $M$  and  $Y$



Exp 2: Manipulating  $M$  and measuring  $Y$  (holding  $X$  constant)



However, the problem is that it assumes that the observational  $M$  in Experiment 1 and the manipulation of  $M$  in Experiment 2 are the same variable.

This assumption is problematic in psychology because mediators of interest in psychology are often latent, continuous psychological constructs, which cannot be equated with the manipulation approaches.

For example, the mediator of interest is social distance. To manipulate the perceived social distance, participants are often asked to complete a priming task such as reading a short story. It's obvious that reading different stories is not equivalent with perceived social distance.

This is the first concern.

In this presentation, we use the subscripts o and m to distinguish between observed and manipulated

variables.

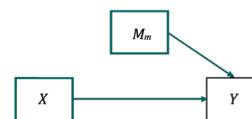
## ► Experimental Control

- Manipulation-of-mediator designs (Pirlott & MacKinnon, 2016)

- Concurrent Double Randomization Design (CDRD, moderation-of-process design)

(Jacoby & Sassenberg, 2011)

Exp 1: Manipulating  $X$  and  $M$ , and measuring  $Y$



Another design is concurrent double randomization design.

In this design, X and M are simultaneously manipulated, and only Y is measured.

It's been argued that the indirect effect, or the "process effect", can be examined by testing the interaction effect of X and Mm on Y.

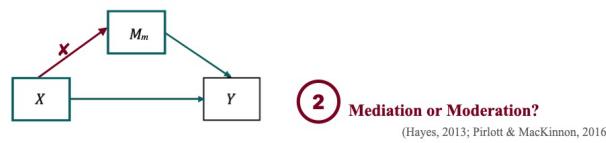
## ► Experimental Control

- Manipulation-of-mediator designs (Pirlott & MacKinnon, 2016)

- Concurrent Double Randomization Design (CDRD, moderation-of-process design)

(Jacoby & Sassenberg, 2011)

Exp 1: Manipulating  $X$  and  $M$ , and measuring  $Y$



The problem is, the X-M relation remains unexamined and the process revealed using this approach is actually **when** or **under what condition** X affects Y, not the mediating mechanism.

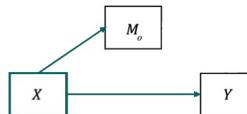
This is the second concern.

## ► Experimental Control

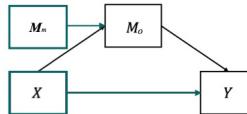
- Manipulation-of-mediator designs (Pirlott & MacKinnon, 2016)

- ▶ Parallel Encouragement Design (PED) (Imai et al., 2013)

Exp 1: Manipulating  $X$  and  $M$ , and measuring  $Y$



Exp 2: Manipulating  $X$  and  $M$ , and measuring  $M$  and  $Y$



Another design is called a parallel encouragement design.

Experiment 1 is a measurement-of-mediation design.

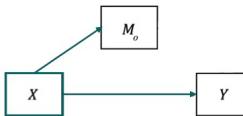
In Experiment 2,  $X$  and  $M$  are manipulated, and both  $M$  and  $Y$  are measured.

## ► Experimental Control

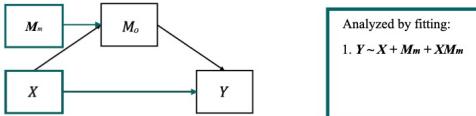
- Manipulation-of-mediator designs (Pirlott & MacKinnon, 2016)

- ▶ Parallel Encouragement Design (PED) (Imai et al., 2013)

Exp 1: Manipulating  $X$  and  $M_o$ , and measuring  $Y$



Exp 2: Manipulating  $X$  and  $M_o$ , and measuring  $M_o$  and  $Y$



Analyzed by fitting:  
1.  $Y \sim X + M_o + XM_o$

This Experiment 2 is often used to test main effects and interaction effects in psychology, typically using regression-based approaches like two-way ANOVA.

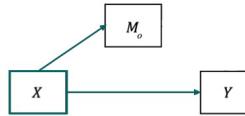
Using such analyses, the potential mediation effect is actually neglected.

## ► Experimental Control

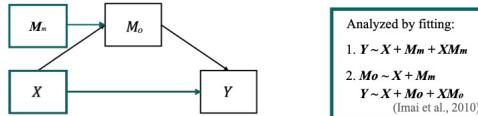
- Manipulation-of-mediator designs (Pirlott & MacKinnon, 2016)

- ▶ Parallel Encouragement Design (PED) (Imai et al., 2013)

Exp 1: Manipulating  $X$  and  $M_o$ , and measuring  $Y$



Exp 2: Manipulating  $X$  and  $M_o$ , and measuring  $M_o$  and  $Y$



Analyzed by fitting:  
1.  $Y \sim X + M_m + XM_m$   
2.  $Mo \sim X + M_m$   
 $Y \sim X + Mo + XM_o$   
(Imai et al., 2010)

Another analytical approach for this design is provided in a package developed by Imai and colleagues, which is based on the observational mediation modeling.

But it currently only allows for discrete Mo and Y.

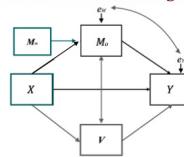
## ► Experimental Control

- An Unresolved Issue

- No confounding assumptions for mediation analysis (VanderWeele & Vansteelandt, 2009)

1. no unmeasured confounding between  $X$  and  $M$
2. no unmeasured confounding between  $X$  and  $Y$
3. no unmeasured confounding between  $M$  and  $Y$
4. no measured or unmeasured confounding between  $M$  and  $Y$  that is affected by  $X$

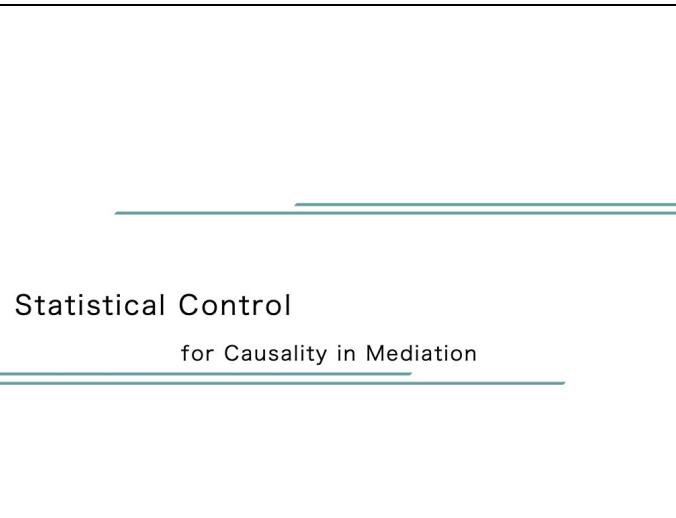
**3 Unmeasured confounding induced by omitted mediators**



In addition, a common issue to experimental control is that manipulations exerted on  $X$  and  $M$  can satisfy the first three no confounding assumptions, but not the fourth.

When there're omitted mediating mechanisms covarying with the mediator of interest, a covariation between the residuals of  $M_o$  and  $Y$  would be induced, leading to biased estimates.

This is the third concern.



Statistical Control  
for Causality in Mediation

Fortunately, unmeasured confounding can be reduced by statistical control.

## ► Statistical Control

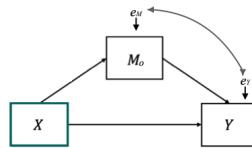
- To Reduce Unmeasured Confounding
  - Instrumental Variable (IV) Regression (Angrist et al., 1996; Angrist & Krueger, 2001)

Instrumental variable regression methods can be used to reduce confounding induced by correlated residuals.

## ► Statistical Control

- To Reduce Unmeasured Confounding
  - ▶ Instrumental Variable (IV) Regression (Angrist et al., 1996; Angrist & Krueger, 2001)

- IV Regression in Mediation

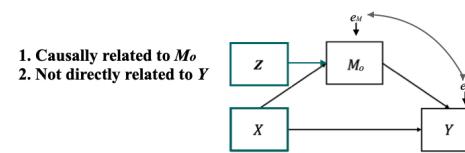


For mediation models with correlated residuals of  $Mo$  and  $Y$ , the relation between  $Mo$  and  $Y$  cannot be estimated because of a shortage of degrees of freedom.

## ► Statistical Control

- To Reduce Unmeasured Confounding
  - ▶ Instrumental Variable (IV) Regression (Angrist et al., 1996; Angrist & Krueger, 2001)

- IV Regression in Mediation



But if we find an instrumental variable that is causally related to the observed M and not directly related to Y, we have enough degrees of freedom to estimate the MY relation.

However, identifying such an appropriate IV can be quite challenging in psychology, which explains why IV regression is much less commonly used in psychology than in other disciplines.

## ► Statistical Control

- Current approaches to Incorporating IV in Mediation

- Baseline covariates as IVs

(Albert, 2008; Dunn & Bentall, 2007; Ten Have et al., 2007)

- External variables as IVs

(Burgess et al., 2015; Dippel et al., 2020; Frolich & Huber, 2017)

- Independent variable  $X$  as an IV

(Bullock & Green, 2021; Maydeu-Olivares et al., 2020)

In other areas, IV regression is often used by treating covariates as IVs or by using variables not originally included in the model as IVs.

However, in psychology, it's often unrealistic to assume no direct effects of covariates on the outcome.

In addition, another approach is to treat  $X$  as an IV, assuming no direct effect of  $X$  on  $Y$ , but this assumption is probably also too strong to hold in psychology.

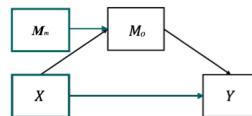


To address the three issues we discussed, we propose a new approach, which combines experimental control and statistical control.

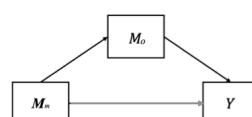
## ► The New Approach

### • Experimental Design

Exp 1: Manipulating  $X$  and  $M$ , and measuring  $M$  and  $Y$



Exp 2: Manipulating  $M$ , and measuring  $M$  and  $Y$



First, we propose an experimental design, with two commonly used experiments.

Experiment 1 is essentially a parallel encouragement design:  $X$  and  $M$  are manipulated, and  $M$  and  $Y$  are measured.

In Experiment 2,  $M$  is manipulated and  $M$  and  $Y$  are measured.

Experiment 1 is used to obtain data for the main mediation analysis. Experiment 2 is used to conduct a sensitivity analysis which will be discussed later.

## ► The New Approach

### • Modeling Strategy

$$\begin{aligned}M_o &= aX + dM_m \\Y &= c'X + bM_o + gXM_o\end{aligned}$$

- 1 ► Separately modeling  $M_m$  and  $M_o$
- 2 ► Modeling the  $XM_o$  interactions

Based on Experiment 1, the indirect effect can be estimated by fitting this model.

The first two concerns are addressed by separately modeling the manipulation and observation of the mediator, and by including the  $XM_o$  interaction to distinguish between mediation and moderation.

## ► The New Approach

### • Modeling Strategy

$$\begin{aligned}M_o &= aX + dM_m \\Y &= c'X + bM_o + gXM_o\end{aligned}$$

- 1 ► Separately modeling  $M_m$  and  $M_o$
- 2 ► Modeling the  $XM_o$  interactions

$$IE = a(b+gX)$$

In the presence of  $XM_o$  interaction, the indirect effect is conditional on levels of X.

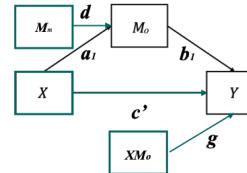
### ► The New Approach

- Treating  $M_m$  as An IV ③

$$\begin{aligned} M_o &= aX + dM_m \\ Y &= c'X + bM_o + gXM_o \end{aligned}$$

- The eligibility

1. Causally related to  $M_o$



Most importantly, to address the concern of unmeasured confounding induced by omitted mediators, we incorporate instrumental variable regression in our model by treating the manipulative variable of the mediator as an IV.

As a manipulation of the mediator, it is natural to expect that  $M_m$  is causally related to  $M_o$ , which satisfies the first requirement.

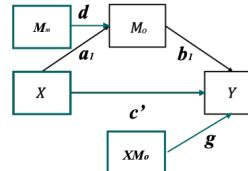
## ► The New Approach

- Treating  $M_m$  as An IV ③

$$M_o = aX + dM_m$$
$$Y = c'X + bM_o + gXM_o$$

► The eligibility

1. Causally related to  $M_o$



This causal relation is also called instrument relevance, which, in our model, can be examined by testing the  $d$  coefficient.

## ► The New Approach

- Treating  $M_m$  as An IV ③

$$M_o = aX + dM_m$$

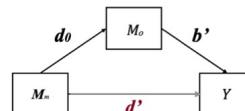
$$Y = c'X + bM_o + gXM_o$$

► The eligibility

1. Causally related to  $M_o$
2. Not directly related to  $Y$

$$M_o = d_0 M_m$$

$$Y = \mathbf{d}' M_m + b' M_o$$



► An assumption:  $M_o$  is the most direct result of the manipulation  $M_m$

However, it's more complicated to test the second requirement.

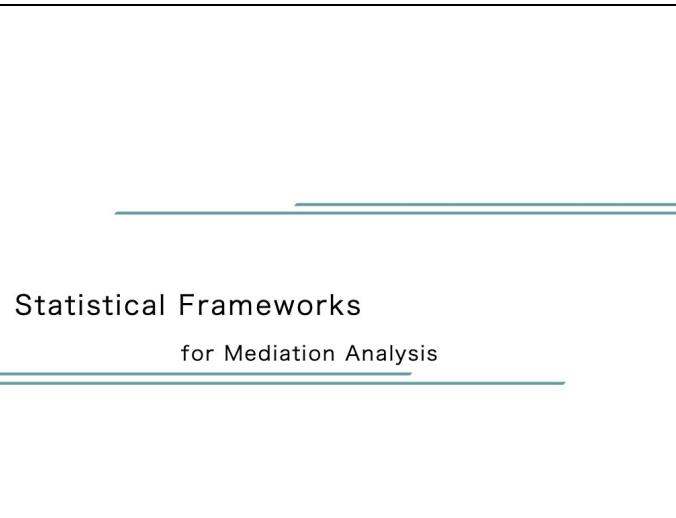
We introduce a sensitivity analysis for the second requirement. Instead of introducing additional IVs to address the shortage of degrees of freedom, we choose to introduce a new, not very excessive assumption.

We assume that  $M_o$  is the most direct result of the manipulation of the mediator. Considering that  $M_m$  is a manipulation approach targeted at varying the mediator, we believe this is a reasonable assumption.

This assumption implies that changes in all other outcome variables caused by  $M_m$  are transmitted through  $M_o$ .

In this way,  $M_m$  is the only possible common cause of  $M_o$  and  $Y$ , so that the second requirement can be tested using data obtained in Experiment 2, by fitting this model and testing the  $d'$  coefficient.

So, if  $d'$  is not significantly nonzero,  $M_m$  can be regarded as an appropriate IV in the mediation model. Therefore, the third concern is addressed.



## Statistical Frameworks for Mediation Analysis

There're two main statistical frameworks that our approach can be built upon.

## ► Statistical Frameworks

The Counterfactual Framework (Pearl, 2001; Robins & Greenland, 1992)	
<b>Definition</b>	$\delta(x) \equiv \mathbb{E}(Y_i(x, M_i(X_1)) - Y_i(x, M_i(X_0)))$
<b>Implementation</b>	mediation (Imai et al., 2010) Model indirect in Mplus (Muthén et al., 2017) Medflex (Steen et al., 2017)

One is the counterfactual framework. The indirect effect under this framework is defined as the average of potential outcome values. The advantage of this definition is that the linearity assumption is not required. But the disadvantage is that instrumental variables are currently not allowed for in the available packages.

(There're other packages under the counterfactual framework, but they do not allow for the XM interaction, thus not listed)



## ► Statistical Frameworks

	The Counterfactual Framework (Pearl, 2001; Robins & Greenland, 1992)	Structural Equation Modeling
<b>Definition</b>	$\delta(x) \equiv \mathbb{E}(Y_i(x, M_i(X_1)) - Y_i(x, M_i(X_0)))$	$M = aX + e_M$ $Y = c'X + bM + e_Y$
<b>Implementation</b>	mediation (Imai et al., 2010) Model indirect in Mplus (Muthén et al., 2017) Medflex (Steen et al., 2017)	AMOS (Arbuckle, 2014) EQS (Bentler, 2004) LISREL (Jöreskog & Sörbom, 2017) Mplus (Muthén & Muthén, 2017) Lavaan (Rosseel, 2012)

On the other hand, SEM is a framework that psychologists may feel more familiar with.

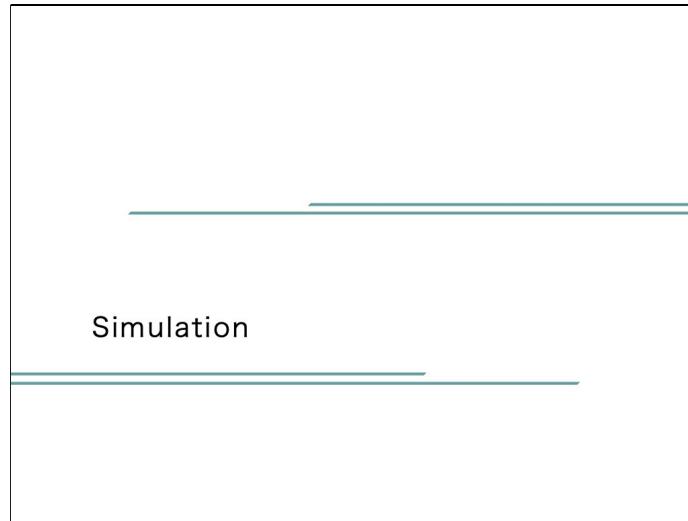
Although the linearity assumption is required for testing the indirect effect, the main advantage of SEM is that IV regression can be easily implemented in SEM, simply by allowing for the estimation of the residual correlation.



## ► Statistical Frameworks

	The Counterfactual Framework (Pearl, 2001; Robins & Greenland, 1992)	Structural Equation Modeling
<b>Definition</b>	$\delta(x) \equiv \mathbb{E}(Y_i(x, M_i(X_1)) - Y_i(x, M_i(X_0)))$	$M = aX + e_M$ $Y = c'X + bM + e_Y$
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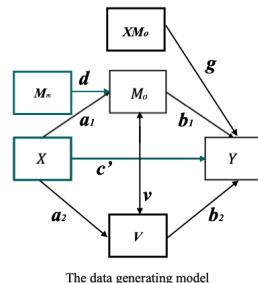
So, our approach is currently implemented under the SEM framework, to make it easier for substantive researchers to use.



To evaluate the proposed approach, we conducted a Monte Carlo simulation study.

## ► Simulation

- Data Generating Model and Parameter Settings

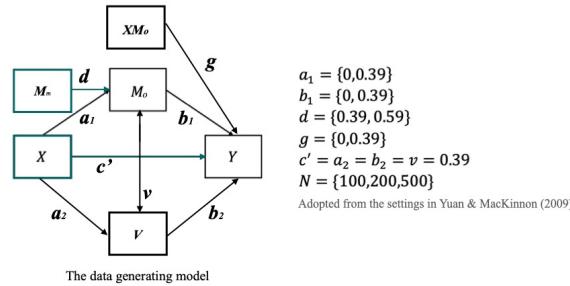


We generated data based on a parallel encouragement design.

Importantly, a confounding variable  $V$  was introduced to simulate omitted mediators that covary with the mediator of interest.

## ► Simulation

- Data Generating Model and Parameter Settings



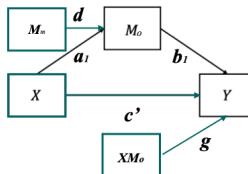
We manipulated the standardized coefficients of  $a_1$  and  $b_1$  paths, the instrument relevance  $d$ , the interaction  $g$ , and the sample size  $N$ .

The settings were adopted from this previous research conducted by Professor MacKinnon.

## ► Simulation

- Analysis Approaches

- ▶ 1.  $M_a$  as the IV (IVSEM)
- ▶ 2.  $M_a$  NOT as the IV ("mediation")



Considering that the analytical approaches for existing manipulation-of-mediator designs (like two-way ANOVA) have differently defined true models, they are not compared in this simulation.

We compared our approach, IVSEM, with the commonly used approach provided in the "mediation" package. Under linearity assumptions, these two approaches mainly differ in that the mediation package makes the fourth no confounding assumption, which is violated in our simulation.

## ► Simulation

- Results

- Without  $XM_o$  interaction

$a_1$	$b_1$	Estimation Bias		Type I Error Rates	
		IVSEM	“mediation” package	IVSEM	“mediation” package
<b>0</b>	<b>0</b>	0.00056	0.00020	0	0.003
	<b>0.39</b>	-0.00081	-0.00095	0.026	0.049
<b>0.78</b>	<b>0</b>	0.00111	<b>0.11706</b>	0.029	<b>0.577</b>
	<b>0.39</b>	-0.00638	<b>0.38897</b>	-	-

$c' = 0.39; d = 0.59; N = 200$

The results showed that our approach can accurately estimate the indirect effect under unmeasured confounding, with well-controlled type I error rates.

Without using IV, the indirect effect estimates of the “mediation” package were biased with inflated type I error rates.

## ► Simulation

- Results

► With  $XMo$  interaction  $IE = ab + cX$

	$a_1$	$b_1$	Estimation Bias		Type I Error Rates	
			IVSEM	"mediation" package	IVSEM	"mediation" package
The Control Group	0	0	-0.00013	0.00045	0.016	0.014
	0.39	0.39	-0.00024	-0.00035	0.000	0.006
	0.78	0.78	0.01651	<b>-0.38023</b>	-	-
The Treatment Group	0	0.39	-0.00046	<b>0.11796</b>	0.043	0.471
	0.39	0.39	-0.000107	-0.00049	0.016	0.033
	0.78	0.78	-0.00220	-0.00237	0.051	0.057

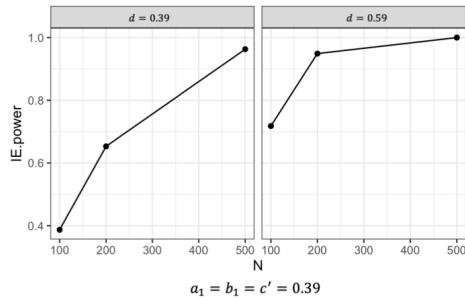
$c' = 0.39; d = 0.59; g = 0.39; N = 200$

Similar results were also found when there's XMo interaction effect in the true model.

## ► Simulation

- Results and Conclusions

- Power of IVSEM



In addition, we examined the statistical power of our approach.

The X-axis is the sample size N, which is naturally an important factor for power.

In addition, the magnitude of instrument relevance d, was also important for power.

Based on these results, we can conclude that with medium effect sizes and a large instrument relevance, N should be at least 200 to have favorable power.

## Summary

- A new approach to address the concerns in mediation analysis
  1. For the mediator: Measurement  $\neq$  Manipulation
  2. The confusion between mediation and moderation
  3. Unmeasured confounding induced by omitted mediators

So, to sum it up. We proposed a new approach to making causal inferences in mediation analysis in psychology, which addressed three major concerns in the field of mediation analysis under manipulation-of-mediator designs.

## Summary

- A new approach to address the concerns in mediation analysis
  1. For the mediator: Measurement  $\neq$  Manipulation
  2. The confusion between mediation and moderation
  3. Unmeasured confounding induced by omitted mediators

Importantly, we combined experimental manipulation with instrumental variable regression to satisfy the fourth no confounding assumption.

## Limitations

- The linearity assumptions
- Cross-sectional designs
- Path models

There're some limitations to the current study. First, our approach is not applicable to non-linear and non-parametric models. Second, we only considered cross-sectional designs. And third, we did not consider measurement error. These issues could be left to future research.

# THANK YOU

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That's all for this presentation. Thank you for your attention. Feel free to ask if there's any questions. And for anyone who's interested in this topic, you're very welcome to contact Zhiming for further discussions.